Macroscopic Field Emission

Effect of Macroscopic Geometry on Microscopic Field Emission

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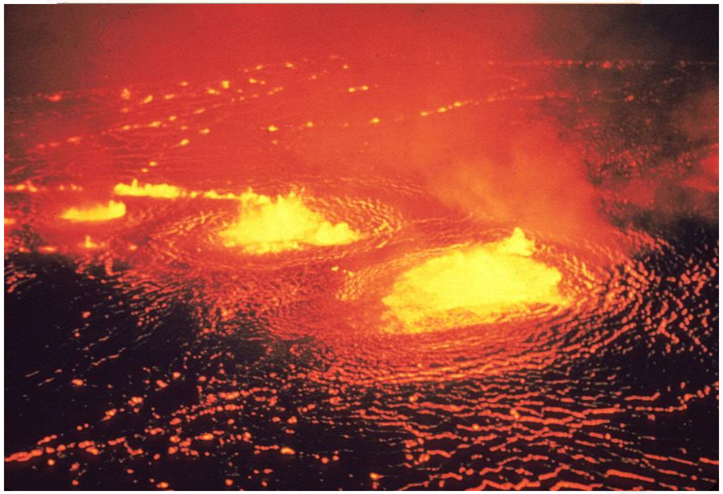
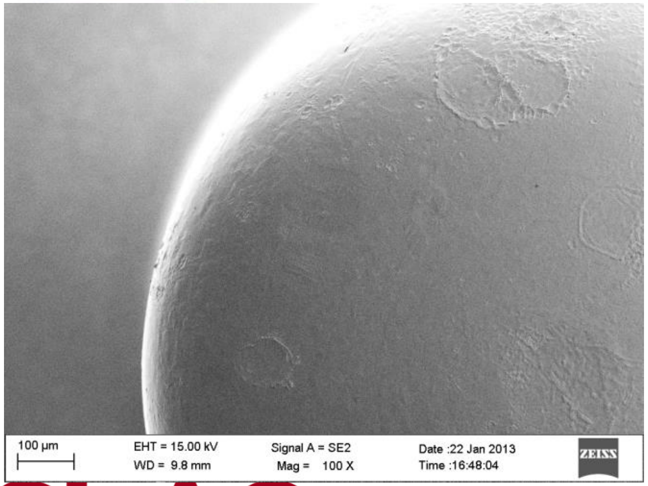
$\omega\uparrow, Q\uparrow(\beta_g\downarrow),\ldots\rightarrow E\uparrow$ All because of damage?

How about the first breakdown? infinite samples.





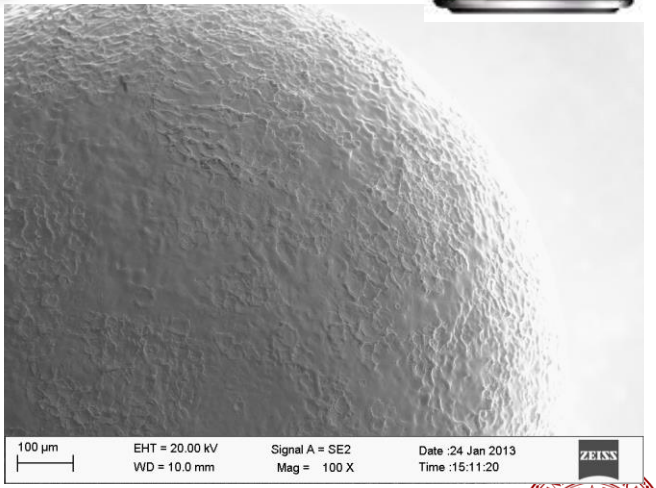
High power

↓

more damage

↓

less gradient



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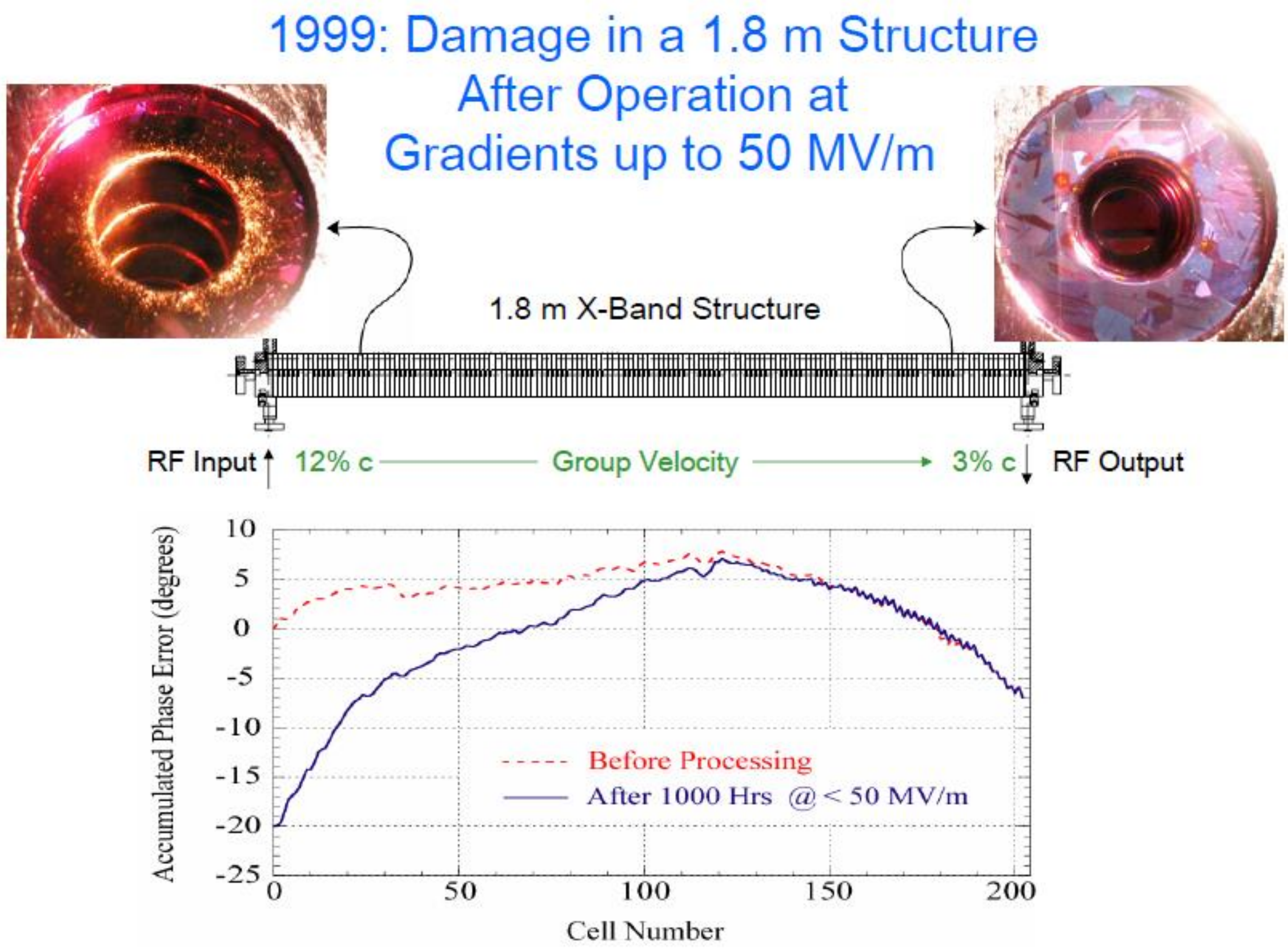
Fascinating, not essential.

Re-used for ECHO ~ 30 MV/m

C. Adolphsen



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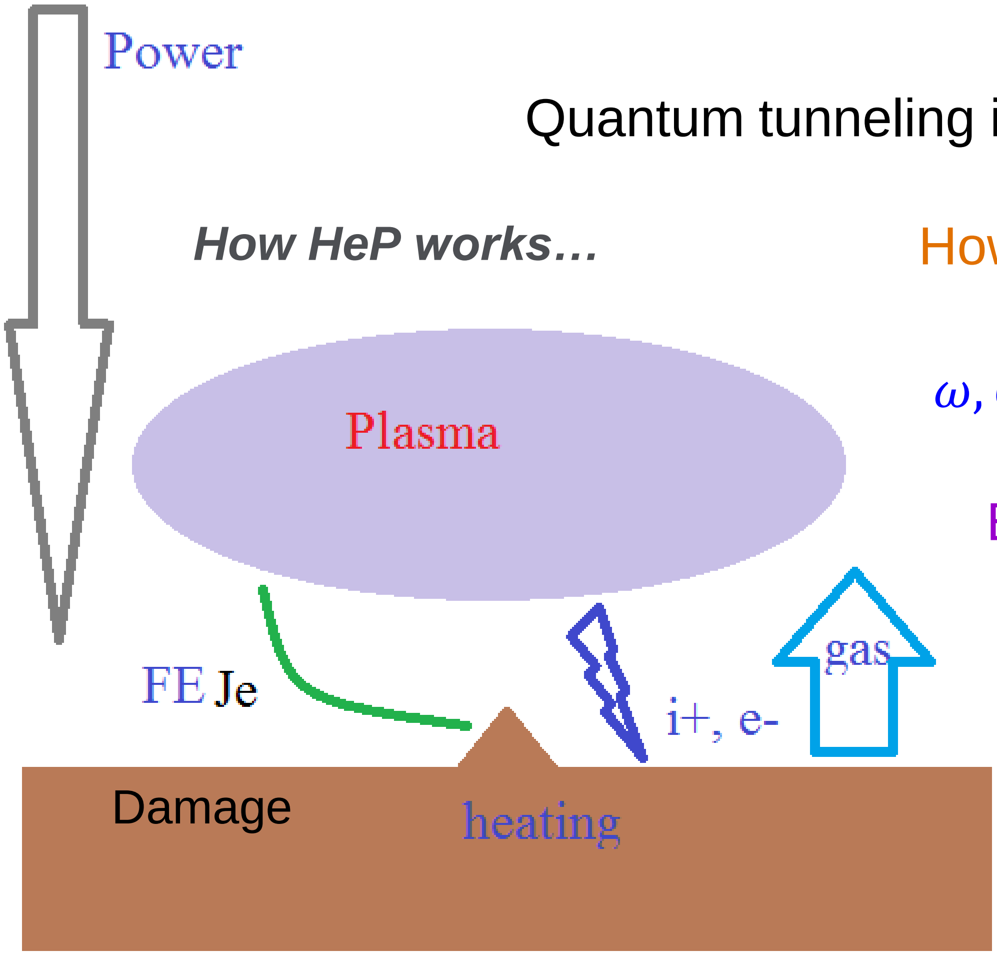
Field emission → Breakdown



Quantum tunneling in the late 1920s (FN): E_s, φ

How about macro parameters?

$\omega, Q(\beta_g), B, number\ of\ cells, \ldots$

Emitters are small, nm ~ um

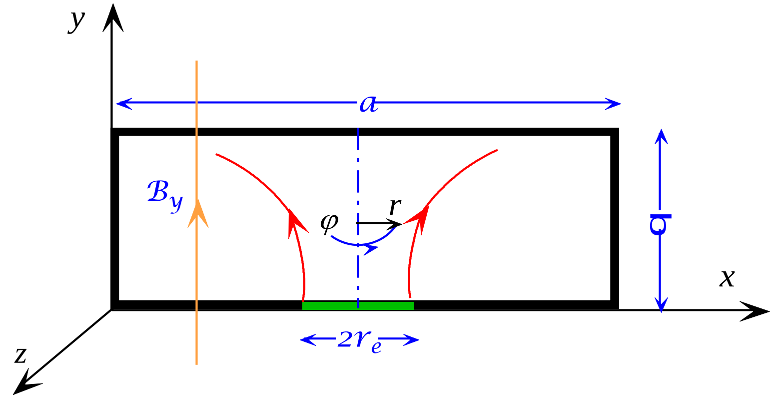
Wavelength, mm ~ cm

Macroscopic FE

Micro FE ➔ macro FE: Setup

TW, Rect. WG, low $\boldsymbol{\varphi}$, a flat disk-like emitter

- $\beta_g(a)$
- dimension $\propto 1/\omega$,
- W/Wo static magnetic field (along y)



Surface field (E_s) = applied field (E_a) + FE induced field (δE_y)

$$J_e = FN(E_s, \boldsymbol{\varphi})$$

$$\delta E_y = f\left(E_a, J_e, \omega, B_y, \beta_g, r_e, t\right)$$

Micro FE ➔ macro FE: How

1. a 3D beam trajectory, the Lorentz equation

$$m_0\gamma\frac{d\vec{v}}{dt} = e\vec{E} + e\left(\vec{v}\times\vec{B}\right) - \frac{e\vec{v}}{c^2}\left(\vec{v}\cdot\vec{E}\right)$$

2. $\boldsymbol{j}(\boldsymbol{r},\boldsymbol{t})$ ➔ $\boldsymbol{\mathcal{J}}(\boldsymbol{r},\omega)$

3. Solving Maxwell equation with the dyadic Green function

$$\delta\boldsymbol{\mathcal{E}}_f(\omega) = -\frac{j\omega}{k^2}\left[k^2\boldsymbol{A} + \nabla(\nabla\cdot\boldsymbol{A})\right]$$

$$\boldsymbol{A} = \int \overline{\boldsymbol{G}}\cdot\boldsymbol{\mathcal{J}}(\boldsymbol{r},\omega)dV'$$

4. IFT: $\delta\boldsymbol{E}_f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta\boldsymbol{\mathcal{E}}_f e^{j\omega t}d\omega$

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Micro FE ➔ macro FE: Green Function

$$G_{yy} = G_{yy1} + G_{yy2},$$

$$G_{yy1} = \frac{\mu_0}{4ab}\sum_{m,n=0}^{\infty}\frac{\varepsilon_m\varepsilon_n}{\gamma_{mn}} f(|z-z'|, \gamma_{mn}, \epsilon) \sin\frac{m\pi x}{a} \sin\frac{m\pi x'}{a} \cos\frac{n\pi y}{b} \cos\frac{n\pi y'}{b},$$

$$G_{yy2} = \frac{\mu_0}{8\pi}\sum_{m,n=-\infty}^{\infty} A_i^{yy}\frac{1}{d_{i,mn}} \cdot \mathbf{Re}\left\{e^{-jkd_{i,mn}}\mathrm{erfc}\left(d_{i,mn}\epsilon - j\,k/2\epsilon\right)\right\},$$

$$\varepsilon_i = \begin{cases} 1, & i = 0 \\ 2, & i \neq 0 \end{cases},$$

$$\gamma_{mn}^2 = (m\pi/a)^2 + (n\pi/b)^2 - k^2$$

the modal and image expansion of the rectangular waveguide

Micro FE → macro FE: simplification

1. A square pulse FE current,

$$J(t) = \begin{cases} J_e, |t| \leq \frac{\pi\tau}{\omega_0} \\ 0, |t| > \frac{\pi\tau}{\omega_0} \end{cases}, \tau = 0.2\ (+/-36^0)$$

2. The initial energy 7 eV (Cu Fermi energy) with 0 trans. momentum.

3. Uniform FE over the emitter surface

4. Beam trajectories not affected by fields generated by FE beam

5. Rf magnetic force is ignored, $F_H/F_E \propto \beta_e \beta_g \ll 1$.

6. Static magnetic field along y axis as a special case

7. Emitter radius nm ~< 10 μm

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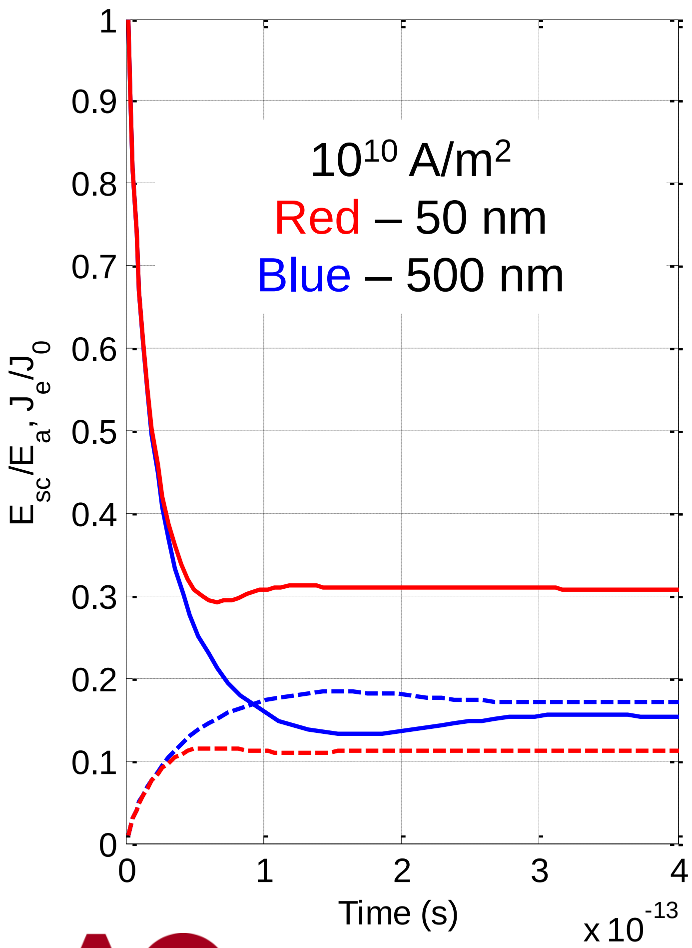
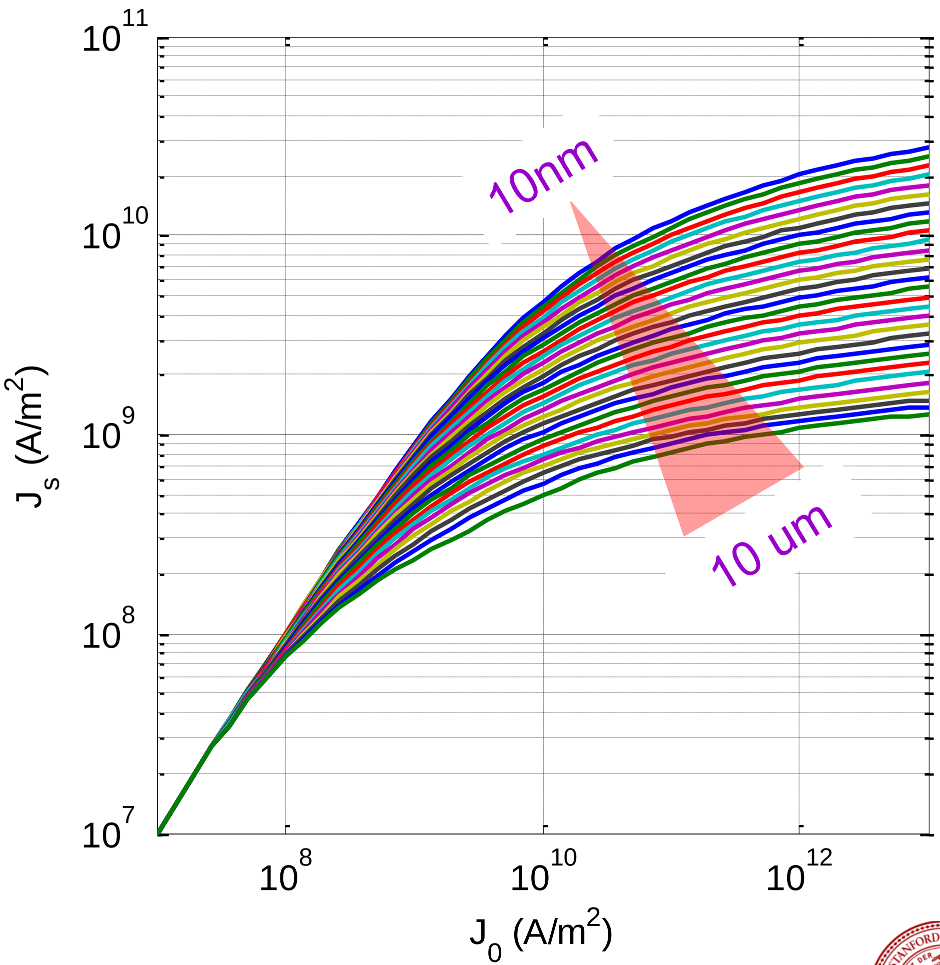
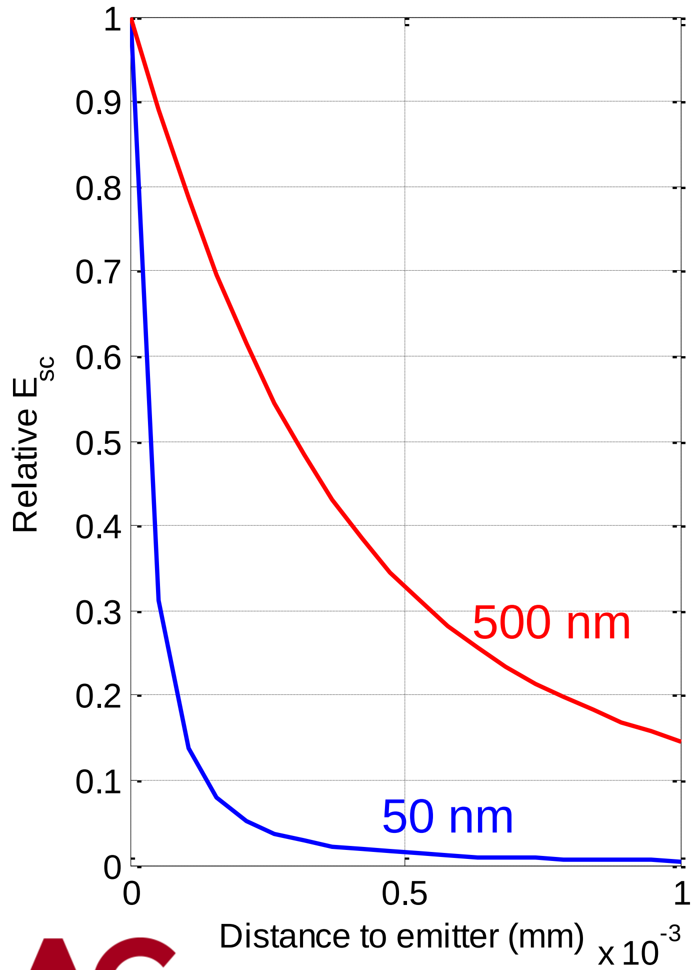
Longitudinal Space Charge (100 MV/m)

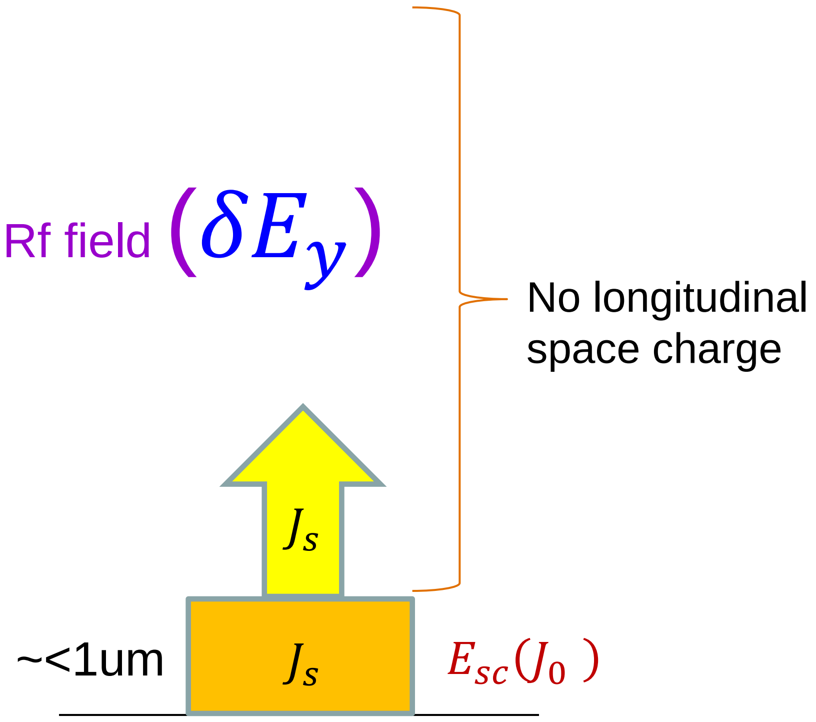




Longitudinal Space Charge (100 MV/m)

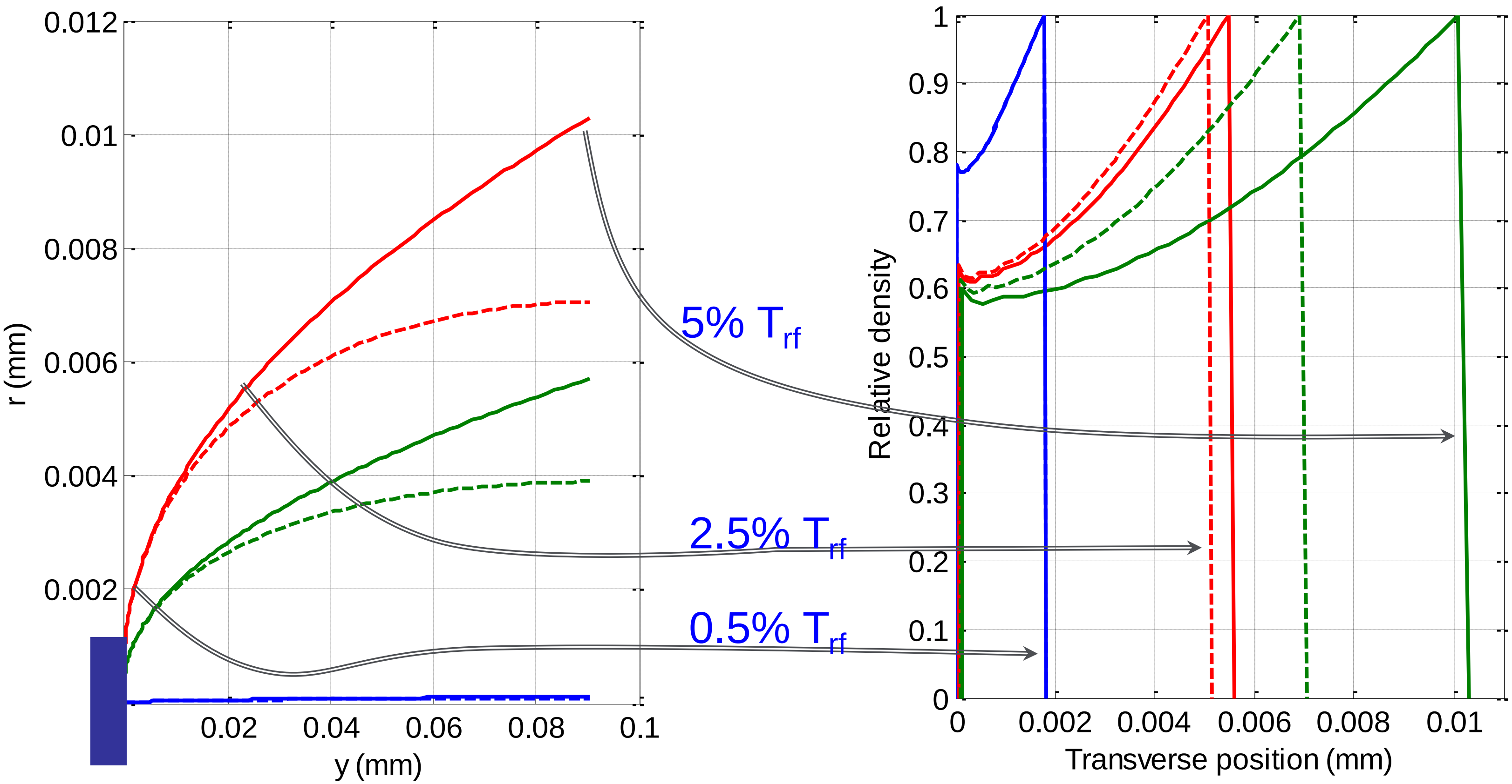
Very short range DC bias (~< 1um)





Micro FE → macro FE: beam trajectory

11. 4 GHz, 50 MV/m,1 um, 10^{10} A/m^2, B_y = 0, 2 Tesla



~radially expands uniformly

Micro FE ➔ macro FE: time domain field

Frequency domain 3D current:

$$\boldsymbol{\mathcal{J}}(r, y, \omega) = \frac{I_e}{\pi R'^2}\frac{2}{\omega}\sin\left(\frac{\pi\tau\omega}{\omega_0}\right)e^{-j\omega t'}\left(\hat{y} + \frac{v_r}{v_y}\hat{r} + \frac{v_\varphi}{v_y}\hat{\varphi}\right)$$

$$\delta E_{yf} = \frac{c\eta I_e}{2\pi^2\omega_0}\int_{g_{min}}^{g_{max}} dg \int_{\sim 1um}^{b} dy' \int_0^{R'} \frac{\sin(\pi\tau g)}{g^2}\sum_{n=1}^{2}\sin(g\omega_0 T_n)\, q_n(d_n, y, y')\frac{r'}{R'^2}dr'$$

$$q_n(d_n, y, y') = \left\{\frac{3\left[y + s(n)y'\right]^2}{d_n^5} - \frac{1}{d_n^3}\right\},\ s(n) = \begin{cases}-1, n = 1\\ \ \ 1, n = 2\end{cases},\ T_{n=1,2} = t - t' - \frac{d_n}{c},\ g = \frac{\omega}{\omega_0},\ \omega_c = \frac{\omega_0}{\sqrt{1-\beta_g^2}}$$

Total field: $g \in [\,0, \infty]$

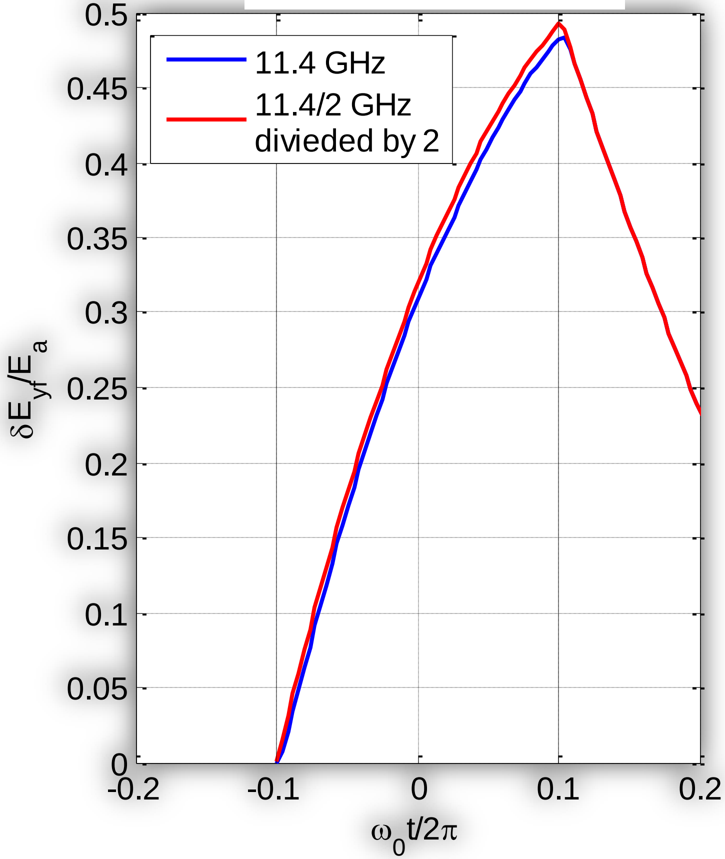
Leftover field (evanescent modes): $g \in [0, \omega_c/\omega_0]$

Example: 60 MV/m,0.5 um, 10^9 A/m^2 , Single burst FE

$$E_{sc}/E_a = -0.08$$

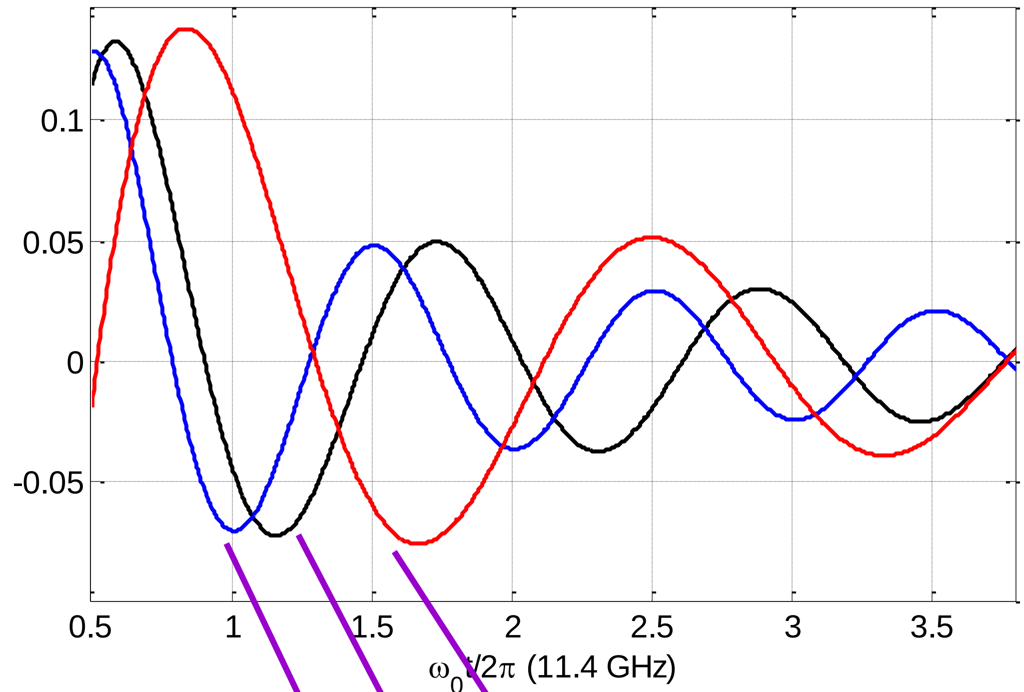
FE self-enhanced!!

Amplitude ~ω^{-1}



Leftover: damped oscillation

Amplitude ~ 1/N

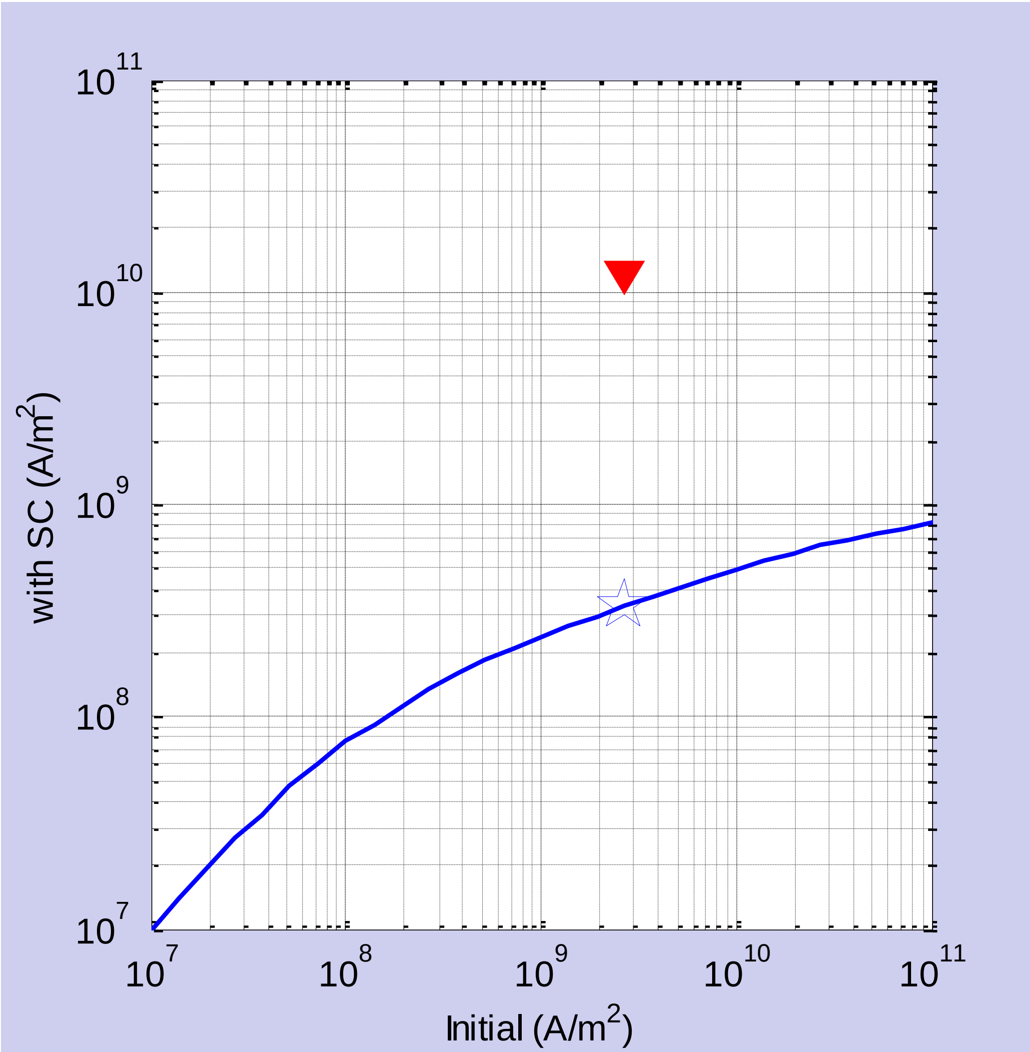


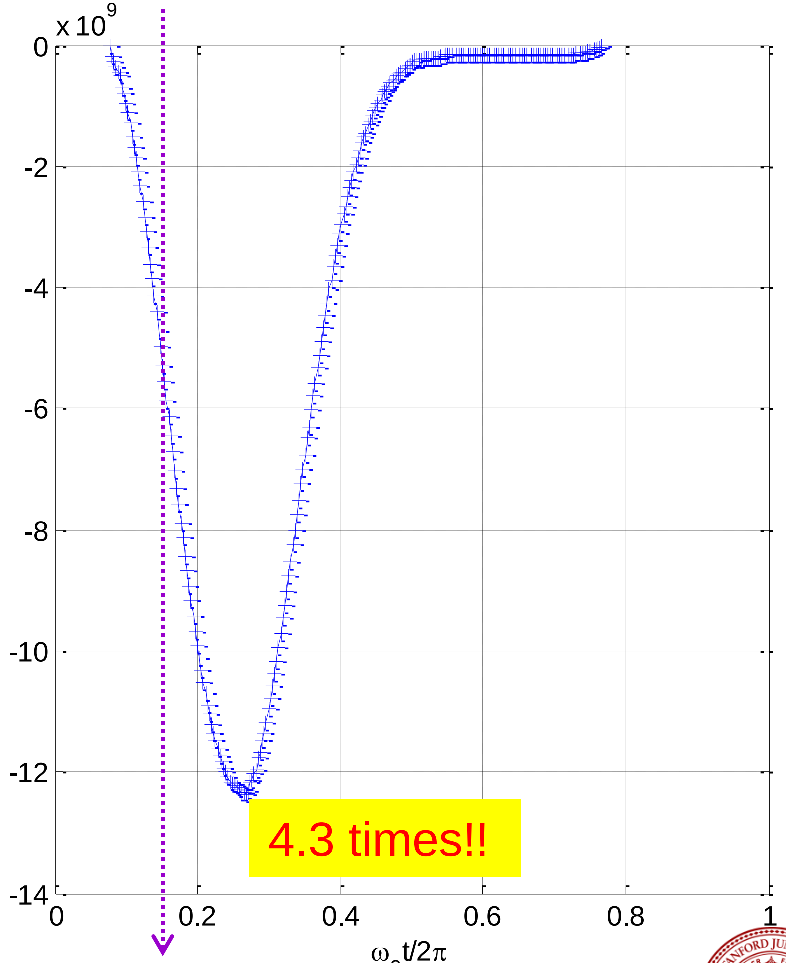
$\beta_g = 0.1, 0.5, 0.8$

Micro FE ➔ macro FE: Pic3P

L. Xiao

WR90, 11.4 GHz, 100 MV/m, -60 Deg, 10 um, $\varphi = 0.43$ eV





Rf crest

Micro FE ➔ macro FE: Stead state

During emission: surface field get enhanced ($\delta E_{yf0} \propto 1/\omega_0$)

Leftover field : dumped oscillation (~1/N, ω_c)

with phase slippage, $\delta\theta = 2n\pi\left(\frac{\omega_0}{\omega_c} - 1\right) \approx n\pi\beta_g^2$

❖ Neglecting the accumulated residual field effects on FE current, at stead state it has

$$\delta E_{ys} = \delta E_{y0}\left[1 - \sum_{n=1}^{\infty} \epsilon \frac{\cos\left(n\pi\beta_g^2\right)}{n}\right] = \delta E_{y0}\left[1 + \varepsilon \mathrm{Ln}\sin\left(\frac{\pi\beta_g^2}{2}\right)\right]$$

$$\epsilon \approx 0.16$$

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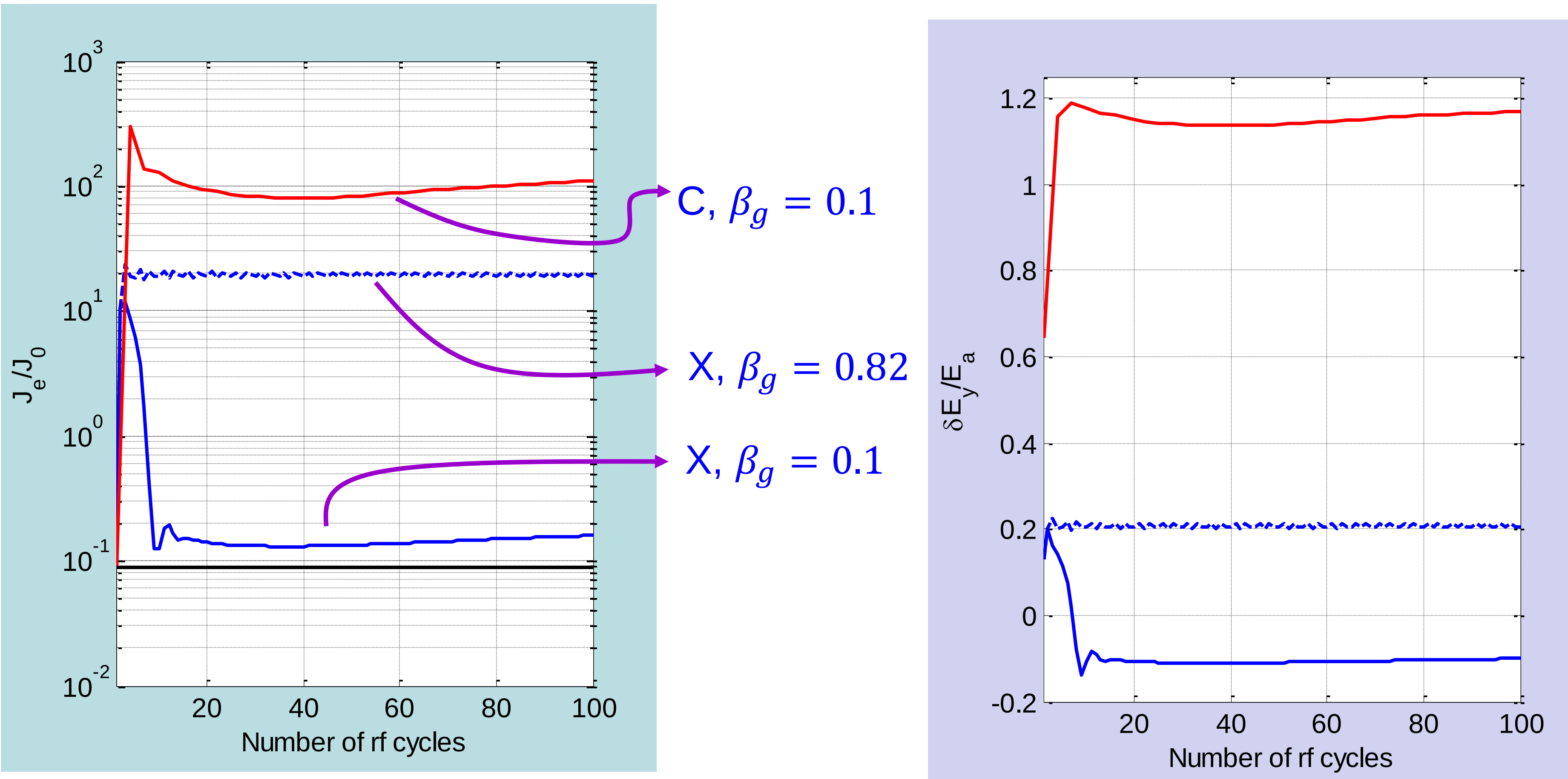
Micro FE → macro FE: ω, β_g

60 MV/m,0.5 um, 10^{10} A/m^2 (10^9 with E_{sc})



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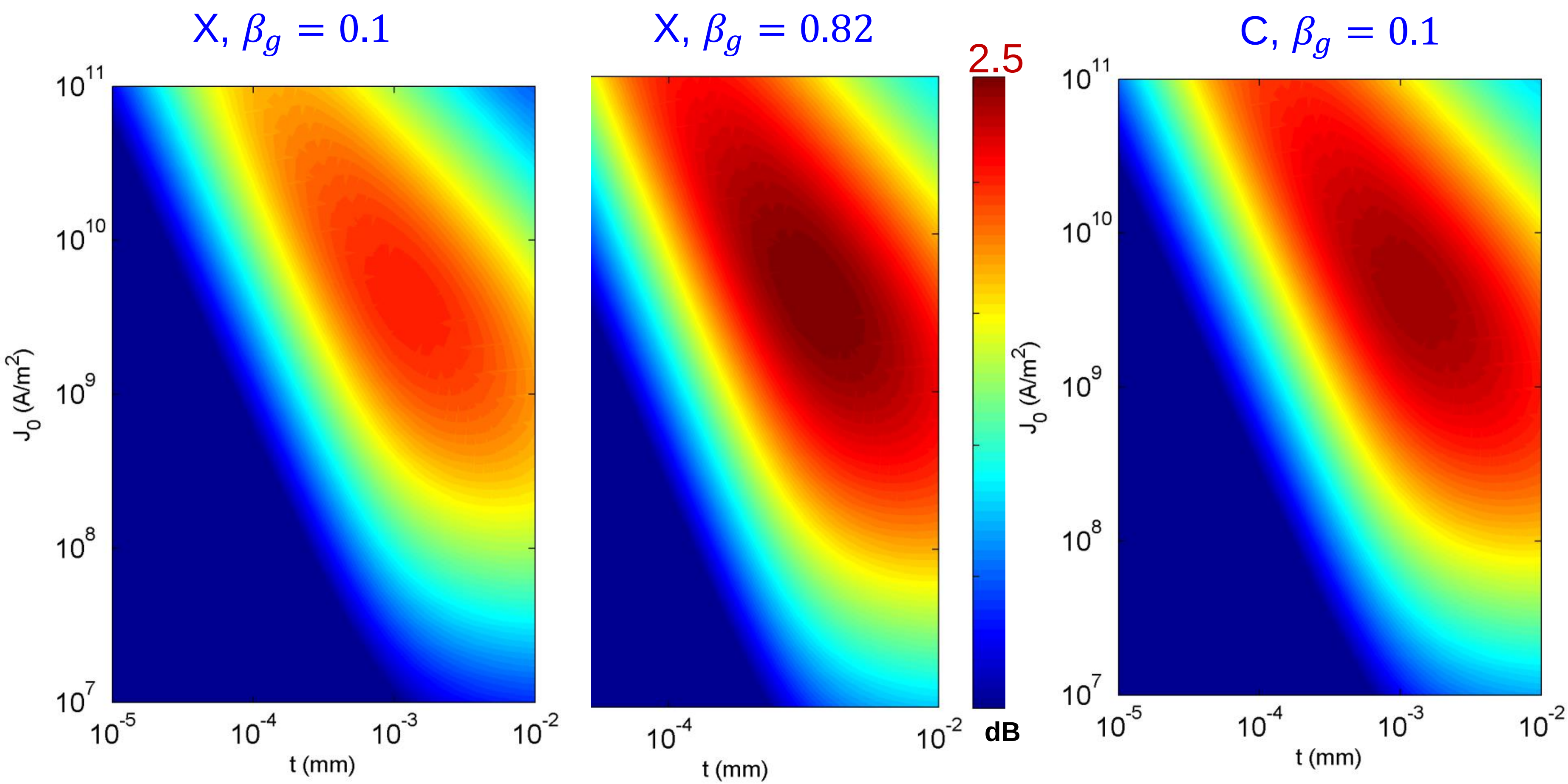
Field enhancement (dB) at 60 MV/m, emitter size and J_e



$$\delta E_{ys} = \delta E_{y0}\left[1 - \sum_{n=1}^{\infty} \epsilon \frac{\cos\left(n\pi\beta_g^2\right)}{n}\right]$$

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Micro FE ➔ macro FE: scaling law

Stability of a system: $\beta_m = max\left(\delta E_{ys}/E_a\right)$

$$E_a[\frac{MV}{m}] \in [30\ 300],\ J_e\left[\frac{A}{m^2}\right] \in \left[10^8\ 10^{11}\right],\ r_e[\mu m] \in [0.01\ 10]$$

$$\beta_m \propto E_a^{\theta-1} J_e^{\alpha} \omega_0^{-1} \beta_g^{\nu}$$

$$J_e \propto E_a^n$$

$$n \in [8\ 15],\ \theta = 1.24,\ \alpha = 0.2,\ \nu = 0.47$$

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Micro FE ➔ macro FE: scaling law

At a given β_m:

$$E_a \propto \omega^{\chi}\beta_g^{-\varsigma}$$

$$\chi = \frac{1}{\alpha n + \theta - 1} \in [0.31, 0.54]$$

$$\varsigma = \frac{\nu}{\alpha n + \theta - 1} \in [0.15, 0.26]$$

$\sim\omega^{1/3}$ -- G.A. Loew and J.W. Wang, Report No. SLAC-PUB-7684, 1997

P/C -- W. Wuensch, CLIC-Note-649, 2006

$$\left.\begin{aligned} P &\propto E_a^2/\omega^2 \\ C &\propto \left(\frac{a}{\lambda}\right)\lambda \propto \beta_g^{\frac{1}{2}}/\omega \end{aligned}\right\} \Rightarrow E_a \propto \omega^{1/2}\beta_g^{-1/4}$$

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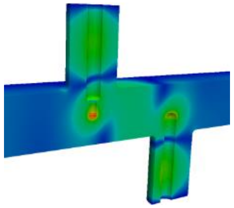


2Pin-waveguide $E_a \propto \beta_g^{-0.17}$

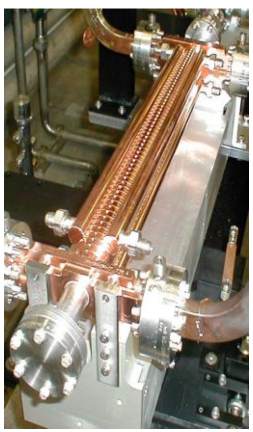
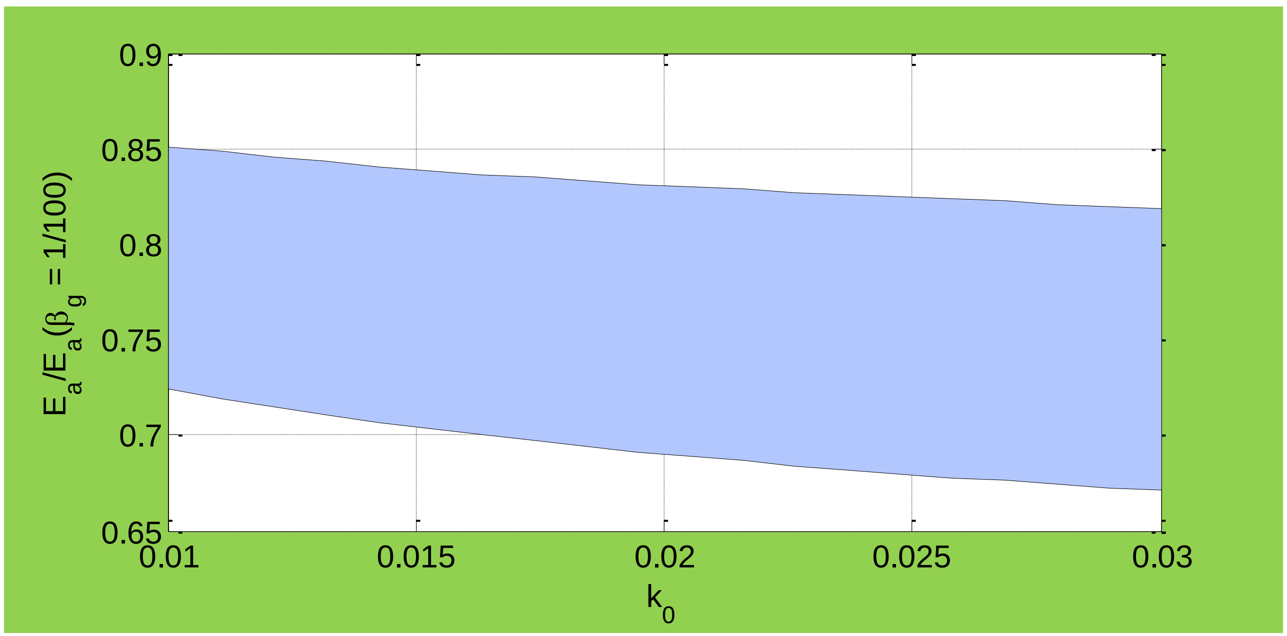
Single cell VS Multi-cell

$$\omega = \omega_0 \left(1 - \frac{k_0}{2} \cos \varphi_N\right)$$

source   cell

According to the phase slippage, the equivalent group velocity of a multi-cell cavity:

$$\beta_{gs} = \sqrt{k_0}$$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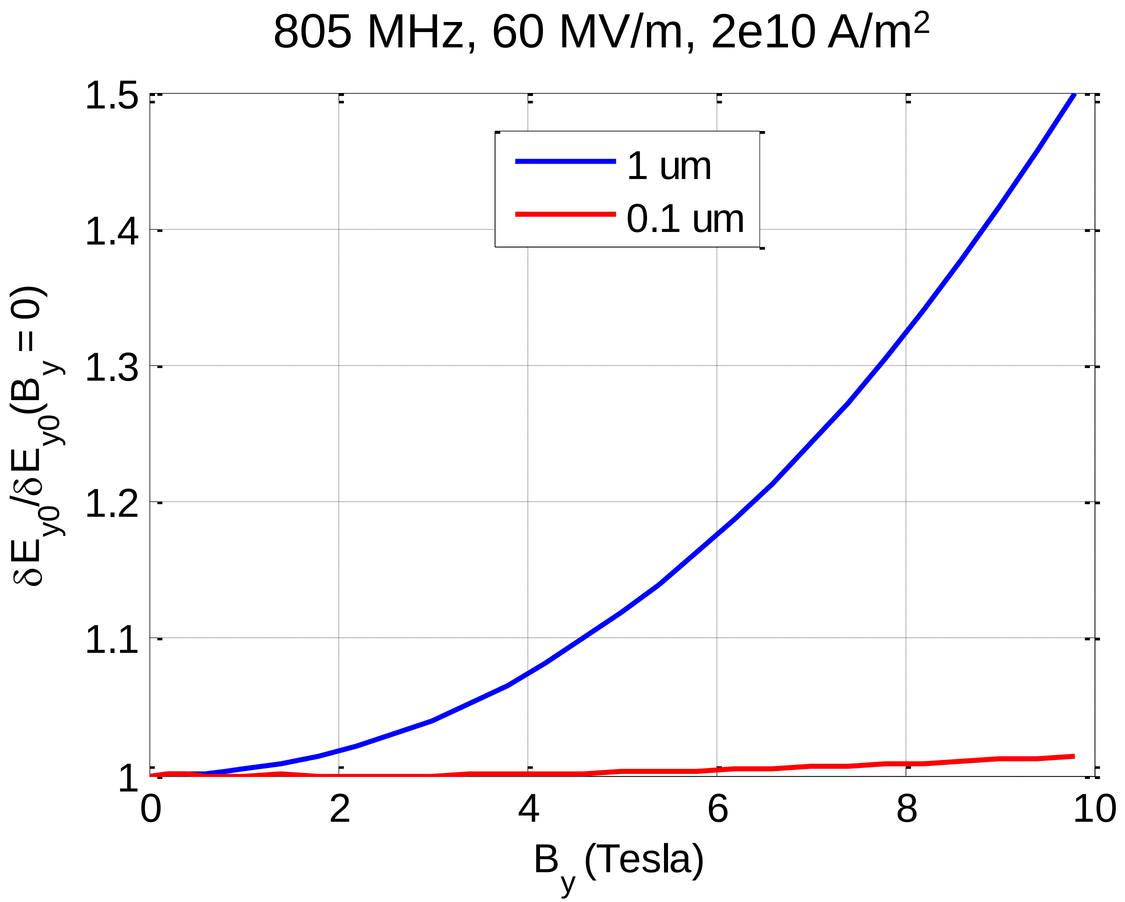
Micro FE ➔ macro FE: Magnetic field



Magnetic field further enhances the field!

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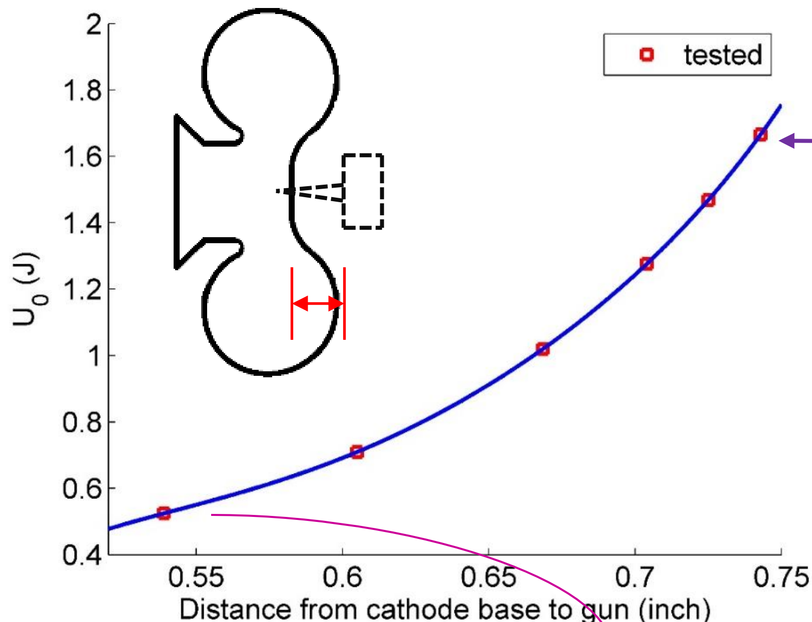
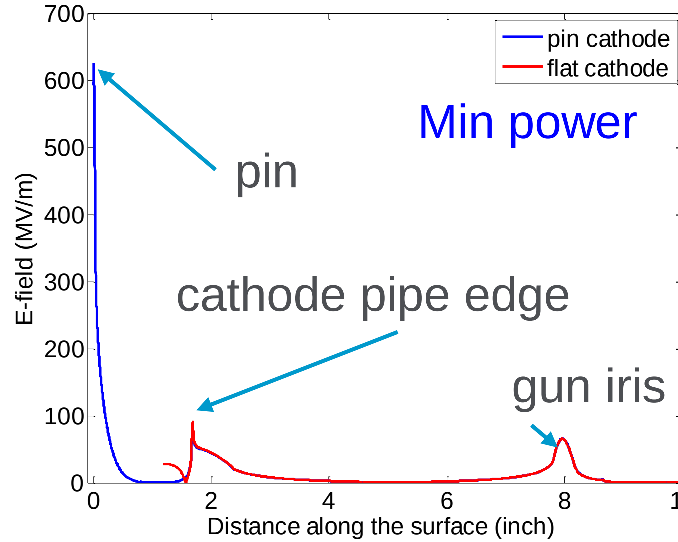
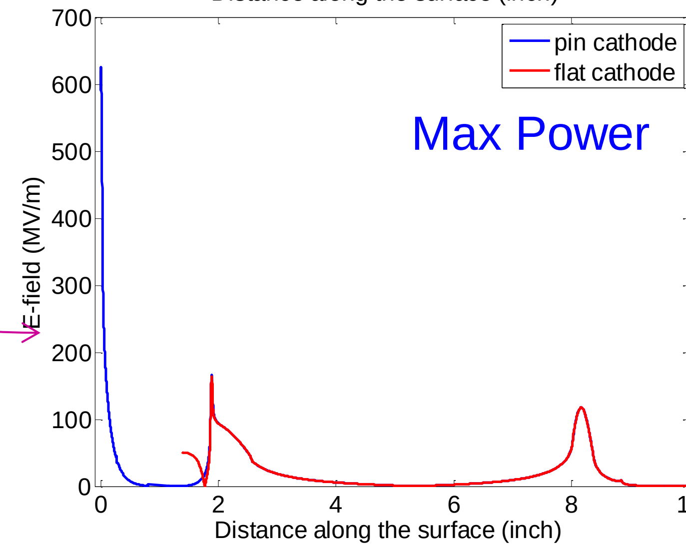
Experiment with an L-band Gun

J. Shao







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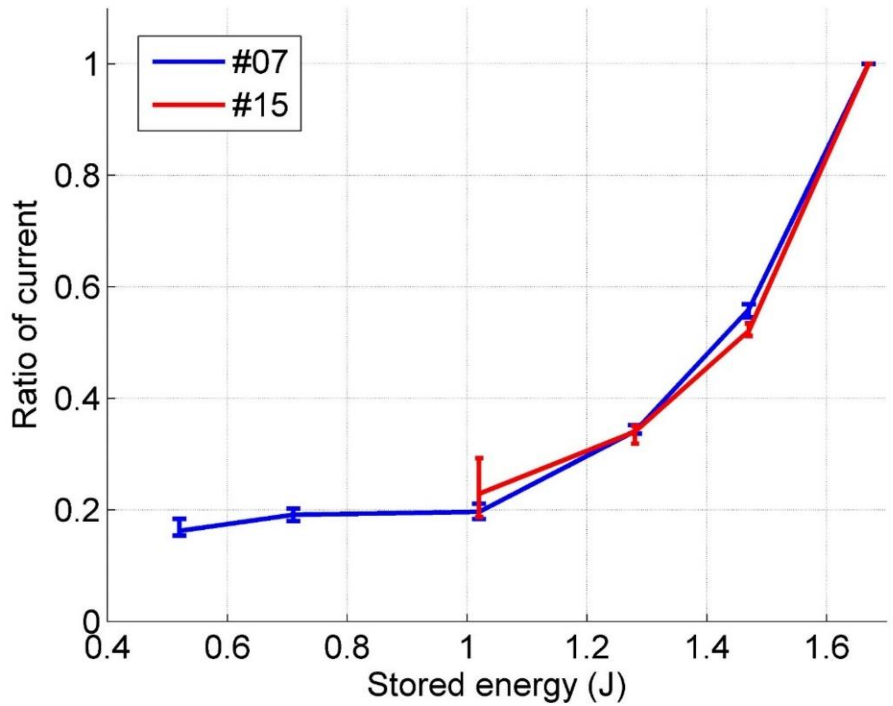
Experiment results

J. Shao



With the same field on the pin cathode, the dark current depends strongly on the stored energy / net rf power flow

Field emission

- A microscopic phenomenon, E_s, φ ,
- the trigger of vacuum breakdown
- **A macroscopic phenomenon,** $\boldsymbol{E_s = E_a + \delta E\left(E_a, J_e, \omega, \beta_g, B \ldots\right)}$
- **The operational field of a system is original.**
- Damage – the consequence of breakdown.
- More power – more damage.
- the damaged surface might not limit the field.

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