Studies of the Resistive Wall Heating at JLAB FEL

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In collaboration with Steve Benson

Acknowledgement

- I thank Steve Benson for his heat deposit measurement on the FEL, and his long time encouragement and discussions which motivate this study

- I thank Dave Douglas, Chris Tennant , Pavel Evtushenko, Steve Benson, and the rest of the FEL team for our collaboration on the experimental studies of dynamics of high brightness e-beams at JLAB FEL

Outline

1. Introduction
2. Mechanism of resistive wall heating
3. Estimation using existing
4. Resistive wall impedance of two parallel plates for non-ultrarelativistic beams
5. Other possible contributing factors to the observed heating
6. Summary

1. Introduction

- Observation: Wiggler chamber heating during high power IR operation at JLAB FEL

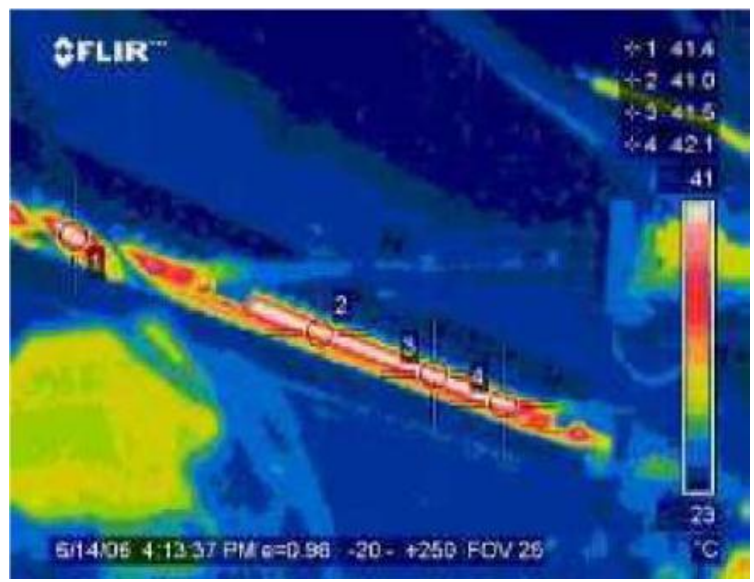
  -As current increase, electron beam seems to move around and mirrors would steer.

  -BPM and viewers move due to heating of wiggler chamber

  -Effect reduced with water cooling

I=3.5 mA (CW e-beam)
σ_z=150 fs or 45μm
E=115 MeV
f_b =37.5 MHz, Q_b=95 pC
⇩ Heat deposit
32 W/m/mA (stainless steel pipe)

I=2.5 mA, Q_b=120 pC
12 W/m/mA (copper coated pipe)



(JLAB Tech note, S. Benson, "Resistive Wall heating in the Jefferson Lab FEL wiggler chambers")

Resistive Wall Effects

- Interaction of the high peak current bunch with waveguide wall of finite conductivity
- Induce longitudinal and transverse resistive wake field, which may cause

  -bunch energy loss

  -wall heating

  -tune shift

  -transverse wake instabilities.
- The effect is more prominent for higher bunch charge density and smaller gap size.

JLAB FEL wiggler waveguide: rectangular geometry

-Wall thickness: 1mm

-tested with uncoated or coated copper (with 1.3um thickness, unpolished)

Different Regimes of Resistive Wall Effects

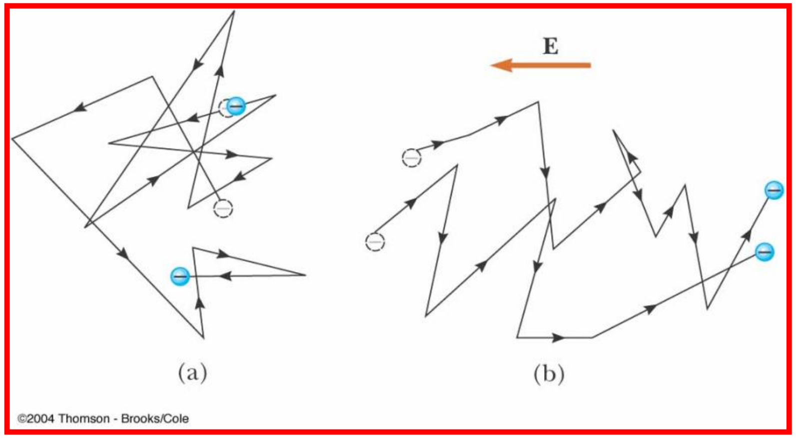
- High current beams with long bunch in storage rings: resistive wake at low frequency
- High peak current beams of short bunch in light sources: resistive wake at high frequency (THz or higher)
- At JLAB, resistive wall heating is the result of both high average current and high peak current of the beam.

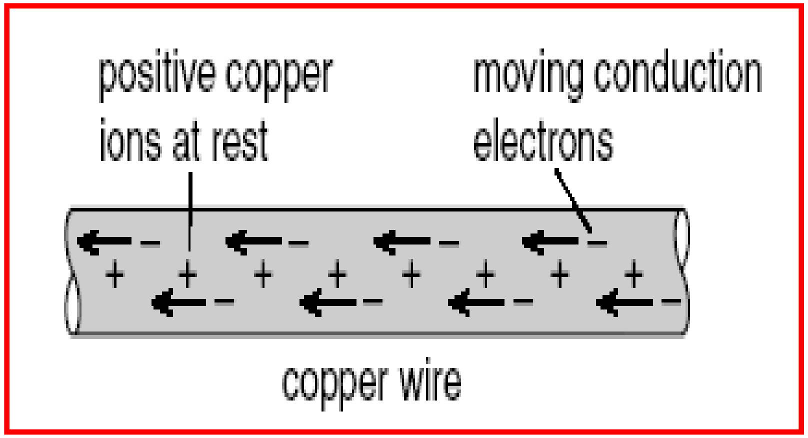
Importance for understanding the observed resistive wall heating

- Important for scale-up designs of higher power FELs
- Shorter wavelength FEL needs either higher e- beam energy or shorter wiggler period. Shorter wiggler period will need narrower gap size, meaning higher resistive wall effects.
- May cause performance limitation to high intensity hadron machine, such as LHC Proton Synchrotron and its booster (also collimators in LHC), JPAC, Fermilab booster, and eRHIC and MEIC (long bunch and high current regime).

2. Mechanism of Resistive Wall Heating

- Drude model (1900, before QM and SR)
  - -Links electrical properties of metal to electron behavior
  - -Immobile ion cores of atoms, classical electron gas
  - -Electrons moving on straight line until collide with ion core





Damping: electron being scattered by ion core

$$m\ddot{\vec{x}} = e\vec{E} - \frac{m\vec{v}}{\tau} = 0, \quad \vec{v} = \frac{e\tau\vec{E}}{m}, \quad \vec{J} = ne\vec{v} = \sigma\vec{E}, \quad \sigma = \frac{ne^2\tau}{m}$$

Relaxation time and Mean Free Path

- Classical model $\sigma = \frac{ne^2\tau}{m}$,

τ – relaxation time
l – mean free path

$$\tau = \frac{l}{v_{rms}}$$

l: inter–atomic spacing, independent of T

v_{rms}: rms thermal velocity

However, measurement show that mean free path is an order of magnitude larger than inter-atom spacing and it depends on T

⇒ the scattering mechanism is not collisions of electrons with ions

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- Quantum model $\sigma = \frac{ne^2\tau}{m^*}$,

$$\tau = \frac{l}{v_F}$$

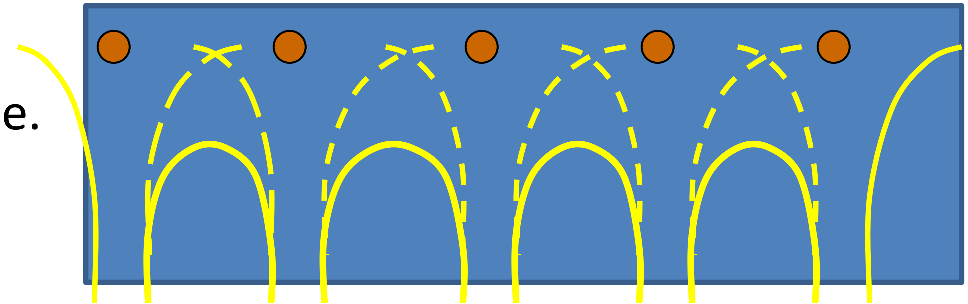
l: path length before scattered by ion thermal vibration or impurity

v_F: Fermi velocity

m^*: effective mass

Quantum Model

Valence electron wave moves freely
in the periodic potentials of the ion lattice.

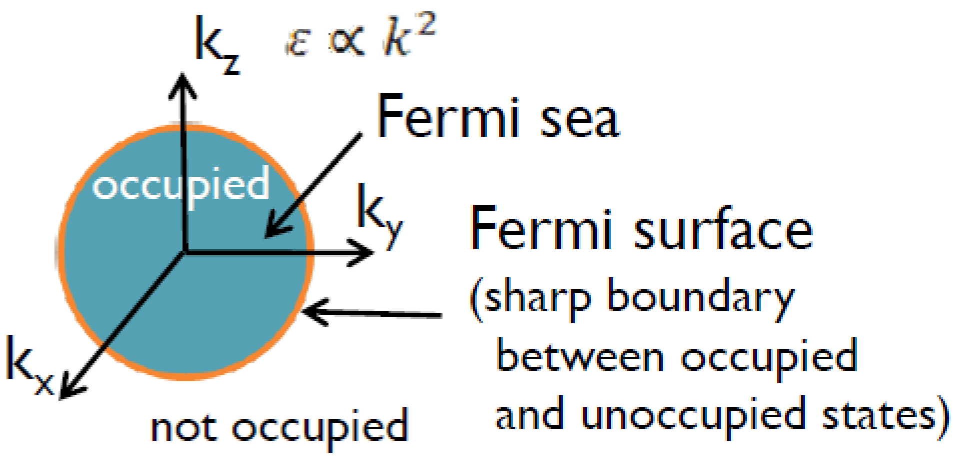


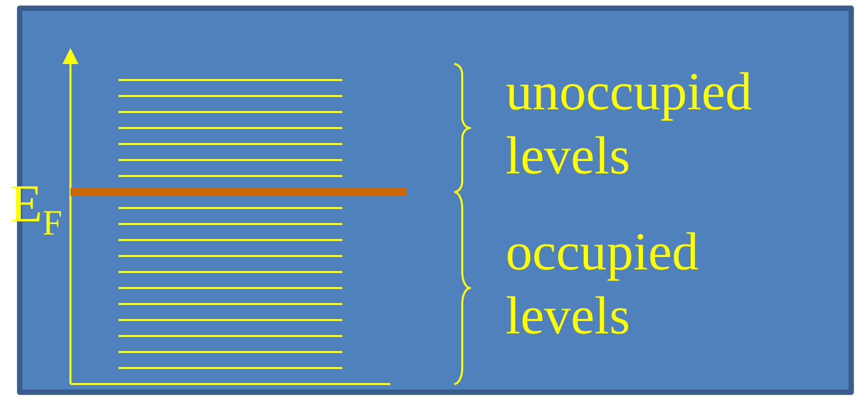
For a typical metal,

$$v_F \approx 0.01\, c \sim 10\, v_{rms}$$

only electrons near E_F contribute to the electrical conductivity

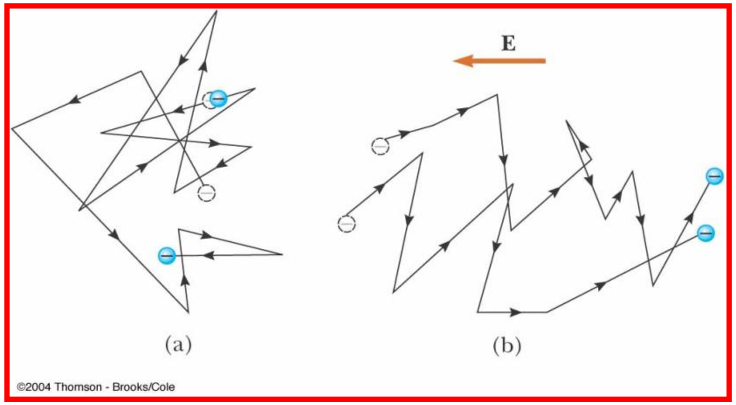
Fermi-Dirac Statistics:





AC Conductivity

When AC field is applied to metal, electron motion is given by



$$m\ddot{\vec{x}} = e\vec{E} - \frac{m\vec{v}}{\tau}$$

$$(\ \vec{E} \sim e^{-i\omega t}, \quad \vec{v} = \dot{\vec{x}} \sim e^{-i\omega t}\ )$$

$$\vec{J} = ne\vec{v} = \bar{\sigma}\vec{E},$$

DC: $\sigma = \frac{ne^2\tau}{m}$

AC: $\bar{\sigma} = \frac{\sigma}{1-i\omega\tau}$

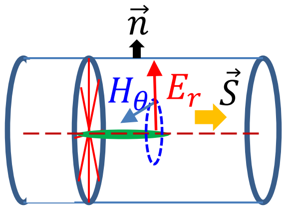
Conductivity $\bar{\sigma}$ is frequency dependent.

Energy Transfer

$$-\int dv\, J\cdot E = \int dv\, \nabla\cdot(E\times H) + \frac{d}{dt}\int dv\, \frac{1}{2}(E\cdot D + H\cdot B) = \int da\, \vec{S}\cdot\vec{n} + \frac{dU}{dt}$$

$(E\times H) \Rightarrow \vec{S}$, $\frac{1}{2}(E\cdot D + H\cdot B) \Rightarrow U$

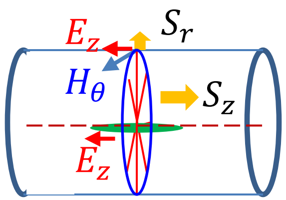
Perfect conducting pipe: Only transverse E_r and H_θ fields (for ultrarelativistic case $v\approx c$)



No loss of energy from beam: $E_z=0$, $J\cdot E=0$

Energy flux going forward: $\vec{S}\cdot\vec{n}=0$

Inside finite conducting pipe: $E_z\neq 0$

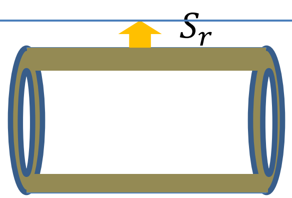


(steady state: dU/dt=0)

Beam kinetic energy loss$=-\int dv\, J\cdot E$

=Energy flux out of the volume$=\int da\, \vec{S}\cdot\vec{n}$ (into the wall and down to the pipe)

Inside conducting pipe wall: $\vec{J}=\sigma\vec{E}$, $E_z\neq 0$



Energy flux into the wall$=-\int da\, \vec{S}\cdot\vec{n}$

= kinetic energy loss of conducting electrons$=\int dv\, J\cdot E=\int dv\, \sigma|E|^2$

3. Estimation using Existing Theories

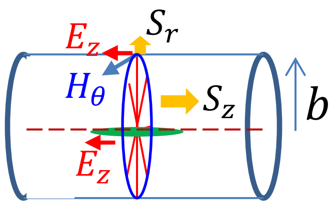
- Long and local range behavior of resistive wake field
- DC and AC conductivity
- Anomalous skin effect
- Non-ultrarelativistic effects
- Circular and flat chamber geometry

Generic FEL parameters are used for estimation:

I=4 mA (CW e-beam)
σ_z=150 fs or 45μm
E=135 MeV
f_b =37 MHz, Q_b=110 pC

Circular pipe

- Resistive wall of a circular pipe for beam with $v = c$ (Bane, Chao)



Approach: $I = qc\,\delta(z - vt)$

In frequency domain $e^{ik(z-vt)}$

-Write out Maxwell equations (8 equations) for

(I) the pipe enclosure ($r < b$)

(II) Inside metal wall ($\mathrm{r} \geq b, \vec{J} = \sigma\vec{E}$)

- Match field components (excluding E_r) at $r = b$

Results:

Inside metal wall , fields exponentially decay. $E_z = E_z(b)e^{i\lambda(r-b)}$

For ω>0, $\lambda = \frac{i+1}{\delta_{skin}}$, with skin depth $\delta_{skin} = \sqrt{\frac{2}{\mu\sigma\omega}}$

On the boundary, $Z_s = -E_z/H_\theta = (k/\lambda)Z_0$, Z_0=120 $\pi\Omega$

For FEL undulator : $\sigma_z \approx 45\ \mu\mathrm{m}$, $\omega \approx c/\sigma_z$,

copper: $\delta_{skin} \approx 63\ \mathrm{nm}$, stainless steel: $\delta_{skin} \approx 400\ \mathrm{nm}$

Resistive Wall Impedance of a Cylindrical Pipe

For r<b, the longitudinal E field is $E_z(r,\omega) = \; E_z(b,\omega) = \frac{Z_0}{2\pi b}\frac{qc}{\frac{ikb}{2} - \lambda/k}$

From Faraday law, $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ At low frequency, $\frac{\partial \vec{D}}{\partial t}$ negligible:

$$2\pi b\, H_\theta(b) = qc, \qquad E_z(b,\omega) = -(k/\kappa) Z_0 H_\theta(b) = \frac{Z_0}{-\lambda/k}\frac{qc}{2\pi b} \sim \frac{1}{b\sqrt{\omega\sigma}}$$

Characteristic length:

$$s_0 = \left(\frac{c\, b^2}{2\,\pi\sigma}\right)^{1/3}$$

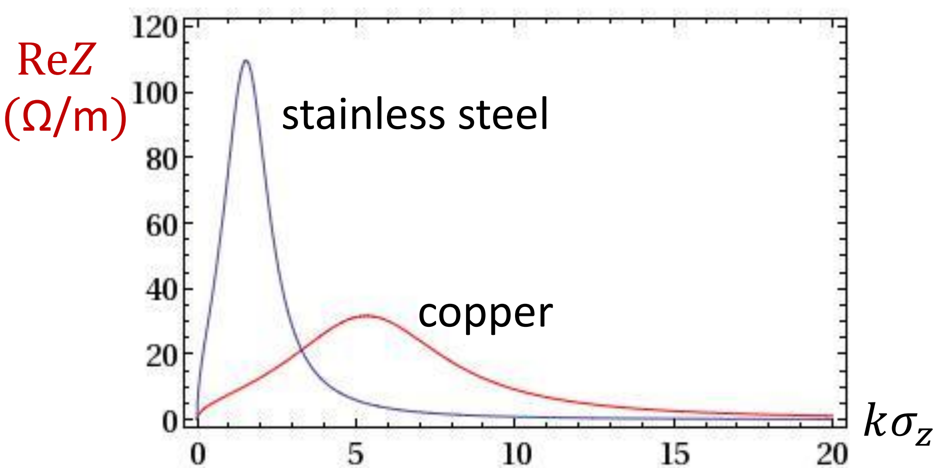
Complete longitudinal Impedance:

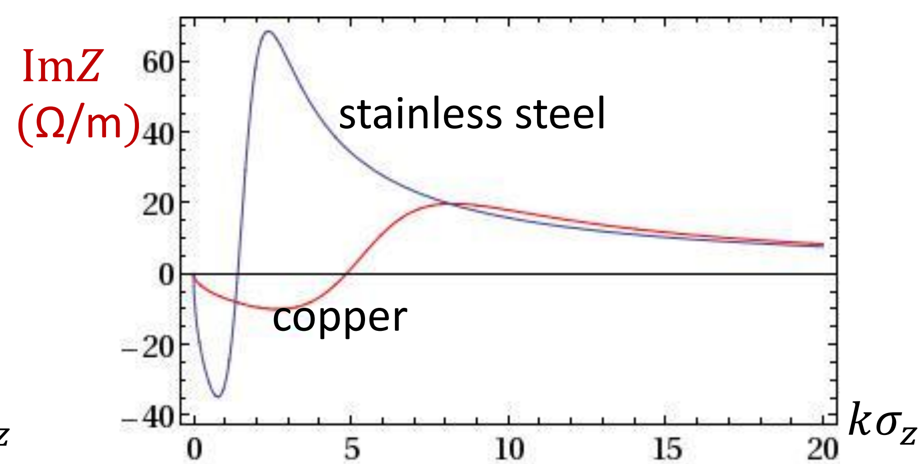
$$Z_{||}(\omega) = -\frac{E_z(r=0,\omega)}{qc} = \frac{Z_0}{2\pi b}\frac{1}{\frac{\lambda}{k} - \frac{ikb}{2}}$$

For b =6 mm, $\sigma_z \approx 45\mu\text{m}$,

Copper: $s_0 = 15\mu\text{m}$, $s_0/\sigma_z \approx 0.3$

Stainless steel: $s_0 = 51\mu\text{m}$, $s_0/\sigma_z \approx 1.1$



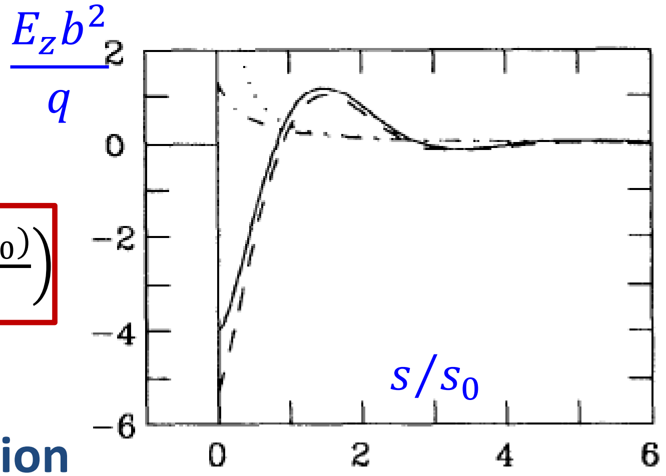


Local and long range Interaction (Bane, 1995)

- **Wake function following the source particle**

  (inverse Fourier transform of impedance)

$$E_z(s) \propto -\frac{q}{b^2}\left(\frac{1}{3}\, e^{-s/s_0} \cos\frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi}\int_0^\infty dx\, \frac{x^2 \exp\left(-x^2 s/s_0\right)}{x^6+8}\right)$$

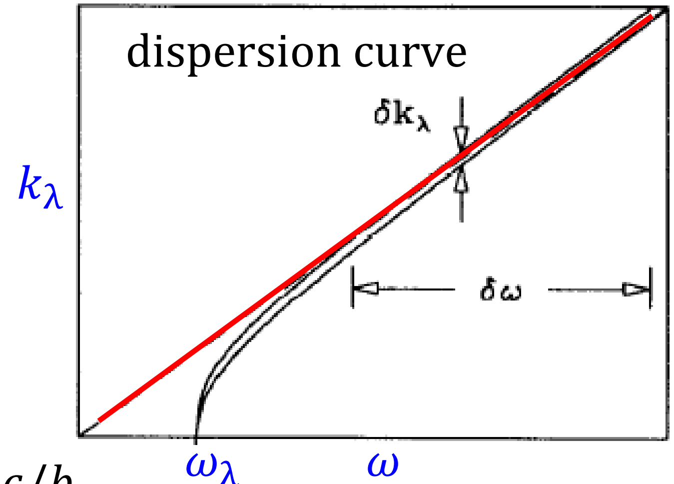


- **Approximate physical picture of damped oscillation**
  - In a perfectly conducting pipe, there is no interaction of the modes with the beam,

dispersion curve for mode:

$$k_\lambda^{(0)} = \frac{\omega}{c}\sqrt{1-(\omega_\lambda/\omega)^2}$$

(never cross the speed of light line)



- For wall with finite conductivity, the dispersion curve is modified by new boundary condition,

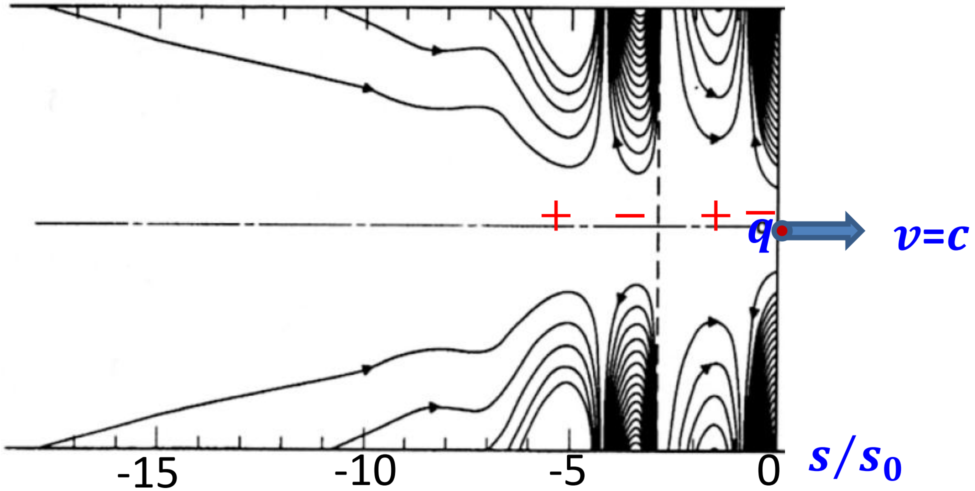
New dispersion curve for TM mode $\omega_\lambda = 2.4\, c/b$

$$\mathrm{Re}(k_\lambda) \approx \left[1 - \frac{1}{2}\left(\frac{\omega_\lambda}{\omega}\right)^2\right] + \frac{1}{2b}\sqrt{\frac{\omega}{2\pi\sigma}}$$

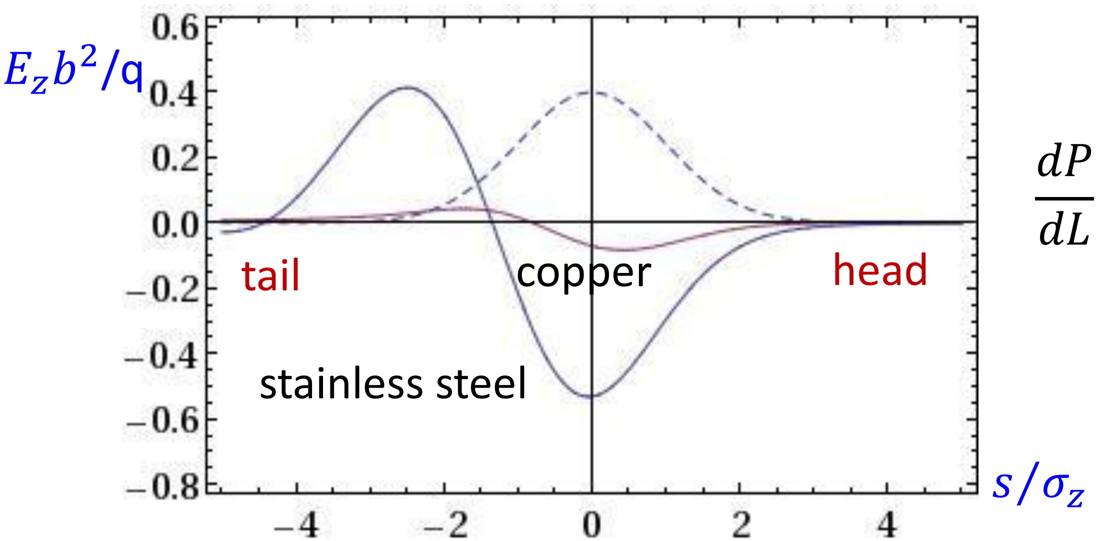
(can cross the speed of light line)

Wake fields on the bunch and in the pipe

- **Field generated inside the pipe (Bane,Chao)**



- **Wake field across the bunch** (b =6 mm, $\sigma_z \approx 45\mu\mathrm{m}$)



Power Loss: Q_b=110 pC, f_b=37 MHz

$$\frac{dP}{dL} = \frac{f_b\,(Q_b)^2 c}{\pi} \int_0^\infty dk\ |\lambda(k)|^2\ \mathrm{Re}\,(Z(k))$$

Copper: 1.9 W/m/mA
Stainless Steel: 16.5 W/m/mA

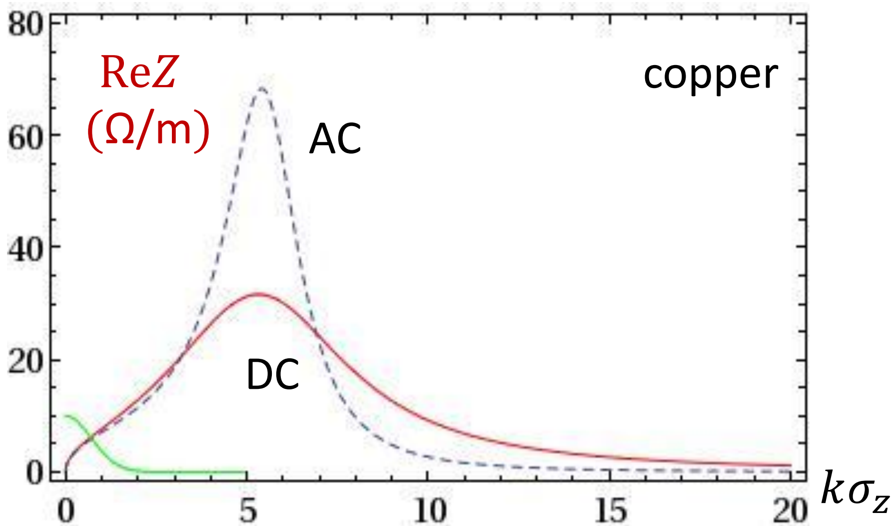
AC Effect

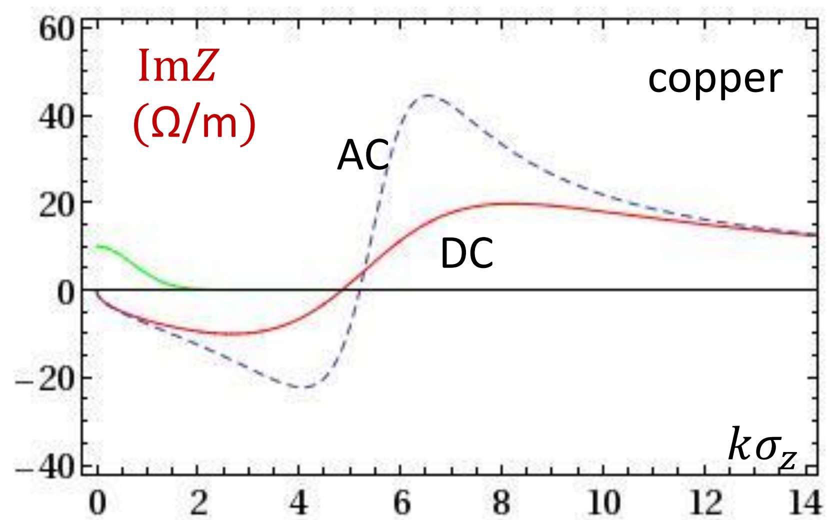
AC Effects:

- At high frequency, the conductivity σ in Ohm's law, $\vec{J} = \sigma \vec{E}$, depends on frequency. This will reduce $\bar{\sigma}$ at higher ω.

$$\bar{\sigma} = \frac{\sigma}{1 - i\,\omega\tau}$$ (DC case when $c\tau/\sigma_z \ll 1$)

JLAB FEL case, 6mm gap copper pipe, $c\tau = 8.1\ \mu\text{m},\ s_0 = 15\mu\text{m},\ c\tau/\sigma_z$=0.18





(b =6 mm, $\sigma_z \approx 45\mu\text{m}$)

Copper	DC loss 1.9 W/m/mA	AC loss 1.8 W/m/mA
Stainless Steel	16.5 W/m/mA	20.5 W/m/mA

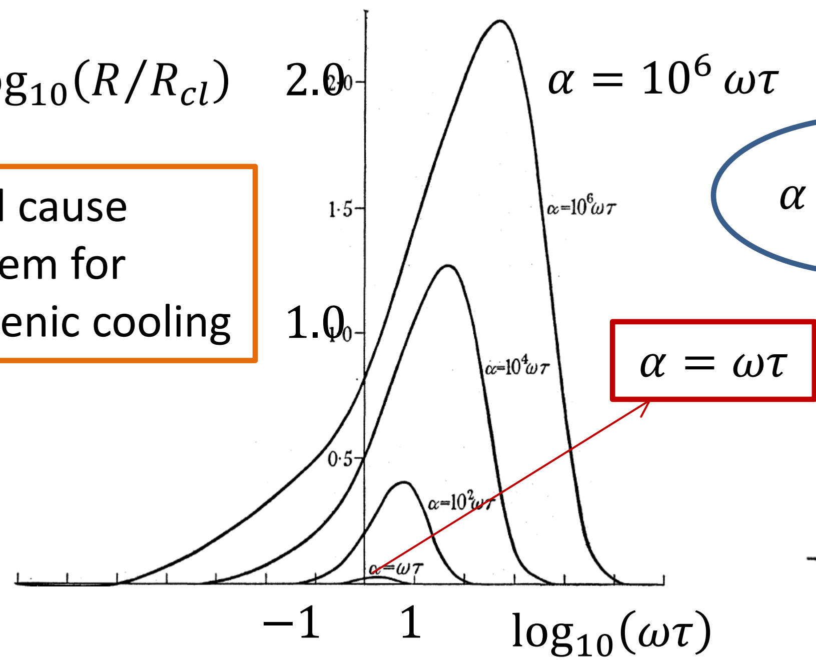
Anomalous Skin Effect

- Pippard (1947), Reuter and Sondheimer (1948)
- Whe the mean free path $\boldsymbol{l \approx v\,\tau} \geq \boldsymbol{\delta_{skin}},$ only a portion of free electrons move under the influence of $\vec{E}$ fields between collisions.
- Breakdown of Ohm's law at low T or high ω. More pronounced at low T.
- For copper at room temperature, $\boldsymbol{l \approx 40\ \mathrm{nm}}$. At $\omega \sim c/\sigma_z$, $\boldsymbol{\delta_{skin} \approx 63\ \mathrm{nm}}$

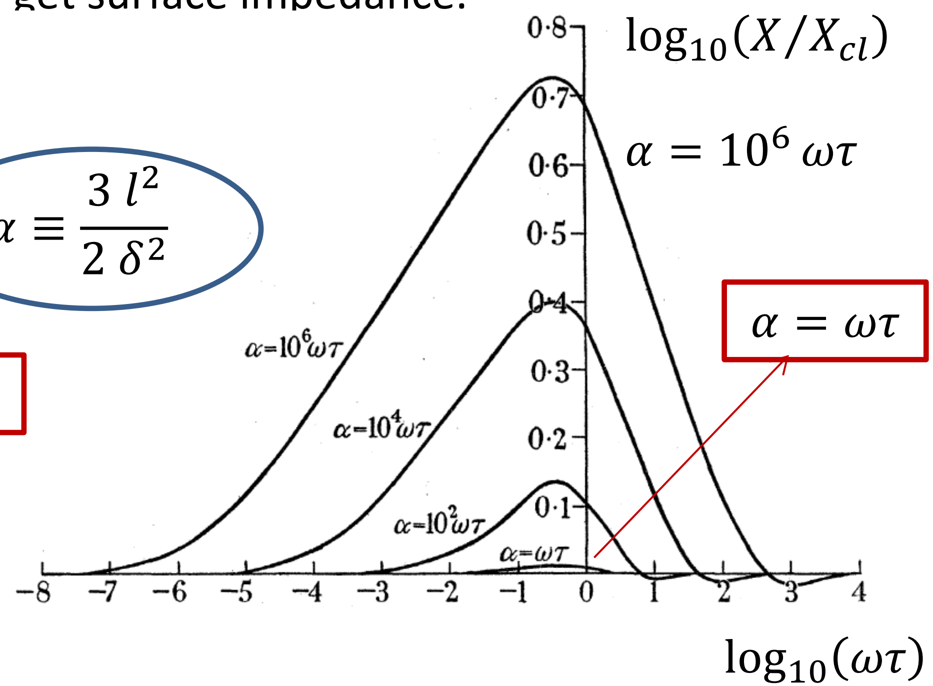
⇒ **ASE has negligible effects in our problem!**

One solves Boltzmann equation for the phase space distribution $f(\vec{r},\vec{v},t)$ of conduction electrons under external $\vec{E}$, $\vec{B}$ fields and get surface impedance.

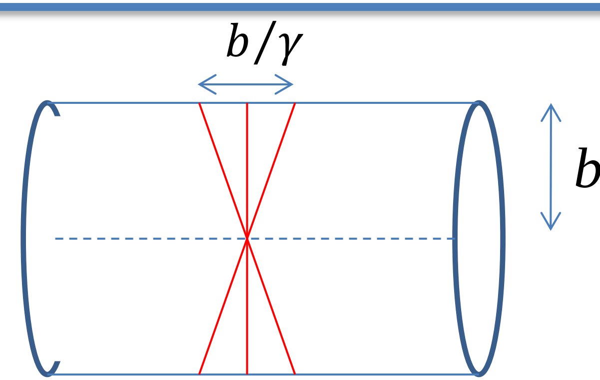
Could cause problem for cryogenic cooling



$$\alpha \equiv \frac{3\,l^2}{2\,\delta^2}$$



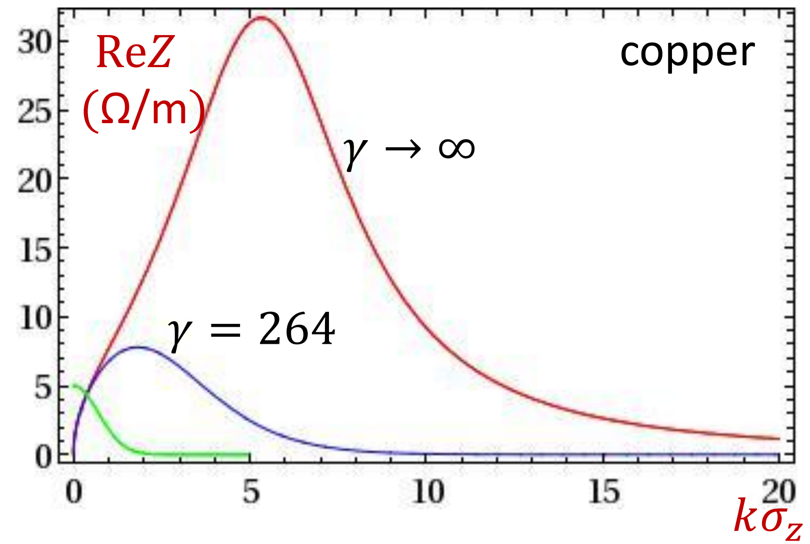
Non-ultrarelativistic Effects



When $v < c$, the high frequency of bunch structure is smeared
On the wall the non-ultrarelativistic spread is b/γ.

For $k = 1/\sigma_z$, E=135MeV, $kb/\gamma \sim 0.5$

- Zimmermann and Oide (2004)
  - -Use Lorentz gauge
  - -Write down solution of wave equations for potentials $(\varphi, \vec{A})$
  - -Express fields in terms of potentials
  - -Match field components at wall boundary
  - -Solve unknown coefficients, get impedance

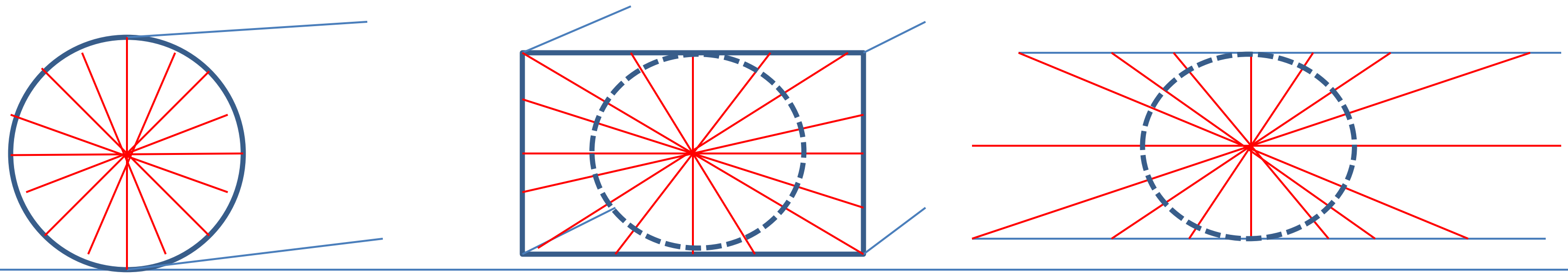


Complete longitudinal Impedance:

$$Z_{||}(\omega) = \frac{Z_0}{2\pi b}\frac{s_0}{b}\left[(I_0(k_r b)^2\left(-i\,\frac{I_1(k_r b)}{I_0(k_r b)} + \frac{(1+i)b}{\gamma\beta\, k^{1/2}(s_0)^{3/2}}\right]^{-1} \quad (\text{ with } k_r = \frac{kb}{\beta\gamma})$$

(b =6 mm, $\sigma_z \approx 45\mu m$)	**Copper**	$v = c$ **1.9 W/m/mA**	**E=135 MeV 1.6 W/m/mA**
	Stainless Steel	**16.5 W/m /mA**	**14 W/m/mA**

Impact of Geometry on Impedance and Wakefield



Piwinski
Bane (1995)
Chao

Gluckstern et. al (1993)
Yokoya (1993)

Henke and Napoly (1991)
Piwinski (1992), Ng (2004)

$v = c$

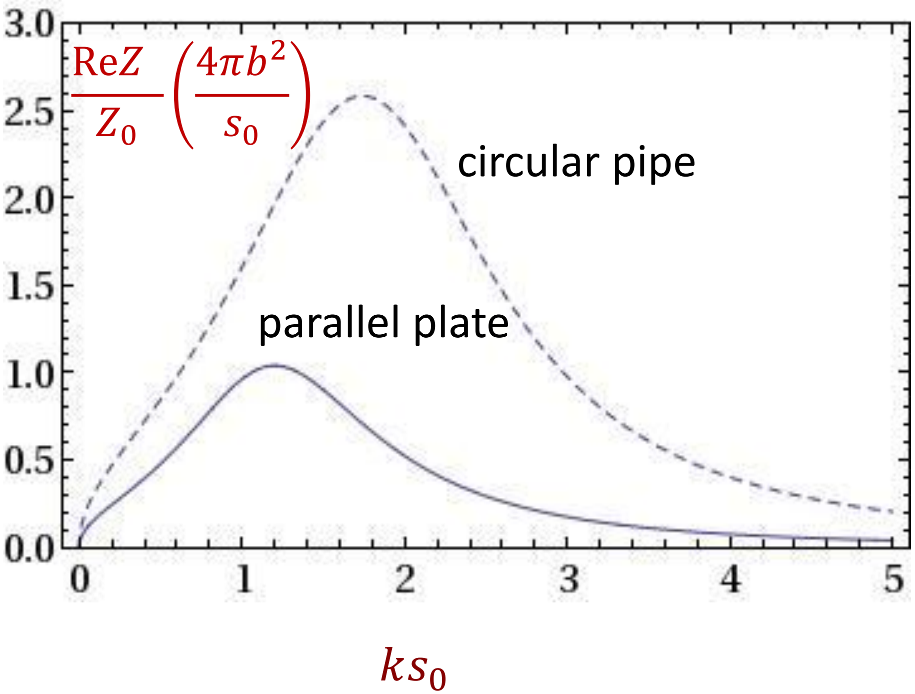
- Boundary geometry breaks the cylindrical symmetry of source field
- Field strength of source charge decreases further away from the charge
  →expecting flat geometry has reduced resistive wall impedance
- Both TE and TM fields exists for the homogeneous solution of the fields

$v < c$ Zimmermann and Oide (2004)

JLAB FEL case

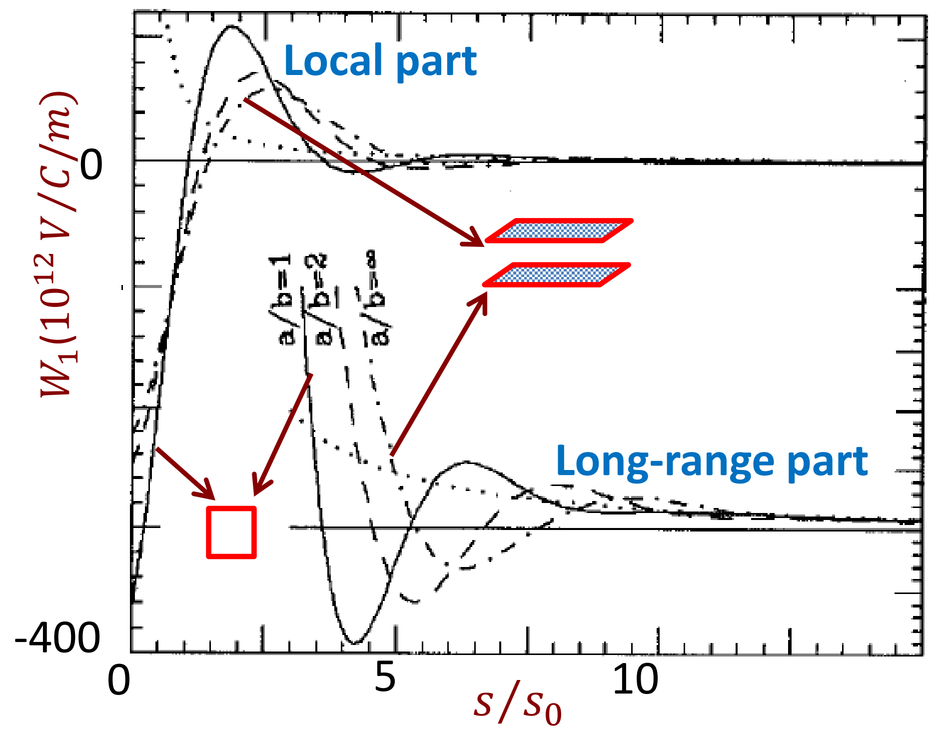
Effect of Geometry in Resistive Wake $(v = c)$

Impedance

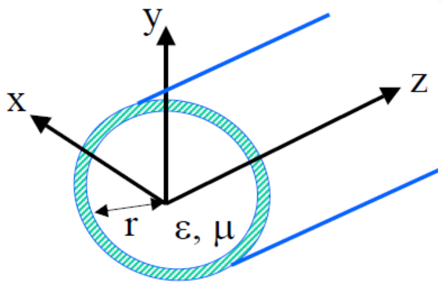


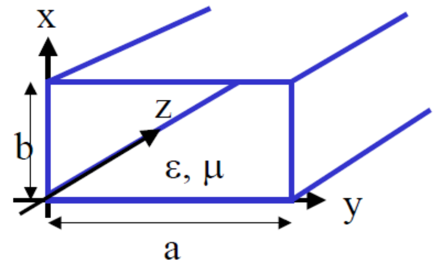
Wake Function

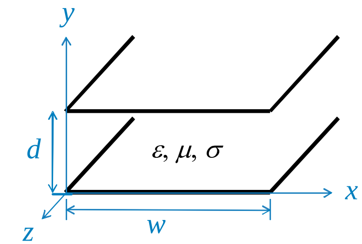
Yokoya (1993)



Impact of Geometry on Impedance and Wakefield







$v = c$

Piwinski
Bane (1995)
Chao

$$Z_{||}(k) = \frac{Z_0}{2\pi b} \frac{1}{\frac{ikb}{2} - \lambda/k}$$

Add space charge field

$v < c$

Zimmermann and Oide (2004)

$$Z_{||}(\omega) = \frac{iZ_0 c k_r^2}{2\pi\omega}\left[K_0(k_r r) + I_0(k_r r)\frac{\omega^2 \lambda K_1(bk_r)K_0(b\lambda) + k_r c^2(\lambda^2 - k^2)K_0(bk_r)K_1(\lambda b)}{\omega^2 \lambda I_1(bk_r)K_0(b\lambda) - k_r c^2(\lambda^2 - k^2)I_0(bk_r)K_1(\lambda b)}\right],$$

$$Z_{||}(k) = \frac{Z_0}{2\pi b}\frac{s_0}{b}\left[(I_0(k_r b)^2\left(-i\,\frac{I_1(k_r b)}{I_0(k_r b)} + \frac{(1+i)b}{\gamma\beta\, k^{1/2}(s_0)^{3/2}}\right]^{-1}$$

Gluckstern et. al (1993)
Yokoya (1993)

Henke and Napoly (1991)
Piwinski (1992)
Ng (2004)

$$\begin{aligned}E_{\pm,z} \simeq\ & \frac{ike}{16\pi^2\epsilon_0} q(q^2 - K^2)\{\cosh(q\Delta)[(K\cosh(qa) + q\sinh(qa))\\ & (k^2(K\sinh(qa) + q\cosh(qa)) + q(q^2 - K^2)\cosh(qa))]^{-1}\\ & \pm \sinh(q\Delta)[(K\sinh(qa) + q\cosh(qa))\\ & (k^2(K\cosh(qa) + q\sinh(qa)) + q(q^2 - K^2)\sinh(qa))]^{-1}\}\end{aligned}$$

$$Z_{||}(k) = \frac{Z_0}{4\pi}\int_{-\infty}^{\infty} d\eta \left[\frac{\lambda}{k}\cosh^2(\eta b) - \frac{ik}{\eta}\cosh(\eta a)\sinh(\eta b)\right]^{-1}$$

Add space charge field

Parallel plate ($v < c$)

We are entering into a new parameter regime

- Near the resonance of the resistive wall impedance
- AC conductivity
- non-ultrarelativistic effect
- $l \sim \delta_{skin}$ (negligible effect)
- Flat geometry
- Possible high frequency structures in bunch
- Bunch length $\sim$ surface roughness characteristic length

⇨ Need impedance theory for flat geometry at $v < c$

4. Parallel Plates with Non-ultrarelativistic Effect

Challenges:

- Additional space charge fields in flat geometry
- Expect $\frac{kb}{\gamma}$ plays a role in impedance reduction from $v = c$ case

Approach:

- Maxwell equations and pseudo Lorentz gauge
- Fields inside the metal wall: Prove Leontovich boundary condition
- Fields in between the plates: solution of the inhomogeneous equation and the homogeneous equations
- Applying boundary condition on the metal wall
- Solve unknown coefficients and get impedance
- Results compared to circular geometry at $v < c$

Analytical Results of Impedance for Parallel Plates ($v < c$)

- New impedance result for parallel plate at $v < c$

$$Z_{||}(k) = \frac{Z_0 \beta^2}{4\pi} \int_{-\infty}^{\infty} \frac{d\eta}{\frac{\lambda}{k}\cosh^2(\alpha b) - \frac{ik}{\alpha}\cosh(\alpha b)\sinh(\alpha b)}$$

for $\alpha^2 = \eta^2 + k^2/\gamma^2$, and $\alpha b = \sqrt{(\eta b)^2 + (kb/\gamma)^2}$

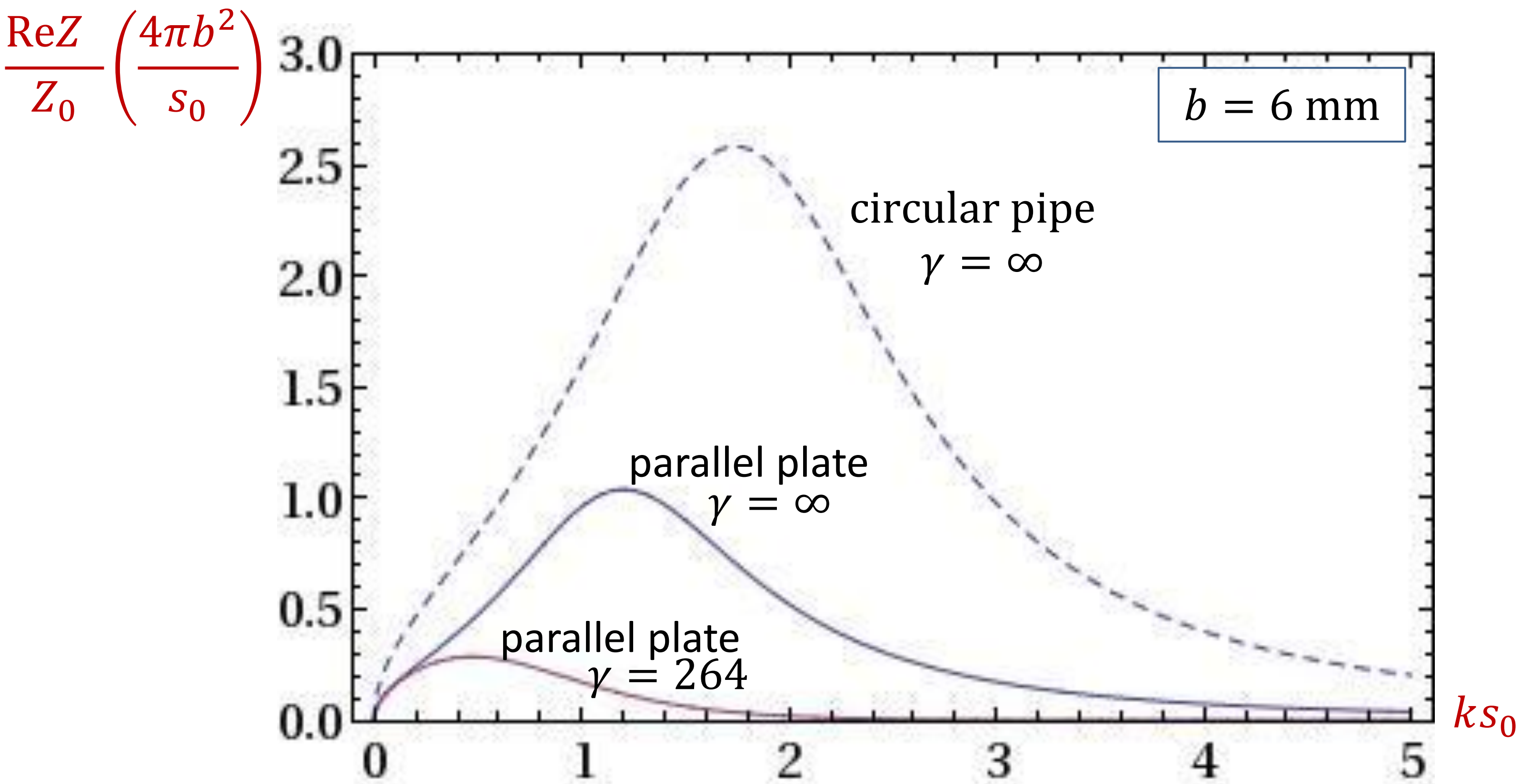
- Compared to Henke and Noplay's result for parallel plate at $v = c$ (simplified by Bane and Stupakov, see also new results by Ng (2004))

$$Z_{||}(k) = \frac{Z_0}{4\pi} \int_{-\infty}^{\infty} \frac{d\eta}{\frac{\lambda}{k}\cosh^2(\eta b) - \frac{ik}{\eta}\cosh(\eta a)\sinh(\eta b)}$$

and Chao and Bane's result for circular pipe at $v = c$

$$Z_{||}(k) = \frac{Z_0}{2\pi b} \frac{1}{\frac{\lambda}{k} - \frac{ikb}{2}}$$

Impedance for Parallel Plates ($v < c$)



- For $v = c$, parallel plate has impedance half of the circular pipe
- For $v < c$, impedance reduction at higher frequency

5. Discussion

- Other possible factors which may cause heating enhancement

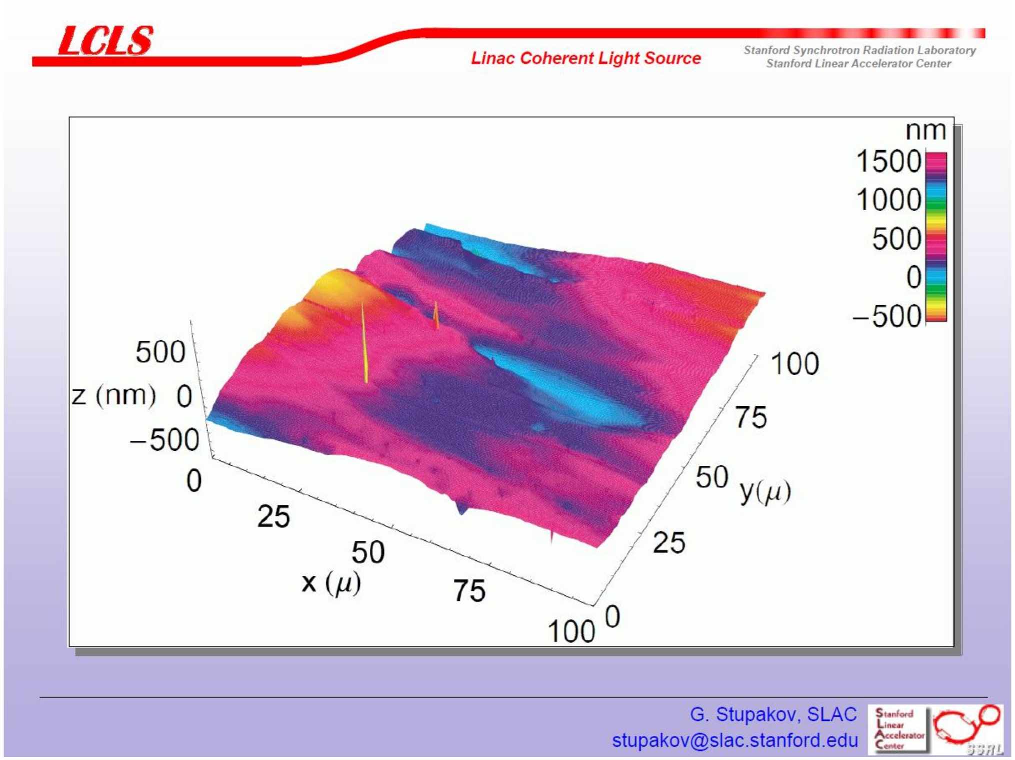
  -Surface roughness (in combination of resistive wall)

  -Higher frequency contents in bunch longitudinal distribution

- Waveguide heating measurement for DarkLight experiment at JLAB FEL (Shukui Zhang)

Surface Roughness

- Surface roughness length scale can be comparable the bunch length
- Usually surface roughness is considered independent of resistive wall effect
- Could there be a joint effect?

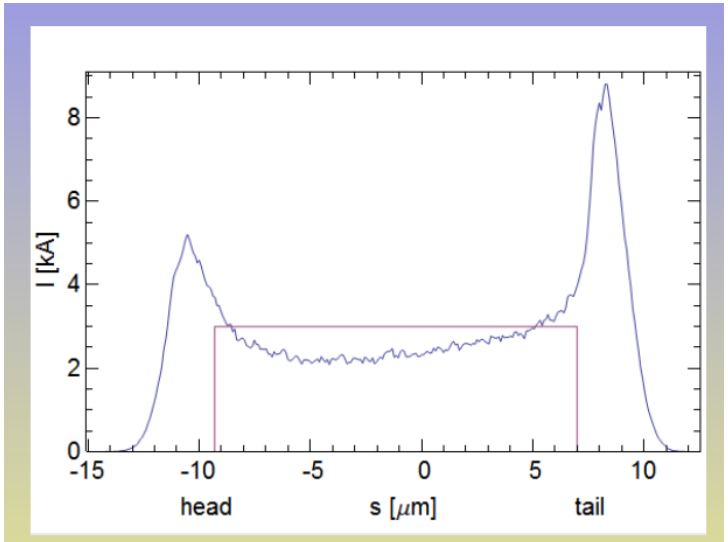


High Frequency Content of the Bunch

- The bunch in FEL undergoes many collective effects and is often rich in phase space structure.

- The high frequency part of the bunch may sample the resonance of resistive wall impedance and enhance the heat deposit.

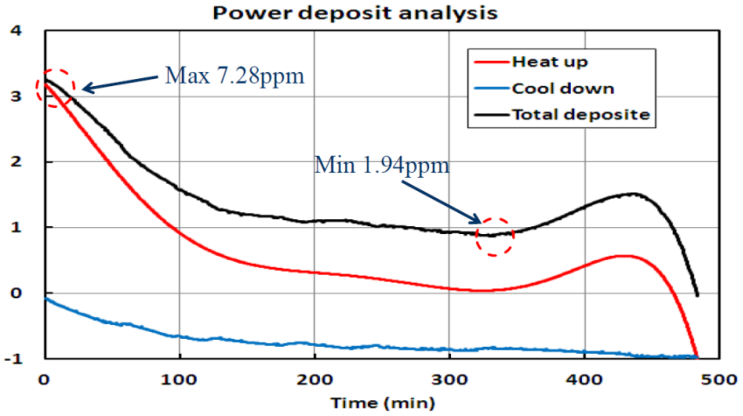
- At LCLS, the estimation (including effect of the double horn bunch distribution) of bunch energy loss due to resistive wall is about ½ of the measured energy loss for beam passing through the undulator waveguide.

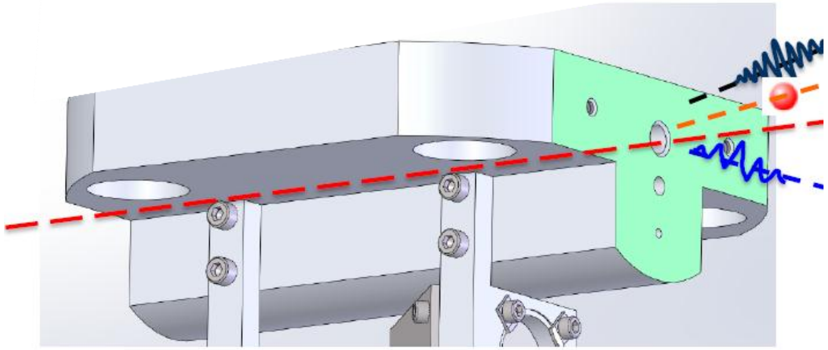


DarkLight Experiment at JLAB FEL (Shukui Zhang)

Demonstrate JLab FEL high current (at 100MeV) electron beam can be *cleanly and stably transmitted through a target* assembly consisting of three 127mm long tubes with different diameters (2 mm, 4 mm and 6 mm), simulating operation of the gaseous hydrogen target proposed for the DarkLight experiment.

- *Halo/resistive wall Heating effect*
- *Machine configuration, Halo management, e-beam characterization*





- Bunch charge: 60pC, Rep rate: 74.85MHz, Average current: 4.49mA
- Beam energy: 100MeV, Beam Power: 0.45MW
- Fraction of average beam power deposited in beam block :3.08 ppm

6. Summary

- Wiggler chamber heating was observed on the JLAB FEL for CW operation
- We are in the regime with AC conductivity, non-ultrarelativistic beam and flat geometry
- Estimation using resistive wall theory shows beam power loss can explain half of the observed heat deposit (for a Gaussian bunch)
- New analysis done for parallel plates with non-ultrarelativistic beams

Future Studies

- More systematic measurements of chamber heating in conjunction with realistic beam characterization
- Thorough evaluation of resistive wall contribution using real parameters
- Consider other possible factors, such as contribution of surface roughness