Measurement of \( ^{16}\text{O}(\gamma,\alpha)^{12}\text{C} \) with a Bubble Chamber and a Bremsstrahlung Beam at Jefferson Lab Injector

Riad Suleiman
January 23, 2014
OUTLINE

• Nucleosynthesis and the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ Reaction
• Time Reversal Reaction: $^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$
• The Bubble Chamber
• Work at HIGS
• Experimental Setup at Jefferson Lab Injector
• Bremsstrahlung Beam and Penfold-Leiss Unfolding
• Statistical and Systematic Errors
• Backgrounds and Ion Energy Distributions
• Safety
• Summary and Outlook
Relative Abundance of Elements by Weight

Universe
- Hydrogen: 73.9%
- Helium: 24.0%
- Oxygen: 1.0%
- Carbon: 0.5%
- Other: 0.6%

Human Body
- Oxygen: 61%
- Carbon: 23%
- Hydrogen: 10%
- Nitrogen: 2.6%
- Calcium: 1.1%
- Phosphorus: 1.1%
- Other: 0.9%

This region is bypassed by $3\alpha$ process.
Stellar Helium Burning

- Helium Reactions:
  I. $\alpha + \alpha \leftrightarrow ^8\text{Be}$
     $(Q = -0.092 \text{ MeV}, T_{1/2} \approx 10^{-16} \text{ s})$
  II. $\alpha + ^8\text{Be} \rightarrow ^{12}\text{C} + \gamma$
      $(Q = +7.367 \text{ MeV}, \text{Hoyle State} = 7.654 \text{ MeV})$
  III. $\alpha + ^{12}\text{C} \rightarrow ^{16}\text{O} + \gamma$
       $(Q = +7.162 \text{ MeV})$
       (slow, otherwise no $^{12}\text{C}$ remains)
  IV. $\alpha + ^{16}\text{O} \rightarrow ^{20}\text{Ne} + \gamma$
      (very slow)

- $\alpha + ^{12}\text{C}$ burning at very small cross section $\sigma \approx 10^{-17} \text{ barn}$

  - Currently, reaction rate error is large ($\pm 35\%$)
  - Goal $< \pm 10\%$

- Thermonuclear reaction rate involving two nuclei is:

$$R = \sqrt[3]{\frac{8}{\pi m (k_BT)^3}} \int_0^\infty E \sigma_{tot}(E) e^{-\frac{E}{k_BT}} dE$$

Only narrow energy range is important (Gamow Peak)
**THE $^{12}$C($\alpha,\gamma$)$^{16}$O Reaction**

- The *holy grail* of nuclear astrophysics

  Affects the synthesis of most of the elements of the periodic table

  Sets the $N(^{12}$C)/$N(^{16}$O) ($\approx 0.4$) ratio in the universe

  Determines the minimum mass a star requires to become a supernova

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The Gamow Peak (Window)

- Narrow energy range where thermonuclear reactions is most likely to occur in stellar plasma is a product of two distributions:
  1. Maxwell-Boltzmann energy distribution with $e^{-E/k_B T}$
  2. Penetration through Coulomb barrier with $e^{-b/E^{3/2}}$

$$E_0 = 1.220 \left( Z_1^2 Z_2^2 A T_6^2 \right)^{1/3} \text{ keV}$$

$$W = 0.2368 \left( Z_1^2 Z_2^2 A T_6^5 \right)^{1/6} \text{ keV}$$

- For $\alpha + ^{12}\text{C}$ ($Z_1=2, Z_2=6, A=3$), and stellar $T=200 \times 10^6$ K:
  - Gamow Peak, $E_0 \approx 300$ keV, $W \approx 50$ keV
    (in Center-of-Mass (CM) of $\alpha + ^{12}\text{C}$ system)
  - Maximum of Maxwell–Boltzmann energy distribution, $k_B T = 17$ keV
**α+^{12}C REACTION**

- α $(J^\pi=0^+)$ + $^{12}$C $(J^\pi=0^+)$ cross section, $\sigma(E_0)$, is dominated by $p$–wave (E1) and $d$–wave (E2) radiative capture to $^{16}$O ground state $(J^\pi=0^+)$

- Two bound states, at 6.92 MeV $(J^\pi=2^+)$ and 7.12 MeV $(J^\pi=1^-)$, with sub–threshold resonances at $E_R=−0.245$ and $−0.045$ MeV, provide most of $\sigma(E_0)$ through their finite widths

- Distinguish E1 and E2 by measuring $\gamma$–angular distributions

<table>
<thead>
<tr>
<th>Transition</th>
<th>$S_{E1}(300)$</th>
<th>$S_{E2}(300)$</th>
<th>$S_{tot}(300)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$^-$ → 0$^+$ (E1)</td>
<td>1–288 keV b</td>
<td>7–120 keV b</td>
<td>40–430 keV b</td>
</tr>
<tr>
<td>2$^+$ → 0$^+$ (E2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Heroic efforts in search of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

Previous Experiments:

A. Direct Measurements:
   I. Helium ions on carbon target: $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$
   II. Carbon ions on helium gas: $^{4}\text{He}(^{12}\text{C}, \gamma)^{16}\text{O}$ or $^{4}\text{He}(^{12}\text{C},^{16}\text{O})\gamma$ (Schürmann)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Beam Current (mA)</th>
<th>Target (nuclei/cm$^2$)</th>
<th>Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redder</td>
<td>0.7</td>
<td>$^{12}\text{C}$, $3\times10^{18}$</td>
<td>900</td>
</tr>
<tr>
<td>Ouellet</td>
<td>0.03</td>
<td>$^{12}\text{C}$, $5\times10^{18}$</td>
<td>1950</td>
</tr>
<tr>
<td>Roters</td>
<td>0.02</td>
<td>$^{4}\text{He}$, $1\times10^{19}$</td>
<td>5000</td>
</tr>
<tr>
<td>Kunz</td>
<td>0.5</td>
<td>$^{12}\text{C}$, $3\times10^{18}$</td>
<td>700</td>
</tr>
<tr>
<td>EUROGAM</td>
<td>0.34</td>
<td>$^{12}\text{C}$, $1\times10^{19}$</td>
<td>2100</td>
</tr>
<tr>
<td>GANDI</td>
<td>0.6 (?)</td>
<td>$^{12}\text{C}$, $2\times10^{18}$</td>
<td>?</td>
</tr>
<tr>
<td>Schürmann</td>
<td>0.01</td>
<td>$^{4}\text{He}$, $4\times10^{17}$</td>
<td>?</td>
</tr>
<tr>
<td>Plag</td>
<td>0.005</td>
<td>$^{12}\text{C}$, $6\times10^{18}$</td>
<td>278</td>
</tr>
</tbody>
</table>

B. Indirect Measurements:
   I. $\beta$–delayed $\alpha$ decay of $^{16}\text{N}$ ($J^\pi=2^-$, $T_{1/2}=7.13$ s, BR=0.12%)
      $^{16}\text{N} \rightarrow \beta^- + ^{16}\text{O}^*(J^\pi=1^-) \rightarrow \alpha + ^{12}\text{C}$
**ASTROPHYSICAL S-FACTOR $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$**

- Define *S-Factor* to remove both 1/E dependence of nuclear cross sections and Coulomb barrier transmission probability:

$$S \equiv E_{CM} \sigma(\alpha, \gamma) e^{2\pi \eta}$$

$$\eta = \frac{1}{137} Z_\alpha Z_\text{C}^{12} \sqrt{\frac{m_{12} C_\alpha}{2E_{CM}}}$$

<table>
<thead>
<tr>
<th>Author</th>
<th>$S_{\text{tot}}(300)$ (keV b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hammer (2005)</td>
<td>162±39</td>
</tr>
<tr>
<td>Kunz (2001)</td>
<td>165±50</td>
</tr>
</tbody>
</table>

R-matrix Extrapolation to stellar helium burning at E = 300 keV
**TIME REVERSAL REACTION**

Bubble Chamber experiment measures total cross section, $E_1 + E_2$

- We can separate $E_1$ and $E_2$ if we use linearly polarized $\gamma$ but cannot measure $\alpha$ and $^{12}\text{C}$ angular distribution

Stellar helium burning at $E = 300$ keV
**Reciprocity Relation: (γ,α) and (α,γ)**

- **A(α, γ)B:**
  \[ \sigma_{Bγ}^{j→i}(E_γ) = \frac{(2J_i + 1)(2J_α + 1)}{2J_j + 1} \frac{m_{Aα}c^2E_{Aα}}{E_γ^2} \sigma_{Aα}^{i→j}(E_{Aα}) \]

- \[ m_{Aα}c^2 = \frac{M(^{12}C) \cdot M(α)}{M(^{12}C) + M(α)} = 2796 \text{ MeV} \]
  \[ J_i = 0, J_j = 0, J_α = 0 \]

- \[ E_{Aα} = E_{CM} \]

- \[ E_{CM} = \sqrt{m_B^2 + 2E_γm_B - m_B - Q} \]

- \[ E_γ ≈ E_{CM} + Q \]

- \[ Q = m_A + m_α - m_B = +7.162 \text{ MeV} \]

- \[ \sigma(γ,α)(E_γ) = \frac{m_{Aα}c^2E_{CM}}{E_γ^2} \sigma(α,γ)(E_{CM}) \]

- \( \sigma(γ,α) \) is over two orders of magnitude larger than \( \sigma(α, γ) \)
**NEW APPROACH: REVERSAL REACTION + BUBBLE CHAMBER**

- Extra gain (factor of 100) by measuring time reversal reaction

\( \gamma + {}^{16}\text{O} \rightarrow {}^{12}\text{C} + \alpha \)

- Target density up to \(10^4\) higher than conventional targets. Number of \(^{16}\text{O}\) nuclei = \(3.5 \times 10^{22}\) /cm\(^2\) (3.0 cm cell)

- Measures total cross section \(\sigma_{\text{tot}}\) (or \(S_{\text{tot}}\))

- Solid Angle and Detector Efficiency = 100%

- Electromagnetic debris (electrons and gammas, or positrons) do NOT trigger nucleation (detector is insensitive to \(\gamma\)-rays by at least 1 part in \(10^{11}\)).

- Monochromatic \(\gamma\) beam at HIGS \(\approx 10^{7-8} \gamma/\text{s}\)

- Bremsstrahlung at JLab \(\approx 10^9 \gamma/\text{s}\) (top 250 keV)
**The Bubble Chamber**

- Donald Glaser won Nobel Prize for inventing chamber to detect particles (1960)
- Now being used in Dark Matter Search Experiments: COUPP, PICASSO, SIMPLE
- **Superheat Preparation:**
  - Liquid is pressurized at ambient temperature (1 to 2)
  - Then pressure is kept constant while temperature is increased to above boiling point (2 to 3)
  - Finally pressure is slowly released while keeping temperature constant (3 to 4)
  - At this point (4), still liquid but now superheated
- **Bubble Formation:**
  - Particle energy loss will induce vaporization
  - Resultant vapor bubble is observable either **visibly** or **audibly**
  - Bubble growth is captured by a digital camera
  - Pressure is increased (4 to 3) to quench bubble. It takes about few seconds for liquid to return to a stable state
  - Superheat is restored by releasing pressure again (3 to 4), and cycle is repeated for each bubble event
**Bubble Growth and Quenching**

$^{19}\text{F}(\gamma, \alpha)^{15}\text{N}$ event in $\text{C}_4\text{F}_{10}$

100 Hz Digital Camera: $\Delta t = 10$ ms

3.0 cm
**Bubble Chamber Principle**

I. For bubble formation, particle must be over thresholds in both $E$ and $dE/dx$.

II. Only bubbles with $r > R_c$ grow to be macroscopic.

   $$R_c = \frac{2s}{(P_v - P_f)}$$

   $s$: Surface tension

III. Bubble requires minimum deposited energy ($E_c$) within minimum distance $L_c$ (=a$R_c$, 10s of nm to a few μm).

   $$\frac{dE}{dx} > \left( \frac{dE}{dx} \right)_c = \frac{E_c}{\alpha R_c}$$

   $\alpha$: Free parameter (to determined experimentally)

   $$E \geq E_c = \frac{4}{3} \pi R_c^3 (\rho h + P_i) + 4 \pi R_c^2 \left( s - T \frac{\partial s}{\partial T} \right)$$
**Efficiency Curve**

N\(_2\)O thresholds
Superheat = 3.3 °C

N\(_2\)O efficiency curve, HIGS April 2013,  \(E_{\gamma} = 9.7\) MeV
ACOUSTIC SIGNAL DISCRIMINATION

I. Bubble growth produces an audible click which is recorded by piezo-electric transducers

II. Neutron Events:
   I. $^{17}\text{O}(\gamma,n)^{16}\text{O}$
   II. Neutron–nucleus elastic scattering:
        $^{16}\text{O}(n,n)$, $^{14}\text{N}(n,n)$

Ions $^{16}\text{O}$ or $^{14}\text{N}$ will generate a single bubble

III. Alpha Events:
   I. $^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$
   II. $^{17}\text{O}(\gamma,\alpha)^{13}\text{C}$
   III. $^{18}\text{O}(\gamma,\alpha)^{14}\text{C}$

Ions $^{12}\text{C}+\alpha$ or $^{13}\text{C}+\alpha$ or $^{14}\text{C}+\alpha$ will generate a combined multi-bubble

Suppress neutron events by 100 using acoustic signal
**Bubble Chamber at HIGS**

I. High Intensity Gamma Source (HIGS) at Duke University

II. $\gamma$-rays generated by Compton backscattering of free-electron-laser (FEL) light from high-energy electron beam bunches
Measuring $^{19}\text{F}(\gamma,\alpha)^{15}\text{N}$ at HIGS

$C_4\text{F}_{10}$ Bubble Chamber
$T = 30^\circ\text{C}$
$P = 3$ atm

2 Fast Digital Cameras
First determination of an astrophysical cross section with a bubble chamber: The $^{15}\text{N}(\alpha, \gamma)^{19}\text{F}$ reaction

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$^f$ Triangle Universities Nuclear Laboratory, Durham, NC 27708, USA
Bremsstrahlung Background at HIGS

Electron Beam Energy: 400 MeV
Electron Beam Current: 41 mA
Interaction Length: 35 m
Vacuum: $2 \times 10^{-10}$ Torr
Residual Gas: $Z = 10$

Strong Bremsstrahlung Background
(when coupled with large cross sections at high energies)
RECENT WORK

N$_2$O Bubble Chamber
T = 5°C
P = 60 atm

First $\gamma + O \rightarrow \alpha + C$ bubble
April 2013
Superheated Targets

I. List of superheated liquids to be used in experiment:

<table>
<thead>
<tr>
<th>N₂O Targets</th>
<th>¹⁶O</th>
<th>¹⁷O</th>
<th>¹⁸O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Target</td>
<td>99.757%</td>
<td>0.038%</td>
<td>0.205%</td>
</tr>
<tr>
<td>¹⁶O Target</td>
<td></td>
<td>Depleted &gt; 5,000</td>
<td>Depleted &gt; 5,000</td>
</tr>
<tr>
<td>¹⁷O Target</td>
<td></td>
<td>Enriched &gt; 80%</td>
<td>&lt;1.0%</td>
</tr>
<tr>
<td>¹⁸O Target</td>
<td></td>
<td>&lt;1.0%</td>
<td>Enriched &gt; 80%</td>
</tr>
</tbody>
</table>

II. Readout:
   I. Fast Digital Camera
   II. Acoustic Signal to discriminate between neutron and alpha events
EXPERIMENTAL SETUP AT JLAB INJECTOR

- BCM
- 5 MeV Dipole
- 5D Spectrometer
- 2D Spectrometer
- Mott Polarimeter
- Bubble Chamber location
5D Spectrometer

Bubble Chamber at HIGS April 2013

Photon Beam Entrance
BEAMLINE

New Fast Valve to protect from vacuum failure in front of ¼ Cryo-unit

Replace Dipole Magnet

2 Superharps to measure beam profile and absolute beam position
Schematics

- Power deposited in radiator (100 µA and 8.5 MeV):
  1. 0.02 mm: Energy loss = 21 keV, P = 2.1 W
  2. 0.10 mm: Energy loss = 112 keV, P = 11 W
- Pure Copper and Aluminum (high neutron threshold):
  1. $^{63}$C($\gamma$,n) threshold = 10.86 MeV
  2. $^{27}$Al($\gamma$,n) threshold = 13.06 MeV

I. Radiator motion and Sweep Dipole current must be in FSD
II. BCM0L02 and Electron Dump in Beam Loss Accounting (BLA)

Use GEANT4 to design this region

Electron K.E.
3.0 – 8.5 MeV
0.01 – 100 µA

Al Beam Pipe

Sweep Magnet

Cu Radiator 0.02 mm

Cu Vacuum Window 10 mil

Cu Photon Collimator

Cu Inner Pipe

Cu Electron Dump 2 kW (200 µA and 10 MeV)

Bubble Chamber Superheated N$_2$O
3 cm long
5°C, 60 atm

Al Photon Dump 10 cm long
20 cm diameter
Beam Requirements

I. Beam Properties at Radiator:

<table>
<thead>
<tr>
<th>Beam Kinetic Energy, (MeV)</th>
<th>7.9–8.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Current (µA)</td>
<td>0.01–100</td>
</tr>
<tr>
<td>Absolute Beam Energy Uncertainty</td>
<td>&lt;0.1%</td>
</tr>
<tr>
<td>Relative Beam Energy Uncertainty</td>
<td>&lt;0.02%</td>
</tr>
<tr>
<td>Energy Resolution (Spread), $\sigma_T/T$</td>
<td>&lt;0.06%</td>
</tr>
<tr>
<td>Beam Size, $\sigma_{x,y}$ (mm)</td>
<td>1–2</td>
</tr>
</tbody>
</table>

II. PEPPo achieved $p=8.25$ MeV/c or K.E. = 7.75 MeV

III. January 2014, achieved $p=9.1$ MeV/c or K.E. = 8.6 MeV for one hour. Still, suffer from trips due to wave guide vacuum faults; may improve with more processing.

IV. We may also need to helium process the ¼-cryounit
MEASURING ABSOLUTE BEAM ENERGY

5 MeV Dipole

Electron Beam Momentum

\[ p = \frac{\int Bdl}{\theta} \]
I. Jay Benesch designed and is now working with Engineering to fabricate a more uniform and higher field dipole

II. New Hall Probe: 0.01% accuracy, resolution to 2 ppm, and a temperature stability of 10 ppm/°C

III. Better shielding of stray magnetic fields

IV. Additional goal: Relative beam energy uncertainty <0.02%
**Bremsstrahlung Beam**

- Use both GEANT4 and FLUKA to calculate Bremsstrahlung spectra (we will not measure Bremsstrahlung spectra)

- Monte Carlo simulation of Bremsstrahlung at radiotherapy energies is well studied, accuracy: ±5%

$^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$ is ideal case for Bremsstrahlung beam and Penfold–Leiss Unfolding:

1. Very steep; only photons near endpoint contribute to yield
2. No-structure (resonances)
GEANT4 SIMULATION

- Both GEANT4 and FLUKA use models that calculate wrong photo-nuclear cross sections. Both do not allow for user’s cross sections. What to do?
  I. Use GEANT4 and FLUKA to produce the photon spectra impinging on the superheated liquid.
  II. Fold the above photon spectra with our cross sections in stand-alone codes.

- Use GEANT4 to design radiator, collimator, and dumps

- Geometry in GEANT4:
Penfold-Leiss Cross Section Unfolding

- Measure yields at: $E = E_1, E_2, \ldots, E_n$ where, $E_i - E_{i-1} = \Delta, i = 2, n$

$$Y(E_i) = \int_{th}^{E_i} n_\gamma(E_i, k)\sigma(k)dk \approx \sum_{j=1}^{i} N_\gamma(E_i, \Delta, E_j)\sigma(E_j)$$

Volterra Integral Equation of First Kind

- The solution can be written in two forms:

$$\sigma_i = \frac{1}{N_{ii}} \left[ y_i - \sum_{j=1}^{i-1} (N_{ij}\sigma_j) \right]$$

- Or, Matrix form:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} N_{\gamma,11} & 0 & \cdots & 0 \\ N_{\gamma,21} & N_{\gamma,22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ N_{\gamma,n1} & N_{\gamma,n2} & \cdots & N_{\gamma,nn} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_n \end{bmatrix}$$

$$[Y] = [N] \bullet [\sigma]$$

$$[\sigma] = [N]^{-1} \bullet [Y]$$
STATISTICAL ERROR PROPAGATION

• Note: 
  \[ \frac{dy_i}{y_i} = \frac{1}{\sqrt{y_i}} \quad \frac{dN_{ij}}{N_{ij}} = \frac{1}{\sqrt{N_{ij}}} \approx 0 \]

  \[ dy_i = \sqrt{y_i} \quad dy_i = \sqrt{y_i + 2y_i^{bg}} \]

• With:
  \[ [B] = [N]^{-1} \]
  \[ [\sigma] = [B] \bullet [Y] \]

• Then:
  \[ [d\sigma^2] = [B] \bullet [dY^2] \bullet [B]^T \]
Where:

\[
\begin{bmatrix}
    y_1 & 0 & \cdots & 0 \\
    0 & y_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & y_n
\end{bmatrix}
\]

\[
\begin{bmatrix}
    d\sigma_1^2 & \text{cov}(\sigma_1, \sigma_2) & \cdots & \text{cov}(\sigma_1, \sigma_n) \\
    \text{cov}(\sigma_2, \sigma_1) & d\sigma_2^2 & \cdots & \text{cov}(\sigma_2, \sigma_n) \\
    \vdots & \vdots & \ddots & \vdots \\
    \text{cov}(\sigma_n, \sigma_1) & \text{cov}(\sigma_n, \sigma_2) & \cdots & d\sigma_n^2
\end{bmatrix}
\]

\[
(d\sigma_i)^2 = \frac{1}{N_{ii}^2} \left[ dy_i^2 + \sum_{j=1}^{i-1} (N_{ij} d\sigma_j)^2 + \sum_{k=1}^{i-1} \sum_{l=1}^{i-1} N_{ik} \text{cov}(\sigma_k, \sigma_l) N_{il} \right]
\]

For mono-chromatic photon beam

\[
\left( \frac{d\sigma_i}{\sigma_i} \right)^2 = \left( \frac{dy_i}{y_i} \right)^2 = \frac{1}{y_i}
\]

\[
\text{var}(y_i, y_i) = y_i \\
\text{cov}(y_i, y_j) = 0 \\
\text{cov}(\sigma_i, \sigma_j) \neq 0
\]
I. Radiator thickness = 0.02 mm

II. Bubble Chamber thickness = 3.0 cm, number of $^{16}\text{O}$ nuclei = $3.474 \times 10^{22} /\text{cm}^2$

III. Background subtraction of $^{18}\text{O}(\gamma, \alpha)^{14}\text{C}$

\[
[N] = \begin{bmatrix}
3.267e14 & 0 & 0 & 0 & 0 & 0 & 0 \\
9.782e13 & 6.439e13 & 0 & 0 & 0 & 0 & 0 \\
5.013e13 & 3.858e13 & 2.539e13 & 0 & 0 & 0 & 0 \\
1.494e13 & 1.236e13 & 9.514e12 & 6.258e12 & 0 & 0 & 0 \\
8.540e12 & 7.369e12 & 6.097e12 & 4.692e12 & 3.086e12 & 0 & 0 \\
3.801e12 & 3.370e12 & 2.908e12 & 2.406e12 & 1.852e12 & 1.217e12 & 0 \\
2.075e12 & 1.875e12 & 1.663e12 & 1.435e12 & 1.187e12 & 9.137e11 & 6.004e11
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>Electron Beam K. E.</th>
<th>Beam Current (µA)</th>
<th>Time (hour)</th>
<th>$y_i$</th>
<th>$dy_i$ (no bg)</th>
<th>$dy_i/y_i$ (no bg, %)</th>
<th>$dy_i$ (with bg)</th>
<th>$dy_i/y_i$ (with bg, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>100</td>
<td>100</td>
<td>545</td>
<td>23</td>
<td>4.2</td>
<td>134</td>
<td>24.6</td>
</tr>
<tr>
<td>8.0</td>
<td>100</td>
<td>20</td>
<td>581</td>
<td>24</td>
<td>4.1</td>
<td>77</td>
<td>13.3</td>
</tr>
<tr>
<td>8.1</td>
<td>80</td>
<td>10</td>
<td>852</td>
<td>29</td>
<td>3.4</td>
<td>60</td>
<td>7.0</td>
</tr>
<tr>
<td>8.2</td>
<td>20</td>
<td>10</td>
<td>634</td>
<td>25</td>
<td>3.9</td>
<td>40</td>
<td>6.3</td>
</tr>
<tr>
<td>8.3</td>
<td>10</td>
<td>10</td>
<td>812</td>
<td>28</td>
<td>3.4</td>
<td>39</td>
<td>4.8</td>
</tr>
<tr>
<td>8.4</td>
<td>4</td>
<td>10</td>
<td>746</td>
<td>27</td>
<td>3.6</td>
<td>36</td>
<td>4.8</td>
</tr>
<tr>
<td>8.5</td>
<td>2</td>
<td>10</td>
<td>763</td>
<td>28</td>
<td>3.7</td>
<td>32</td>
<td>4.2</td>
</tr>
</tbody>
</table>
SYSTEMATIC ERROR PROPAGATION

• For absolute beam energy uncertainty of $\delta E$ (= 0.1%) and zero relative beam energy uncertainty:

$$\frac{dy_i}{y_i} = \frac{y_i(E_i + \delta E) - y_i(E_i)}{y_i(E_i)}$$

$$\frac{dN_{ij}}{N_{ij}} = \frac{N_{ij}(E_i + \delta E) - N_{ij}(E_i)}{N_{ij}(E_i)}$$

$$E_0 = 7.8 + \delta E$$

$$E_i = E_0 + i\Delta$$

<table>
<thead>
<tr>
<th>$E_i$ (MeV)</th>
<th>$dy_i/y_i$ (%)</th>
<th>$d\sigma_i/\sigma_i$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>12.5</td>
<td>12.6</td>
</tr>
<tr>
<td>8.0</td>
<td>10.8</td>
<td>10.5</td>
</tr>
<tr>
<td>8.1</td>
<td>9.3</td>
<td>9.1</td>
</tr>
<tr>
<td>8.2</td>
<td>8.0</td>
<td>7.1</td>
</tr>
<tr>
<td>8.3</td>
<td>7.0</td>
<td>6.3</td>
</tr>
<tr>
<td>8.4</td>
<td>6.3</td>
<td>5.8</td>
</tr>
<tr>
<td>8.5</td>
<td>5.6</td>
<td>5.2</td>
</tr>
</tbody>
</table>

• Accounted for $dN_{ij}$ due to energy error when calculating $dy_i$

This is the cross section dependence on energy
\[
\[dN_{ij} / N_{ij}\] = \begin{bmatrix}
0.100 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.058 & 0.050 & 0 & 0 & 0 & 0 & 0 \\
0.041 & 0.039 & 0.033 & 0 & 0 & 0 & 0 \\
0.031 & 0.031 & 0.029 & 0.025 & 0 & 0 & 0 \\
0.025 & 0.025 & 0.025 & 0.023 & 0.020 & 0 & 0 \\
0.021 & 0.021 & 0.021 & 0.021 & 0.020 & 0.017 & 0 \\
0.018 & 0.018 & 0.018 & 0.018 & 0.018 & 0.017 & 0.022 \\
\end{bmatrix}
\]

- With:

\[
[B] = [N]^{-1}
\]

\[
[\sigma] = [B] \bullet [Y]
\]

- Then:

\[
[d\sigma^2] = [B] \bullet \left( [dY^2] + [dN^2] \bullet [\sigma^2] \right) \bullet [B]^T
\]
• **Where:**

Note: Correlation Coefficient \((\rho_{ij}) = 1\)

\[
[dY^2] = \begin{bmatrix}
(dy_1)^2 & dy_1 dy_2 & \cdots & dy_1 dy_n \\
(dy_2 dy_1) & (dy_2)^2 & \cdots & dy_n dy_n \\
\vdots & \vdots & \ddots & \vdots \\
(dy_n dy_1) & dy_n dy_2 & \cdots & (dy_n)^2
\end{bmatrix}
\]

\[
[d\sigma^2] = \begin{bmatrix}
d\sigma_1^2 & \text{cov}(\sigma_1, \sigma_2) & \cdots & \text{cov}(\sigma_1, \sigma_n) \\
\text{cov}(\sigma_2, \sigma_1) & d\sigma_2^2 & \cdots & \text{cov}(\sigma_2, \sigma_n) \\
\vdots & \vdots & \ddots & \vdots \\
\text{cov}(\sigma_n, \sigma_1) & \text{cov}(\sigma_n, \sigma_2) & \cdots & d\sigma_n^2
\end{bmatrix}
\]

\[
[dN^2] = \begin{bmatrix}
(dN_{11})^2 & 0 & \cdots & 0 \\
(dN_{21})^2 & (dN_{22})^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
(dN_{n1})^2 & (dN_{n2})^2 & \cdots & (dN_{nn})^2
\end{bmatrix}
\]

\[
[\sigma^2] = \begin{bmatrix}
\sigma_1^2 & 0 & \cdots & 0 \\
0 & \sigma_2^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_n^2
\end{bmatrix}
\]

\[
\text{var}(y_i, y_i) = (dy_i)^2 \\
\text{cov}(y_i, y_j) = \rho_{ij} dy_i dy_j
\]

No energy-to-energy change in systematic error
**Systematic Error Propagation**

\[
(d\sigma_i)^2 \approx \frac{1}{N_{ii}^2} \left[ dy_i^2 - 2dy_i \sum_{j=1}^{i-1} N_{ij} d\sigma_j \right. \\
+ \sum_{j=1}^{i-1} \left( N_{ij} d\sigma_j \right)^2 + \sum_{k=1}^{i-1} \sum_{l=1}^{i-1} N_{ik} \text{cov}(\sigma_k, \sigma_l) N_{il} \\
+ \sum_{j=1}^{i-1} \left( dN_{ij} \sigma_j \right)^2 + \left( dN_{ii} \sigma_i \right)^2 \left] \right.
\]

\[\text{cov}(y_i, y_j) \neq 0, \quad \text{cov}(\sigma_i, \sigma_j) \neq 0\]
### Other Systematic Errors

<table>
<thead>
<tr>
<th>Description</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Current, $\delta I/I$</td>
<td>3%</td>
</tr>
<tr>
<td>Photon Flux, $\delta \varphi/\varphi$</td>
<td>5%</td>
</tr>
<tr>
<td>Radiator Thickness, $\delta R/R$</td>
<td>3%</td>
</tr>
<tr>
<td>Bubble Chamber Thickness, $\delta T/T$</td>
<td>3%</td>
</tr>
<tr>
<td>Bubble Chamber Efficiency, $\varepsilon$</td>
<td>5%</td>
</tr>
</tbody>
</table>

Then:

\[
(dy_i)^2 = (dy_i (\delta E))^2 + \left[ \left( \frac{\delta I}{I} \right)^2 + \left( \frac{\delta R}{R} \right) + \left( \frac{\delta T}{T} \right)^2 + \varepsilon^2 \right] y_i^2
\]

\[
(dN_{ij})^2 = \left( \frac{\delta \phi}{\phi} \right)^2 N_{ij}^2
\]
<table>
<thead>
<tr>
<th>Electron Beam K. E.</th>
<th>Cross Section (nb)</th>
<th>Stat Error (no bg, %)</th>
<th>Stat Error (with bg, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>0.046</td>
<td>4.4</td>
<td>24.5</td>
</tr>
<tr>
<td>8.0</td>
<td>0.185</td>
<td>6.0</td>
<td>20.7</td>
</tr>
<tr>
<td>8.1</td>
<td>0.58</td>
<td>6.3</td>
<td>14.7</td>
</tr>
<tr>
<td>8.2</td>
<td>1.53</td>
<td>8.2</td>
<td>13.8</td>
</tr>
<tr>
<td>8.3</td>
<td>3.49</td>
<td>9.1</td>
<td>13.3</td>
</tr>
<tr>
<td>8.4</td>
<td>7.2</td>
<td>10.6</td>
<td>13.8</td>
</tr>
<tr>
<td>8.5</td>
<td>13.6</td>
<td>12.2</td>
<td>14.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Electron Beam K. E.</th>
<th>Cross Section (nb)</th>
<th>Sys Error (Energy, %)</th>
<th>Sys Error (Total, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>0.046</td>
<td>12.5</td>
<td>15.3</td>
</tr>
<tr>
<td>8.0</td>
<td>0.185</td>
<td>10.2</td>
<td>13.5</td>
</tr>
<tr>
<td>8.1</td>
<td>0.58</td>
<td>8.3</td>
<td>12.2</td>
</tr>
<tr>
<td>8.2</td>
<td>1.53</td>
<td>7.0</td>
<td>11.4</td>
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<tr>
<td>8.3</td>
<td>3.49</td>
<td>6.0</td>
<td>10.7</td>
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<tr>
<td>8.4</td>
<td>7.2</td>
<td>5.3</td>
<td>10.5</td>
</tr>
<tr>
<td>8.5</td>
<td>13.6</td>
<td>4.7</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Note: Relative systematic errors do not get magnified in PL Unfolding
JLab PROJECTED $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ S-Factor

- Statistical Error: dominated by background subtraction from $^{18}\text{O}(\gamma,\alpha)^{14}\text{C}$ (depletion = 5,000)

<table>
<thead>
<tr>
<th>Electron Beam K. E.</th>
<th>Gamma Energy (MeV)</th>
<th>$E_{CM}$ (MeV)</th>
<th>Cross Section (nb)</th>
<th>$S_{tot}$ Factor (keV b)</th>
<th>Stat Error (%)</th>
<th>Sys Error (Total, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>7.85</td>
<td>0.69</td>
<td>0.046</td>
<td>62.2</td>
<td>24.5</td>
<td>15.3</td>
</tr>
<tr>
<td>8.0</td>
<td>7.95</td>
<td>0.79</td>
<td>0.185</td>
<td>48.7</td>
<td>20.7</td>
<td>13.5</td>
</tr>
<tr>
<td>8.1</td>
<td>8.05</td>
<td>0.89</td>
<td>0.58</td>
<td>41.8</td>
<td>14.7</td>
<td>12.2</td>
</tr>
<tr>
<td>8.2</td>
<td>8.15</td>
<td>0.99</td>
<td>1.53</td>
<td>35.5</td>
<td>13.8</td>
<td>11.4</td>
</tr>
<tr>
<td>8.3</td>
<td>8.25</td>
<td>1.09</td>
<td>3.49</td>
<td>32.0</td>
<td>13.3</td>
<td>10.7</td>
</tr>
<tr>
<td>8.4</td>
<td>8.35</td>
<td>1.19</td>
<td>7.2</td>
<td>28.8</td>
<td>13.8</td>
<td>10.5</td>
</tr>
<tr>
<td>8.5</td>
<td>8.45</td>
<td>1.29</td>
<td>13.6</td>
<td>26.3</td>
<td>14.8</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Bubble Chamber experiment measures total S-Factor, $S_{E1} + S_{E2}$
BACKGROUNDs

I. Background from oxygen isotopes and nitrogen in $N_2O$:
   - $^{18}O(\gamma,\alpha)^{14}C$
   - $^{17}O(\gamma,\alpha)^{13}C$
   - $^{14}N(\gamma,p)^{13}C$

- Natural Abundance:
  I. $^{17}O$: 0.038%
  II. $^{18}O$: 0.205%

- Expected Rates:
  I. $^{17}O(\gamma,\alpha)^{13}C$, depletion=5,000
  II. $^{18}O(\gamma,\alpha)^{14}C$, depletion=5,000
  III. $^{14}N(\gamma,p)^{13}C$, Chamber eff. = $10^{-8}$
II. Background from:
   – $^{17}\text{O}(\gamma,n)^{16}\text{O}$ and secondary (n,n) neutron–nucleus elastic scattering

III. Background from Chamber glass:
   – Neutron–nucleus elastic scattering from $^{29}\text{Si}(\gamma,n)^{28}\text{Si}$

IV. Cosmic–ray background:
   – $\mu^+$–nuclear
   – neutron–nuclear elastic scattering

➢ Reject neutron events using acoustic signal (100 suppression factor)
**ION ENERGY DISTRIBUTIONS**

- Use depleted N\(_2\)O:
  - I. \(^{17}\)O depletion = 5,000
  - II. \(^{18}\)O depletion = 5,000

- Suppress background with Bubble Chamber thresholds

\[ E_{CM} \approx E_{\gamma} - Q \]
\[ E_{CM} = T_{\alpha} + T_{C} \]

\[ T_{\alpha,lab} \approx \frac{m_{c}}{m_{\alpha} + m_{c}} E_{CM} \]
\[ T_{C,lab} \approx \frac{m_{\alpha}}{m_{\alpha} + m_{c}} E_{CM} \]

- Threshold Efficiency (function of superheat):

<table>
<thead>
<tr>
<th>Particle</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(^{\pm})</td>
<td>&lt;10(^{-11})</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>&lt;10(^{-11})</td>
</tr>
<tr>
<td>(^{14})N((\gamma,p))(^{13})C</td>
<td>&lt;10(^{-8})</td>
</tr>
</tbody>
</table>

No acoustic cut
SAFETY

- Superheated liquid: $\text{N}_2\text{O}$, Nitrous oxide (laughing gas)
  1. At room temperature, it is colorless, non-flammable gas, with slightly sweet odor and taste

- High pressure system:
  1. Design Authority: Dave Meekins
  2. $T = 5^\circ\text{C}$
  3. $P = 60\ \text{atm}$

- Buffer liquid: Mercury
  1. Closed system
  2. Volume: 135 mL
SUMMARY AND OUTLOOK

• More testing of N$_2$O Bubble Chamber at HIGS (Spring 2014)
• Measure cross sections of $^{18}$O($\gamma$, $\alpha$)$^{14}$C and $^{17}$O($\gamma$, $\alpha$)$^{13}$C at HIGS (Fall 2014)
• Test Bubble Chamber at JLab with Bremsstrahlung beam (2015)
• Run depleted N$_2$O bubble chamber at JLab to measure $^{16}$O($\gamma$, $\alpha$)$^{12}$C

• Beam issues:
  – Design radiator, collimator, and dumps with GEANT4
  – Simulate photon spectra with GEANT4 and FLUKA
  – Deliver 8.5 MeV K.E. beam to 5D Spectrometer with <0.1% absolute energy uncertainty

• Bubble Chamber issues:
  – Study acoustic signal and measure neutron events suppression factor
  – Deadtime measurements (now $\tau \pm \delta\tau = 10.0 \pm 0.9$ sec)
  – Measure O-isotopes depletion

• Background tests:
  – Measure cosmic-ray background
  – Study chamber thresholds efficiency vs. superheat and measure $\gamma$-rays suppression factor
Cost Estimate

I. New beamline components:
   I. New Dipole Magnet and Hall Probe
   II. 2 Super Harps
   III. Fast Valve

II. Summary of labor cost by group:

<table>
<thead>
<tr>
<th>Group</th>
<th>Labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey &amp; Alignment</td>
<td>3 wks x 2</td>
</tr>
<tr>
<td>Magnet Test</td>
<td>1 wk x 2</td>
</tr>
<tr>
<td>Engineering Design</td>
<td>16 wks</td>
</tr>
<tr>
<td>Software</td>
<td>3 wks x 2</td>
</tr>
<tr>
<td>EES</td>
<td>6 wk x 2</td>
</tr>
<tr>
<td>EH&amp;Q</td>
<td>4 wks</td>
</tr>
<tr>
<td>Item</td>
<td>Material Procurement</td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>New Dipole Magnet</td>
<td>Dipole Magnet ($8,000) Hall Probe System ($10,000)</td>
</tr>
<tr>
<td>New Beamline</td>
<td>2 Super Harps (20,000) Fast Valve ($23,000)</td>
</tr>
<tr>
<td>Radiator (cooled ladder, FSD)</td>
<td>0.02 and 0.10 mm Cu foils ($2,000)</td>
</tr>
<tr>
<td>Sweep Dipole</td>
<td></td>
</tr>
<tr>
<td>Electron Dump</td>
<td>Pure Cu ($5,000)</td>
</tr>
<tr>
<td>Cu Collimator</td>
<td>Pure Cu ($5,000)</td>
</tr>
<tr>
<td>Photon Dump &amp; Stand</td>
<td>Pure Al ($3,000)</td>
</tr>
<tr>
<td>Safety Review</td>
<td></td>
</tr>
<tr>
<td>Install</td>
<td></td>
</tr>
<tr>
<td>Bubble Chamber</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$76,000</td>
</tr>
<tr>
<td>Indirect G&amp;A (55.65%)</td>
<td>$42,300</td>
</tr>
<tr>
<td>Indirect Stat &amp; Fringe (57.15%)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$118,300</td>
</tr>
</tbody>
</table>
CO2 SUPERHEATED LIQUID?

- Similar Bubble Chamber operational parameters as N₂O
- Natural Abundance: ¹³C: 1.07%
- Depletion: ¹³C depletion=1,000
- ¹³C(γ,n)¹²C Background

For comparison, ¹⁷O(γ,n)¹⁶O

- ¹²C(γ,2α)α Background
WATER SUPERHEATED LIQUID?

- Etching of glass vessel by superheated H₂O
- T = 250°C
- P = 75 atm
- Background from secondary neutron–nucleus elastic scattering by neutrons from d(γ,n)p
ALL DIMENSIONS ARE IN mm
Rolfs and Rodney, 1988