The Paul Trap Simulator Experiment: Studying Transverse Beam Dynamics in a Compact Laboratory Experiment*

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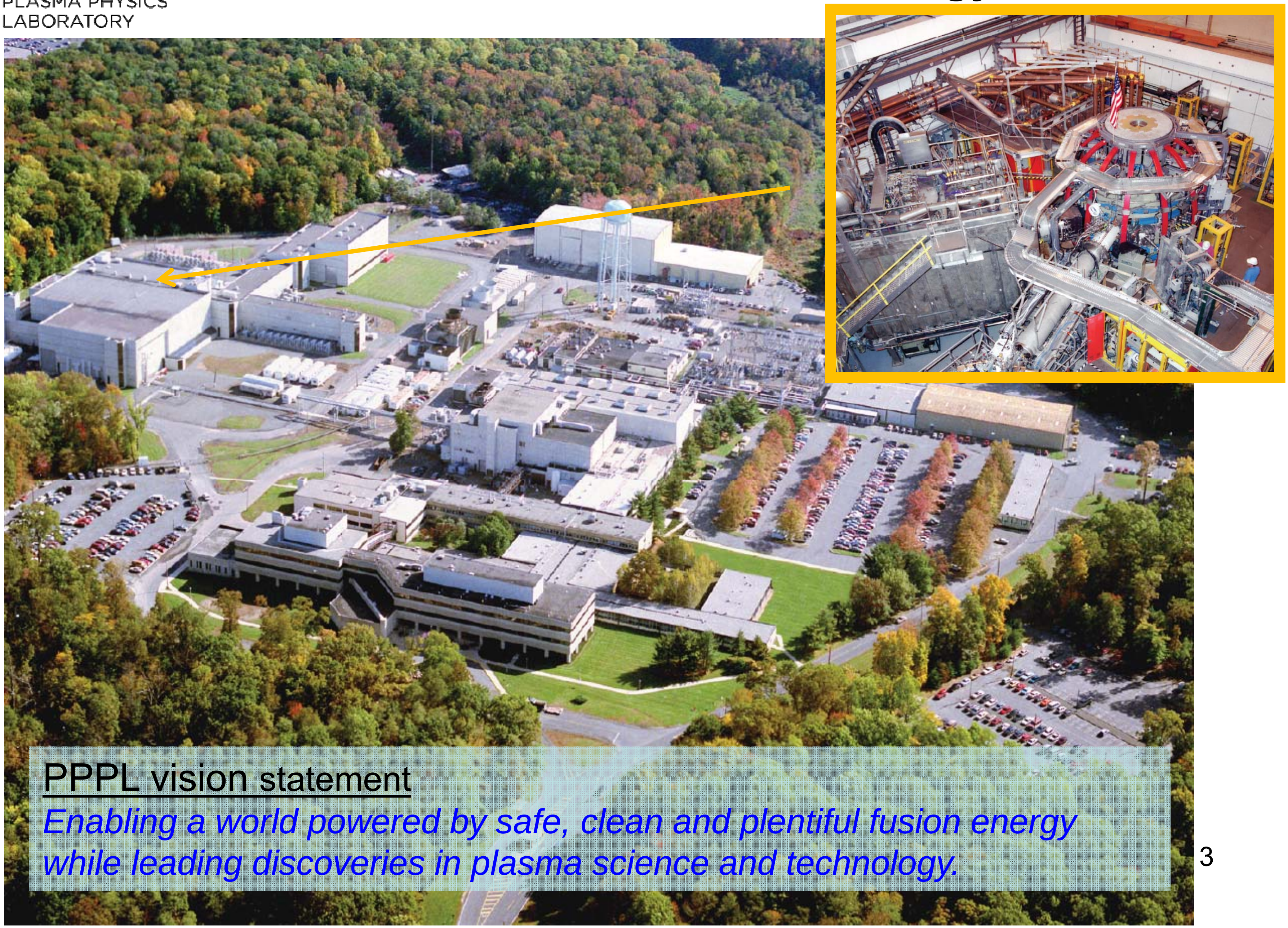
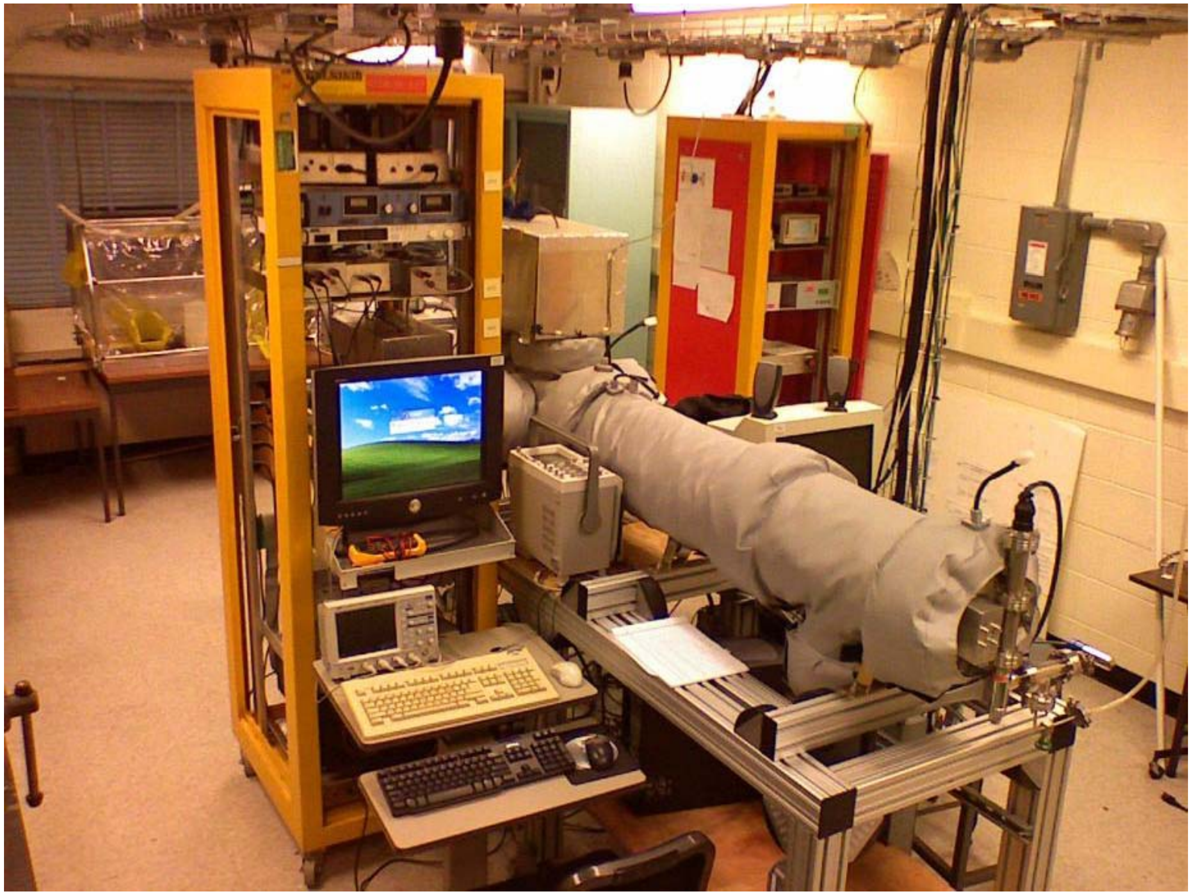


PPPL vision statement

Enabling a world powered by safe, clean and plentiful fusion energy while leading discoveries in plasma science and technology.

PTSX Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

- Purpose: PTSX simulates, in a compact experiment, the transverse nonlinear dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems.
- Applications: Accelerator systems for high energy and nuclear physics applications, heavy ion fusion, spallation neutron sources, and high energy density physics.



The Goal of PTSX is to Study Key Issues in the Physics of Intense Beams

Issues:

- Beam mismatch and envelope instabilities;
- Collective wave excitations;
- Chaotic particle dynamics and production of halo particles;
- Mechanisms for emittance growth;
- Compression techniques; and
- Effects of distribution function on stability properties.

Today: transverse beam compression, the effects of random lattice noise, and transverse beam modes.

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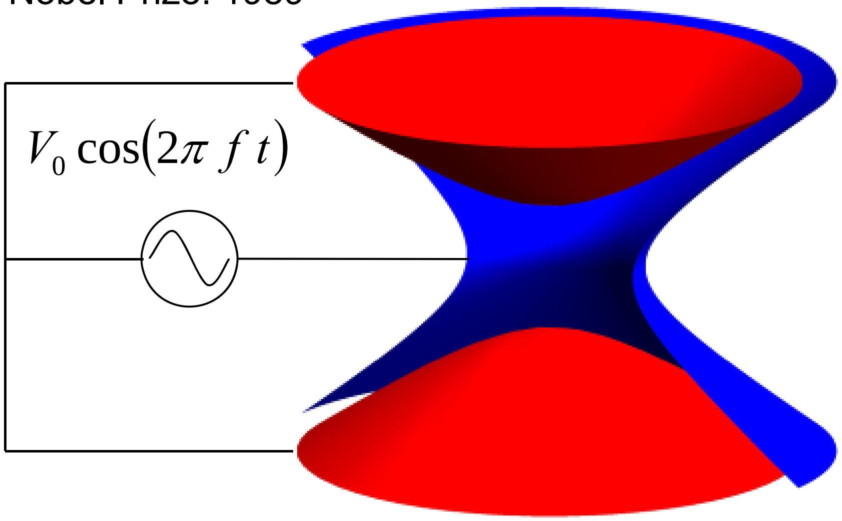
Paul Traps

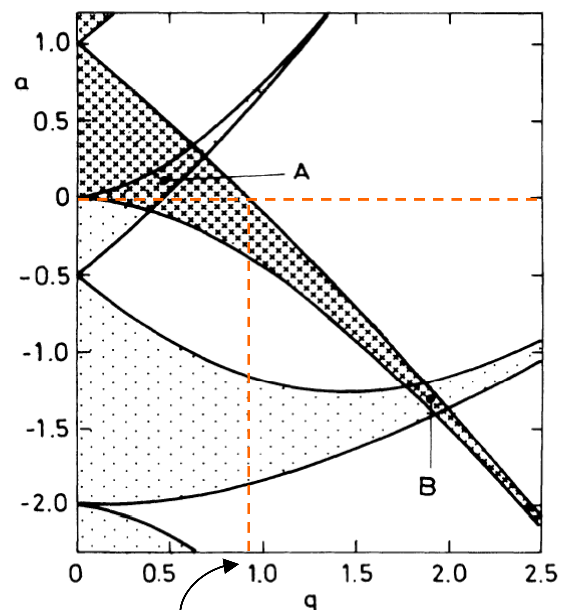
Paul W., Steinwedel H. (1953).
"Ein neues Massenspektrometer ohne Magnetfeld",
R Zeitschrift für Naturforschung A 8 (7): 448-450

Nobel Prize: 1989



$$\phi_{trap}(\boldsymbol{x}) = \frac{V_0 \cos(2\pi f t)}{r_0^2 + 2z_0^2}\left(x^2 + y^2 - 2z^2\right)$$

R. Blümel, et al., Phys Rev A, **40** 808 (1989)

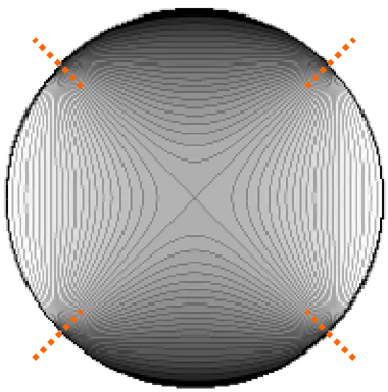


$q = 0.908$

$$\frac{d^2}{d\tau^2}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \left[2q\cos(2\tau)\right]\begin{pmatrix} x \\ y \\ -2z \end{pmatrix} = 0$$

$$q = \frac{4eV_0}{m(2\pi f)^2\left(r_0^2 + 2z_0^2\right)} \qquad \tau = \pi f t$$

A Hyperbolic Potential in Any Pair of Variables Will Do – r-z or x-y



$$\phi_{ap}(x,y,t) = \frac{4V_0(t)}{\pi}\sum_{\ell=1}^{\infty}\frac{\sin(\ell\pi/2)}{\ell}\left(\frac{r}{r_w}\right)^{2\ell}\cos(2\ell\theta)$$

$$e_b\phi_{ap}(x,y,t) = \frac{1}{2}\kappa_q(t)(x^2-y^2)$$

$$\kappa_q(t) = \frac{8e_bV_0(t)}{m_b\pi\ r_w^2}$$

$$V_0(t) = V_{0\ \max}\sin(\omega t)$$

The ponderomotive force…

$$\vec{F}_p = -\frac{\omega_p^2}{\omega^2}\vec{\nabla}\frac{\langle\varepsilon_0E^2\rangle}{2}$$

…can be written as…

$$\vec{F}_p = -m_b\omega_q^2\vec{r}$$

…where…

$$\omega_q = \frac{8e_bV_{0\max}}{m_b\pi r_w^2 f}\xi$$

…and

$$\xi = \frac{1}{2\sqrt{2}\pi}$$

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Analogy Between AG System and Paul Trap

AG System		Paul Trap
$\mathbf{B}_q^{foc}(\mathbf{x}) = B_q'(z)(y\hat{\mathbf{e}}_x + x\hat{\mathbf{e}}_y)$ $\mathbf{F}_{foc}(\mathbf{x}) = -\kappa_q(z)(x\hat{\mathbf{e}}_x - y\hat{\mathbf{e}}_y)$	Quadrupolar Focusing	$e\phi_{ap}(x,y,t) = \frac{1}{2} m\kappa_q'(t)(x^2 - y^2)$
$\kappa_q(z) = \frac{ZeB_q'(z)}{\gamma m\beta c^2}$		$\kappa_q'(t) = \frac{8eV_0(t)}{m\pi r_w^2}$
$\psi = \frac{Ze}{\gamma m\beta^2 c^2}\left[\phi(x,y,s) - \beta A_z(x,y,s)\right]$	Self-Forces	usual $\phi_{self}(x,y,t)$
$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi = -\frac{2\pi K}{N}\int dx'dy'\, f_b$	Field Equations	Poisson's Equation

Vlasov Equation

$$\left\{\frac{\partial}{\partial s} + x'\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} - \left(\kappa_q(s)x + \frac{\partial\psi}{\partial x}\right)\frac{\partial}{\partial x'} - \left(-\kappa_q(s)y + \frac{\partial\psi}{\partial y}\right)\frac{\partial}{\partial y'}\right\} f_b = 0$$

The resulting ponderomotive force is a radial linear restoring force with characteristic frequency ω_q.

$$\omega_q = \frac{8eV_{0\max}}{m\pi r_w^2 f}\xi$$

$$\xi = \frac{1}{2\sqrt{2}\pi}$$ for a sinusoidal waveform $V(t)$.

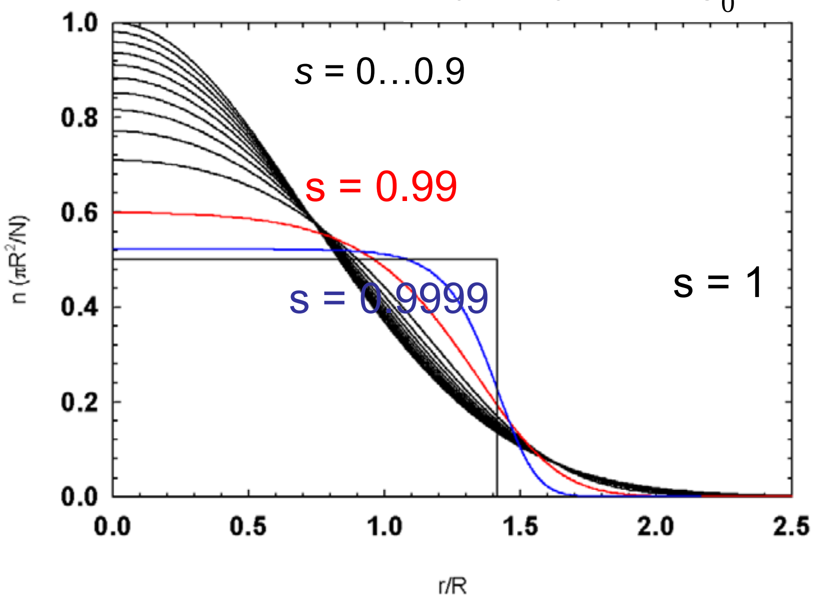
$$\xi = \frac{\eta\sqrt{3 - 2\eta}}{4\sqrt{3}}$$ for a periodic step function waveform $V(t)$ with fill factor η.

$$\sigma_v = \frac{\omega_q}{f} < \sigma_{v\max}$$

Smooth Focusing Equilibria are Parameterized by the Normalized Intensity *s*

In thermal equilibrium, $$n(r)=n(0)\exp\left[-\frac{m\omega_q^2 r^2+2q\phi^s(r)}{2kT}\right]$$

Poisson's equation $$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \phi^s}{\partial r}=\frac{qn(r)}{\varepsilon_0}$$ becomes a nonlinear equation for ϕ^s that must be solved numerically.



Solutions are characterized by the normalized intensity parameter *s*.

$$s \equiv \frac{\omega_p^2}{2\omega_q^2} < 1$$

$$\frac{\upsilon}{\upsilon_0}=(1-s)^{1/2}$$ for a flat-top radial density distribution

PTSX-accessible (s = 0.1 to 0.6)

s	v/v_0
0.1	0.95
0.2	0.90
0.3	0.84
0.4	0.77
0.5	0.71
0.6	0.63
0.7	0.55
0.8	0.45
0.9	0.32
0.99	0.10
0.999	0.03

If $p = n\,kT$, then the statement of local force balance on a fluid element can be manipulated to give a global force balance equation.

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

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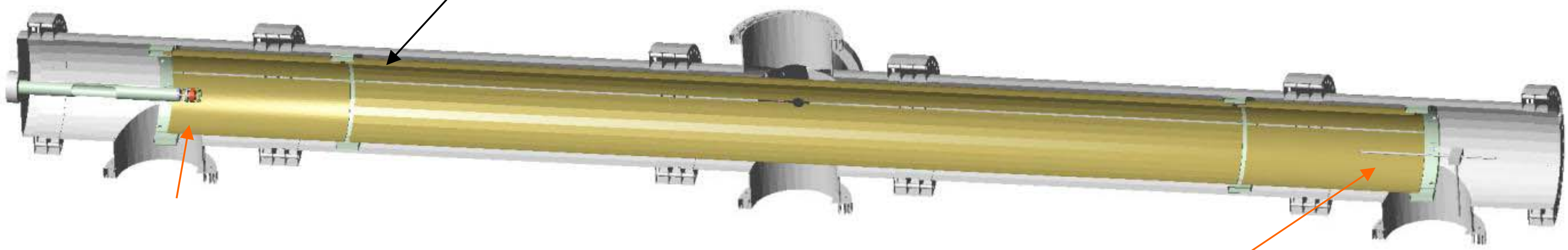
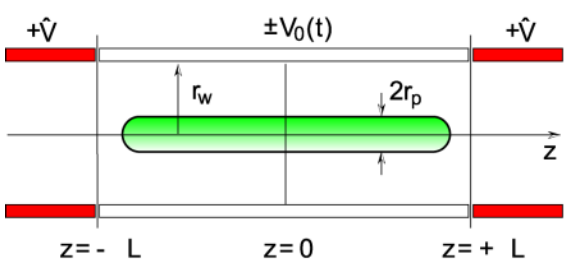
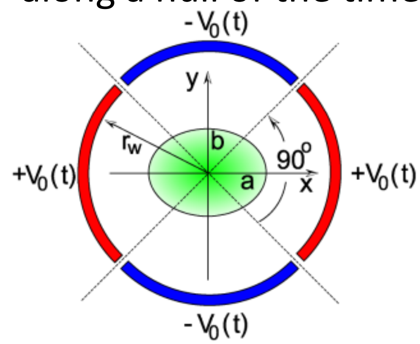
PTSX Apparatus



$$e\phi_{ap}(x,y,t) = \frac{1}{2}\kappa_q'(t)(x^2 - y^2)$$

The PTSX collector disk is a 5 mm diameter copper disk, held at ground, that is mounted to a linear motion feedthrough and moves along a null of the time-dependent oscillating potential $\pm V_0(t)$.





$$\kappa_q'(t) = \frac{8eV_0(t)}{m\pi r_w^2}$$

$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$

Plasma length	2 m	Wall voltage	140 V
Wall radius	10 cm	End electrode voltage	20 V
Plasma radius	~ 1 cm	Frequency	60 kHz
Cesium ion mass	133 amu	Pressure	$5\text{x}10^{-10}$ Torr
Ion source grid voltages	< 10 V	Trapping time	100 ms

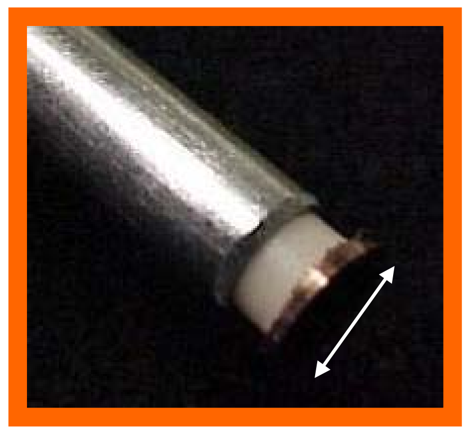
Electrodes, Ion Source, and Collector

Broad flexibility in applying V(t) to electrodes with arbitrary function generator.

Increasing source current creates plasmas with intense space-charge.

1.25 in

Large dynamic range using sensitive electrometer.



5 mm

Measures average Q(r).

Plasmas are Manipulated Using an Inject-Hold-Dump Cycle

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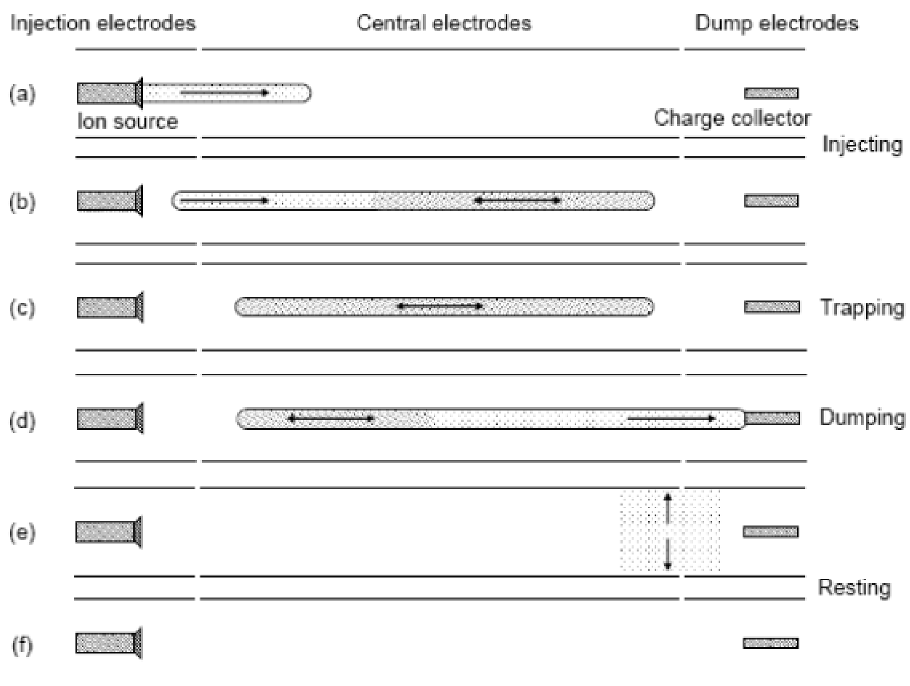
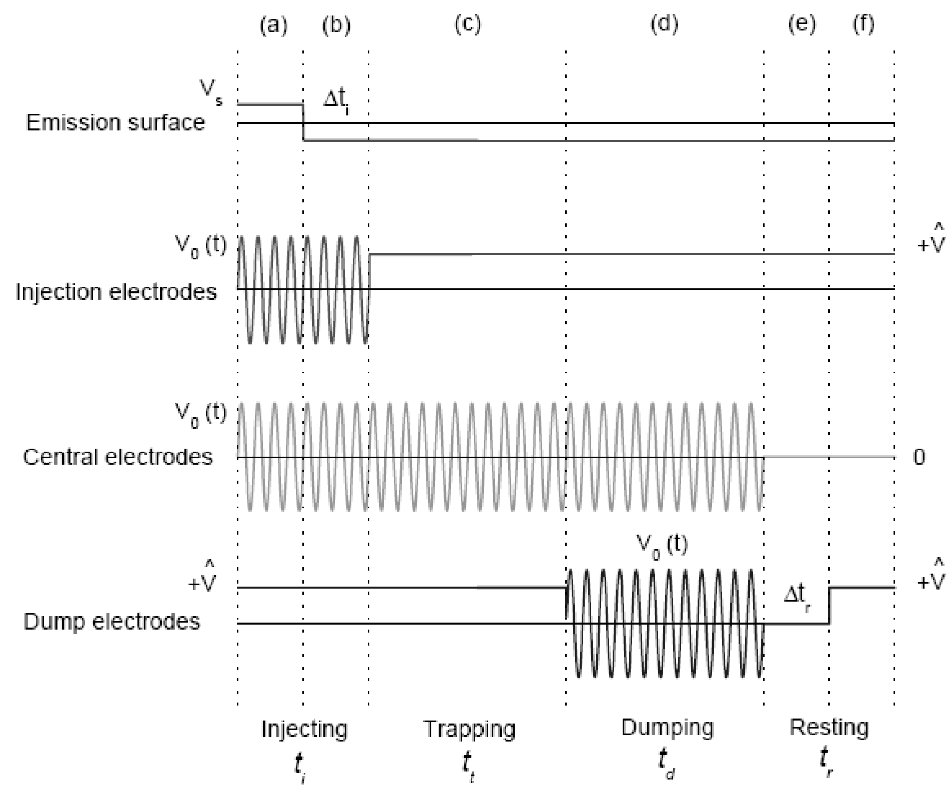


f = 75 kHz
T = 13.3 μs
V_s = 3 V
v_z = 2 m/ms



1.7 ms $<$ 300 ms $>$ 5 ms $>$ 1 ms

Figures: M. Chung, Ph.D. Thesis, Princeton University (2008).

Transverse Dynamics are the Same – Including Self-Field Effects

If…

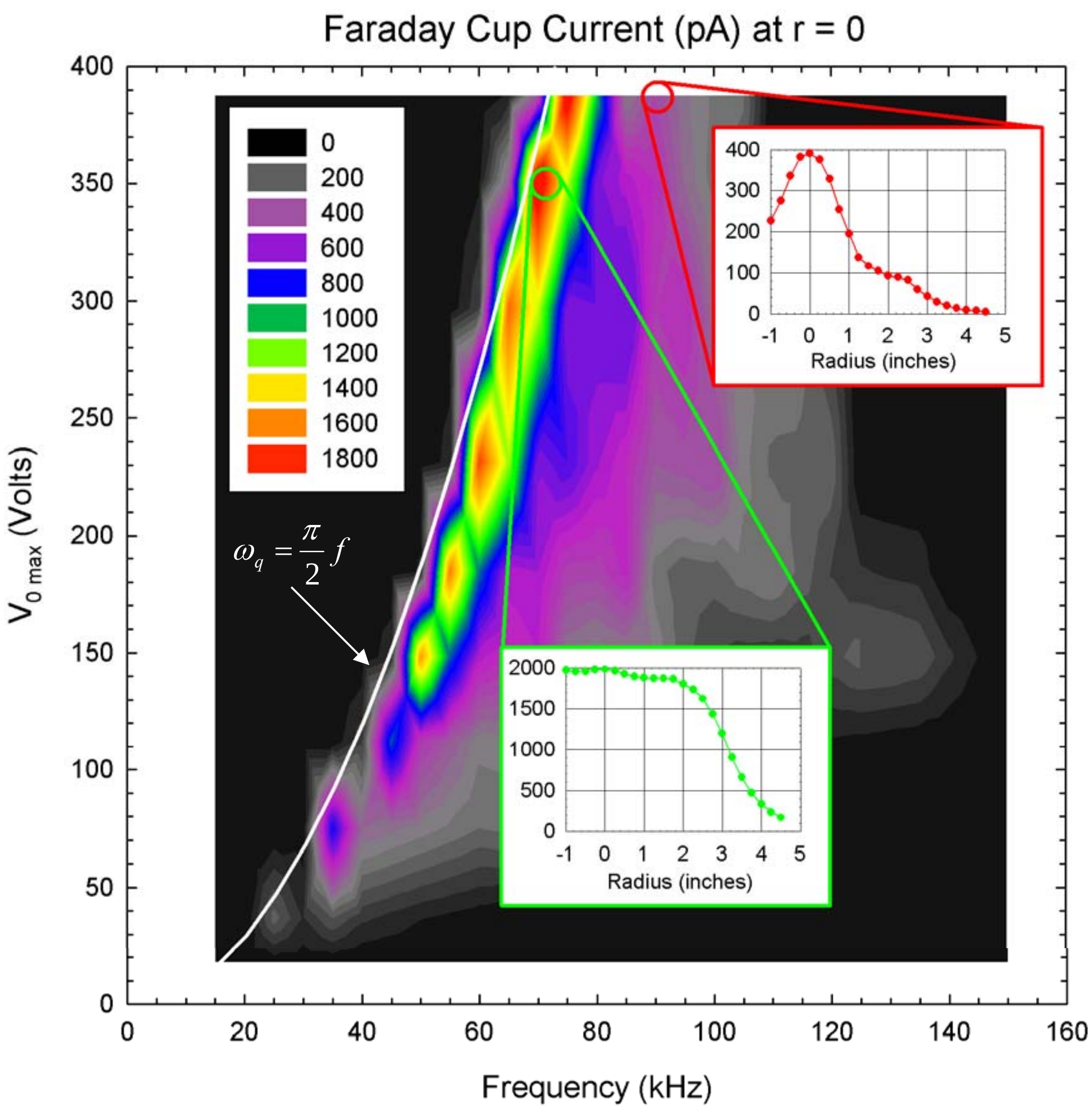
- Long coasting beams
- Beam radius << lattice period
- Motion in beam frame is nonrelativistic

Then, when in the beam frame, both systems have…

- Quadrupolar external forces
- Self-forces governed by a Poisson-like equation
- Distributions evolve according to nonlinear Vlasov-Maxwell equation

Ions in PTSX have the same transverse equations of motion as ions in an alternating-gradient system ***in the beam frame***.

Instability of Single Particle Orbits – An Early Experiment



- Experiment - stream Cs^+ ions from source to collector without axial trapping of the plasma.

Electrode parameters:

- $V_0(t) = V_{0\,\max} \sin(2\pi f t)$
- $V_{0\,\max} = 387.5$ V
- $f = 90$ kHz

Ion source parameters:

- $V_{accel} = -183.3$ V
- $V_{decel} = -5.0$ V

Mismatch Between Ion Source and Focusing Lattice Creates Halo Particles

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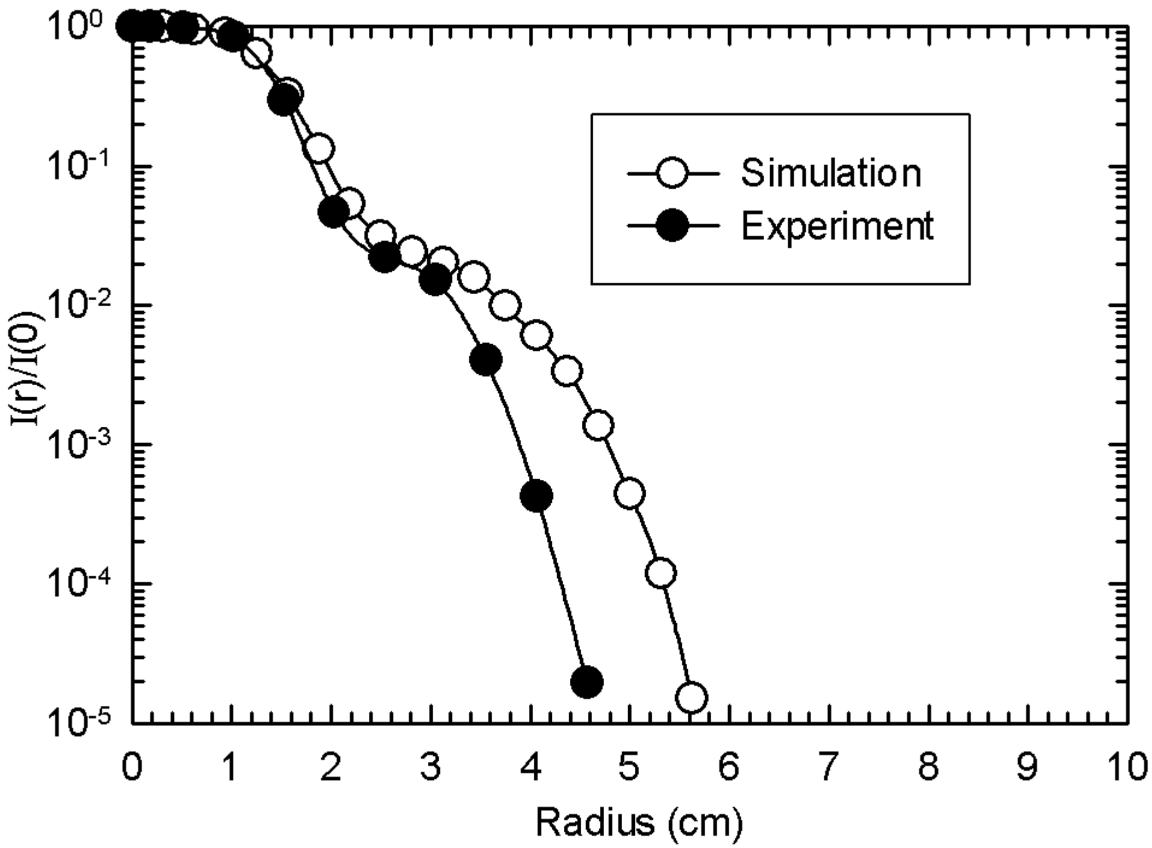
Streaming-mode experiment



"Simulation" is a 3D WARP simulation that includes injection from the ion source.

- $s = \omega_p^2/2\omega_q^2 = 0.6$.
- $\nu/\nu_0 = 0.63$
- $V_{0\ max} = 235$ V
  $f = 75$ kHz
  $\sigma_v = 49^o$

Qualitatively similar to C. K. Allen, *et al*., Phys. Rev. Lett. **89** (2002) 214802 on the Los Alamos low-energy demonstration accelerator (LEDA).

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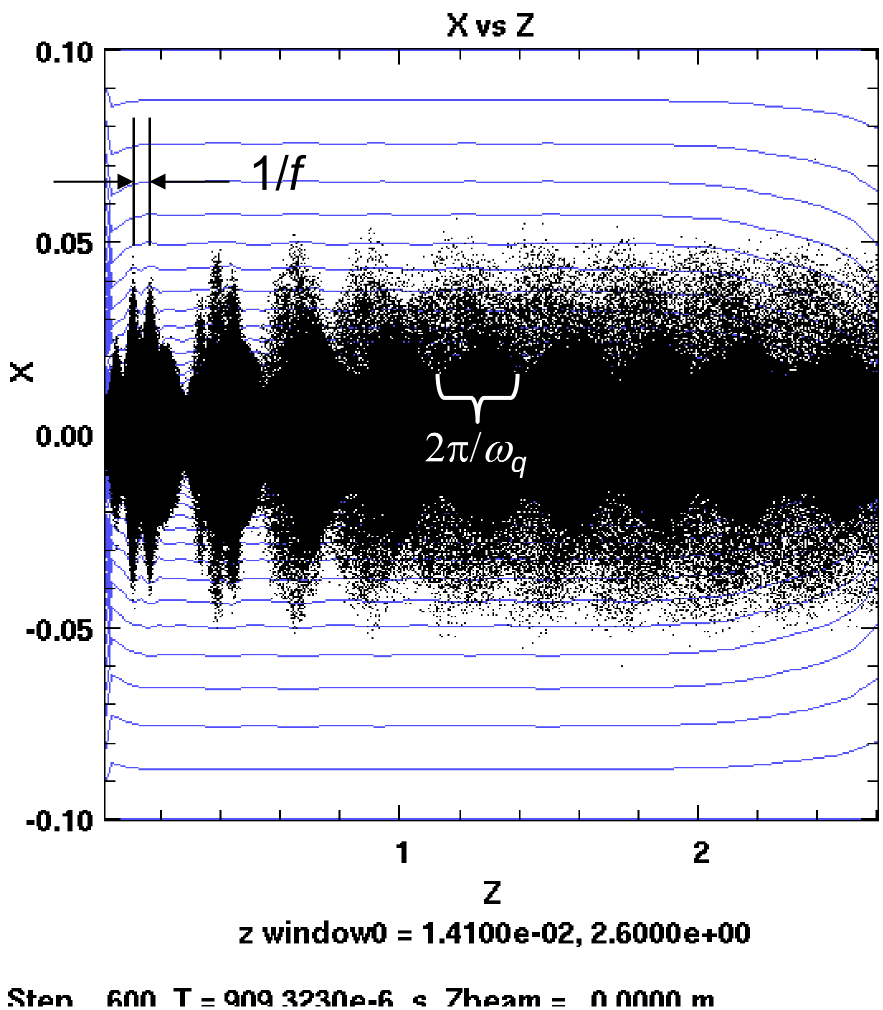
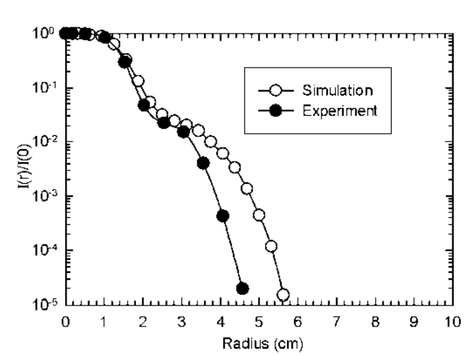
WARP Simulations Reveal the Evolution of the Halo Particles in PTSX



Oscillations can be seen at both f and ω_q near z = 0.

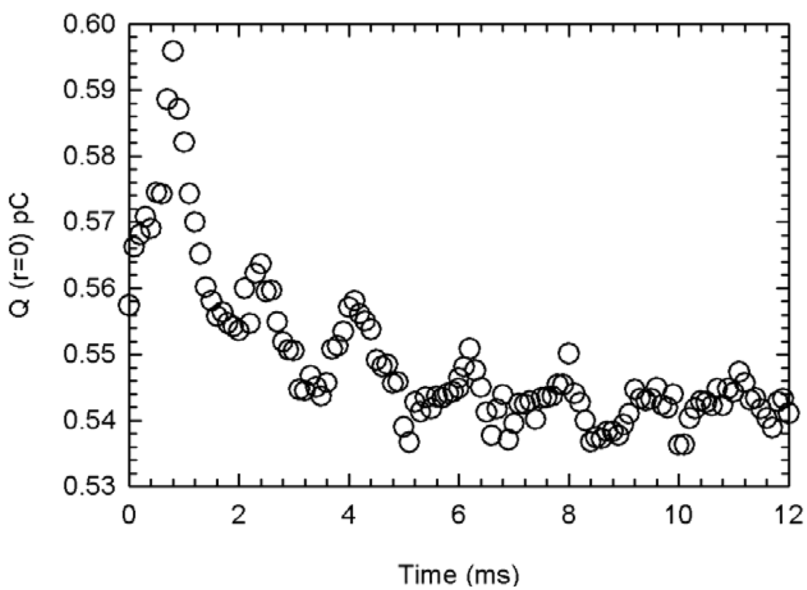
Downstream, the transverse profile relaxes to a core plus a broad, diffuse halo.



Go back to s ~ 0.2.

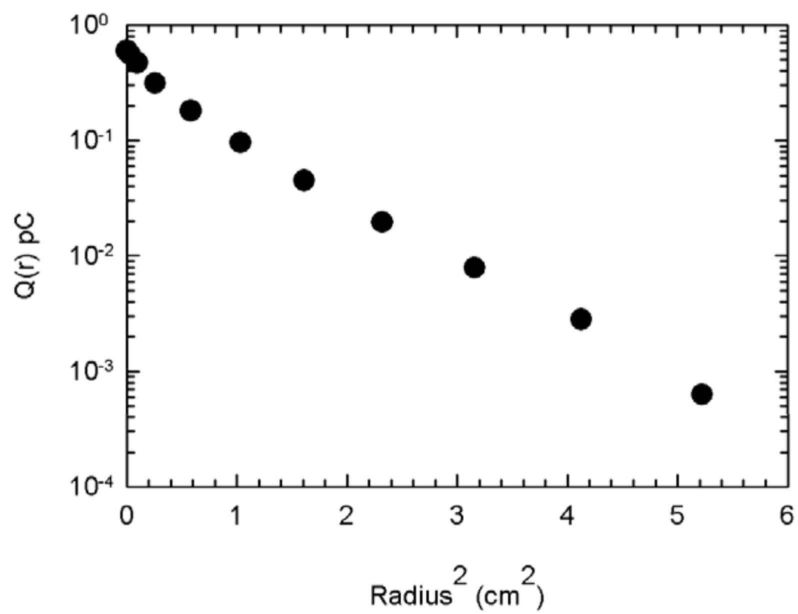
- $s = \omega_p^2/2\omega_q^2$ = 0.6.
- ν/ν_0 = 0.63
- $V_{0\,max}$ = 235 V
  f = 75 kHz
  $\sigma_v = 49^o$

Oscillations From Residual Ion Source Mismatch Damp Away in PTSX



The injected plasma is still mismatched because a circular cross-section ion source is coupling to an oscillating quadrupole transport system.

Over 12 ms, the oscillations in the on-axis plasma density damp away…



… and leave a plasma that is nearly Gaussian.

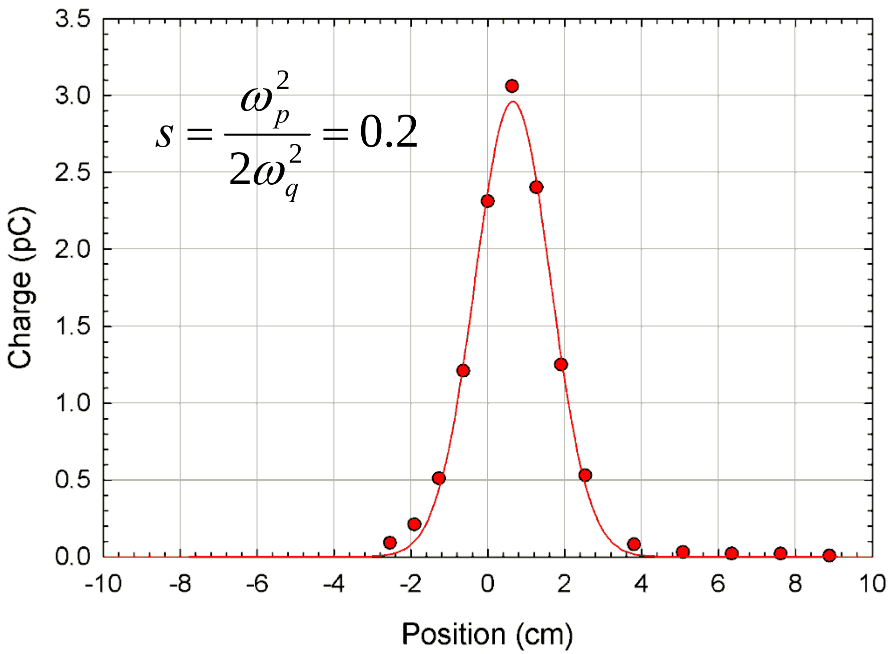
$kT = 0.12\text{eV}$

nearly the thermal temperature of the ion source.

- $s = \omega_p^2/2\omega_q^2 = 0.2$.
- $\nu/\nu_0 = 0.88$
- $V_{0\ max} = 150$ V
  $f = 60$ kHz
  $\sigma_v = 49^o$

Radial Profiles of Trapped Plasmas are Gaussian – Consistent with Thermal Equilibrium

• I_b = 5 nA	• t_{hold} = 1 ms
• V_0 = 235 V	• σ_v = 49°
• f = 75 kHz	• $\omega_q = 6.5 \times 10^4$ s^{-1}

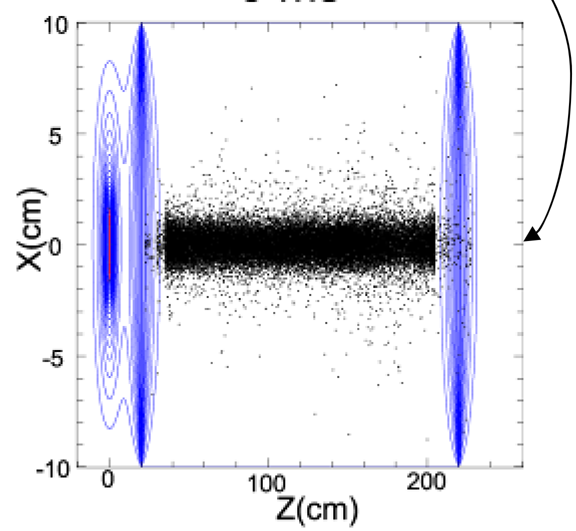


- $n(r_0) = 1.4 \times 10^5$ cm^{-3}
- R_b = 1.4 cm
- s = 0.2

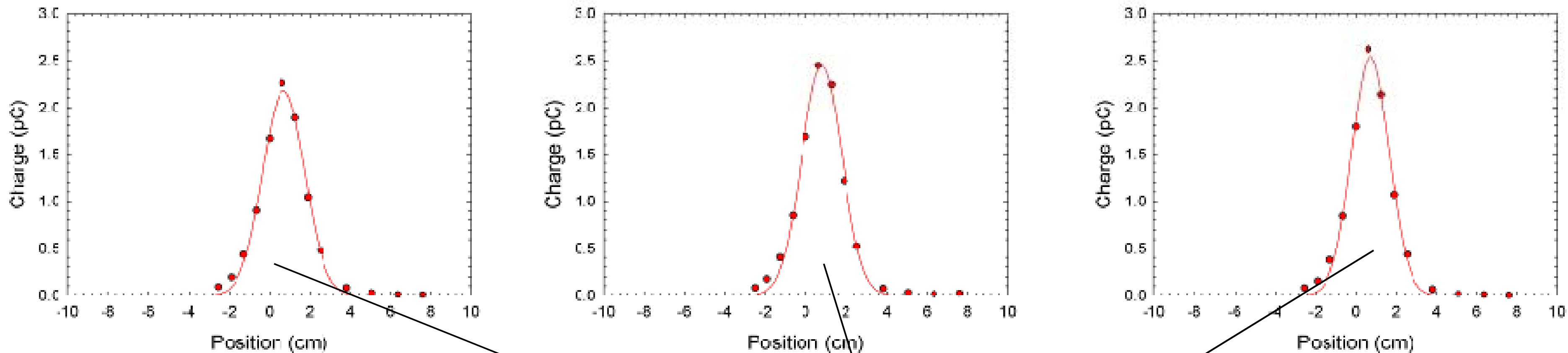
The charge $Q(r)$ is collected through the 1 cm aperture is averaged over 2000 plasma shots.

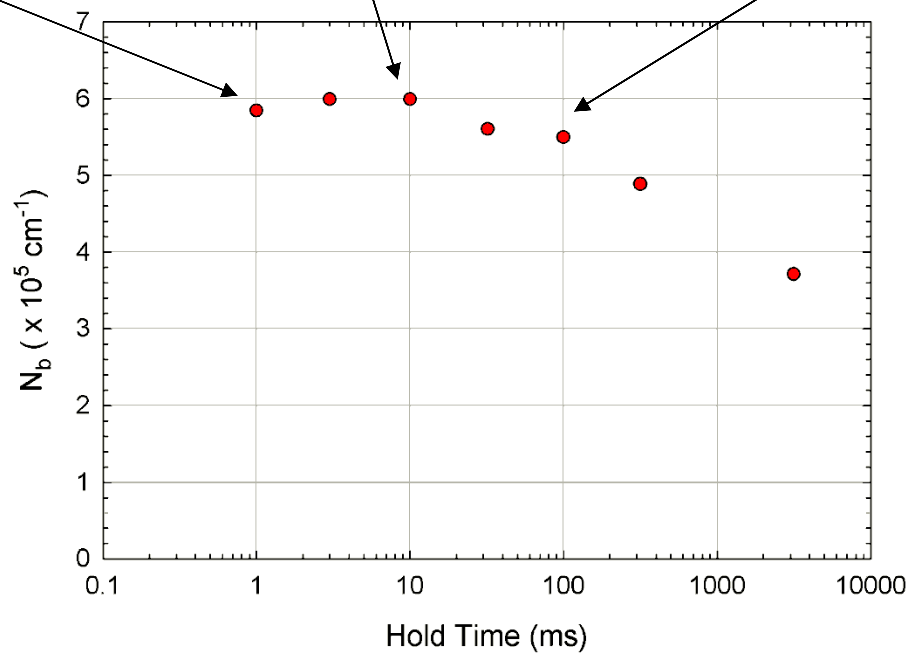
The density is calculated from

$$n(r) = \frac{Q(r)}{e\pi r_{aperture}^2 l_{plasma}}$$



PTSX Simulates Equivalent Propagation Distances of 7.5 km





- At f = 75 kHz, a lifetime of 100 ms corresponds to **7,500 lattice periods**.
- If S is 1 m, the PTSX simulation experiment would correspond to a **7.5 km beamline**.

- $s = \omega_p^2/2\omega_q^2 = 0.18$.
- V_0 = 235 V
  f = 75 kHz
  σ_v = 49°

Phys. Rev. Lett. **92**, 155002 (2004).

Transverse Bunch Compression by Increasing ω_q

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

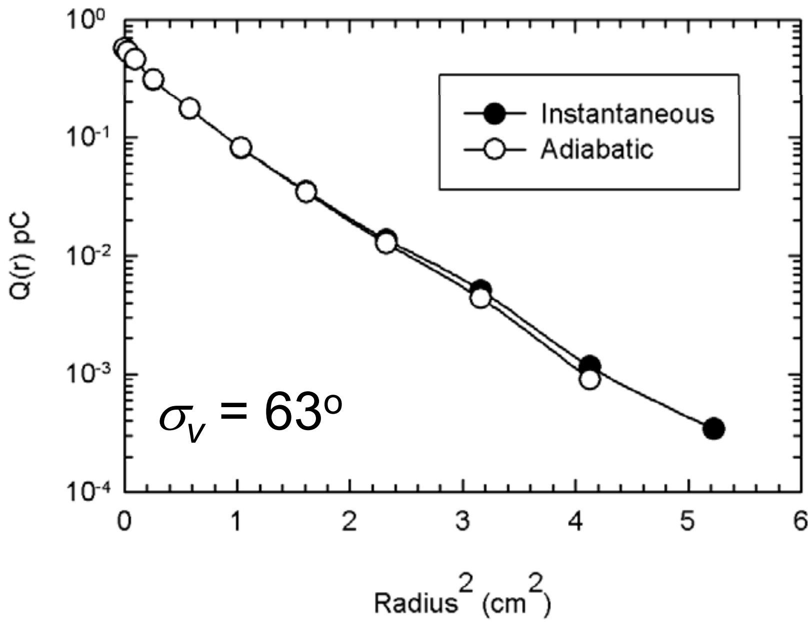
If line density N is constant and kT doesn't change too much, then increasing ω_q decreases R, and the bunch is compressed.

$$\omega_q = \frac{8eV_{0\max}}{m\pi r_w^2 f}\xi$$

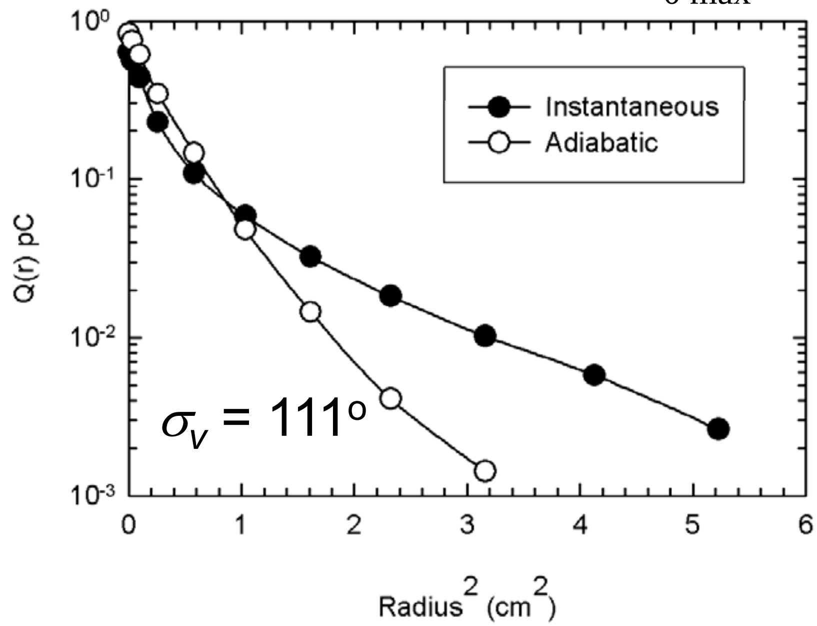
Either
1.) increasing $V_{0\max}$ (increasing magnetic field strength) or
2.) decreasing f (increasing the magnet spacing)
increases ω_q

Adiabatic Amplitude Increases Transversely Compress the Bunch

20% increase in $V_{0\,max}$



90% increase in $V_{0\,max}$

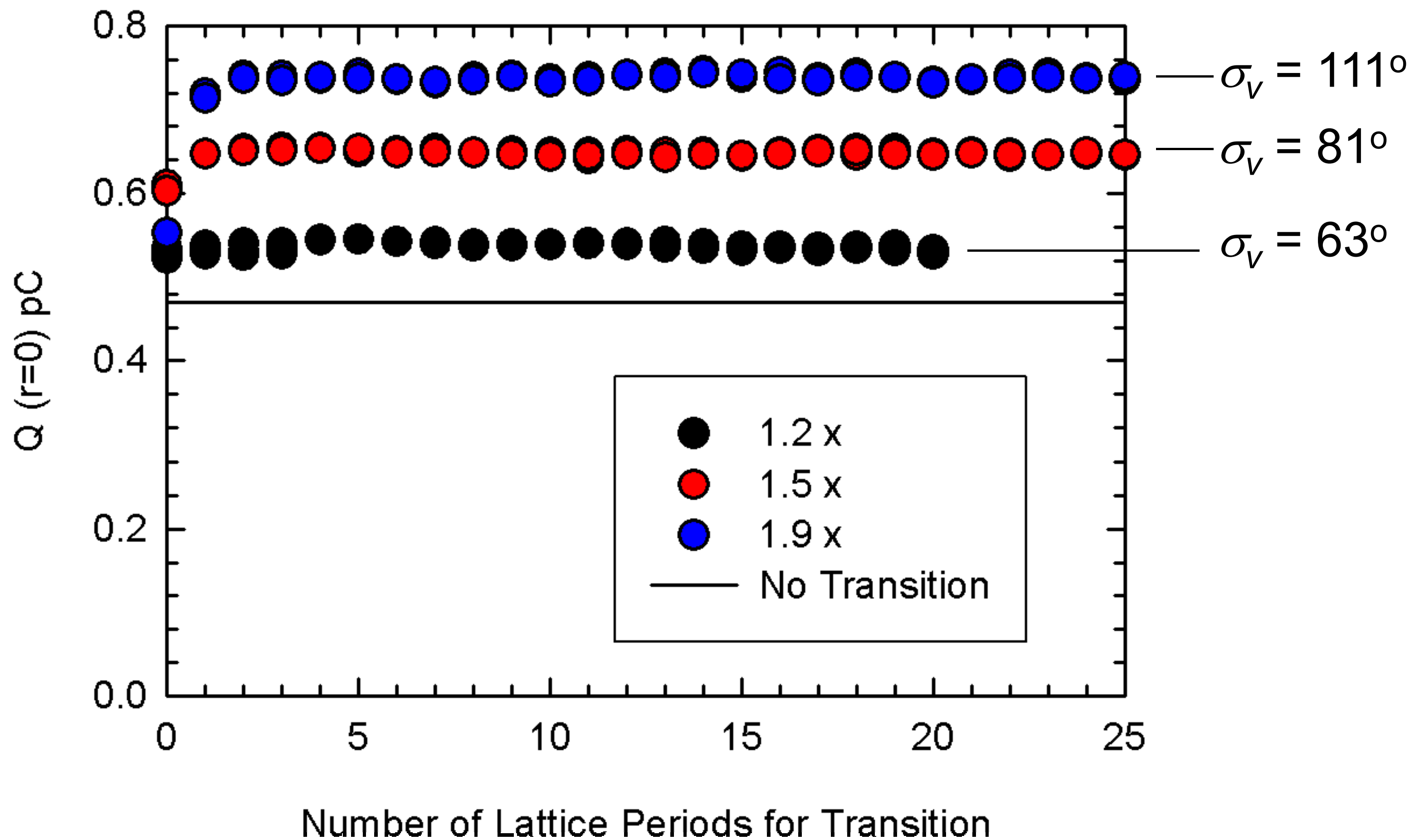


Baseline		Adiabatic	Instantaneous
R = 0.83 cm	R = 0.79 cm	R = 0.63 cm	R = 0.93 cm
kT = 0.12 eV	kT = 0.16 eV	kT = 0.26 eV	kT = 0.58 eV
s = 0.20	s = 0.18	s = 0.10	s = 0.08
$\varepsilon \sim R\sqrt{kT} \rightarrow$	$\Delta\varepsilon$ = 10%	$\Delta\varepsilon$ = 10%	$\Delta\varepsilon$ = 140%

- $s = \omega_p^2/2\omega_q^2 = 0.20$
- $v/v_0 = 0.88$
- $V_{0\,max}$ = 150 V
- f = 60 kHz
- σ_v = 49°

Nucl. Inst. and Meth. in Phys. Res. **A 577**, 117 (2007).

Less Than Four Lattice Periods Adiabatically Compress the Bunch

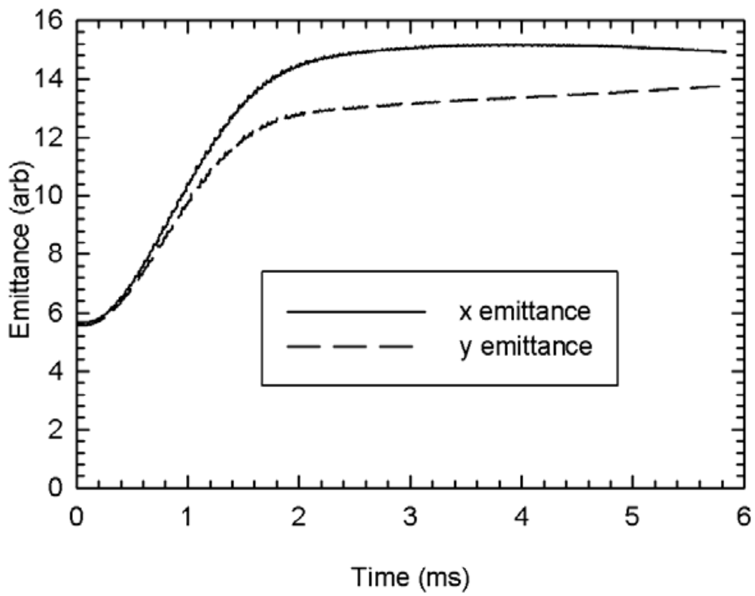


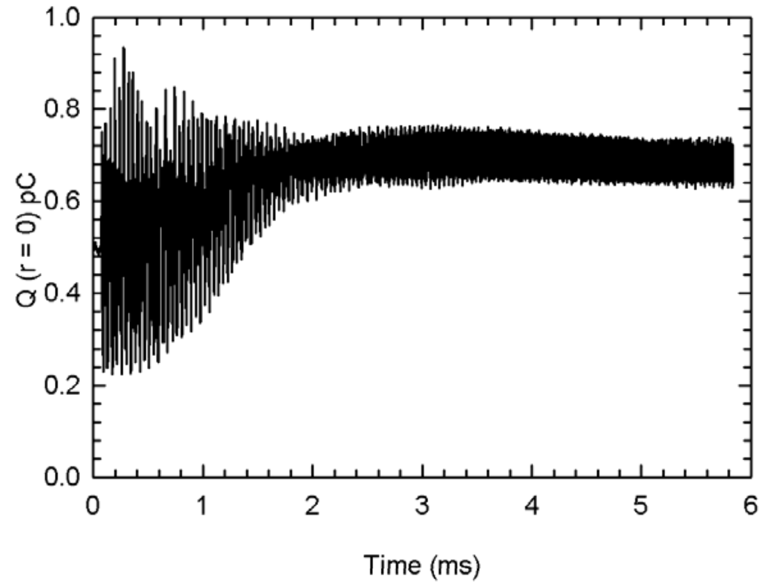
- $s = \omega_p^2/2\omega_q^2 = 0.20$
- $v/v_0 = 0.88$
- $V_{0\ max}$ = 150 V
- f = 60 kHz
- $\sigma_v = 49°$

Phys. Rev. ST Accel. Beams **10**, 064202 (2007).

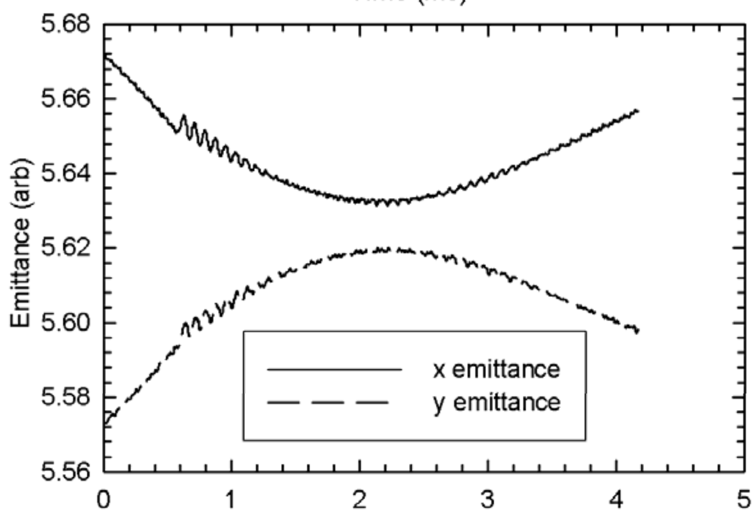
2D WARP PIC Simulations Corroborate Adiabatic Transitions in Only Four Lattice Periods

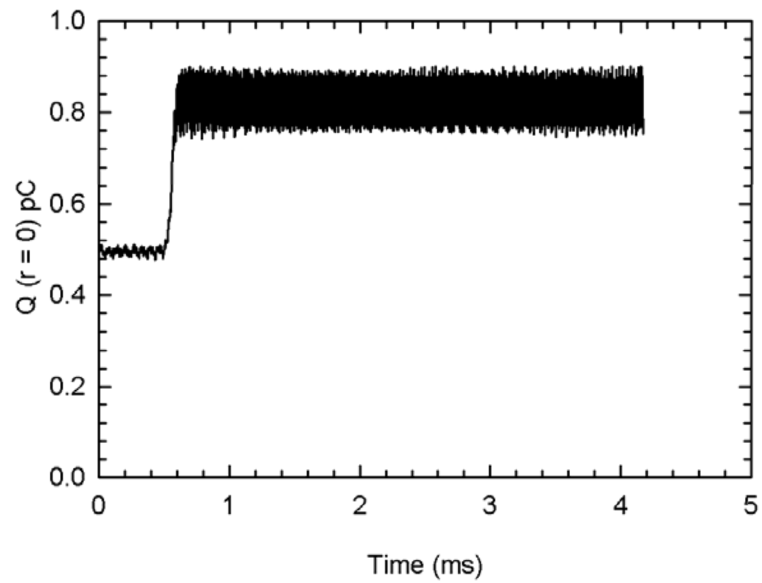
Instantaneous Change.





Change Over Four Lattice Periods.





Nucl. Inst. and Meth. in Phys. Res. **A 577**, 117 (2007).

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Peak Density Scales Linearly with ω_q

$$m\omega_q^2 R^2 \sim 2kT$$

$$\varepsilon \sim R\sqrt{kT}$$

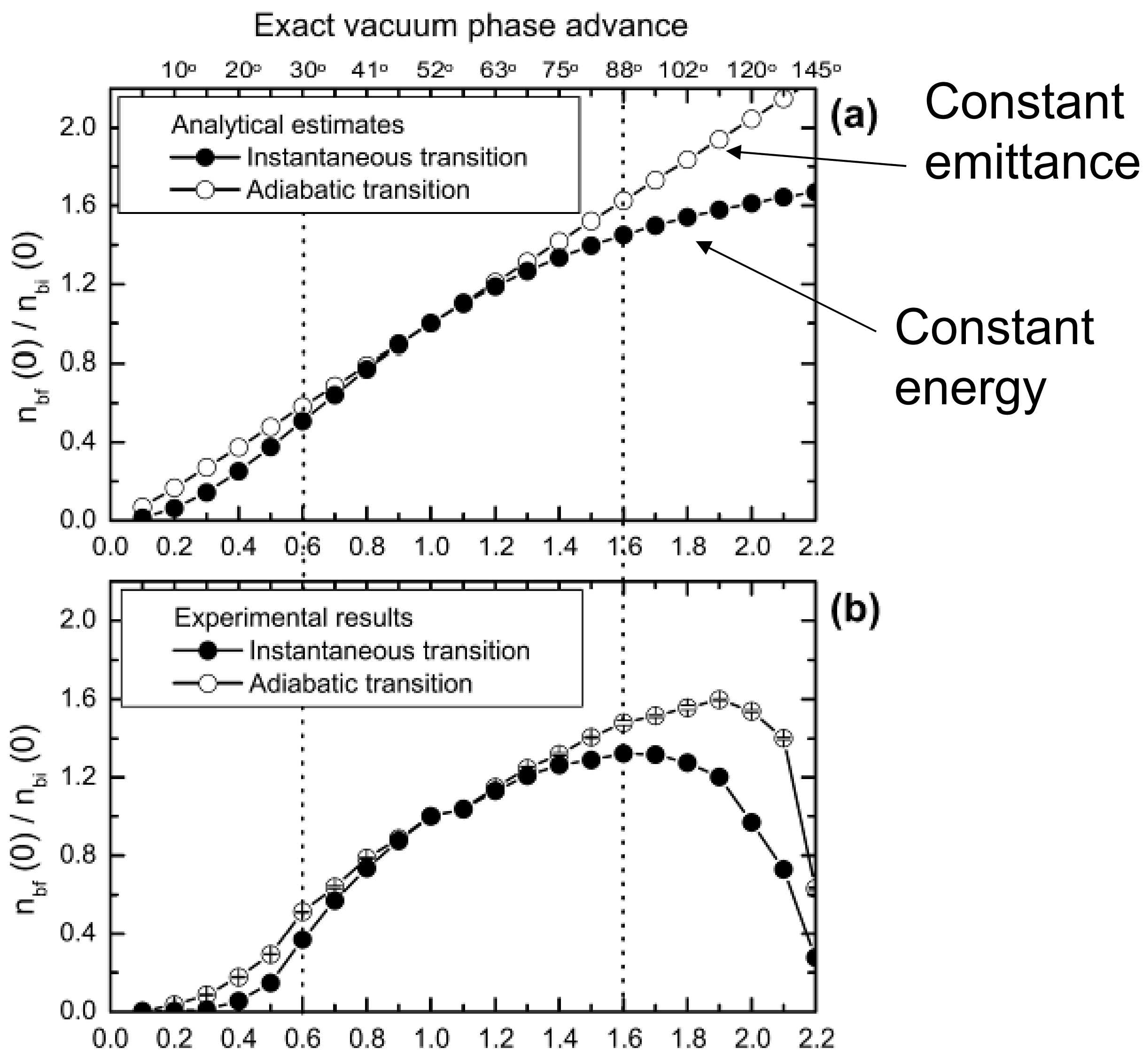
$$\Downarrow$$

$$\omega_q R^2 \sim const.$$

$$n(0)R^2 \sim N = const.$$

$$n(0) \sim \omega_q$$

$$\omega_q = \frac{8eV_{0\max}}{m\pi r_w^2 f}\xi$$



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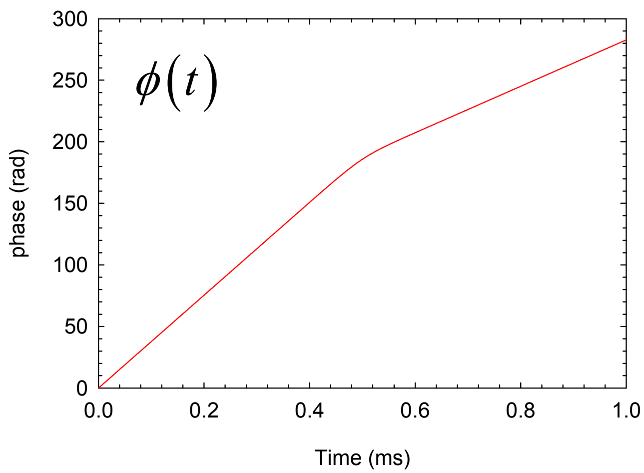
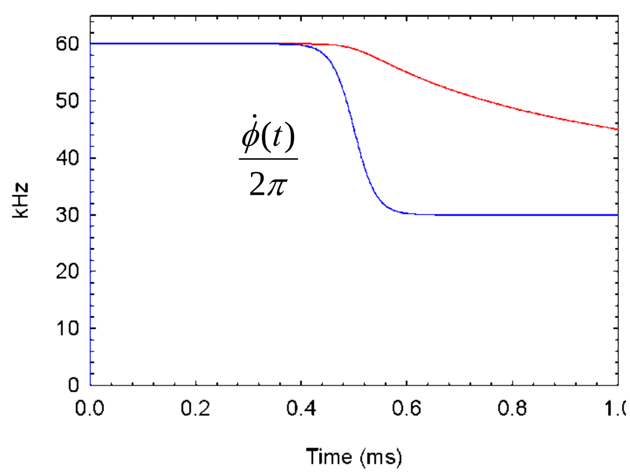
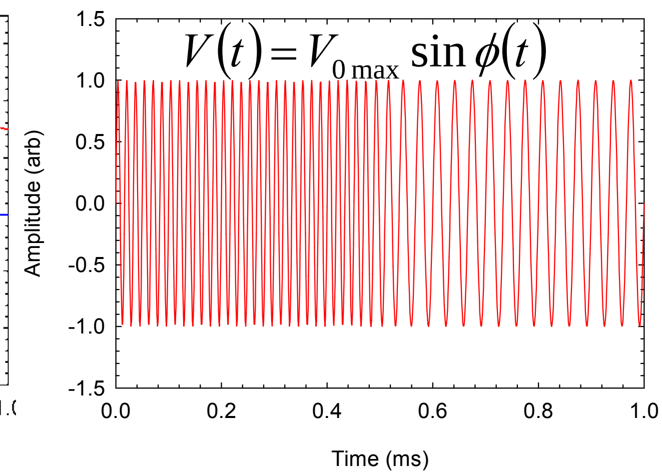
Increasing ω_q adiabatically by decreasing f

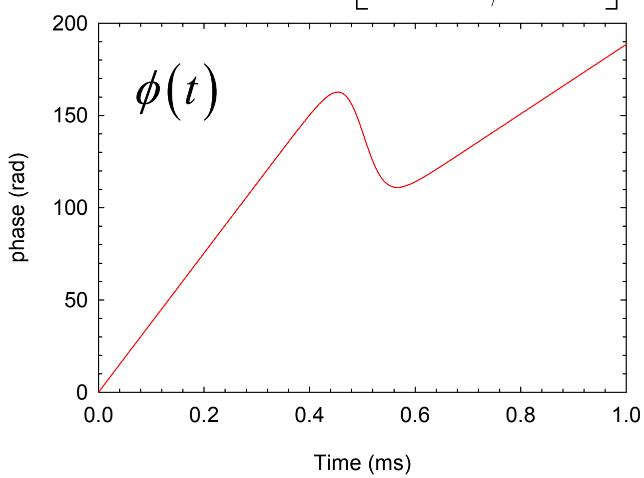
$$V(t) = V_{0\,\max} \sin\phi(t)$$

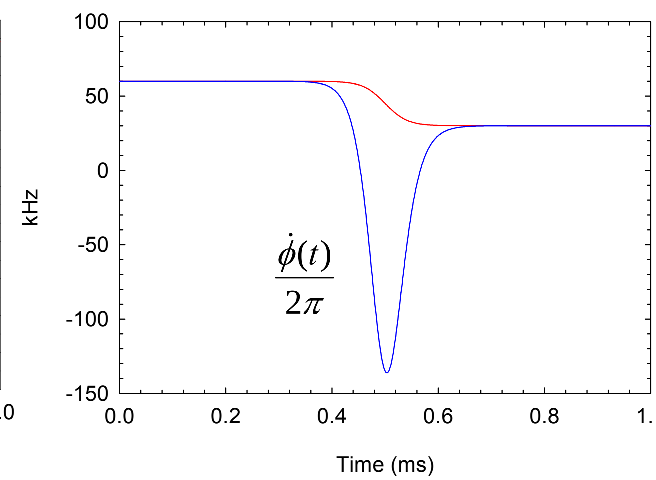


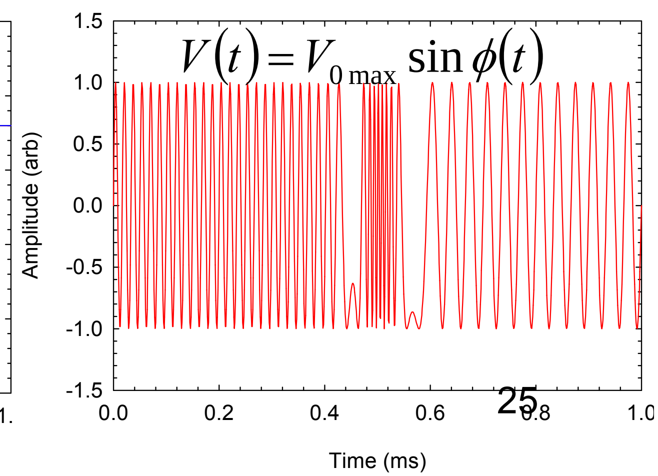




$$\frac{\phi(t)}{2\pi} = f_f t + \frac{f_i - f_f}{2} t \left[\tanh \frac{-(t - t_{1/2})}{\tau/2} + 1 \right]$$

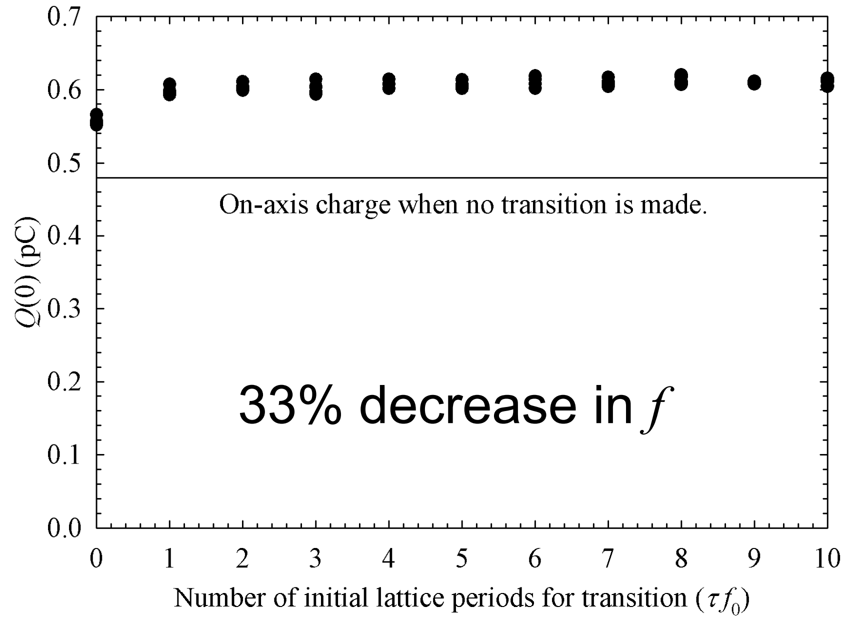




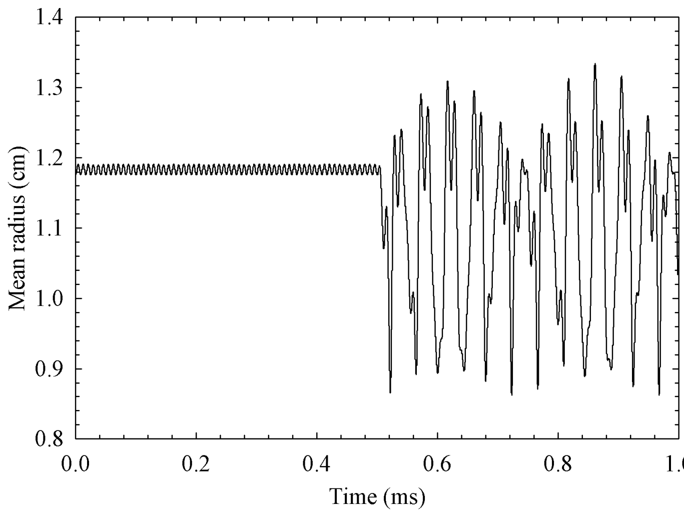


Adiabatically Decreasing f Compresses the Bunch

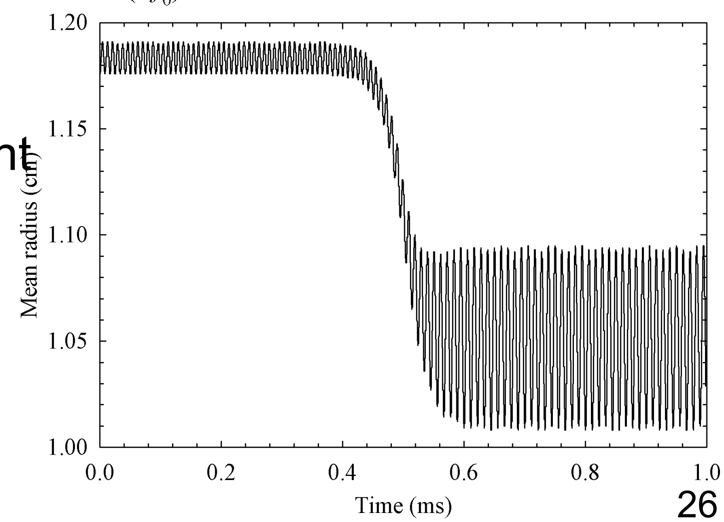
- $s = \omega_p^2/2\omega_q^2$ = 0.2.
- ν/ν_0 = 0.88
- $V_{0\,max}$ = 150 V
  f = 60 kHz
  σ_v = 49°



$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$

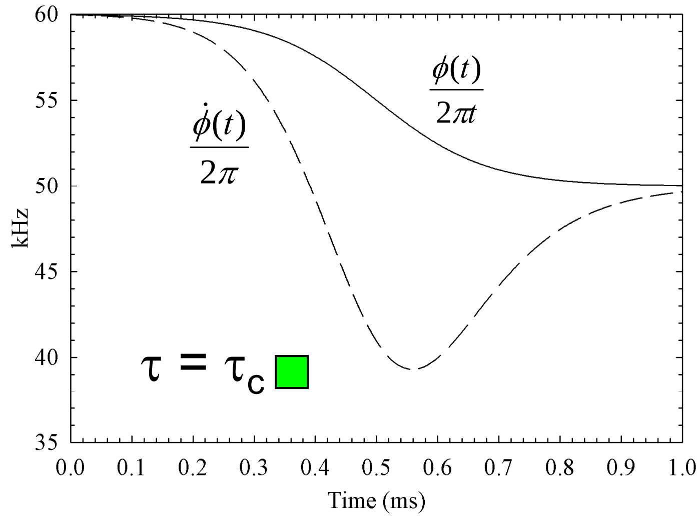


Good agreement with KV-equivalent beam envelope solutions.



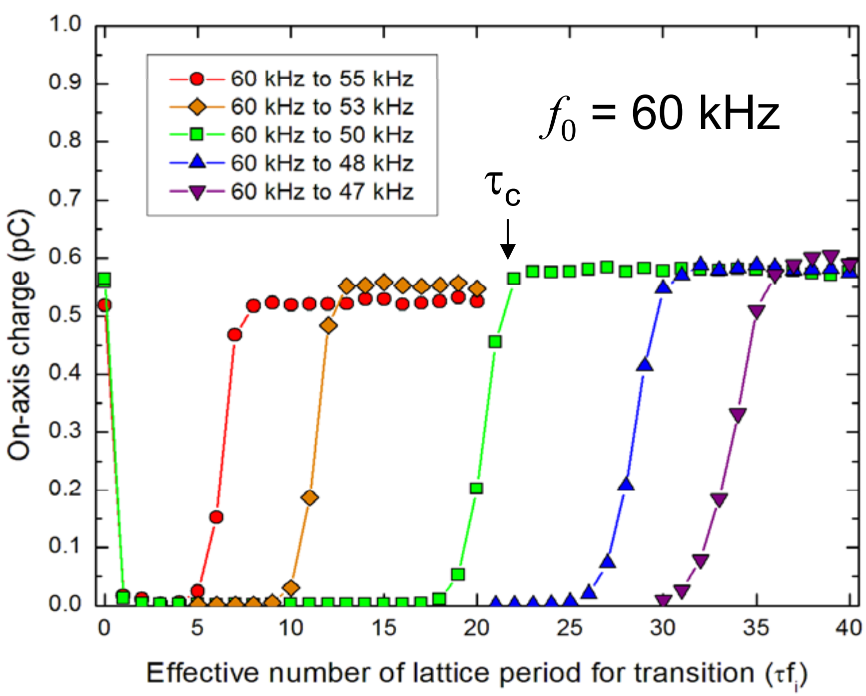
Phys. Rev. ST Accel. Beams **10**, 124201 (2007).

Transverse Confinement is Lost When Single-Particle Orbits are Unstable



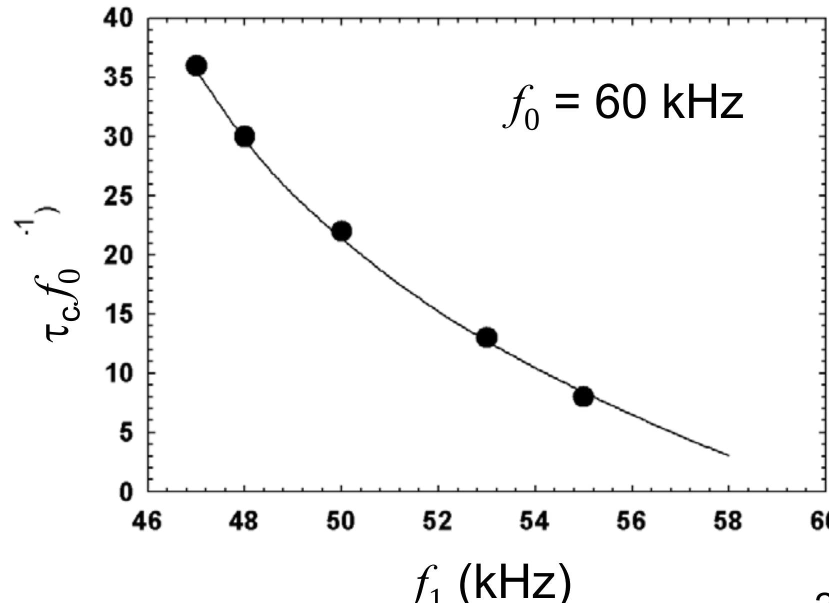
$$\frac{\phi(t)}{2\pi} = f_f t + \frac{f_i - f_f}{2} t \left[\tanh \frac{-(t - t_{1/2})}{\tau/2} + 1 \right]$$

$$\sigma_v = \frac{\omega_q}{f} \propto \frac{1}{f^2}$$



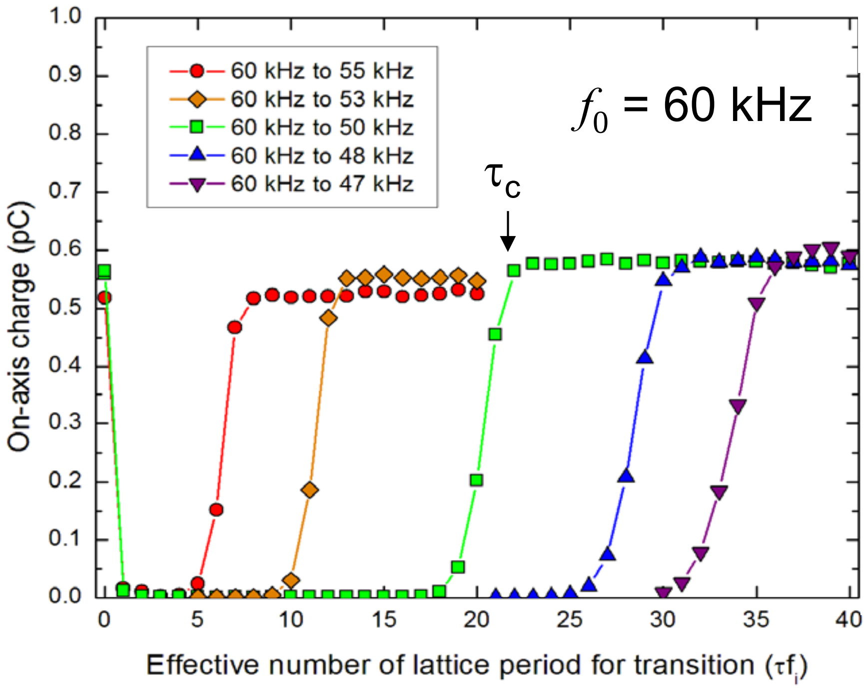
Measured τ_c (dots)

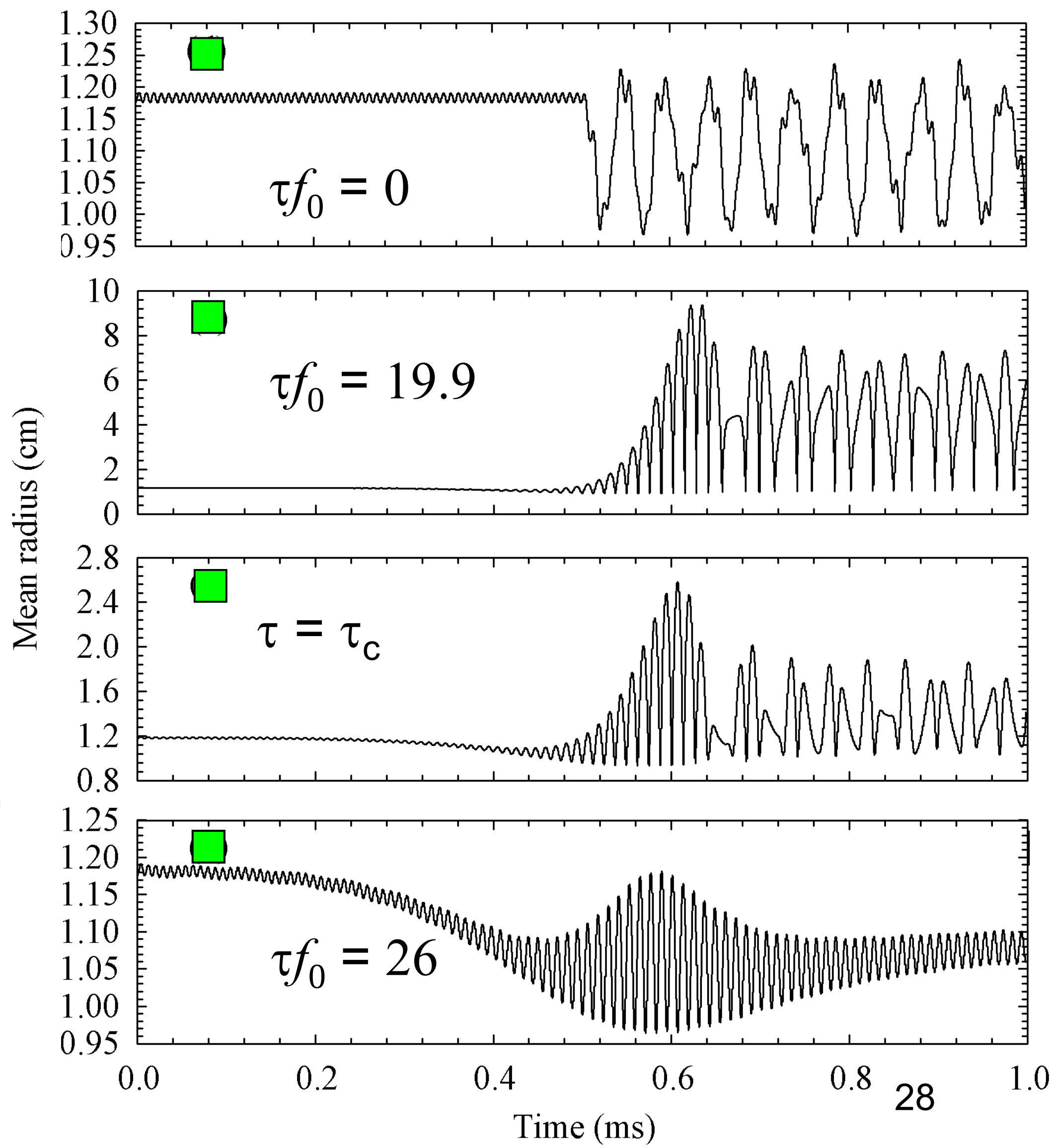
Set $\sigma_{v\ max}$ = 180º and solve for τ_c (line)



Phys. Rev. ST Accel. Beams **10**, 124201 (2007).

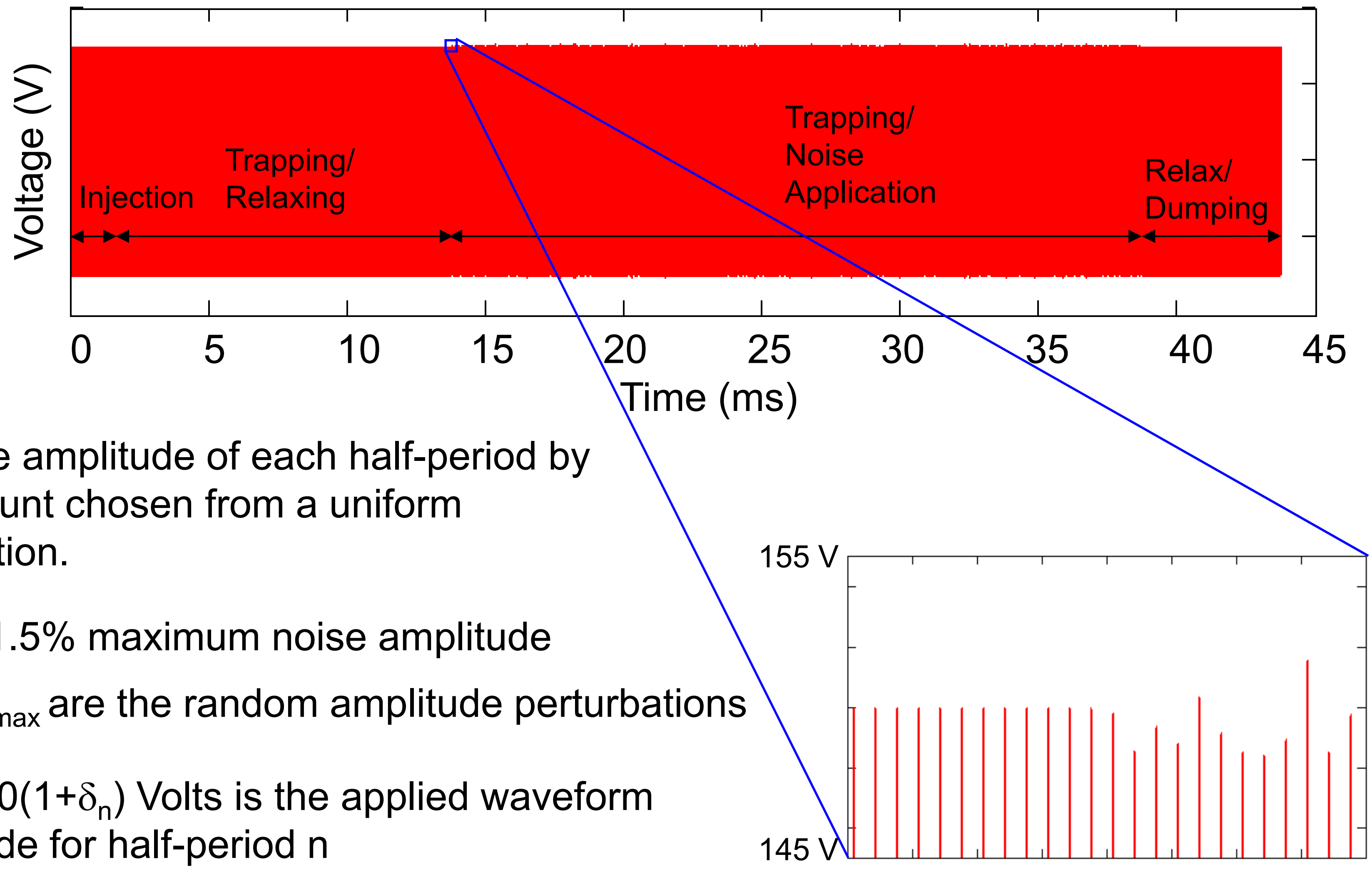
Good Agreement Between Data and KV-Equivalent Beam Envelope Solutions





Phys. Rev. ST Accel. Beams **10**, 124201 (2007).

The Effects of Noise on Beam Propagation



Vary the amplitude of each half-period by an amount chosen from a uniform distribution.

Δ_{max} = 1.5% maximum noise amplitude

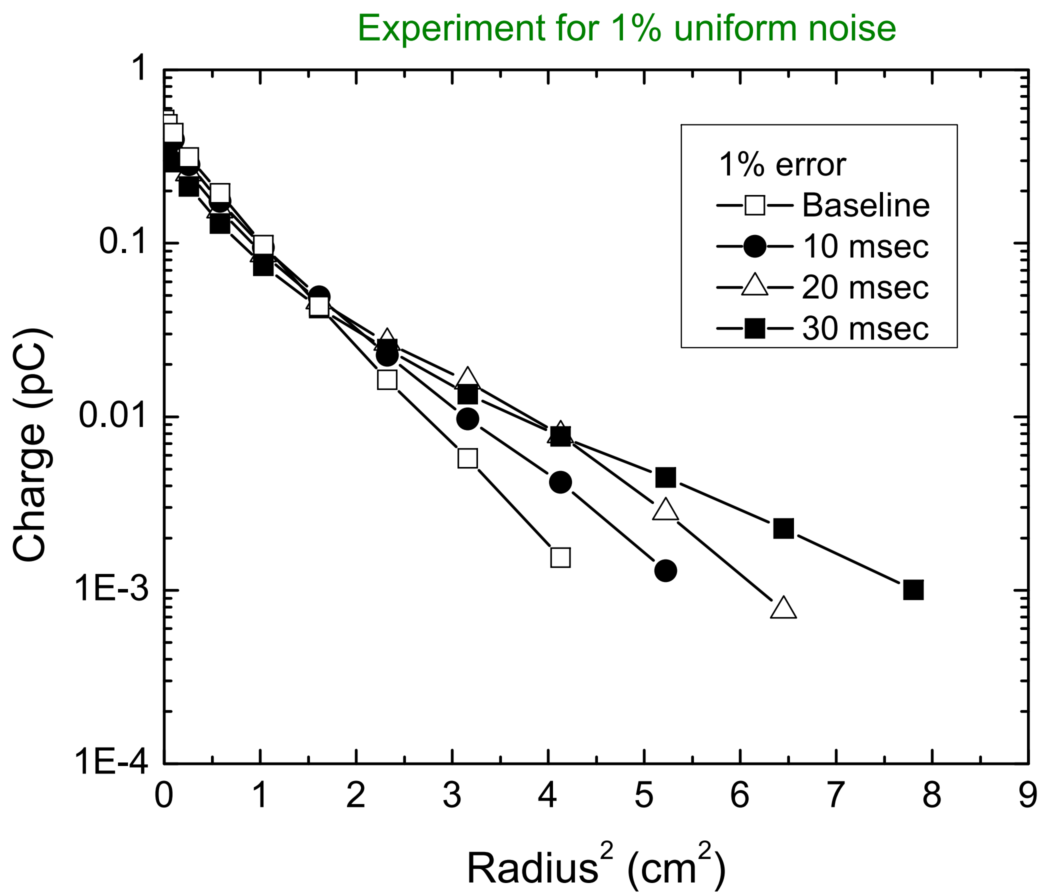
$|\delta_n| < \Delta_{max}$ are the random amplitude perturbations

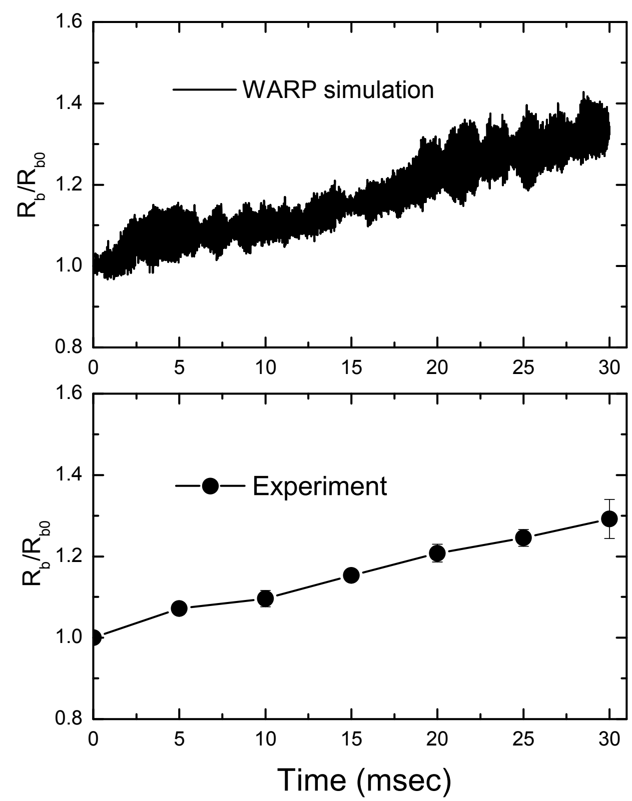
$V_n = 150(1+\delta_n)$ Volts is the applied waveform amplitude for half-period n

Noise Drives a Continuous Increase in RMS Radius

$$N_b = \int_0^{r_w} n_b(r) 2\pi r dr$$

$$R_b = \sqrt{\int_0^{r_w} r^2 n_b(r) 2\pi r dr / N_b}$$





Phys. Rev. ST Accel. Beams **12,** 054203 (2009).

Noise Drives Continuous Emittance Growth

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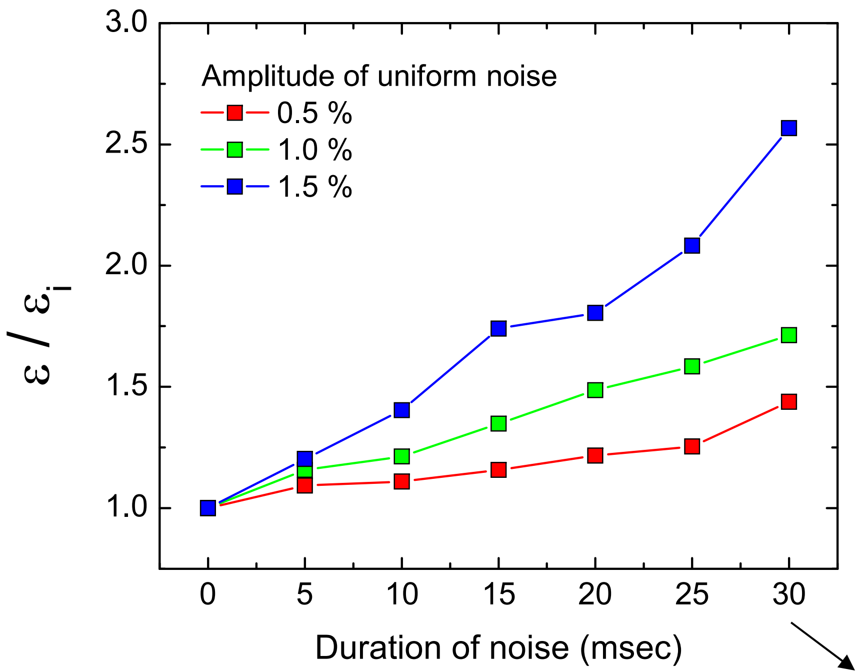
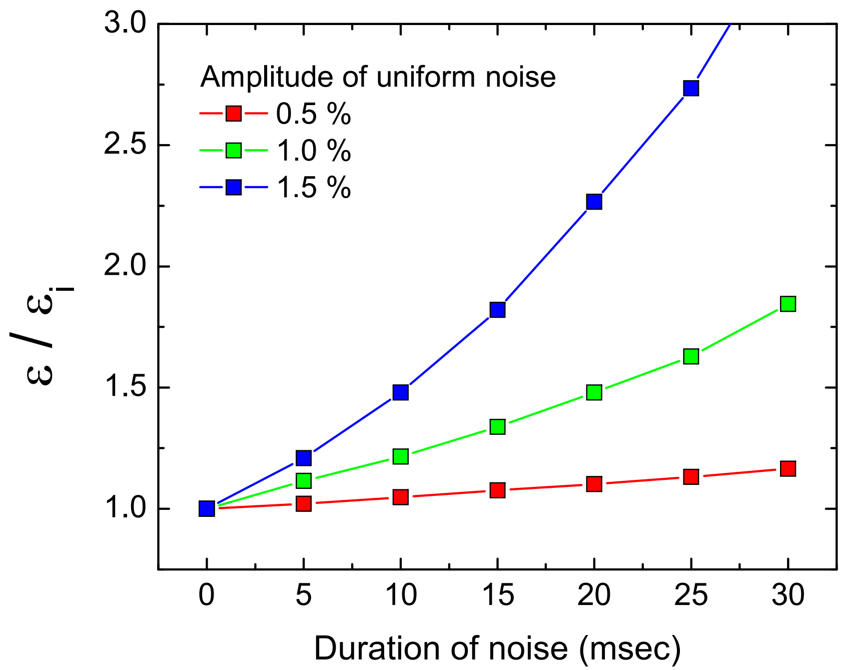
- Continuous emittance growth ~ linear with the noise duration

$$\frac{\varepsilon}{\varepsilon_i} = \frac{R_b\sqrt{T_\perp}}{R_{bi}\sqrt{T_{\perp i}}},\ \ m\omega_q^2 R_b^2 = 2k_B T_\perp + \frac{N_b q^2}{4\pi\varepsilon_o}$$

Experiments



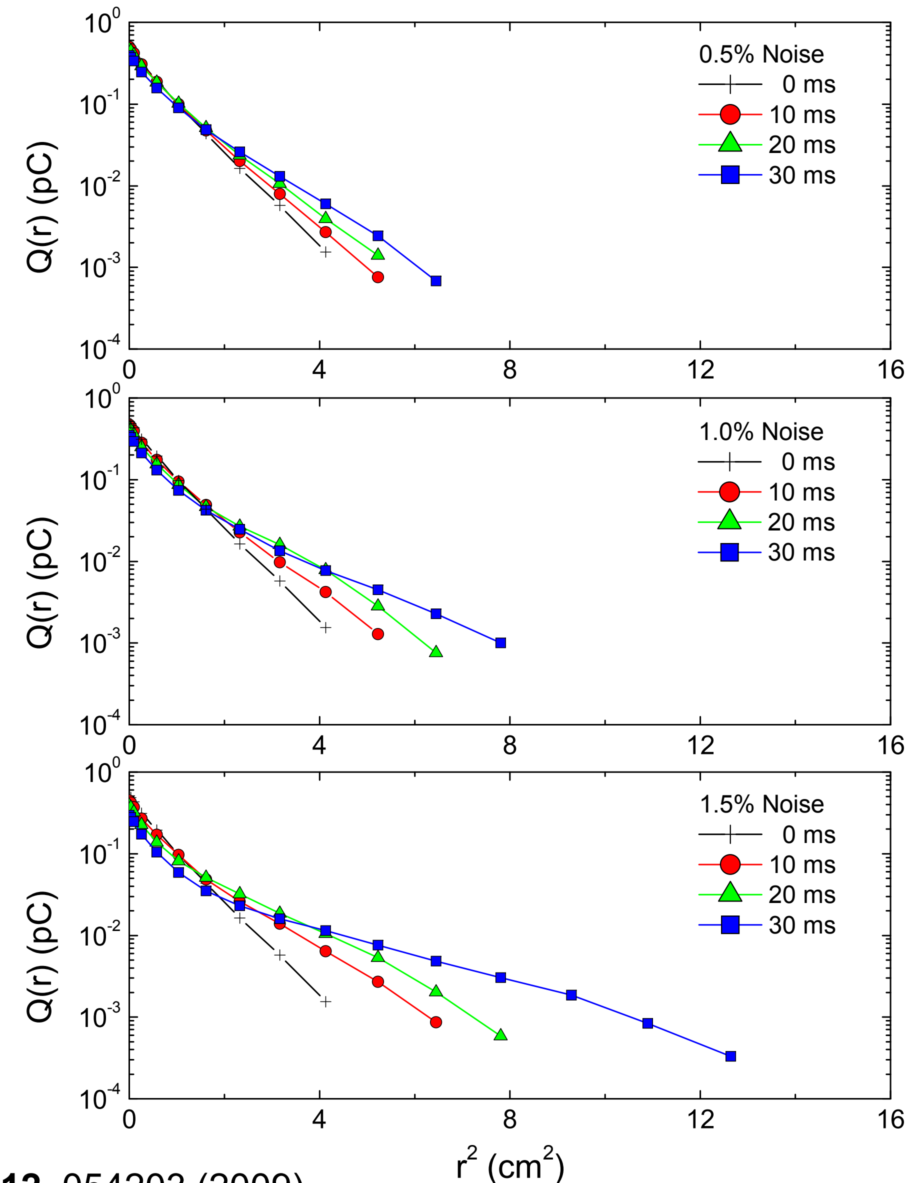
WARP 2D PIC Simulations



1800 lattice periods

Phys. Rev. ST Accel. Beams **12,** 054203 (2009).

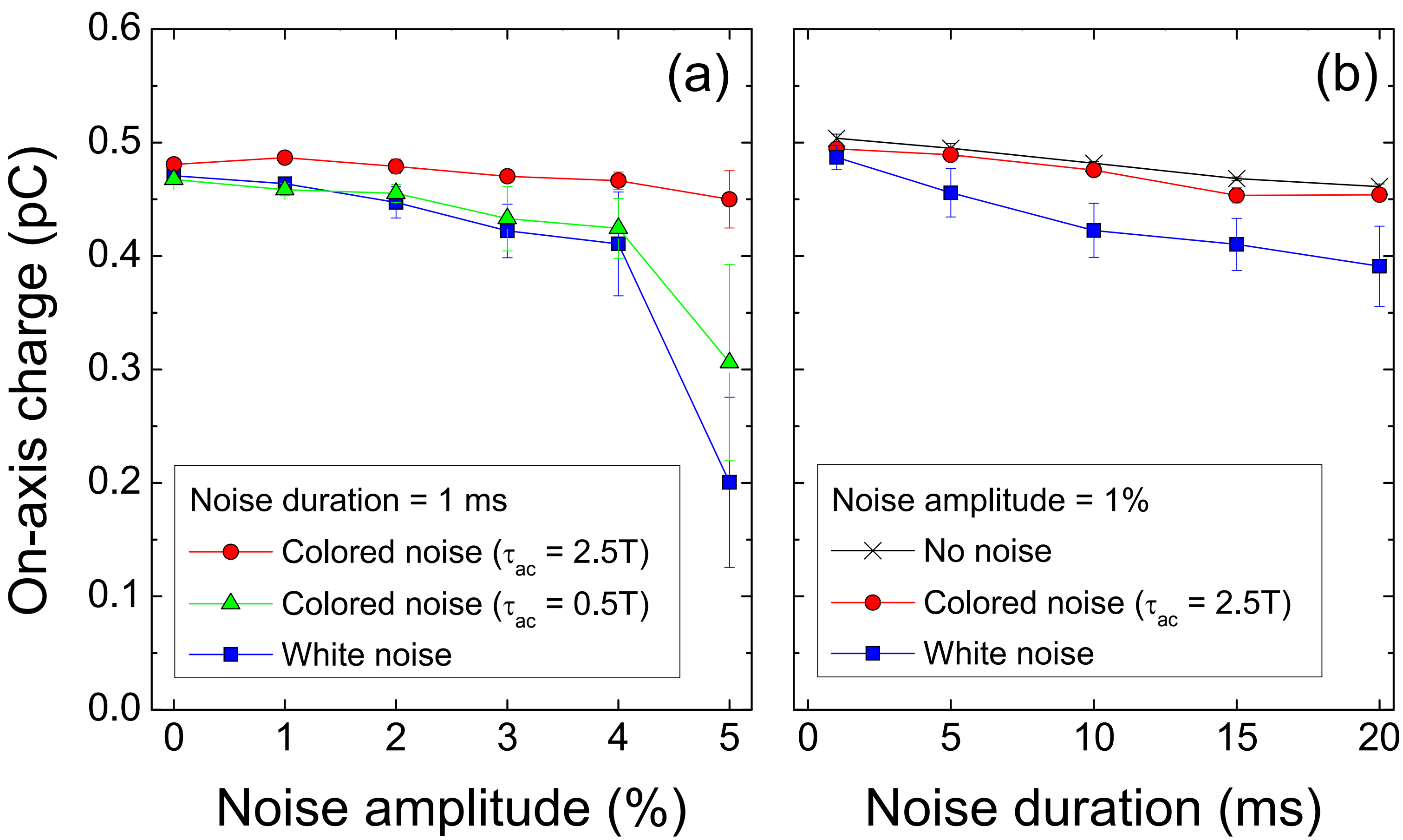
Noise Drives the Continuous Development of a Non-Thermal Tail Distribution



A straight line in the log of Q(r) versus r^2 plot indicates that the radial profile is a Gaussian function of r

Phys. Rev. ST Accel. Beams **12,** 054203 (2009).

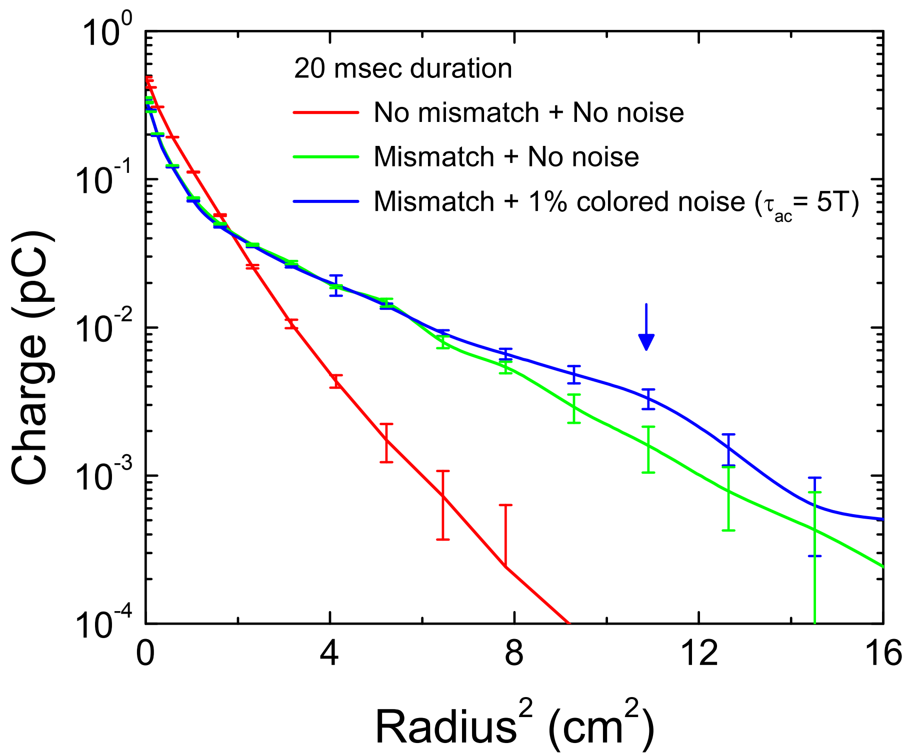
Colored Noise with Finite Autocorrelation Time is Less Detrimental Than White Noise



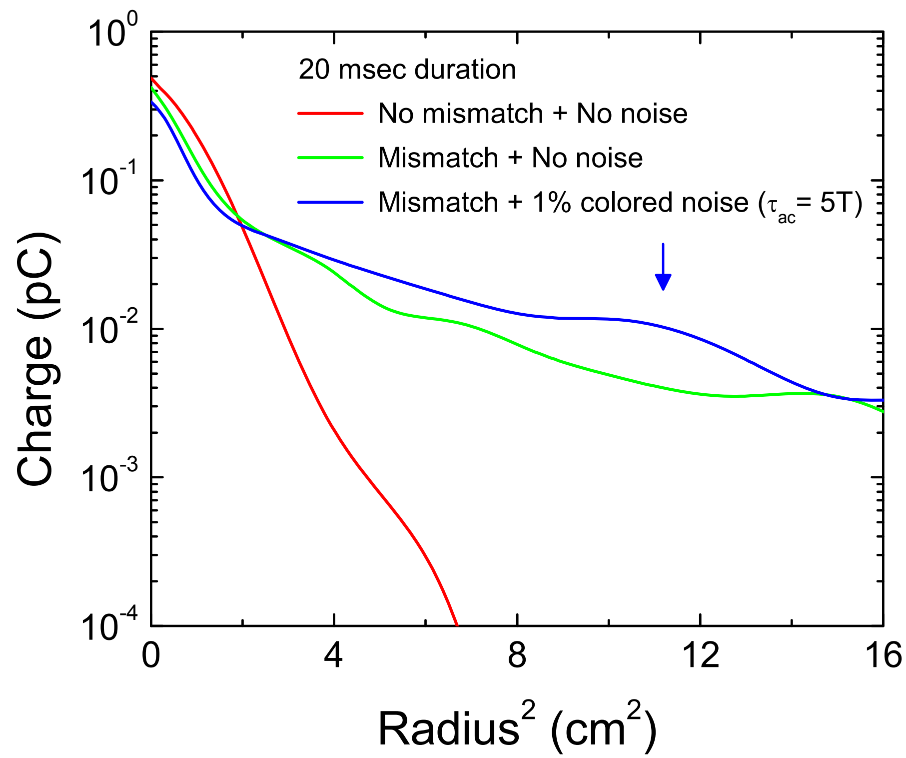
Phys. Rev. ST Accel. Beams **12,** 054203 (2009).

Colored Noise Can Enhance Halo Formation During Beam Mismatch

Experiments



WARP 2D PIC Simulations



Beam mismatch is induced by instantaneously increasing the voltage amplitude by 1.5 times, and switching back to the original value after one focusing period.

Phys. Rev. ST Accel. Beams **12,** 054203 (2009).

C. L. Bohn and I.V. Sideris, Phys. Rev. Lett. **91**, 264801 (2003).

Studies of Beam Modes Begins with Simple Expressions for Two Particular Modes

Breathing mode:

$$\omega_B = 2\omega_q \left(1 - \frac{1}{2}\hat{s}\right)^{1/2}$$

Quadrupole mode:

$$\omega_Q = 2\omega_q \left(1 - \frac{3}{4}\hat{s}\right)^{1/2}$$

Using KV distribution and smooth focusing approximation.

$$\hat{s} = \frac{\omega_p^2}{2\omega_q^2}$$

$$s \sim 0.23$$

$$f_0 = 60kHz$$

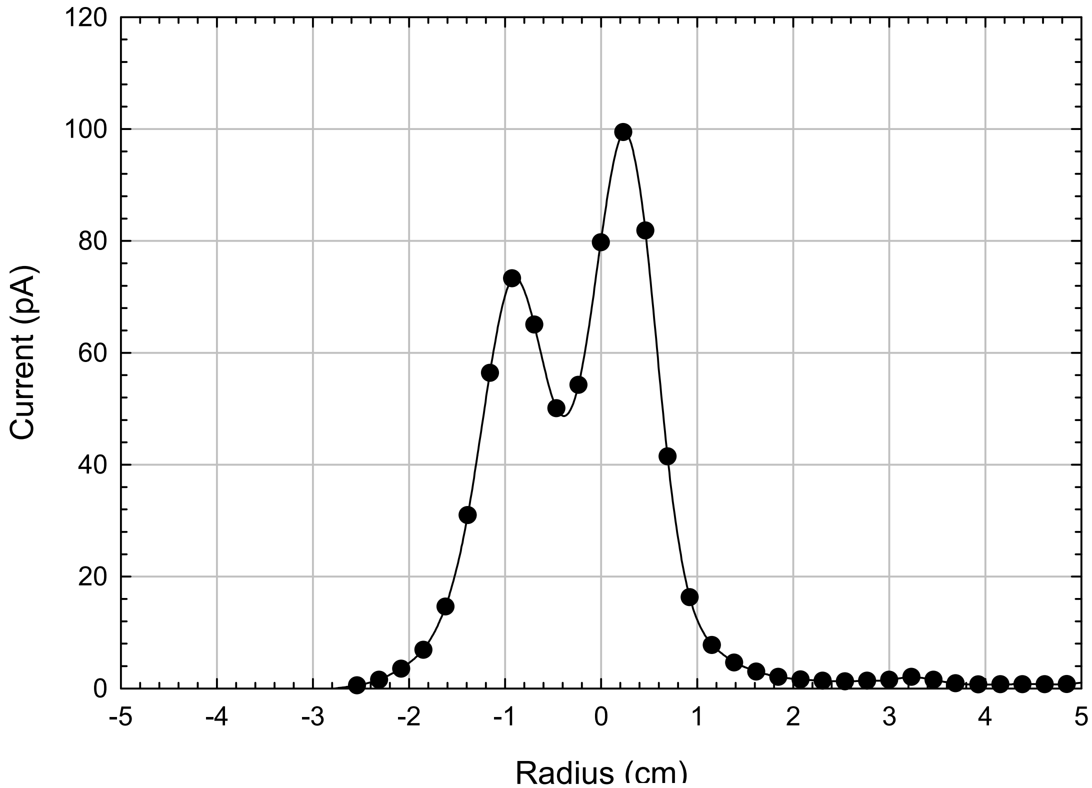
$$\sigma_v \sim 48.6\deg$$

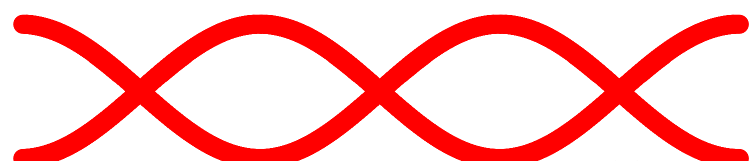
$$2f_q = \frac{2\omega_q}{2\pi} \sim 16.08kHz$$

$$f_B = \frac{\omega_B}{2\pi} \sim 15.13kHz$$

$$f_Q = \frac{\omega_Q}{2\pi} \sim 14.63kHz$$

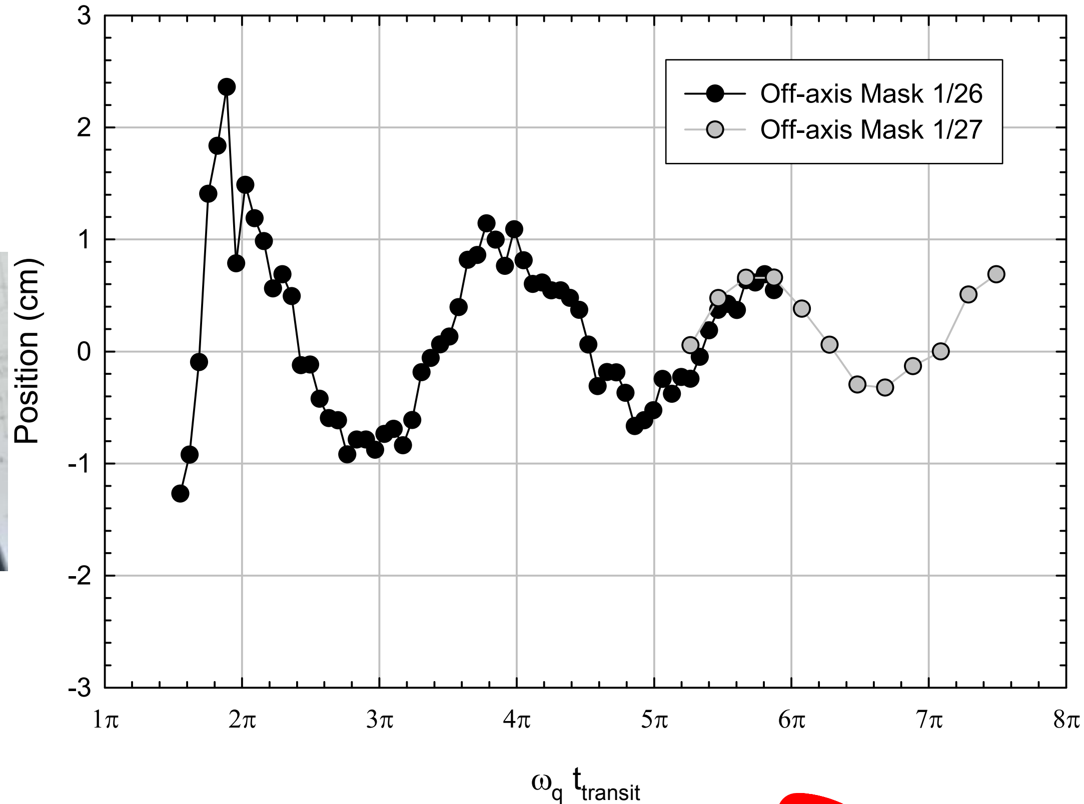
An Initially Hollow Beam Changes from Hollow to Peaked and Back Again as it Streams From Source to Collector

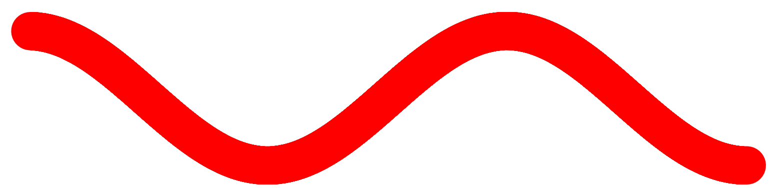




$\omega_q t_{transit} = 1.5\pi$

An ℓ = 1 Dipole Mode Can Be Excited By Masking the Ion Source





PTSX operated in “streaming” mode.

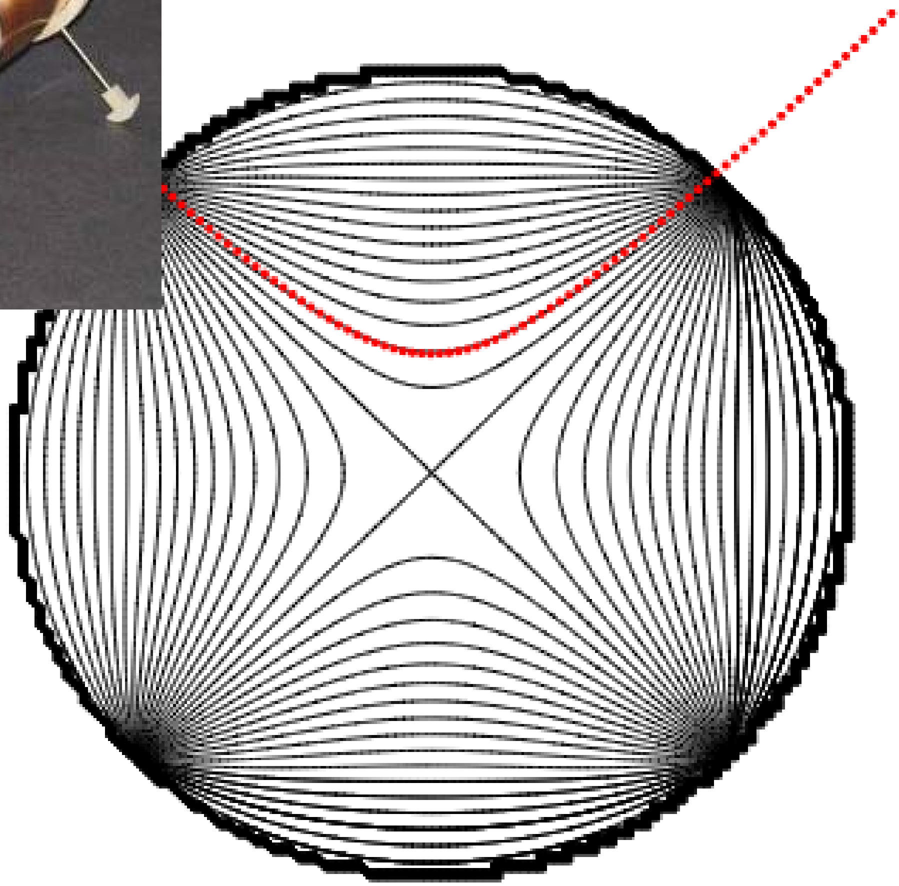
$t_{transit}$ decreased by increasing Injection voltage from 3 V to 3(

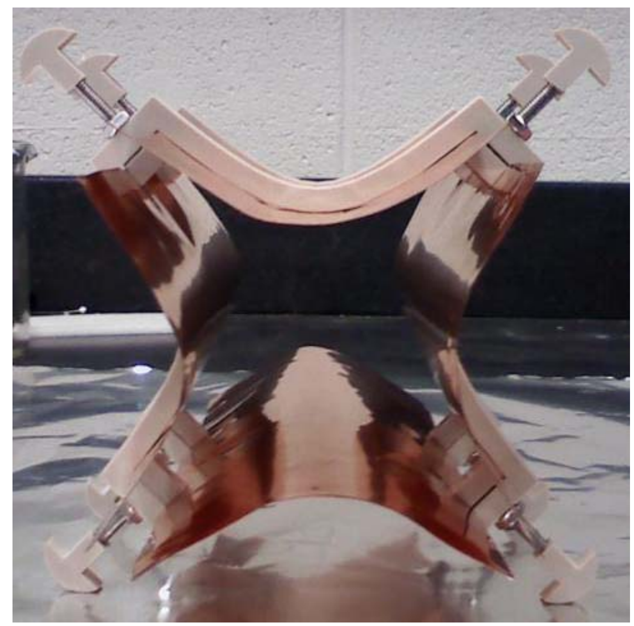
ω_q decreased by raising f from 60 kHz to 90 kHz

V_0 is varied to vary ω_q

$\omega_q t_{transit} = 1.5\pi$

Collective Mode Diagnostics Were Installed But Were Not Sensitive to the Modes and Degraded Confinement





The Modes Can Be Excited Using Externally Applied Perturbations Near the Mode Frequency

- Sum-of-sines is applied to arbitrary function generator

$$V(t) = \widehat{V}_0 \sin(2\pi f_0 t) + \delta V \sin(2\pi f_1 t)$$

where f_1 is near the mode frequency

- Typical Operating Parameters

$$\widehat{V}_0 \sim 140V \qquad \delta V \sim 0.7V (0.5\% \widehat{V}_0)$$

$$f_0 \sim 60KHz \qquad f_1 \sim \text{varying}$$

$$2 f_q \sim 16.083kHz$$

The Modes Can ALSO Be Excited Using the Beat Frequency Between f_0 and f_1

- Sum-of-sines is applied to arbitrary function generator

$$V(t) = \hat{V}_0 \sin(2\pi f_0 t) + \delta V \sin(2\pi f_1 t)$$

where f_1 is near $f_0 \pm f_{mode}$

- Typical Operating Parameters

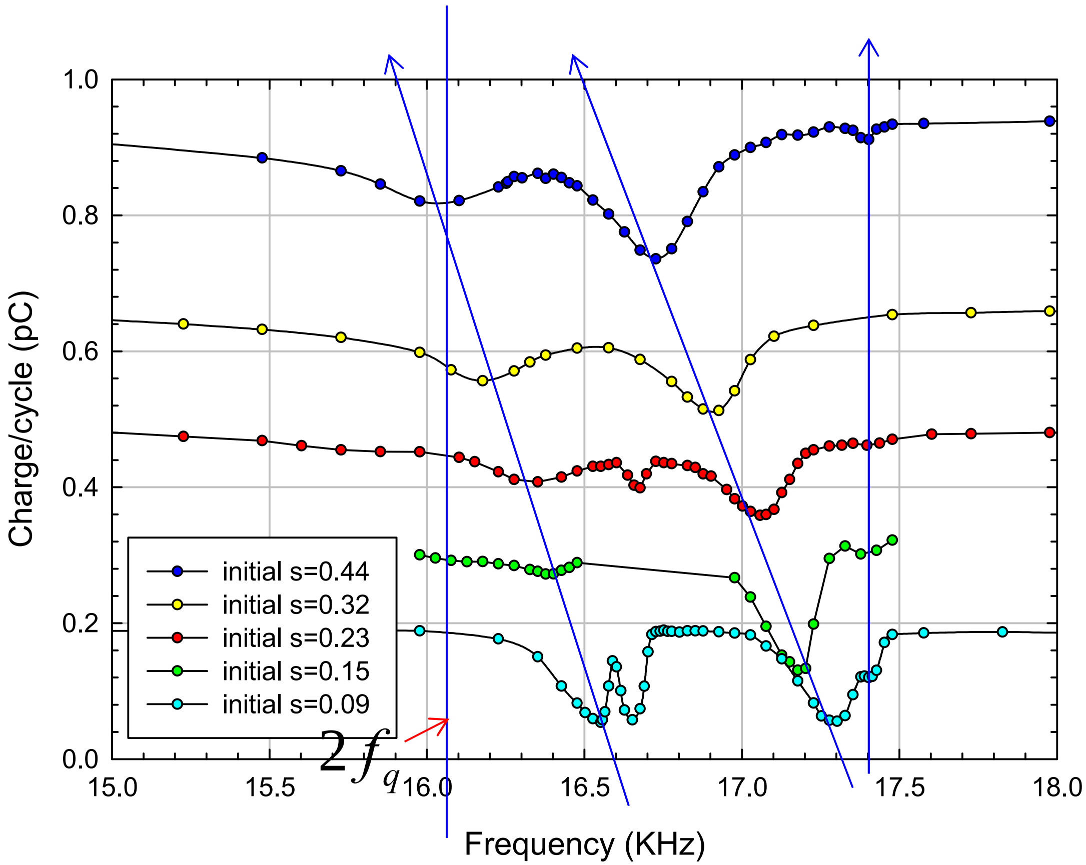
$$\hat{V}_0 \sim 140V \qquad \delta V \sim 0.7V(0.5\%\hat{V}_0)$$

$$f_0 \sim 60KHz \qquad f_1 \sim \text{varying}$$

$$2f_q \sim 16.083kHz \qquad t_{perturbation} = 30ms$$

Beat-Method Frequency Scans With Different Initial Amounts of Space-Charge Attempt to Find the Space-Charge Dependence

Change of on-axis density under different perturbation amplitudes

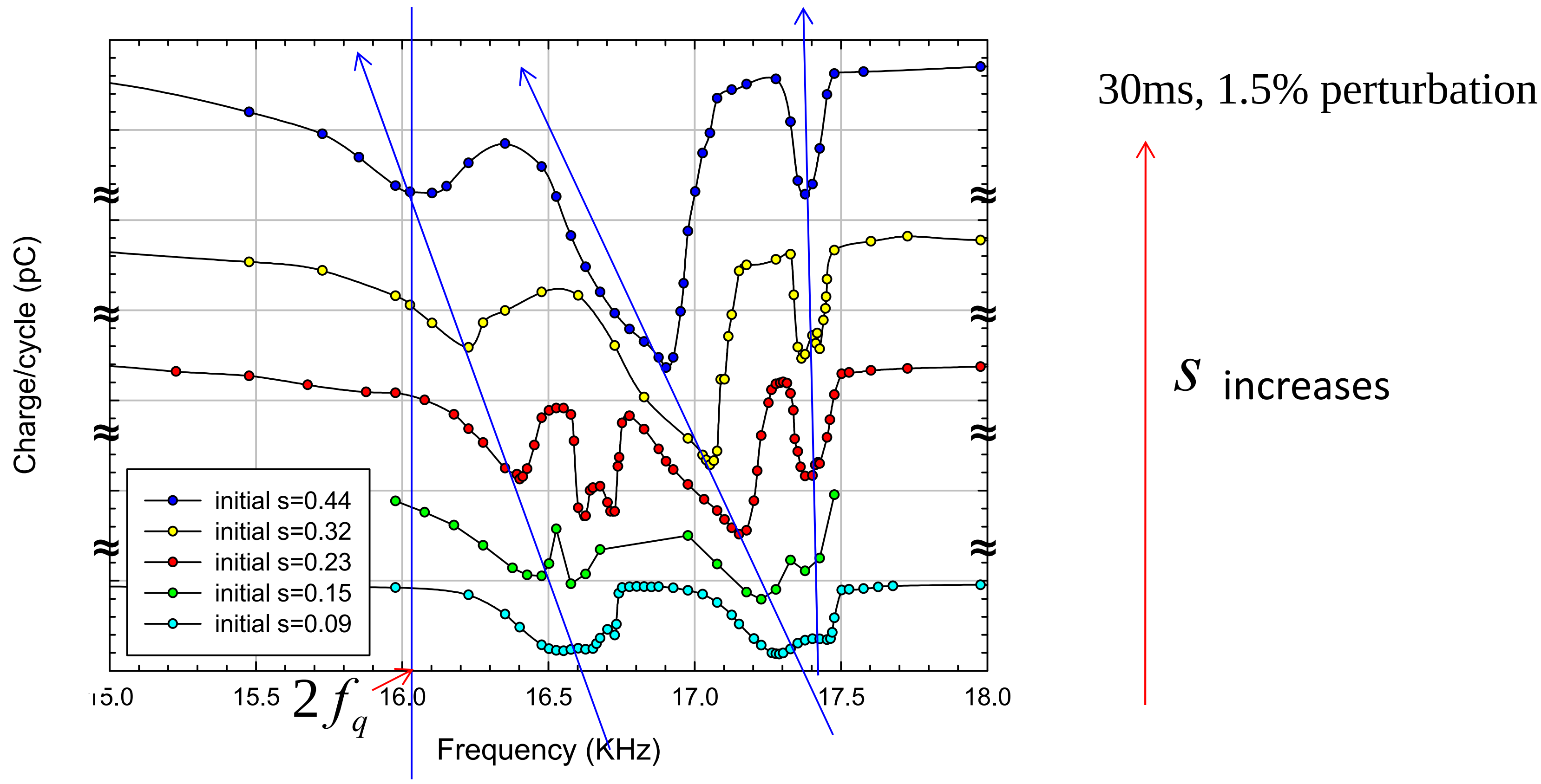


30ms, 1.0% perturbation

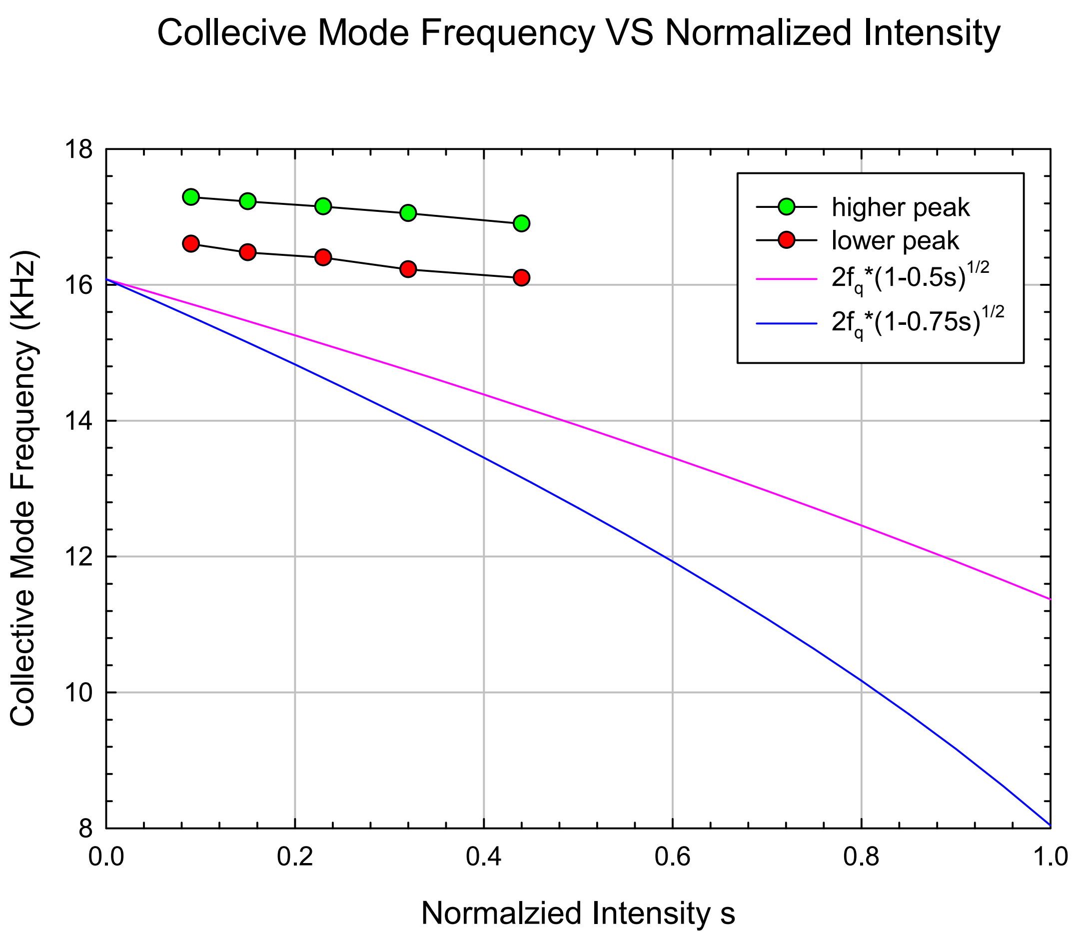
s increases

Beat-Method with Larger Amplitude Perturbations

Change of on-axis density under different perturbation amplitudes

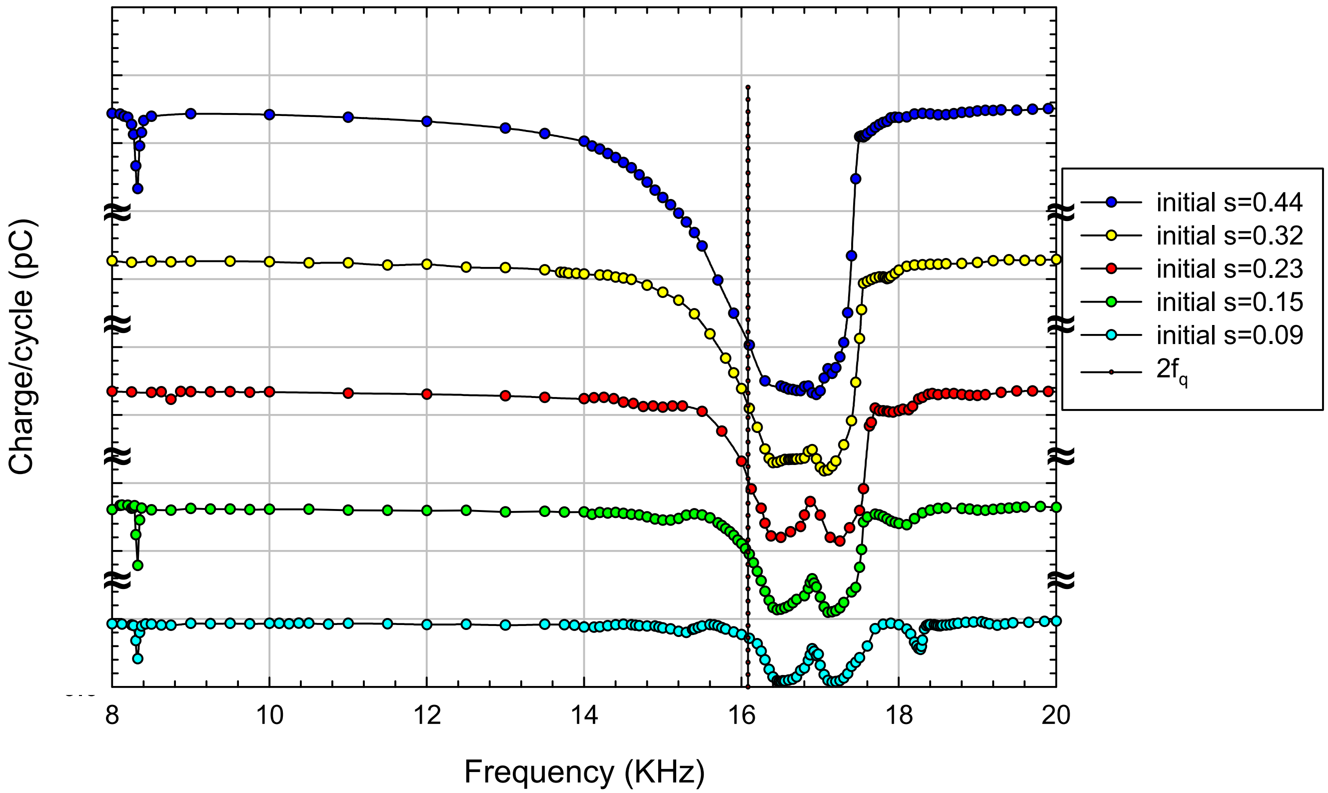


The Measured Frequencies Are Larger Than Those Computed in the Simple Model

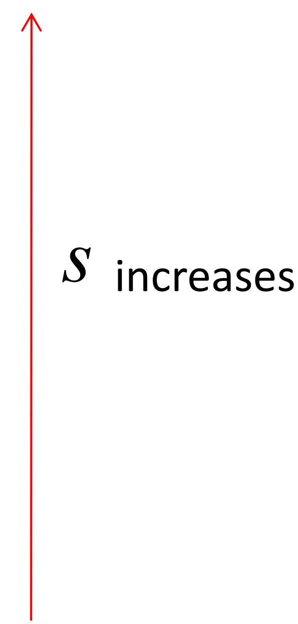


The Linear Drive is Stronger But Does Not Exhibit a Clear Dependence on Space-Charge

Change of on-axis density under different perturbation amplitudes

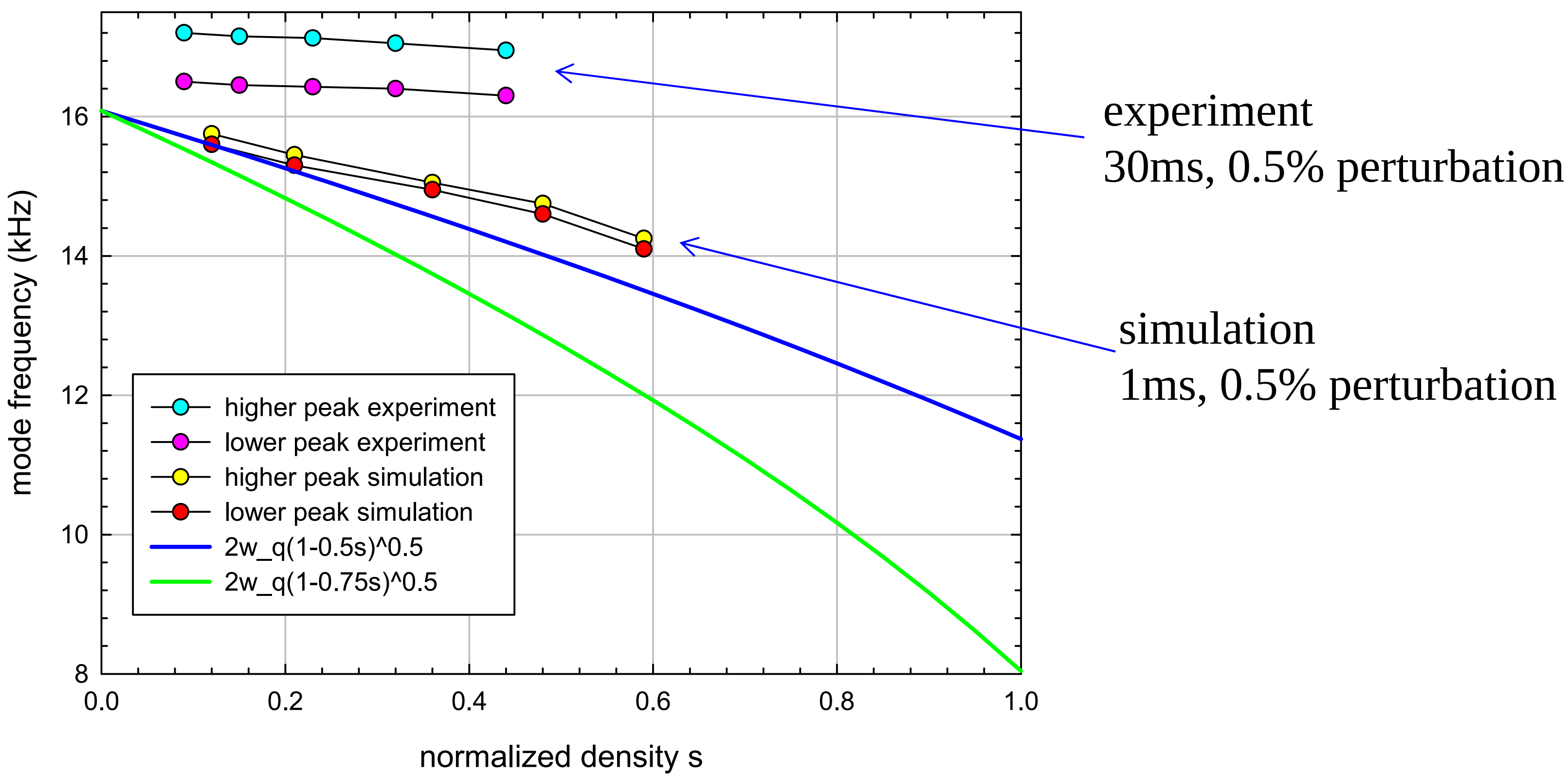


30ms, 0.5% perturbation



Observed Mode Frequencies are Larger Than Those in Warp or the Simple Model

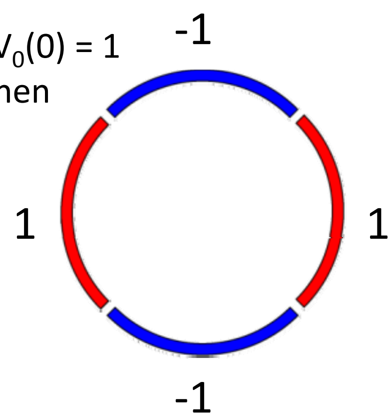
Mode Frequency vs Normalized Density s



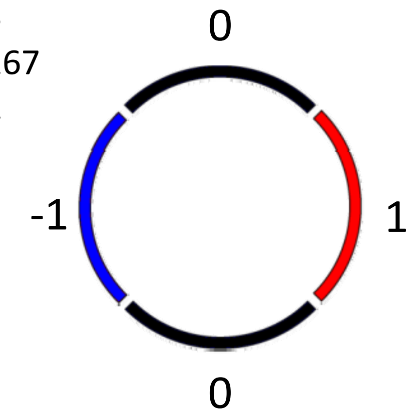
Using a Second Arbitrary Function Generator to Break the Quadrupole Symmetry

$$\phi(r,\theta)=\sum_n C_n\left(\frac{r}{r_w}\right)^n\cos(n\theta)\qquad A_n=\frac{1}{4}\int_0^{2\pi}V(\theta)\cos(n\theta)$$

At t = 0, with $V_0(0)$ = 1
(1, -1, 1, -1), then
A_2 = 1
A_6 = 0.333

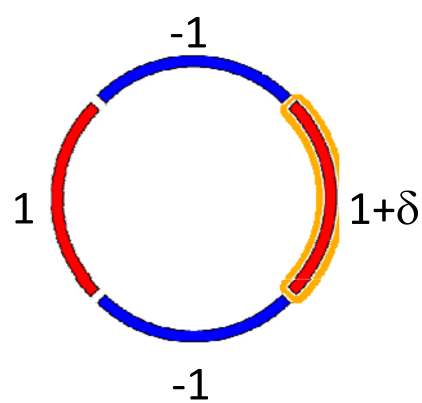


If a dipole is applied as (1, 0, -1, 0), then
A_1 = 0.5
A_3 = 0.167
A_5 = 0.1



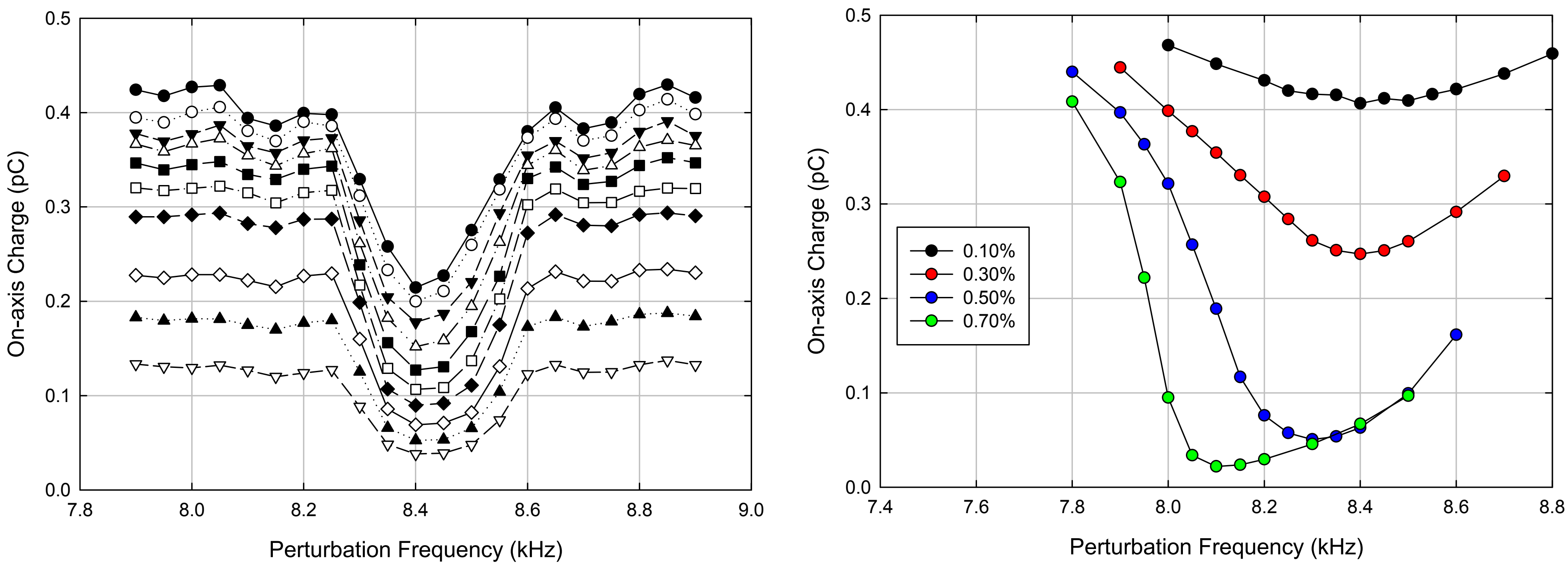
A perturbation (1+δ, -1, 1, -1), can be decomposed as
(1+δ/4, -1-δ/4, 1+δ/4, -1-δ/4) + (δ/2, 0, -δ/2, 0) + (δ/4, δ/4, δ/4, δ/4) and then

$A_0 = \delta\pi/8$
$A_1 = \delta/4$
$A_2 = 1+\delta/4$
$A_3 = \delta/12$
$A_5 = \delta/20$
$A_6 = 1/3 + \delta/12$



The higher-order terms are less significant because the contribution of each term is proportional to $(r/r_w)^n A_n$.

ℓ= 1 Perturbations Excite Dipole Modes Near the Dipole Mode Frequency



As expected, the ℓ= 1 dipole mode frequency does not depend on the amount of space-charge.

Stronger perturbations lead to an unexplained shift in the peak frequency.

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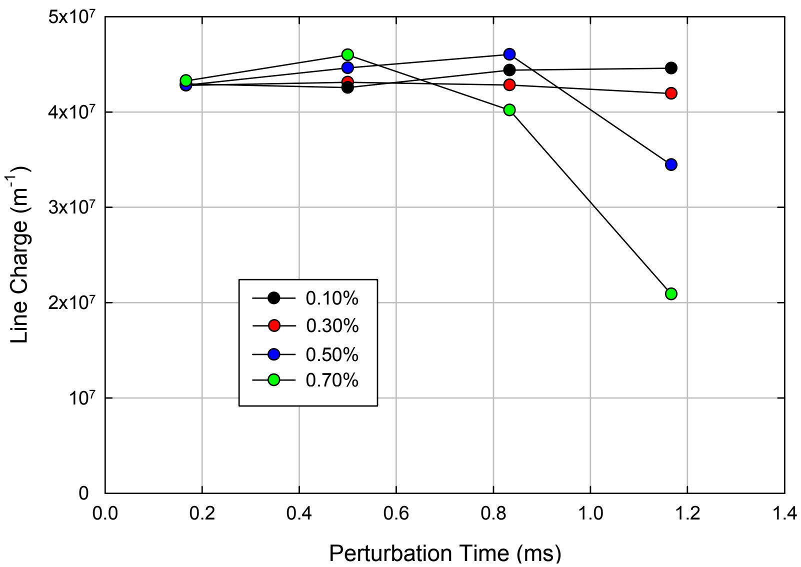
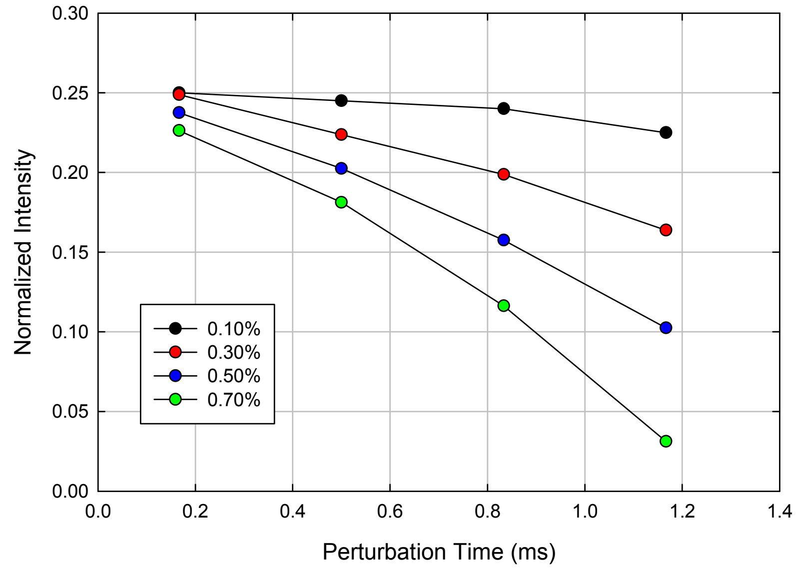
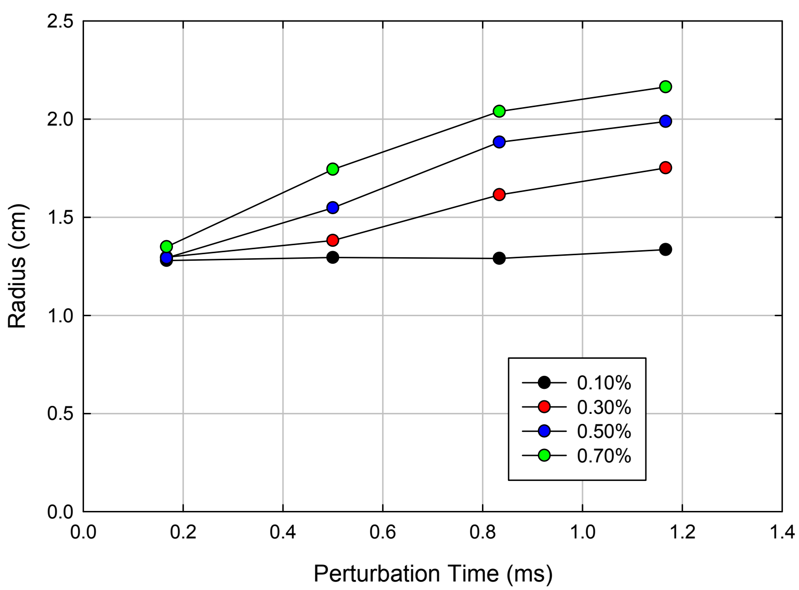
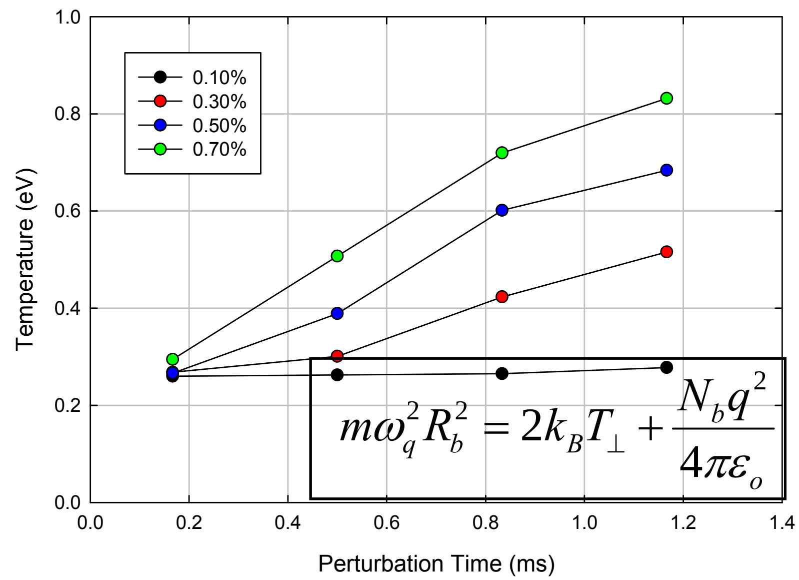
Dipole Gaussian White Noise Leads to Radius Temperature and, Thus, Emittance Growth



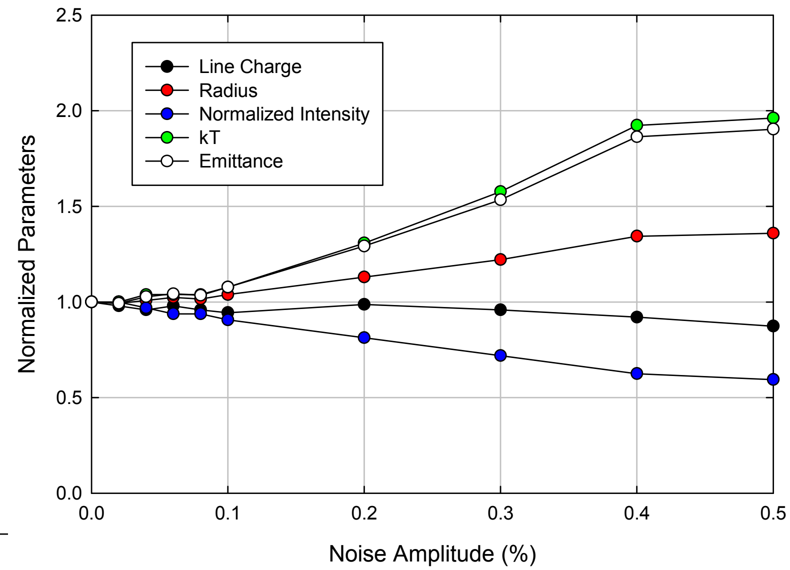


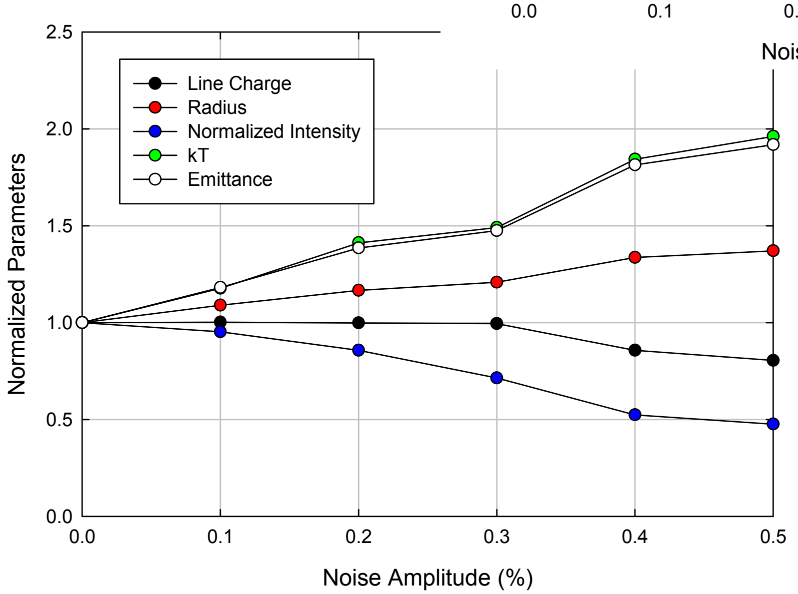


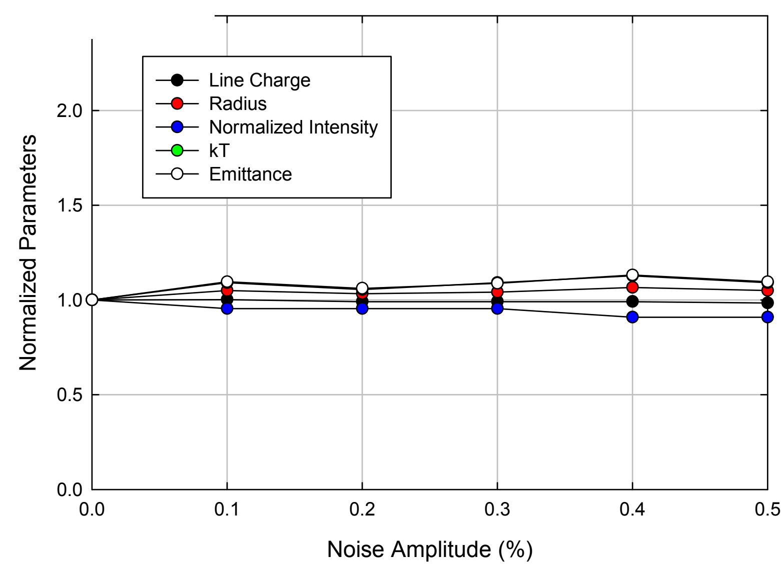


$$m\omega_q^2 R_b^2 = 2k_B T_\perp + \frac{N_b q^2}{4\pi\varepsilon_o}$$

As Expected the Dipole Noise has a Larger Effect Than the Quadrupole Noise

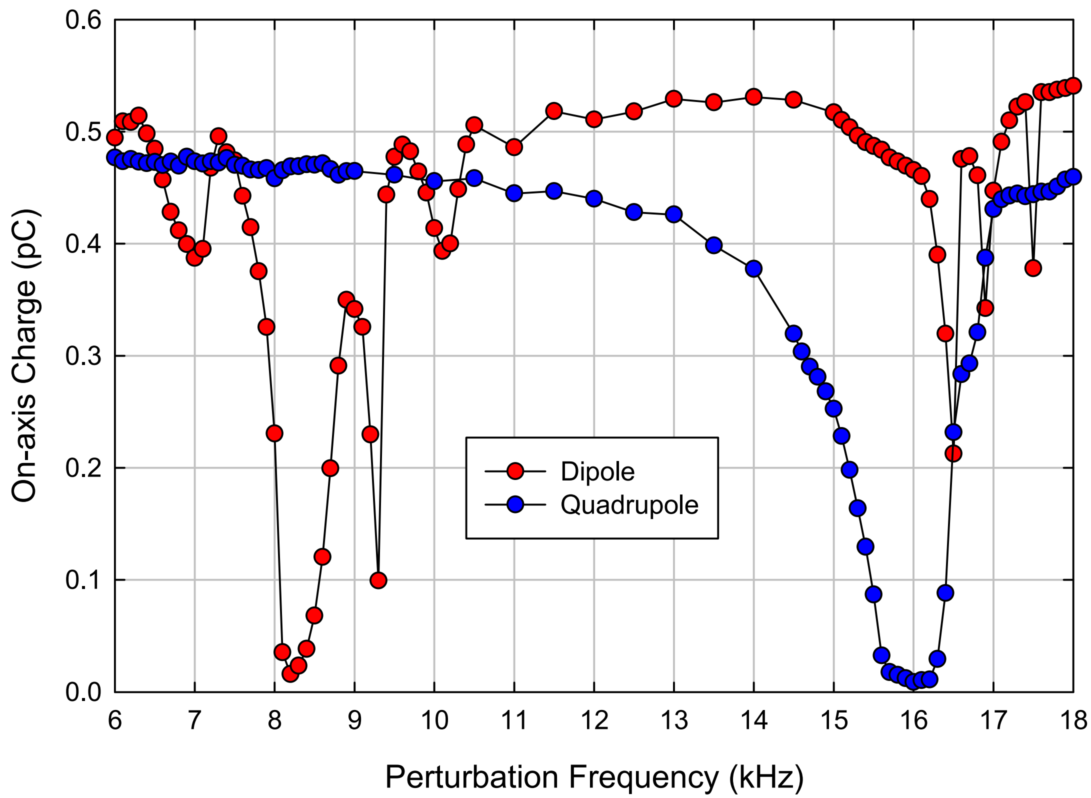






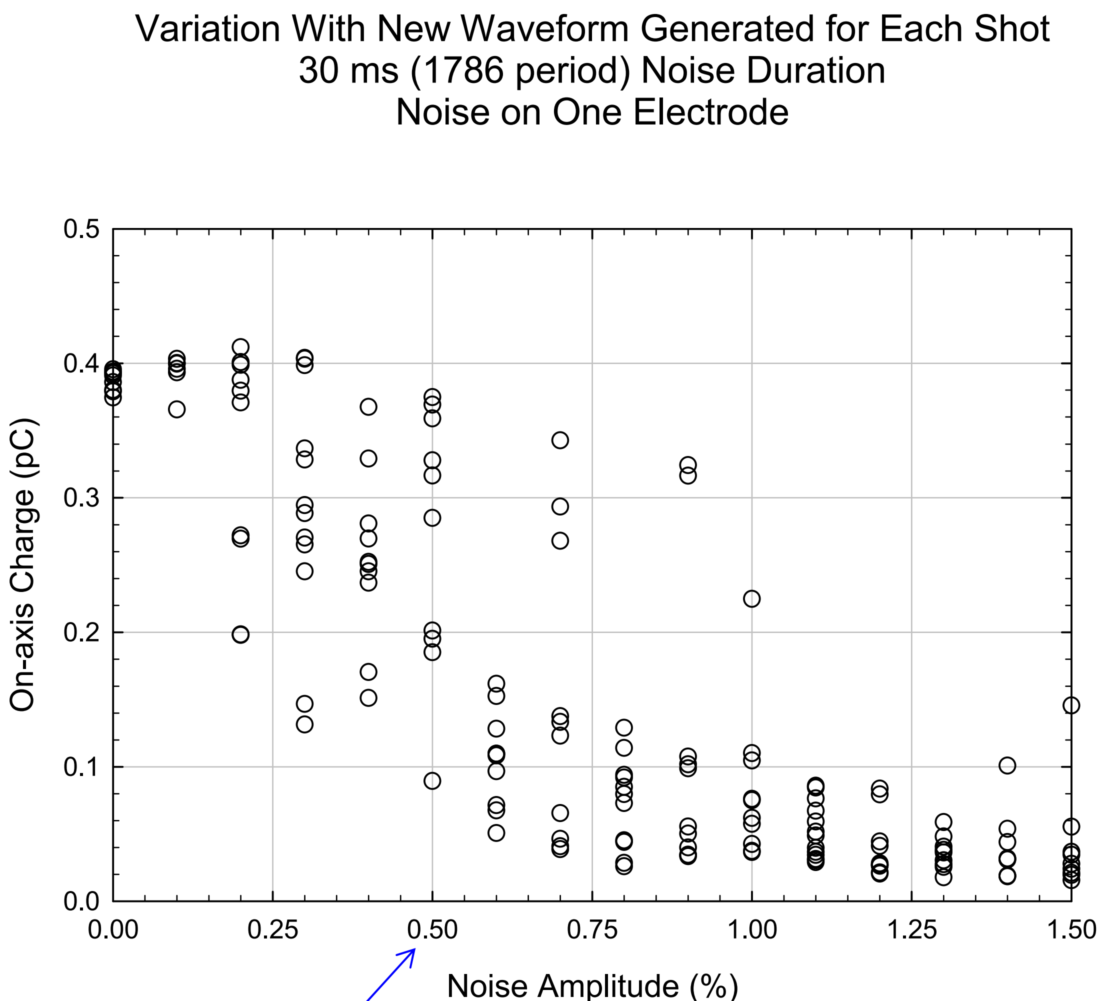
Using the arbitrary function generators, the dipole noise and the quadrupole noise can be considered separately.

The Right Mode Structure and Mode Frequency Excite Dipole and Quadrupole Modes



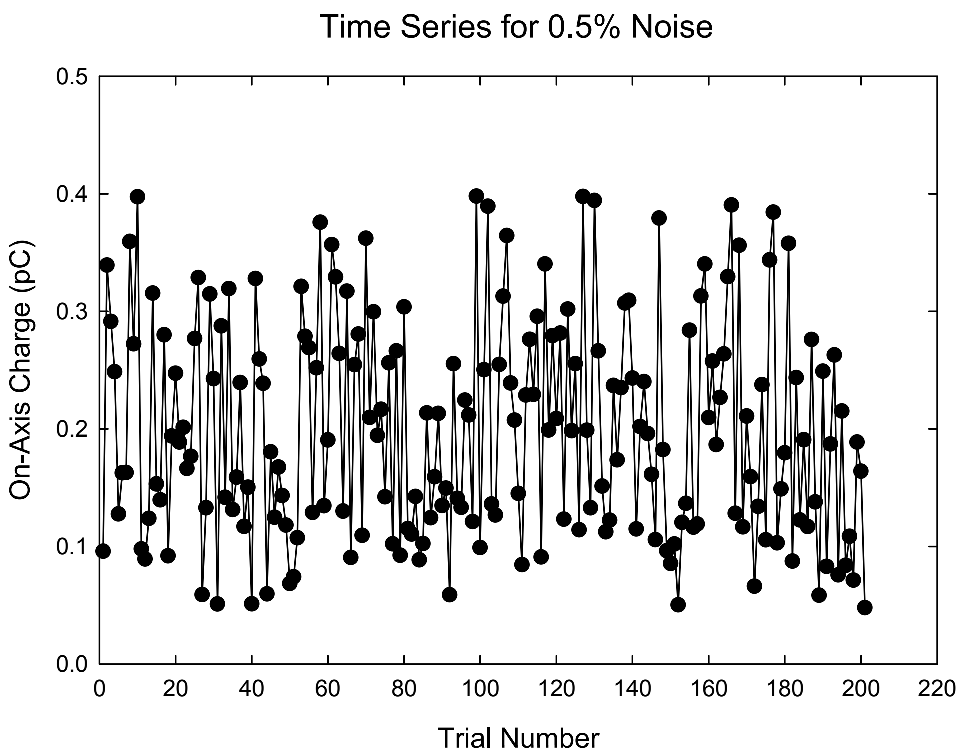
Dipole perturbations do not strongly excite the quadrupole mode, an quadrupole perturbations no not strongly excite the dipole mode.

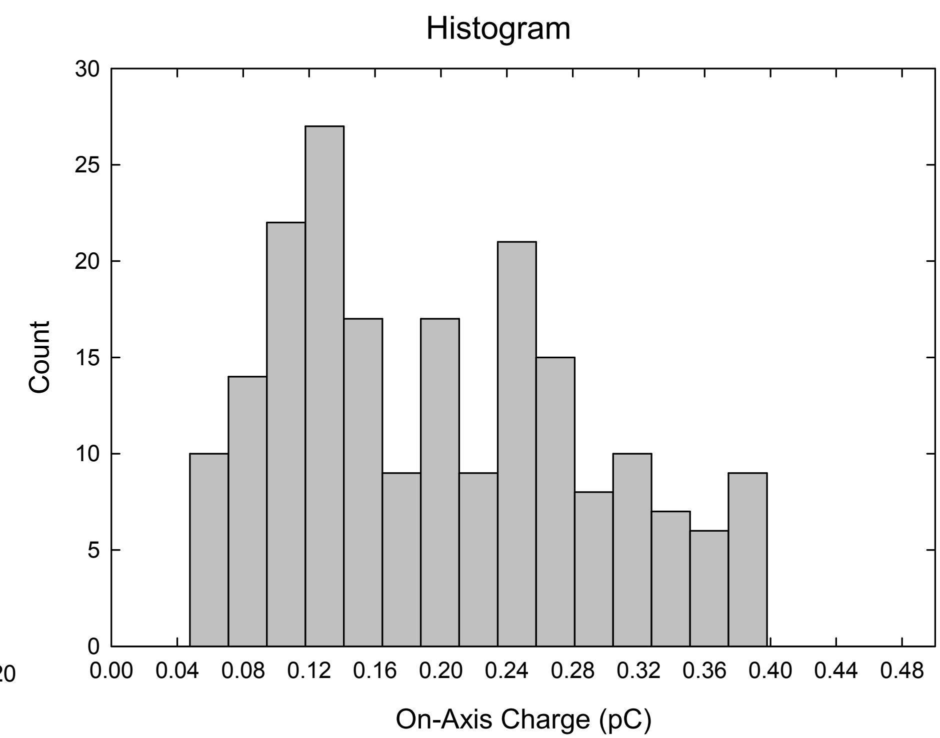
Understanding the Statistical Nature of Noise Applied to One Electrode



Focus on 0.50% noise amplitude

Time Series and Histogram of On-axis Charge Measurements for 200 Sets of Random Numbers

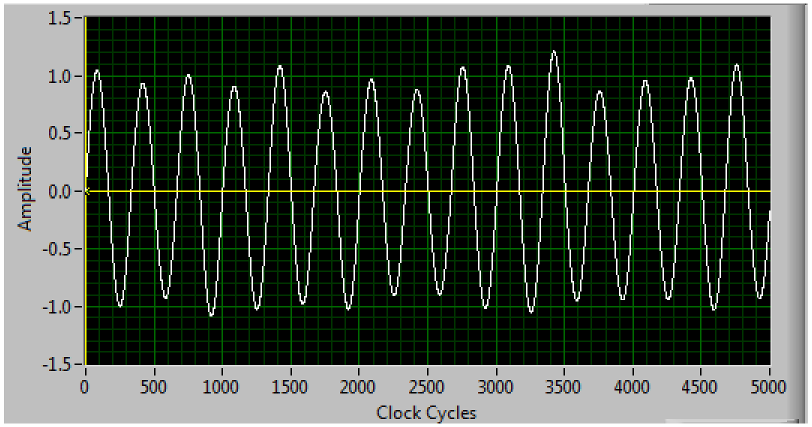




As before, this is a predominantly the result of the dipole perturbation.

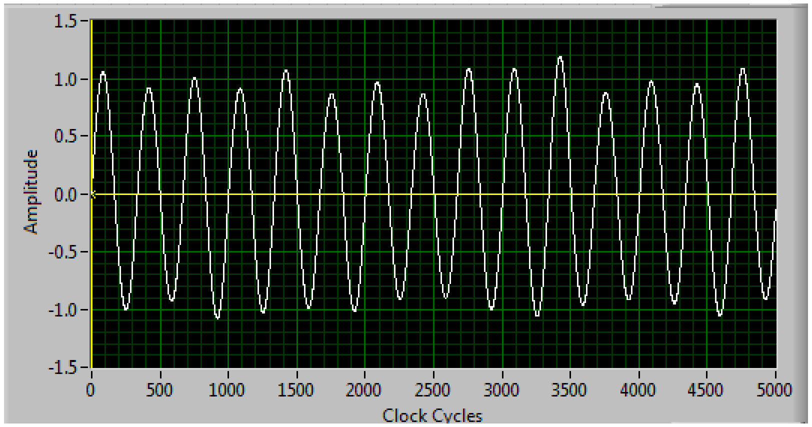
Fix the Waveform and Manipulate the Spectrum to See How the Noise Acts By Coupling to the Modes

Before



First 15 of 1000 periods

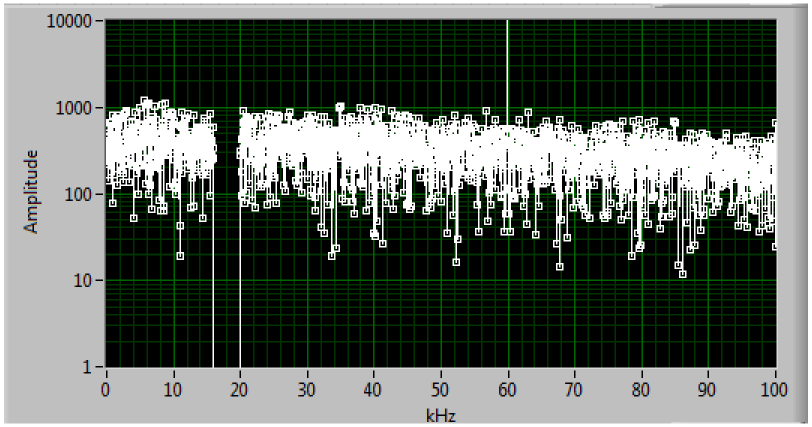
After



First 15 of 1000 periods

Example with 10% noise

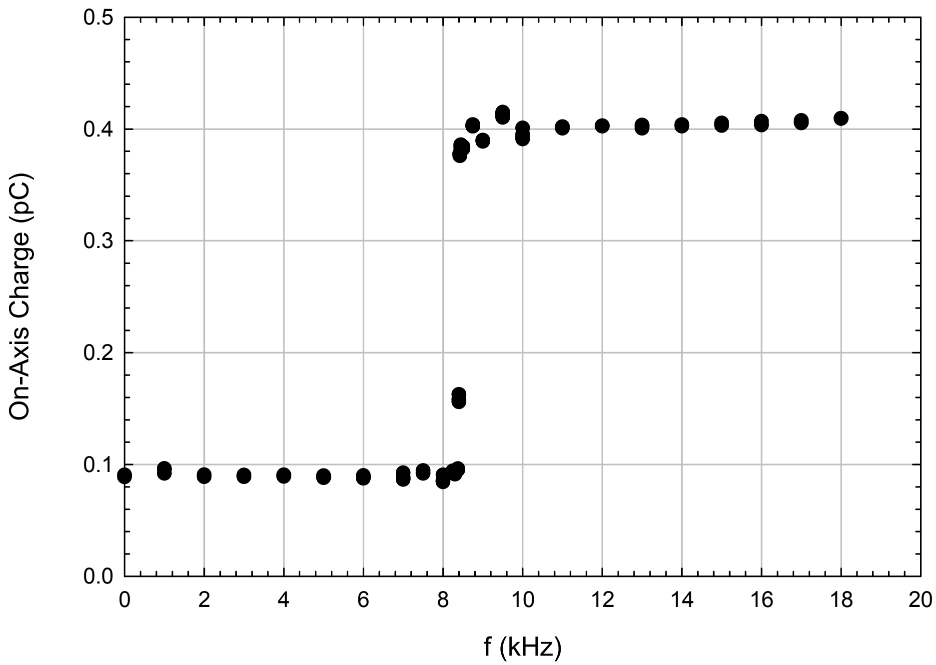
Manipulate



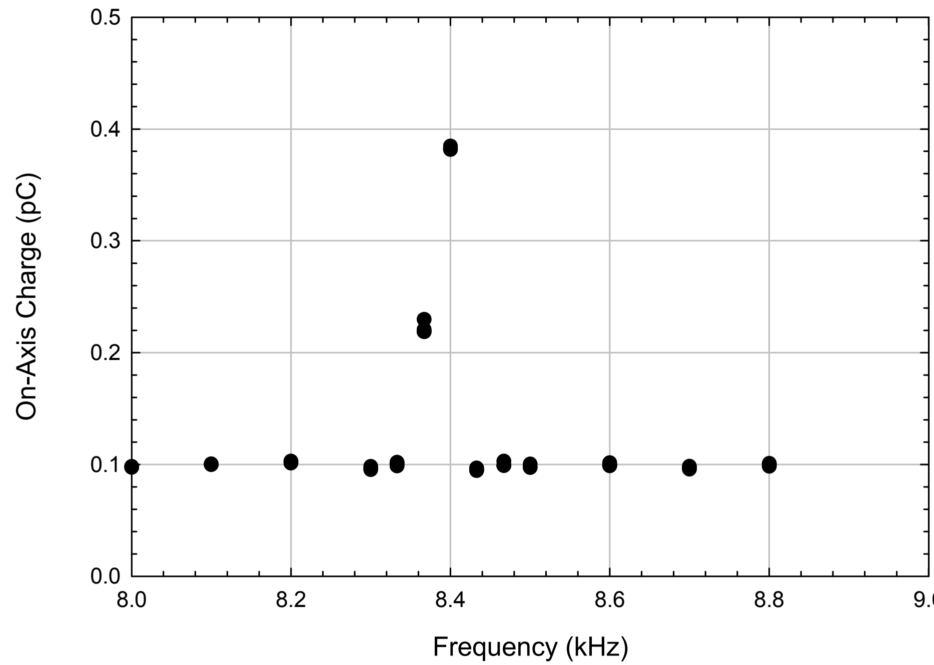
First 100 kHz of the FFT of the Waveform

Noise Applied to One Electrode Damages the Beam Through Its Interaction with the ℓ = 1 Dipole Mode

One Set of 0.5% Noise Applied to One Electrode
A Notch Filter Removes Frequencies from Zero to f kHz



One Set of 0.5% Noise Applied to One Electrode
A Notch Filter Removes One Frequency Component



PTSX is a Compact Experiment for Studying the Propagation of Beams Over Large Distances

- PTSX is a versatile research facility in which to simulate collective processes and the transverse dynamics of intense charged particle beam propagation over large distances through an alternating-gradient magnetic quadrupole focusing system using a compact laboratory Paul trap.
- PTSX explores important beam physics issues such as:
  - Beam mismatch and envelope instabilities;
  - Collective wave excitations;
  - Chaotic particle dynamics and production of halo particles;
  - Mechanisms for emittance growth;
  - Compression techniques; and
  - Effects of distribution function on stability properties.