The Paul Trap Simulator Experiment:
Studying Transverse Beam Dynamics in a Compact Laboratory Experiment*

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PPPL vision statement
Enabling a world powered by safe, clean and plentiful fusion energy while leading discoveries in plasma science and technology.
PTSX Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

• **Purpose:** PTSX simulates, in a compact experiment, the transverse nonlinear dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems.

• **Applications:** Accelerator systems for high energy and nuclear physics applications, heavy ion fusion, spallation neutron sources, and high energy density physics.
The Goal of PTSX is to Study Key Issues in the Physics of Intense Beams

Issues:

• Beam mismatch and envelope instabilities;
• Collective wave excitations;
• Chaotic particle dynamics and production of halo particles;
• Mechanisms for emittance growth;
• Compression techniques; and
• Effects of distribution function on stability properties.

Today: transverse beam compression, the effects of random lattice noise, and transverse beam modes.
Paul Traps


Nobel Prize: 1989

\[ \phi_{\text{trap}}(x) = \frac{V_0 \cos(2\pi f t)}{r_0^2 + 2z_0^2} \left( x^2 + y^2 - 2z^2 \right) \]

\[ \frac{d^2}{d\tau^2} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \left[ 2q \cos(2\tau) \right] \begin{pmatrix} x \\ y \\ -2z \end{pmatrix} = 0 \]

\[ q = \frac{\frac{4eV_0}{m(2\pi f)^2 \left( r_0^2 + 2z_0^2 \right)}}{\tau = \pi ft} \]

\[ q = 0.908 \]
A Hyperbolic Potential in Any Pair of Variables Will Do – r-z or x-y

\[
\phi_{ap}(x, y, t) = \frac{4V_0(t)}{\pi} \sum_{\ell=1}^{\infty} \frac{\sin(\ell \pi / 2)}{\ell} \left( \frac{r}{r_w} \right)^{2\ell} \cos(2\ell \theta)
\]

\[
e_b \phi_{ap}(x, y, t) = \frac{1}{2} \kappa_q(t)(x^2 - y^2)
\]

\[
\kappa_q(t) = \frac{8e_b V_0(t)}{m_b \pi r_w^2}
\]

\[
V_0(t) = V_{0 \text{ max}} \sin(\omega t)
\]

The ponderomotive force can be written as...

\[
\vec{F}_p = -\frac{\omega_p^2}{\omega^2} \nabla \left( \frac{\varepsilon_0 E^2}{2} \right)
\]

\[
\vec{F}_p = -m_b \omega_q^2 \vec{r}
\]

...where...

\[
\omega_q = \frac{8e_b V_{0 \text{ max}}}{m_b \pi r_w^2} \xi
\]

...and

\[
\xi = \frac{1}{2\sqrt{2\pi}}
\]
Analogy Between AG System and Paul Trap

Quadrupolar Focusing

\[ B_{q\text{foc}}^f (\mathbf{x}) = B_q'(z) \left( y\hat{e}_x + x\hat{e}_y \right) \]

\[ F_{\text{foc}} (\mathbf{x}) = -\kappa_q (z) \left( x\hat{e}_x - y\hat{e}_y \right) \]

\[ \kappa_q (z) = \frac{ZeB_q'(z)}{\gamma m \beta c^2} \]

\[
\psi = \frac{Ze}{\gamma m \beta^2 c^2} \left[ \phi(x, y, s) - \beta A_z(x, y, s) \right]
\]

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi = -\frac{2\pi K}{N} \int dx' dy' f_b
\]

Self-Forces

Vlasov Equation

\[
\left\{ \frac{\partial}{\partial s} + x' \frac{\partial}{\partial x} + y' \frac{\partial}{\partial y} - \left( \kappa_q(s) x + \frac{\partial \psi}{\partial x} \right) \frac{\partial}{\partial x'} - \left( -\kappa_q(s) y + \frac{\partial \psi}{\partial y} \right) \frac{\partial}{\partial y'} \right\} f_b = 0
\]

Field Equations

The resulting ponderomotive force is a radial linear restoring force with characteristic frequency \( \omega_q \).

\[
\omega_q = \frac{8eV_0 \text{max}}{m \pi r_w^2 f}
\]

\[
\xi = \frac{1}{2\sqrt{2\pi}}
\]

for a sinusoidal waveform \( V(t) \).

\[
\xi = \frac{\eta \sqrt{3} - 2\eta}{4\sqrt{3}}
\]

for a periodic step function waveform \( V(t) \) with fill factor \( \eta \).

Poisson’s Equation

\[
e\phi_{ap} (x, y, t) = \frac{1}{2} m \kappa_q'(t)(x^2 - y^2)
\]

\[
\kappa_q'(t) = \frac{8eV_0(t)}{m \pi r_w^2}
\]

usual \( \phi_{\text{self}}(x,y,t) \)

\[
\sigma_v = \frac{\omega_q}{f} < \sigma_{v\text{max}}
\]
Smooth Focusing Equilibria are Parameterized by the Normalized Intensity $s$

In thermal equilibrium, 

$$n(r) = n(0) \exp \left[ -\frac{m \omega_q^2 r^2 + 2q \phi^s(r)}{2kT} \right]$$

becomes a nonlinear equation for $\phi^s$ that must be solved numerically.

Poisson’s equation becomes a nonlinear equation for $\phi^s$ that must be solved numerically.

Solutions are characterized by the normalized intensity parameter $s$.

$$s = \frac{\omega_p^2}{2 \omega_q^2} < 1$$

for a flat-top radial density distribution.

If $p = n kT$, then the statement of local force balance on a fluid element can be manipulated to give a global force balance equation.

$$m \omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\epsilon_0}$$

If $p = n kT$, then the statement of local force balance on a fluid element can be manipulated to give a global force balance equation.
The PTSX collector disk is a 5 mm diameter copper disk, held at ground, that is mounted to a linear motion feedthrough and moves along a null of the time-dependent oscillating potential ±V₀(t).

\[ e\phi_{ap}(x, y, t) = \frac{1}{2}\kappa_q'(t)(x^2 - y^2) \]

\[ \kappa_q'(t) = \frac{8eV_0(t)}{m\pi r_w^2} \]

\[ \omega_q = \frac{8eV_{0\text{max}}}{m\pi r_w^2 f \xi} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma length</td>
<td>2 m</td>
</tr>
<tr>
<td>Wall voltage</td>
<td>140 V</td>
</tr>
<tr>
<td>Wall radius</td>
<td>10 cm</td>
</tr>
<tr>
<td>Plasma radius</td>
<td>~ 1 cm</td>
</tr>
<tr>
<td>Cesium ion mass</td>
<td>133 amu</td>
</tr>
<tr>
<td>Ion source grid voltages</td>
<td>&lt; 10 V</td>
</tr>
<tr>
<td>End electrode voltage</td>
<td>20 V</td>
</tr>
<tr>
<td>Frequency</td>
<td>60 kHz</td>
</tr>
<tr>
<td>Pressure</td>
<td>5x10^{-10} Torr</td>
</tr>
<tr>
<td>Trapping time</td>
<td>100 ms</td>
</tr>
</tbody>
</table>
Electrodes, Ion Source, and Collector

Broad flexibility in applying $V(t)$ to electrodes with arbitrary function generator.

Increasing source current creates plasmas with intense space-charge.

Large dynamic range using sensitive electrometer.

Measures average $Q(r)$. 
Plasmas are Manipulated Using an Inject-Hold-Dump Cycle

\[ f = 75 \text{ kHz} \]
\[ T = 13.3 \mu \text{s} \]
\[ V_S = 3 \text{ V} \]
\[ v_z = 2 \text{ m/ms} \]

Transverse Dynamics are the Same – Including Self-Field Effects

If…

• Long coasting beams
• Beam radius << lattice period
• Motion in beam frame is nonrelativistic

Then, when in the beam frame, both systems have…

• Quadrupolar external forces
• Self-forces governed by a Poisson-like equation
• Distributions evolve according to nonlinear Vlasov-Maxwell equation

Ions in PTSX have the same transverse equations of motion as ions in an alternating-gradient system in the beam frame.
Instability of Single Particle Orbits – An Early Experiment

• Experiment - stream Cs⁺ ions from source to collector without axial trapping of the plasma.

Electrode parameters:
• \( V_0(t) = V_{0\text{ max}} \sin (2\pi f t) \)
• \( V_{0\text{ max}} = 387.5 \text{ V} \)
• \( f = 90 \text{ kHz} \)

Ion source parameters:
• \( V_{\text{accel}} = -183.3 \text{ V} \)
• \( V_{\text{decel}} = -5.0 \text{ V} \)
Mismatch Between Ion Source and Focusing Lattice Creates Halo Particles

"Simulation" is a 3D WARP simulation that includes injection from the ion source.

- $s = \omega_p^2/2\omega_q^2 = 0.6$
- $\nu/\nu_0 = 0.63$
- $V_{0\text{ max}} = 235\text{ V}$
- $f = 75\text{ kHz}$
- $\sigma_v = 49^\circ$

WARP Simulations Reveal the Evolution of the Halo Particles in PTSX

Oscillations can be seen at both $f$ and $\omega_q$ near $z = 0$.

Downstream, the transverse profile relaxes to a core plus a broad, diffuse halo.

- $s = \frac{\omega_p^2}{2\omega_q^2} = 0.6$
- $\nu / \nu_0 = 0.63$
- $V_{\text{max}} = 235 \text{ V}$
- $f = 75 \text{ kHz}$
- $\sigma_v = 49^\circ$

Go back to $s \sim 0.2$. 
Oscillations From Residual Ion Source Mismatch Damp Away in PTSX

The injected plasma is still mismatched because a circular cross-section ion source is coupling to an oscillating quadrupole transport system.

Over 12 ms, the oscillations in the on-axis plasma density damp away...

... and leave a plasma that is nearly Gaussian.

\( kT = 0.12 \text{eV} \)

- \( s = \frac{\omega_p^2}{2\omega_q^2} = 0.2 \)
- \( \nu / \nu_0 = 0.88 \)
- \( V_{\text{max}} = 150 \text{ V} \)
- \( f = 60 \text{ kHz} \)
- \( \sigma_v = 49^\circ \)
Radial Profiles of Trapped Plasmas are Gaussian – Consistent with Thermal Equilibrium

- $I_b = 5 \text{nA}$
- $V_0 = 235 \text{ V}$
- $f = 75 \text{ kHz}$
- $t_{\text{hold}} = 1 \text{ ms}$
- $\sigma_v = 49^\circ$
- $\omega_q = 6.5 \times 10^4 \text{ s}^{-1}$

The charge $Q(r)$ is collected through the 1 cm aperture is averaged over 2000 plasma shots.

The density is calculated from

$$n(r) = \frac{Q(r)}{e \pi r_{\text{aperture}}^2 I_{\text{plasma}}}$$

- $n(r_0) = 1.4 \times 10^5 \text{ cm}^{-3}$
- $R_b = 1.4 \text{ cm}$
- $s = 0.2$


• At $f = 75$ kHz, a lifetime of $100$ ms corresponds to $7,500$ lattice periods.

• If $S$ is $1$ m, the PTSX simulation experiment would correspond to a $7.5$ km beamline.

• $s = \frac{\omega_p^2}{2\omega_q^2} = 0.18$.

• $V_0 = 235$ V
  $f = 75$ kHz
  $\sigma_v = 49^\circ$

Transverse Bunch Compression by Increasing $\omega_q$

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

If line density $N$ is constant and $kT$ doesn’t change too much, then increasing $\omega_q$ decreases $R$, and the bunch is compressed.

$$\omega_q = \frac{8eV_{0\text{ max}}}{m\pi r_w f}$$

Either
1.) increasing $V_{0\text{ max}}$ (increasing magnetic field strength) or
2.) decreasing $f$ (increasing the magnet spacing) increases $\omega_q$
Adiabatic Amplitude Increases Transversely
Compress the Bunch

20% increase in $V_{0\text{max}}$

![Graph showing 20% increase in $V_{0\text{max}}$]

$\sigma_v = 63^\circ$

Baseline

$R = 0.83 \text{ cm}$

$kT = 0.12 \text{ eV}$

$s = 0.20$

$\varepsilon \sim R \sqrt{kT} \Delta \varepsilon = 10\%$

$\bullet \ s = \omega_p^2/2 \omega_q^2 = 0.20$

$\bullet \ \nu/\nu_0 = 0.88$

$\bullet \ V_{0\text{max}} = 150 \text{ V}$

$\bullet \ f = 60 \text{ kHz}$

$\bullet \ \sigma_v = 49^\circ$

Adiabatic

$R = 0.63 \text{ cm}$

$kT = 0.26 \text{ eV}$

$s = 0.10$

$\Delta \varepsilon = 10\%$

$\bullet \ V_{0\text{max}} = 150 \text{ V}$

$\bullet \ f = 60 \text{ kHz}$

$\bullet \ \sigma_v = 49^\circ$

Instantaneous

$R = 0.93 \text{ cm}$

$kT = 0.58 \text{ eV}$

$s = 0.08$

$\Delta \varepsilon = 140\%$

$\bullet \ V_{0\text{max}} = 150 \text{ V}$

$\bullet \ f = 60 \text{ kHz}$

$\bullet \ \sigma_v = 49^\circ$

Less Than Four Lattice Periods Adiabatically Compress the Bunch

\[ s = \frac{\omega_p^2}{2\omega_q^2} = 0.20 \]
\[ \nu / \nu_0 = 0.88 \]
\[ V_{0 \max} = 150 \text{ V} \]
\[ f = 60 \text{ kHz} \]
\[ \sigma_v = 49^\circ \]

2D WARP PIC Simulations Corroborate Adiabatic Transitions in Only Four Lattice Periods

Instantaneous Change.

Change Over Four Lattice Periods.

Peak Density Scales Linearly with $\omega_q$

\[ m\omega_q^2 R^2 \sim 2kT \]
\[ \varepsilon \sim R \sqrt{kT} \]
\[ \omega_q R^2 \sim \text{const.} \]
\[ n(0) R^2 \sim N = \text{const.} \]

\[ \omega_q = \frac{8eV_0 \max}{m\pi r_w^2 f} \]

Constant emittance

Constant energy

Increasing $\omega_q$ adiabatically by decreasing $f$

$$V(t) = V_{0,\text{max}} \sin \phi(t)$$

$$\phi(t) = \frac{f_i t}{2\pi} + \frac{f_i - f_f}{2} \left[ \tanh \left( \frac{t - t_{1/2}}{\tau/2} \right) + 1 \right]$$

$$\frac{\phi(t)}{2\pi} = f_i t + \frac{f_i - f_f}{2} \left[ \tanh \left( \frac{t - t_{1/2}}{\tau/2} \right) + 1 \right]$$

$$\frac{\dot{\phi}(t)}{2\pi}$$

$$V(t) = V_{0,\text{max}} \sin \phi(t)$$
Adiabatically Decreasing $f$ Compresses the Bunch

- $s = \omega_p^2/2\omega_q^2 = 0.2$.
- $\nu/\nu_0 = 0.88$
- $V_{0\text{ max}} = 150$ V
- $f = 60$ kHz
- $\sigma_v = 49^\circ$

$33\%$ decrease in $f$

Good agreement with KV-equivalent beam envelope solutions.

Transverse Confinement is Lost When Single-Particle Orbits are Unstable

\[ \phi(t) = f_i t + \frac{f_i - f_f}{2} \left[ \tanh \left( \frac{t - t_{i/2}}{\tau/2} \right) + 1 \right] \]

\[ \sigma_v = \frac{\omega_q}{f} \propto \frac{1}{f^2} \]

Measured \( \tau_c \) (dots)
Set \( \sigma_{v \text{ max}} = 180^\circ \) and solve for \( \tau_c \) (line)

Good Agreement Between Data and KV-Equivalent Beam Envelope Solutions


\[ f_0 = 60 \text{ kHz} \]
Vary the amplitude of each half-period by an amount chosen from a uniform distribution.

\[ \Delta_{\text{max}} = 1.5\% \text{ maximum noise amplitude} \]

\[ |\delta_n| < \Delta_{\text{max}} \] are the random amplitude perturbations

\[ V_n = 150(1+\delta_n) \text{ Volts is the applied waveform amplitude for half-period } n \]
Noise Drives a Continuous Increase in RMS Radius

\[ N_b = \frac{\int_0^n n_b(r)2\pi r dr}{R_b^2} \]

\[ R_b = \sqrt{\frac{\int_0^n r^2 n_b(r)2\pi r dr}{N_b}} \]

Experiment for 1% uniform noise

Noise Drives Continuous Emittance Growth

- Continuous emittance growth ~ linear with the noise duration

\[
\frac{\varepsilon}{\varepsilon_i} = \frac{R_b \sqrt{T_{\perp}}}{R_{bi} \sqrt{T_{\parallel i}}} , \quad m\omega_p^2 R_b^2 = 2k_B T_{\perp} + \frac{N_b q^2}{4\pi\varepsilon_o}
\]

**Experiments**

WARP 2D PIC Simulations

Amplitude of uniform noise
- 0.5 %
- 1.0 %
- 1.5 %

1800 lattice periods

Noise Drives the Continuous Development of a Non-Thermal Tail Distribution

A straight line in the log of $Q(r)$ versus $r^2$ plot indicates that the radial profile is a Gaussian function of $r$.

Colored Noise with Finite Autocorrelation Time is Less Detrimental Than White Noise

![Graph A](a) On-axis charge (pC) vs. Noise amplitude (%)

- Red circles: Colored noise ($\tau_{ac} = 2.5T$)
- Green triangles: Colored noise ($\tau_{ac} = 0.5T$)
- Blue squares: White noise

- Noise duration = 1 ms

![Graph B](b) Noise duration (ms) vs. Noise amplitude (%)

- Red circles: Colored noise ($\tau_{ac} = 2.5T$)
- Blue squares: White noise

- Noise amplitude = 1%

Colored Noise Can Enhance Halo Formation During Beam Mismatch

**Experiments**

20 msec duration
- No mismatch + No noise
- Mismatch + No noise
- Mismatch + 1% colored noise ($\tau_{ac} = 5T$)

**WARP 2D PIC Simulations**

20 msec duration
- No mismatch + No noise
- Mismatch + No noise
- Mismatch + 1% colored noise ($\tau_{ac} = 5T$)

Beam mismatch is induced by instantaneously increasing the voltage amplitude by 1.5 times, and switching back to the original value after one focusing period.


Studies of Beam Modes Begins with Simple Expressions for Two Particular Modes

Breathing mode:

$$\omega_B = 2\omega_q \left(1 - \frac{1}{2}\hat{s}\right)^{1/2}$$

Quadrupole mode:

$$\omega_Q = 2\omega_q \left(1 - \frac{3}{4}\hat{s}\right)^{1/2}$$

Using KV distribution and smooth focusing approximation.

$$\hat{s} = \frac{\omega_p^2}{2\omega_q^2}$$

$$s \sim 0.23$$

$$f_0 = 60\text{kHz}$$

$$\sigma_v \sim 48.6\text{deg}$$

$$2f_q = \frac{2\omega_q}{2\pi} \sim 16.08\text{kHz}$$

$$f_B = \frac{\omega_B}{2\pi} \sim 15.13\text{kHz}$$

$$f_Q = \frac{\omega_Q}{2\pi} \sim 14.63\text{kHz}$$
An Initially Hollow Beam Changes from Hollow to Peaked and Back Again as it Streams From Source to Collector

\[ \omega q t_{\text{transit}} = 1.5 \pi \]
An $\ell = 1$ Dipole Mode Can Be Excited By Masking the Ion Source

PTSX operated in “streaming” mode.

$t_{\text{transit}}$ decreased by increasing Injection voltage from 3 V to 30 V.

$\omega_q$ decreased by raising f from 60 kHz to 90 kHz.

$V_0$ is varied to vary $\omega_q$. 

$\omega_q t_{\text{transit}} = 1.5\pi$
Collective Mode Diagnostics Were Installed But Were Not Sensitive to the Modes and Degraded Confinement
The Modes Can Be Excited Using Externally Applied Perturbations Near the Mode Frequency

- Sum-of-sines is applied to arbitrary function generator
  \[ V(t) = \hat{V}_0 \sin(2\pi f_0 t) + \delta V \sin(2\pi f_1 t) \]
  where \( f_1 \) is near the mode frequency

- Typical Operating Parameters
  \[ \begin{align*}
  \hat{V}_0 & \sim 140V \\
  f_0 & \sim 60\text{KHz} \\
  2f_q & \sim 16.083\text{kHz} \\
  \delta V & \sim 0.7V (0.5\% \hat{V}_0) \\
  f_1 & \sim \text{varying}
  \end{align*} \]
The Modes Can ALSO Be Excited Using the Beat Frequency Between $f_0$ and $f_1$

- Sum-of-sines is applied to arbitrary function generator

$$V(t) = \tilde{V}_0 \sin(2\pi f_0 t) + \delta V \sin(2\pi f_1 t)$$

where $f_1$ is near $f_0 \pm f_{\text{mode}}$

- Typical Operating Parameters

$$\tilde{V}_0 \sim 140V$$
$$\delta V \sim 0.7V (0.5\% \tilde{V}_0)$$
$$f_0 \sim 60KHz$$
$$2f_q \sim 16.083kHz$$
$$f_1 \sim \text{varying}$$
$$t_{\text{perturbation}} = 30ms$$
Beat-Method Frequency Scans With Different Initial Amounts of Space-Charge Attempt to Find the Space-Charge Dependence

Change of on-axis density under different perturbation amplitudes

30ms, 1.0% perturbation

$S$ increases
Beat-Method with Larger Amplitude Perturbations

Change of on-axis density under different perturbation amplitudes

30ms, 1.5% perturbation

$S$ increases
The Measured Frequencies Are Larger Than Those Computed in the Simple Model

Collective Mode Frequency VS Normalized Intensity

![Graph showing collective mode frequency vs. normalized intensity with data points and theoretical lines for higher and lower peaks, as well as equations for two different cases: $2f_q^*(1-0.5s)^{1/2}$ and $2f_q^*(1-0.75s)^{1/2}$.](image-url)
The Linear Drive is Stronger But Does Not Exhibit a Clear Dependence on Space-Charge

Change of on-axis density under different perturbation amplitudes

30ms, 0.5% perturbation

S increases
Observed Mode Frequencies are Larger Than Those in Warp or the Simple Model

Mode Frequency vs Normalized Density $s$

- higher peak experiment
- lower peak experiment
- higher peak simulation
- lower peak simulation

$2w_q(1-0.5s)^{0.5}$
$2w_q(1-0.75s)^{0.5}$

experiment
30ms, 0.5% perturbation

simulation
1ms, 0.5% perturbation
Using a Second Arbitrary Function Generator to Break the Quadrupole Symmetry

\[ \phi(r, \theta) = \sum_n C_n \left( \frac{r}{r_w} \right)^n \cos(n\theta) \]

\[ A_n = \frac{1}{4} \int_0^{2\pi} V(\theta) \cos(n\theta) \]

At \( t = 0 \), with \( V_0(0) = 1 \)
(1, -1, 1, -1), then
\( A_2 = 1 \)
\( A_6 = 0.333 \)

A perturbation \( (1+\delta, -1, 1, -1) \), can be decomposed as
\( (1+\delta/4, -1-\delta/4, 1+\delta/4, -1-\delta/4) + (\delta/2, 0, -\delta/2, 0) + (\delta/4, \delta/4, \delta/4, \delta/4) \) and then

\[ A_0 = \delta\pi/8 \]
\[ A_1 = \delta/4 \]
\[ A_2 = 1+\delta/4 \]
\[ A_3 = \delta/12 \]
\[ A_5 = \delta/20 \]
\[ A_6 = 1/3 + \delta/12 \]

If a dipole is applied as (1, 0, -1, 0), then
\( A_1 = 0.5 \)
\( A_3 = 0.167 \)
\( A_5 = 0.1 \)

The higher-order terms are less significant because the contribution of each term is proportional to \( (r/r_w)^n A_n \).
ℓ = 1 Perturbations Excite Dipole Modes Near the Dipole Mode Frequency

As expected, the ℓ = 1 dipole mode frequency does not depend on the amount of space-charge. Stronger perturbations lead to an unexplained shift in the peak frequency.
Dipole Gaussian White Noise Leads to Radius Temperature and, Thus, Emittance Growth

\[
m\omega_q^2 R_b^2 = 2k_B T_\perp + \frac{N_b q^2}{4\pi \varepsilon_o}
\]
As Expected the Dipole Noise has a Larger Effect Than the Quadrupole Noise

Using the arbitrary function generators, the dipole noise and the quadrupole noise can be considered separately.
The Right Mode Structure and Mode Frequency Excite Dipole and Quadrupole Modes

Dipole perturbations do not strongly excite the quadrupole mode, and quadrupole perturbations do not strongly excite the dipole mode.
Understanding the Statistical Nature of Noise Applied to One Electrode

Variation With New Waveform Generated for Each Shot
30 ms (1786 period) Noise Duration
Noise on One Electrode

Focus on 0.50% noise amplitude
Time Series and Histogram of On-axis Charge Measurements for 200 Sets of Random Numbers

As before, this is a predominantly the result of the dipole perturbation.
Fix the Waveform and Manipulate the Spectrum to See How the Noise Acts By Coupling to the Modes

Before

First 15 of 1000 periods

Example with 10% noise

After

First 15 of 1000 periods

Manipulate

First 100 kHz of the FFT of the Waveform
Noise Applied to One Electrode Damages the Beam Through Its Interaction with the $\ell = 1$ Dipole Mode

One Set of 0.5% Noise Applied to One Electrode
A Notch Filter Removes Frequencies from Zero to f kHz

One Set of 0.5% Noise Applied to One Electrode
A Notch Filter Removes One Frequency Component
PTSX is a Compact Experiment for Studying the Propagation of Beams Over Large Distances

- PTSX is a versatile research facility in which to simulate collective processes and the transverse dynamics of intense charged particle beam propagation over large distances through an alternating-gradient magnetic quadrupole focusing system using a compact laboratory Paul trap.

- PTSX explores important beam physics issues such as:
  - Beam mismatch and envelope instabilities;
  - Collective wave excitations;
  - Chaotic particle dynamics and production of halo particles;
  - Mechanisms for emittance growth;
  - Compression techniques; and
  - Effects of distribution function on stability properties.