Stability of Ion Polarization in Figure-8 MEIC

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Outline

1. Spin Motion in “Figure-8” Storage Rings
2. Ion polarization control with 3D rotators in MEIC collider
3. Strength of the zero-integer spin resonance in MEIC
   - Coherent part of the resonance strength. Response function
   - Compensation of the coherent part of the resonance strength
   - Incoherent part of the resonance strength
4. Summary
5. Future plans
Spin Motion in “Figure-8” Storage Rings

- Properties of a figure-8 structure
  - Spin precessions in the two arcs are exactly cancelled
  - In an ideal structure (without perturbations) all solutions are periodic
  - The spin tune is zero independent of energy

- A figure-8 ring provides unique capabilities for ion polarization control
  - It allows for stabilization and control of the polarization by small field integrals
  - Spin rotators are compact, easily rampable and have little or no orbit distortion
  - It eliminates depolarization problem during acceleration
  - It provides efficient polarization control of any particles including deuterons
  - It is currently the only practical way to accommodate polarized deuterons
  - It allows for a spin flipping system with a spin reversal time of ~1 s
Polarization Control Concept

- Local spin rotator determines spin tune and local spin direction

- Polarization is stable if \( \nu >> \omega_0 \)
  
  - \( \omega_0 \) is the zero-integer spin resonance strength
  - \( B_{||}L \) of only 3 Tm provides deuteron polarization stability up to 100 GeV
  - A conventional ring at 100 GeV would require \( B_{||}L \) of 1200 Tm or \( B_{\perp}L \) of 400 Tm
Control of Ion Polarization with Small Solenoids

- Universal 3D spin rotator

- $n_x$ control module (constant radial orbit bump)
  \[ \phi_{z1} = \pi v \frac{n_x}{\sin \phi_y} \]
  \[ L \approx 8 \text{m} \quad \Delta x \approx 1.6 \text{ cm} \quad B_{y_{\text{max}}} = 3 \text{T} \]

- $n_y$ control module (constant vertical orbit bump)
  \[ \phi_{z2} = \pi v \frac{n_y}{\sin \phi_x} \]
  \[ L \approx 8 \text{m} \quad \Delta y \approx 1.6 \text{ cm} \quad B_{x_{\text{max}}} = 3 \text{T} \]

- $n_z$ control module
  \[ \phi_{z3} = \pi v n_z \]

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Control of the Ion Polarization in Collider Ring

- Placement of the 3D spin rotator in the collider ring

- Integration of the 3D spin rotator into the collider ring’s lattice
  - Seamless integration into virtually any lattice

Spin-control solenoids

Vertical-field dipoles

Radial-field dipoles

Lattice quadrupoles

\[ \Delta x = \Delta y = 16 \text{ mm} \]
\[ L_x = L_y = 0.6 \text{ m} \]
\[ L_z = 2 \text{ m} \]
Control Solenoid Fields

- Momentum dependence of \( B_{zi} = \varphi_{zi} B \rho / (1 + G) L \) for deuterons, \( \nu = 10^{-4} \)

- Momentum dependence of the solenoid fields for protons, \( \nu = 0.01 \)
Strength of Zero-Integer Spin Resonance

\[ \vec{e}_x, \vec{e}_y, \vec{e}_z \] unit vectors of accelerator coordinate frame

Thomas-BMT equation in the accelerator frame

\[
\frac{d\vec{S}}{d\theta} = [\vec{W} \times \vec{S}], \quad \vec{W} = \gamma G \vec{K} + \vec{w}.
\]

\( \theta = 2\pi z / L \) generalized azimuthal angle (normalized distance along reference orbit)

\( \vec{K} \) curvature of the design orbit

Spin perturbation components in linear approximation \((\gamma G >> 1)\)

\[
w_x = -\gamma G \tau'_y, \quad w_y = \gamma G \tau'_x, \quad w_z = (1 + G) h_z.
\]

\( \vec{\tau} = (\tau_x, \tau_y, \tau_z) \) velocity direction, \( \vec{\tau}' = d\vec{\tau} / d\theta \)

\[
h_z = \frac{B_z L}{2\pi B\rho} \quad \text{Normalized strength of the control solenoids, which set the}
\]

\( \text{polarization direction} \ \vec{n} \ \text{and the spin tune} \ \nu. \)

The resonance strength is determined by the spin perturbation component transverse to \( \vec{n} \)

\[
w_0 = |\vec{w} - (\vec{w}\vec{n})\vec{n}| \]
Spin Field without Control Solenoids

\[ \begin{align*}
\vec{e}_1 + i \vec{e}_3 &= \left( \vec{e}_x + i \vec{e}_z \right) e^{-i\Psi_y} \\
\vec{e}_2 &= \vec{e}_y
\end{align*} \]

relation of unit vectors of the «spin» reference frame \( \vec{e}_1, \vec{e}_2, \vec{e}_3 \) to unit vectors of the accelerator system

Here \( \Psi_y = \gamma G \int_0^\theta K_y d\theta \) is the spin rotation angle in the collider’s bending dipoles.

In the “spin” reference frame, spin components of a particle moving along the design orbit remain constant.

Averaging the spin perturbation gives the following spin field components in the spin reference frame

\[ \begin{align*}
\omega_1 + i \omega_3 &= -\gamma G \langle \tau'_y e^{i\Psi_y} \rangle = \gamma G \frac{d}{d\theta} e^{i\Psi_y} \\
\omega_2 &= \gamma G \langle \tau'_x \rangle = 0
\end{align*} \]

\[ \vec{\omega} = \vec{\omega}_{\text{coherent}} + \vec{\omega}_{\text{incoherent}}; \quad \omega_{\text{incoherent}} \ll \omega_{\text{coherent}}. \]

\( \omega_{\text{coherent}} \) caused by closed orbit distortions

\( \omega_{\text{incoherent}} \) caused by emittances of the synchrotron and betatron oscillations

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Spin Response Function

- Spin field induced by perturbing radial field $h_x(\theta)$ can be calculated using response function $F(\theta)$:

  $$\omega_i + i \omega_3 = \gamma G\langle h_x(\theta)F(\theta) \rangle$$

- Perturbing radial fields arise, for example, due to dipole roll errors, vertical quadrupole misalignments, etc. Such perturbing fields result in vertical closed orbit distortion, are periodic and determine the coherent part of the spin field.

- For a flat figure-8 design orbit, the response function is given by

  $$F(\theta) = \frac{f_y^*(\theta)}{2} \int_{-\infty}^{\theta} \left( \frac{de^{-i\nu_y}}{d\theta} \right) df_y d\theta - \frac{f_y(\theta)}{2} \int_{-\infty}^{\theta} \left( \frac{de^{-i\nu_y}}{d\theta} \right) df_y^* d\theta$$

  $$f_y = \sqrt{\frac{\beta_y}{R}} \exp\left( \int_0^\theta \frac{R}{\beta_y} d\theta \right)$$

  is the Floke function

  $\beta_y$ and $\nu_y$ are the vertical betatron function and betatron tune

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Response Function in the MEIC Collider Ring

$F(z)$ is the periodic response function describing effect of any radial fields and allowing one to calculate the coherent part of zero-integer resonance strength.
Statistical Model of Quadrupole Misalignments

$$|w_{0}^{(k)}| = \frac{\partial B_y}{\partial x} \frac{L_{quad}}{B\rho} \Delta y \gamma G F(z_k)$$

is the coherent part of zero-integer resonance strength due to quadrupole misalignment

$\Delta y$ is the vertical quadrupole misalignment

**rms vertical orbit distortion for random same-sigma misalignments of all quadrupoles**

orbit distortion in the arcs does not exceed $\pm 100 \, \mu m$
2 T control solenoids allow setting proton and deuteron spin tunes to $n_p=10^{-2}$ and $n_d=10^{-4}$.

This is sufficient for stabilization and control of polarization in MEIC.
Additional Improvement of Polarized Beam Parameters by Compensation of Coherent Part of Resonance Strength

The first 3D rotator located in the straight containing the interaction point directly controls the polarization.

The second 3D rotator with constant solenoid fields is located in the other straight and is used to compensate the coherent part of the zero-integer spin resonance strength.

This allows one to significantly improve the polarized beam parameters as well as to greatly reduce the field integrals of the solenoids used for polarization control in the first rotator. In particular, the spin reversal time of the spin-flipping system of the MEIC ion collider ring will be on the order of 1 ms instead of 1 s.
Incoherent Part of Resonance Strength in MEIC

The incoherent part of the spin field in the linear approximation is determined by particle energy spread.

In an ideal MEIC lattice, the vertical dispersion $D_y = 0$.

In MEIC, the incoherent part of the spin field to second order is vertical and given by

$$\omega = \frac{\gamma^2 G^2}{2} \left\langle \tau_y^2 \frac{d}{d\theta} \Psi_y + \text{Im} \tau_y \left( \frac{d}{d\theta} e^{-i\Psi_y} \right) \int_{-\infty}^{\theta} \tau_y \left( \frac{d}{d\theta} e^{i\Psi_y} \right) d\theta \right\rangle$$

Expressing the velocity direction through Floke function $f$ and beam emittance $\varepsilon_y$, we get a formula for the incoherent part of the spin field:

$$\omega = \frac{\varepsilon_y \gamma^2 G^2}{8\pi} \left[ \int_{0}^{2\pi} \frac{d\Psi(x)}{dx} \left| f'(x) \right|^2 + \right.$$  

$$\left. + \frac{1}{2} \text{Im} \int_{0}^{x} \int_{-\infty}^{x} \frac{d}{dx} e^{-i\Psi(x)} \left( \frac{d}{dy} e^{-i\Psi(y)} \right) \left( f'(x)f''(y) + f''(x)f'(y) \right) \right]$$
Incoherent Part of Resonance Strength vs Momentum in MEIC

Assuming normalized vertical beam emittance of 0.07 μm rad
Summary

- For stability of ion polarization in MEIC, the spin tune induced by the 3D spin rotator must significantly exceed the strength of the zero-integer spin resonance.
- Calculations of resonance strength for MEIC show that its coherent part related to closed orbit distortion is a few orders of magnitude greater than its incoherent part related to beam emittances.
- 3D rotators with 2 T solenoids provide control of proton and deuteron polarizations in MEIC.
- Polarized beam quality can be additionally significantly improved and the field strengths of the control solenoids in the 3D rotator can be significantly reduced by compensating the coherent part of the resonance strength.
What is next? Future Plans

- Minimization of the response function at the interaction point by optimizing magnetic lattice parameters of MEIC
- Development of a scheme for compensation of coherent part of the resonance strength using small solenoids
- Optimization of parameters and control modes of spin-flipping system in MEIC to minimize required fields and beam polarization reversal time
- Development of efficient numerical techniques for spin calculations in MEIC
- Spin tracking using existing established codes to verify the ion polarization control schemes proposed for MEIC
- Systematic comparison of figure-8 and racetrack rings for MEIC
Converting Racetrack Collider to Figure-8 Collider ($\nu=0$)

Analogous control scheme in a Racetrack Collider

Snake axes are parallel to each other

$\nu = 0$

Beam momentum:
$p > 20$ GeV/c

Deviation of closed orbit $\sim 1-4$ cm

Betatron tune shift $\sim 1/\gamma^2$

Two Helical (Dipole) Siberian Snakes.

$(BL)_{\text{max}} \sim 2 \times (25)$ T·m (protons)

$(BL)_{\text{max}} \sim 2 \times (600)$ T·m (deuterons)

Such a snake scheme is not feasible for deuterons

- In a scheme without snakes, depolarization of deuteron beam is unavoidable due to slow field ramp rate in the collider ring (slow spin resonance crossing rate).

- Practical experience with preservation of deuteron polarization exists in accelerators only up to 10 GeV (5 GeV/u). Experimental polarization maintenance time is $\sim 1$ s.

Polarized deuteron beam can only be obtained at high energy in a figure-8 collider.
Using Racetrack Booster

\[ \nu = \frac{1}{2} \]

**Solenoidal snake** does not perturb closed orbit.

Solenoidal Siberian Snakes \((p = 8 \text{ GeV}/c)\)

\[ (BL)_{\text{max}} \approx 30 \text{ T} \cdot \text{m (protons)} \]
\[ (BL)_{\text{max}} \approx 100 \text{ T} \cdot \text{m (deuterons)} \]

Solenoidal Siberian Snakes \((p = 3 \text{ GeV}/c)\)

\[ (BL)_{\text{max}} \approx 10 \text{ T} \cdot \text{m (protons)} \]
\[ (BL)_{\text{max}} \approx 35 \text{ T} \cdot \text{m (deuterons)} \]

It may be worthwhile to analyze the option of a racetrack booster from the point of view of collider luminosity optimization.