Muon Collider Ring Lattice Design

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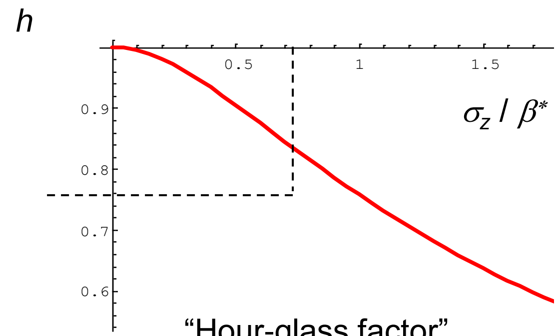
The Road to High Luminosity



"Hour-glass factor"

$$\langle \mathcal{L} \rangle = f_0 \frac{n_b N_\mu^2}{4\pi\varepsilon_\perp \beta^*} h \times \frac{1}{2} \tau f_{rep} \sim \frac{P_\mu \xi}{C\beta^*} h\tau$$

P_μ – average muon beam power (limited by the P-driver power)

$\xi = \dfrac{r_\mu N_\mu}{4\pi\gamma\varepsilon_\perp}$ – beam-beam parameter (limited by particle stability, $\xi < 0.1$?)

C – collider circumference (limited from below by available B-field)

τ – muon lifetime

β^* – beta-function at IP (limited from below by chromaticity of final focusing and aperture restrictions in IR magnets),

small β^* requires small $\sigma_z \Rightarrow$ large $\sigma_p / p = \varepsilon_{||} / \sigma_z$

The recipe:

- Pack as many muons per bunch as the beam-beam effect allows (in practice this means 1 bunch/beam)
- Make beams round to maximize the beam-beam limit
- Develop new chromatic correction scheme to reduce β^*
- Do not leave free spaces to reduce C (also good for neutrino radiation)

MC Lattice Design Challenges

What we would like to achieve compared to other machines:

	MC	Tevatron	LHC
Beam energy (TeV)	0.75	0.98	7
β^* (cm)	1	28	55
Momentum spread (%)	>0.1	<0.01	0.0113
Bunch length (cm)	1	50	15
Momentum compaction factor (10^-3)	0.05	2.3	0.322
Geometric r.m.s. emittance (nm)	3.5	3	0.5
Particles / bunch (10^11)	20	2.7	1.15
Beam-beam parameter, ξ	0.1	0.025	0.01

Muon collider is by far more challenging:

- much larger momentum acceptance with much smaller β^*
- ~ as large Dynamic Aperture (DA) with much stronger beam-beam effect

\- **New ideas for IR magnets chromaticity correction needed!**

Various MC lattice designs studied

- 1996 by Carol J., A. Garren
- 1996 by K.Oide
- "Dipole first" (2007)
- Eliana's "synthetic" (2009)
- Asymmetric dispersion
- "Flat top"

~ satisfy the requirements

1996 designs (especially by K.Oide) had extremely high sensitivity to field errors

Chromatic Correction Basics

Montague chromatic functions :

$$A_x = \frac{\partial}{\partial\delta_p}\alpha_x - \frac{\alpha_x}{\beta_x}\frac{\partial}{\partial\delta_p}\beta_x, \ \ B_x = \frac{1}{\beta_x}\frac{\partial}{\partial\delta_p}\beta_x,$$
$$W_x = \sqrt{A_x^2 + B_x^2},$$

$\alpha_{x,y} = -\beta_{x,y}'/2$, $\beta_{x,y}$ are Twiss lattice functions,
δ_p is relative momentum deviation.

$B_{x,y}$ are most important since they determine modulation of phase advance $\varphi_{x,y}$

$$\varphi_x(s,\delta_p) = \frac{1}{1+\delta_p}\int_0^s \frac{ds'}{\beta_x(s',\delta_p)}$$

Equations for chromatic functions

$$A_x' = 2\varphi_x' B_x - \beta_x(K_1 - D_x K_2),$$
$$B_x' = -2\varphi_x' A_x$$

$A_{x,y}$ are created first, and then converted into $B_{x,y}$ as phase advances $\varphi_{x,y}$ grow

K_1 , K_2 are normalized quadrupole and sextupole gradients,
D_x is dispersion function: $D_x = dx_{c.o.}/d\delta_p$

The receipt:

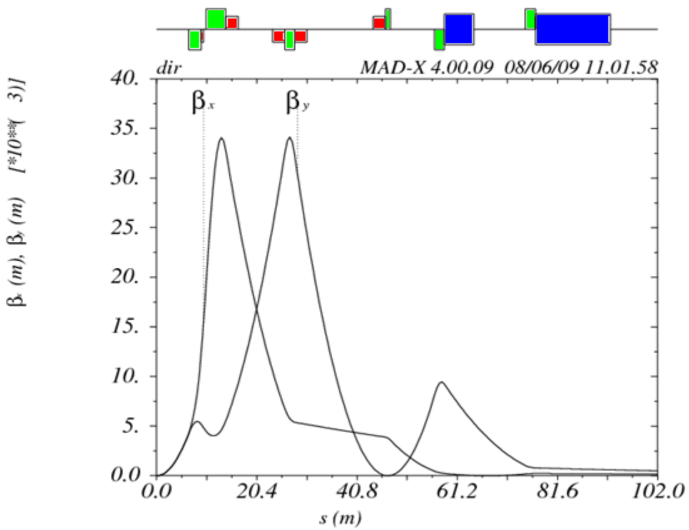
Kill *A*'s before they transform into *B*'s !

- difficult to achieve in both planes

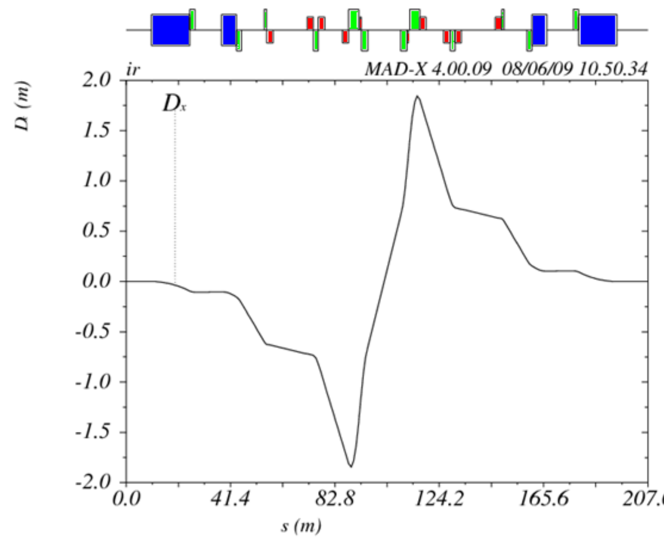
- horizontal correction requires 2 sextupoles 180° apart to cancel spherical aberrations

Anti-Symmetric Dispersion Function

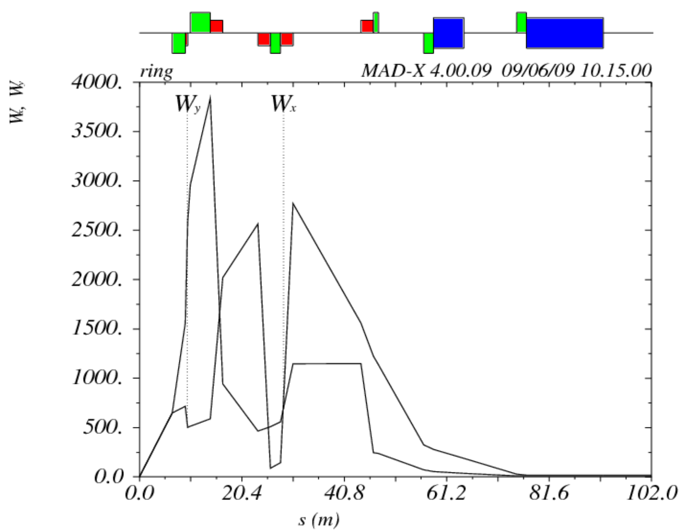
Half-IR optics



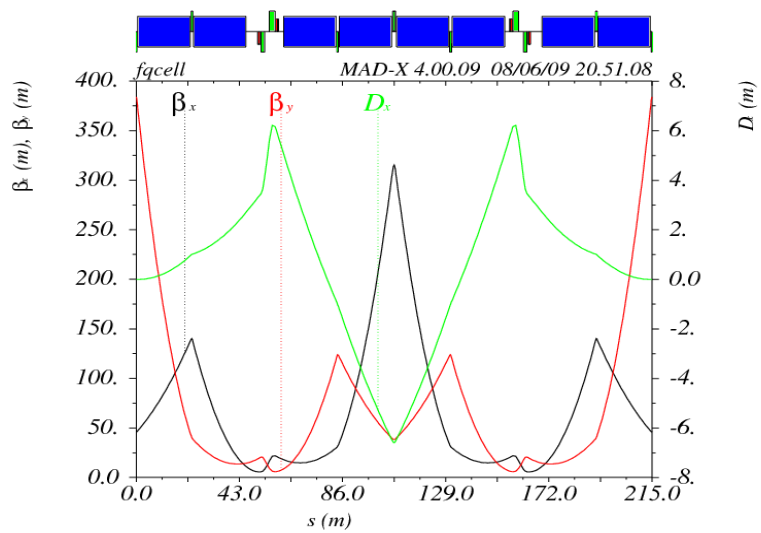
IR dispersion function



Chromaticity correction

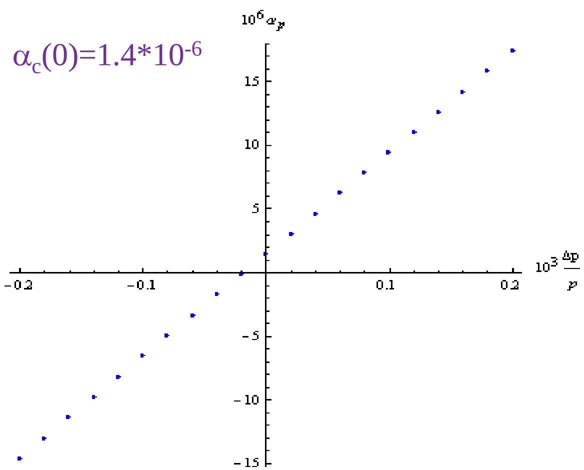


Arc cell

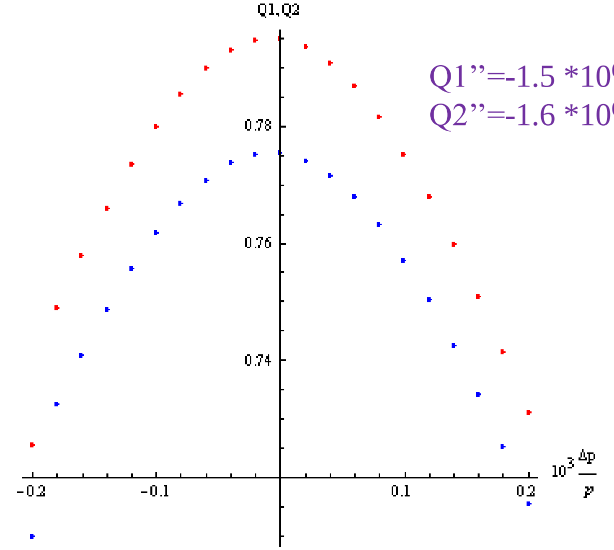


Tunes and Compaction Factor

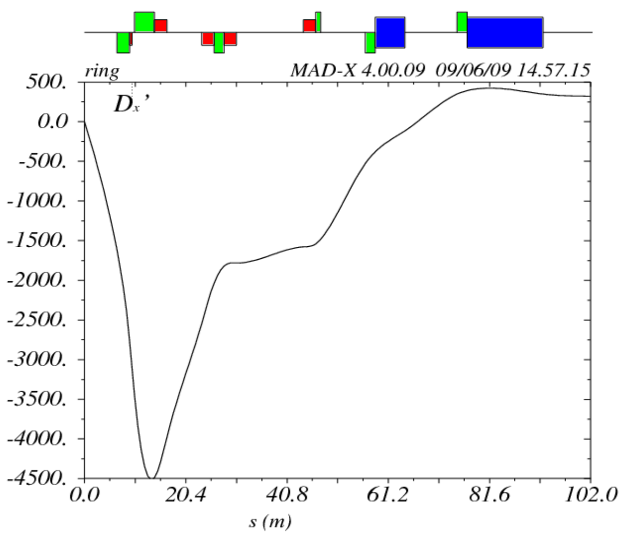
Momentum compaction factor



Tunes momentum depandance



Second order dispersion



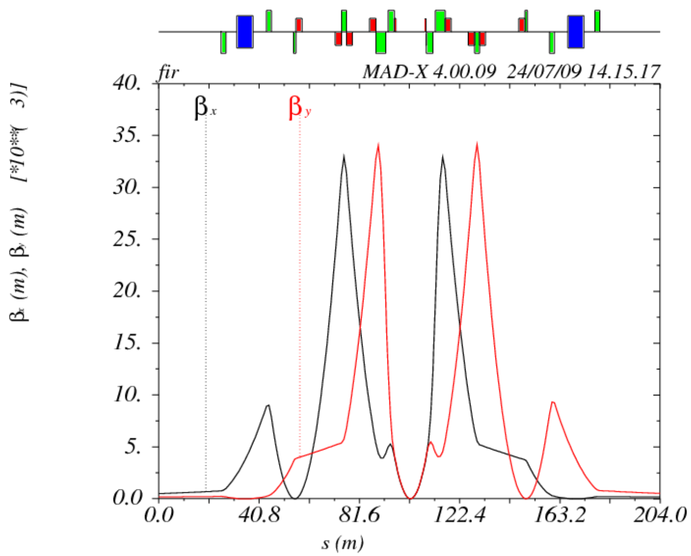
$$\xi_2 = \frac{1}{8\pi}\int_0^C [S_0(s)D_0(s) - K_0]\Delta\beta_1^C(s)\,ds + \frac{1}{4\pi}\int_0^C \beta_0(s)S_0(s)\Delta D_1^C(s)\,ds - \xi_1$$

$$\Delta D_1^C(s) = -\sqrt{\beta_0(s)}\int_s^{s+C}\frac{\sqrt{\beta_0(s')}}{\sin(\mu_0/2)}[S_0(s')D_0(s') - K_0(s')] \times D_0(s')\cos\left[\frac{\mu_0}{2} - |\mu(s') - \mu(s)|\right]ds'$$

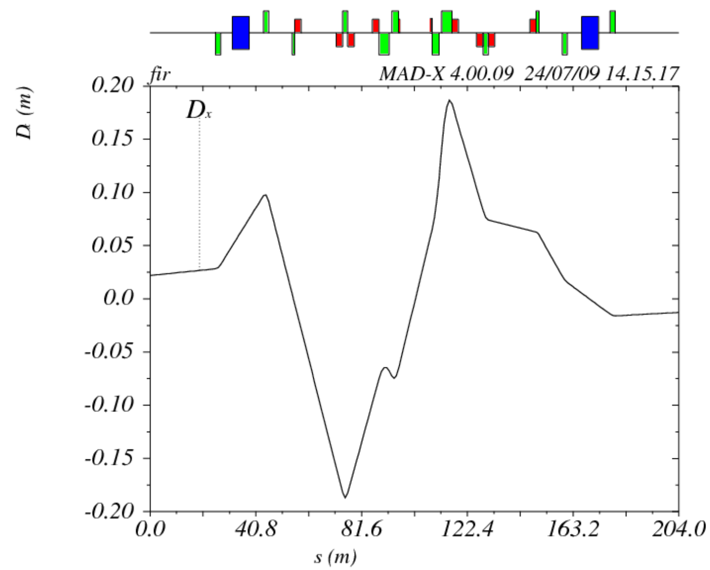
$$\frac{\Delta\beta_1^C(s)}{\beta_0(s)} = \frac{-1}{2\sin\mu_0}\int_s^{s+C}[S_0(s')D_0(s') - K_0(s')]\beta_0(s') \times \cos[\mu_0 - 2|\mu(s') - \mu(s)|]\,ds'$$

Reverse-Sign Quadrupoles IR

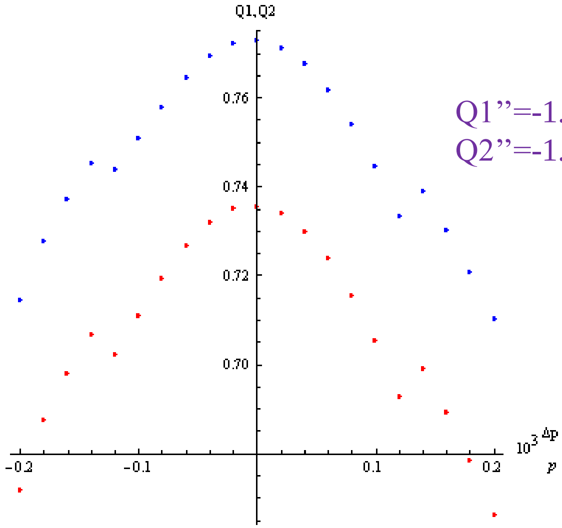
IR beta-functions



Dispersion function



Tunes vs Momentum Deviation



Q1''=-1.4 $*10^6$
Q2''=-1.5 $*10^6$

Second order chromaticity calculated using numerical integration

Q1''=-1.33 $*10^6$
Q2''=-1.39 $*10^6$

$$\xi_2 = \frac{1}{8\pi}\int_0^C [S_0(s)D_0(s) - K_0]\Delta\beta_1^C(s)\, ds + \underline{\frac{1}{4\pi}\int_0^C \beta_0(s)S_0(s)\Delta D_1^C(s)\, ds - \xi_1}$$

New paradigm

- Chromaticity of the larger β-function should be corrected first (before φ is allowed to change) – and in one kick to reduce sensitivity to errors!

$$\boxed{\frac{dQ_y}{dE_y} \sim b_2^2 \beta_x \beta_y^2, \quad \frac{dQ_x}{dE_x} \sim b_2^2 \beta_x^3}$$

- To avoid spherical aberrations it must be $\beta_y \Rightarrow$ then small β_x will kill all detuning coefficients and RDTs (this will not happen if $\beta_y \leftrightarrow \beta_x$)
- Chromaticity of β_x should be corrected with a pair of sextupoles separated by -I section to control DDx (smallness of β_y is welcome but not sufficient)
- Placing sextupoles in the focal points of the other β-function separated from IP by $\varphi = \pi \times$integer reduces sensitivity to the beam-beam interaction.

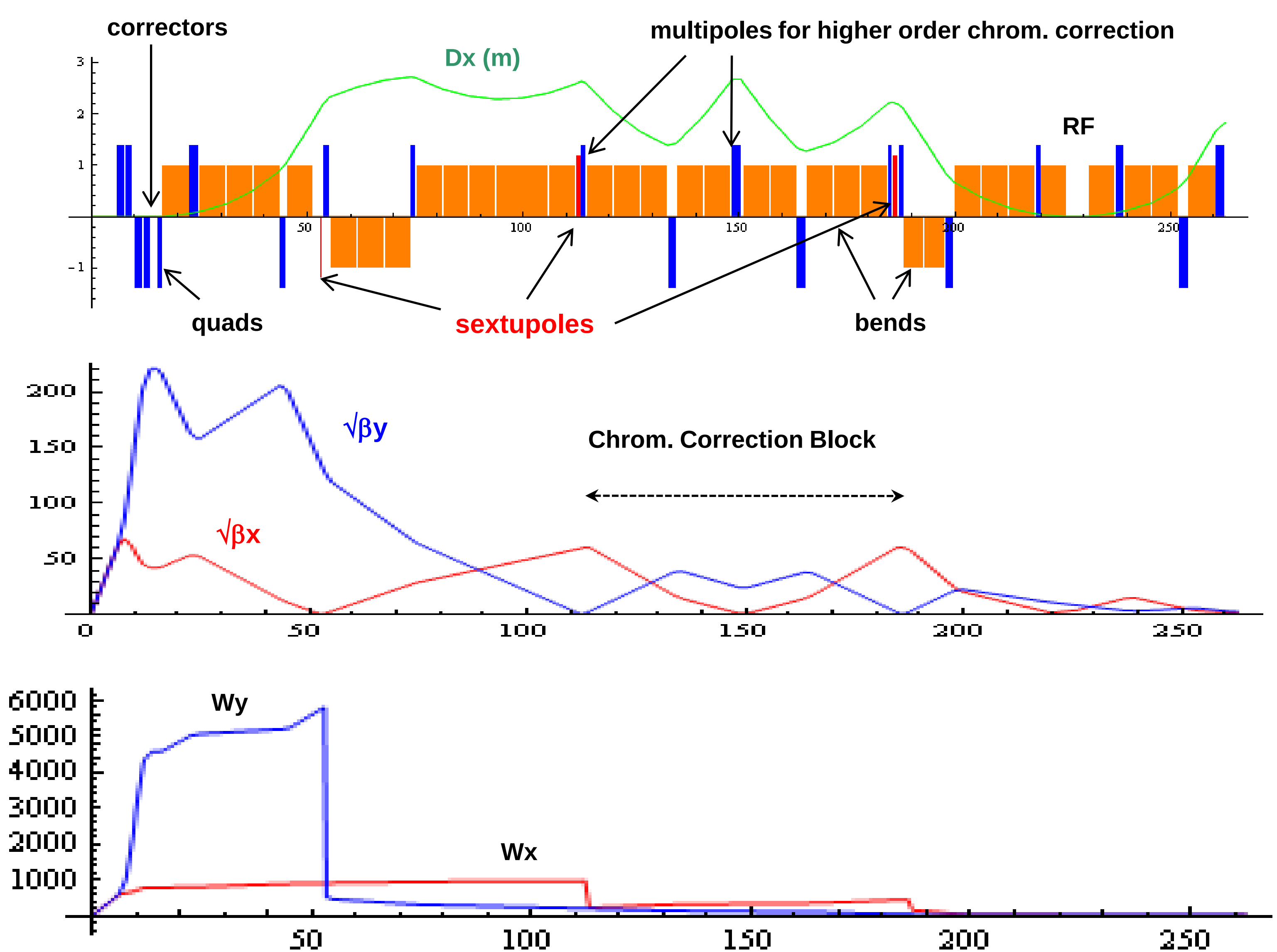
These considerations uniquely determine the IR layout.

Eliana came very close to it, just minor corrections were needed.

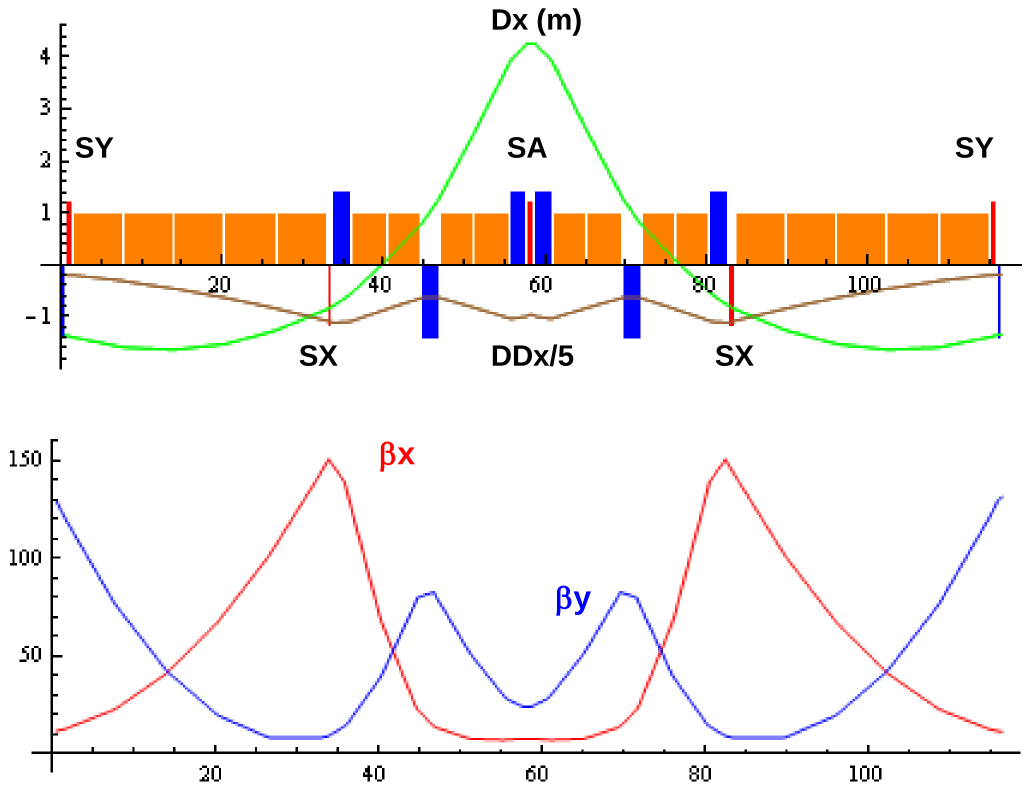
Requirements adopted for the latest version:

- full aperture A = 10sigma_max + 2cm (A.Zlobin adds 1cm on top of that)
- maximum quad gradient 12% below quench limit at 4.5°K as calculated by A.Zlobin
- bending field 8T in large-aperture open-midplane magnets, 10T in the arcs
- IR quad length < 2m (split in parts if necessary!) – no shielding from inside
- Sufficient space for magnet interconnects (typically 30-40cm)

3 Sextupoles Chromatic Correction Scheme



One More Innovation: the Arc Cell



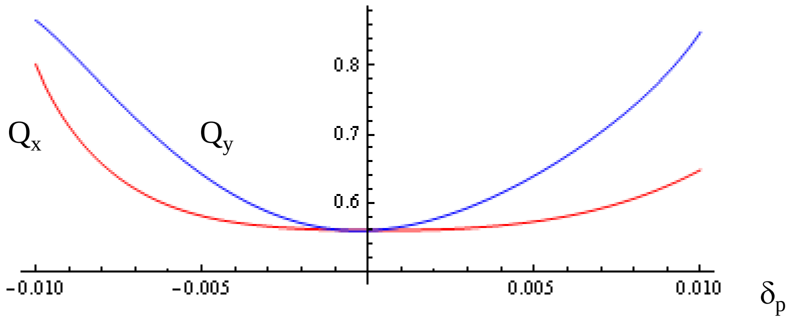
$$\alpha_c = \frac{1}{C}\int_0^C \frac{D_x}{\rho} ds,$$

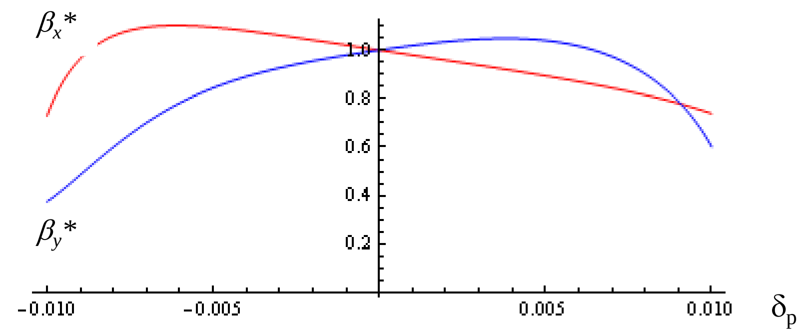
$$\frac{d\alpha_c}{d\delta_p} = \frac{1}{C}\int_0^C \left[\frac{DD_x}{\rho} + \frac{1}{2}(D'_x)^2\right] ds$$

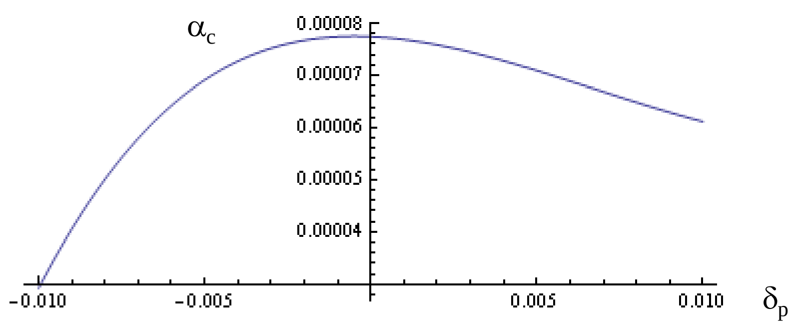
- Central quad and sextupole SA control the momentum compaction factor and its derivative (via Dx and DDx) w/o significant effect on chromaticity
- Large β-functions ratios at SX and SY sextupole locations simplify chromaticity correction
- Phase advance 300°/ cell $\Rightarrow$ spherical aberrations cancelled in groups of 6 cells
- Large dipole packing factor $\Rightarrow$ small circumference (C=2.6 km with 9.2T dipole field)

Momentum Acceptance

Fractional parts of the tunes





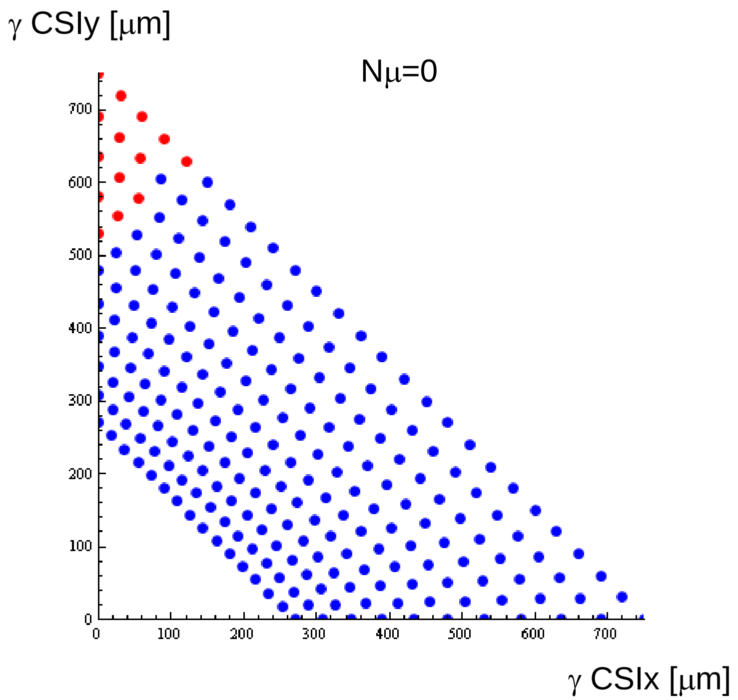


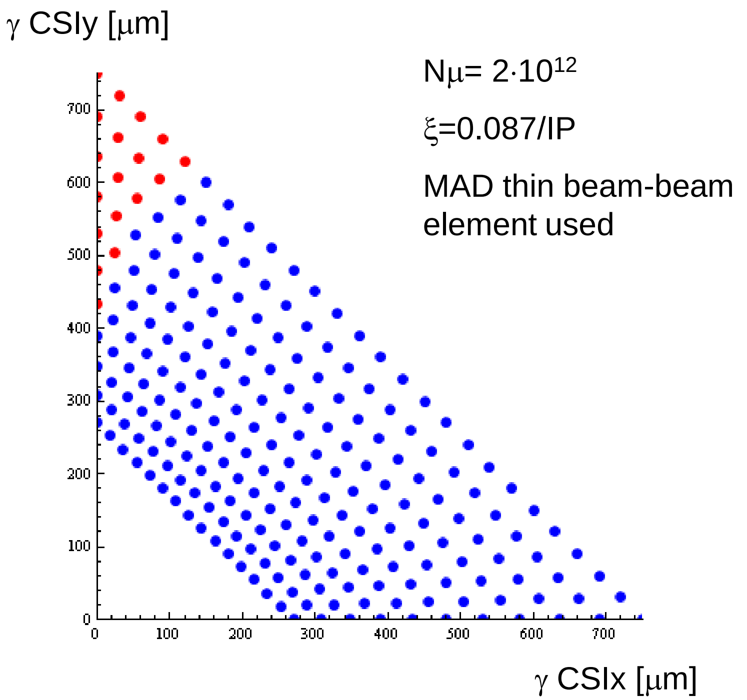
With 2 IPs the central tunes are 18.56, 16.56

- neutral for beam-beam effect
- good for the orbit stability and DA

- Static momentum acceptance is ±1%, but the baseline scheme calls for only ±0.3%
- The momentum compaction factor can be lowered to ~ $5 \cdot 10^{-5}$, or made even smaller negative

Dynamic Aperture





1024 turns DA (Lie3 tracking method): blue – Courant-Snyder Invariants of stable muons, red – lost muons

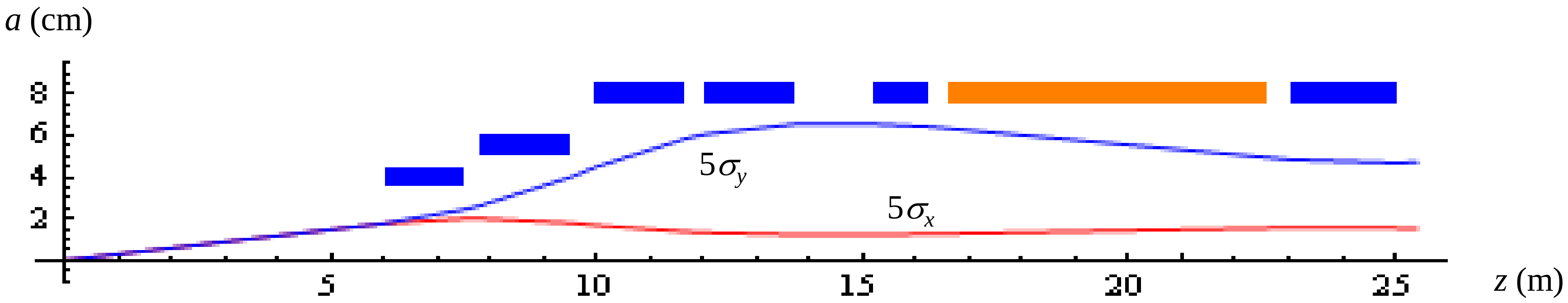
DA= $(\gamma\, CSI / \varepsilon_{\perp N})^{1/2}$= 4.5σ for $\varepsilon_{\perp N}$=25 μm

- Dynamic Aperture is marginally sufficient for $\varepsilon_{\perp N}$=50 μm
- DA can be further increased with vertical nonlinear correctors
- With chosen tunes, 18.56, 16.57, beam-beam increases β^* from 1cm to 1.27cm – with thin beam-beam element, For a long bunch $\beta^* \rightarrow$0,8cm w/o increase in the FF quads

Final Focus Quads

Requirements adopted for this design:

- full aperture 2A = 10sigma_max + 2cm (Sasha Zlobin wants + 1cm more)
- maximum tip field in quads = 10T (G=200T/m for 2A=10cm)
- bending field 8T in large-aperture open-midplane magnets, 10T in the arcs
- IR quad length < 2m (split in parts if necessary!)



Gradient (T/m)	250	187	-131	-131	-89	82
Quench @ 4.5°K	282	209	146	146	(with inner radius 5mm larger)	
Quench @ 1.9°K	308	228	160	160		
Margin @ 4.5°K	1.13	1.12	1.12			
Margin @ 1.9°K	1.23	1.22	1.22			

- Is the margin sufficient? If not lower beam energy or increase β^* to allow for smaller aperture
- We don't need 5sigma+ half-aperture, 3sigma+ is enough: can accommodate $\varepsilon_{\perp N}$=50 μm!
- **No dipole field from 6 to 16.5m, is it worthwhile to create ~2T by displacing the quads?**

Detector Background Simulations (Vadim A.)

Fears:

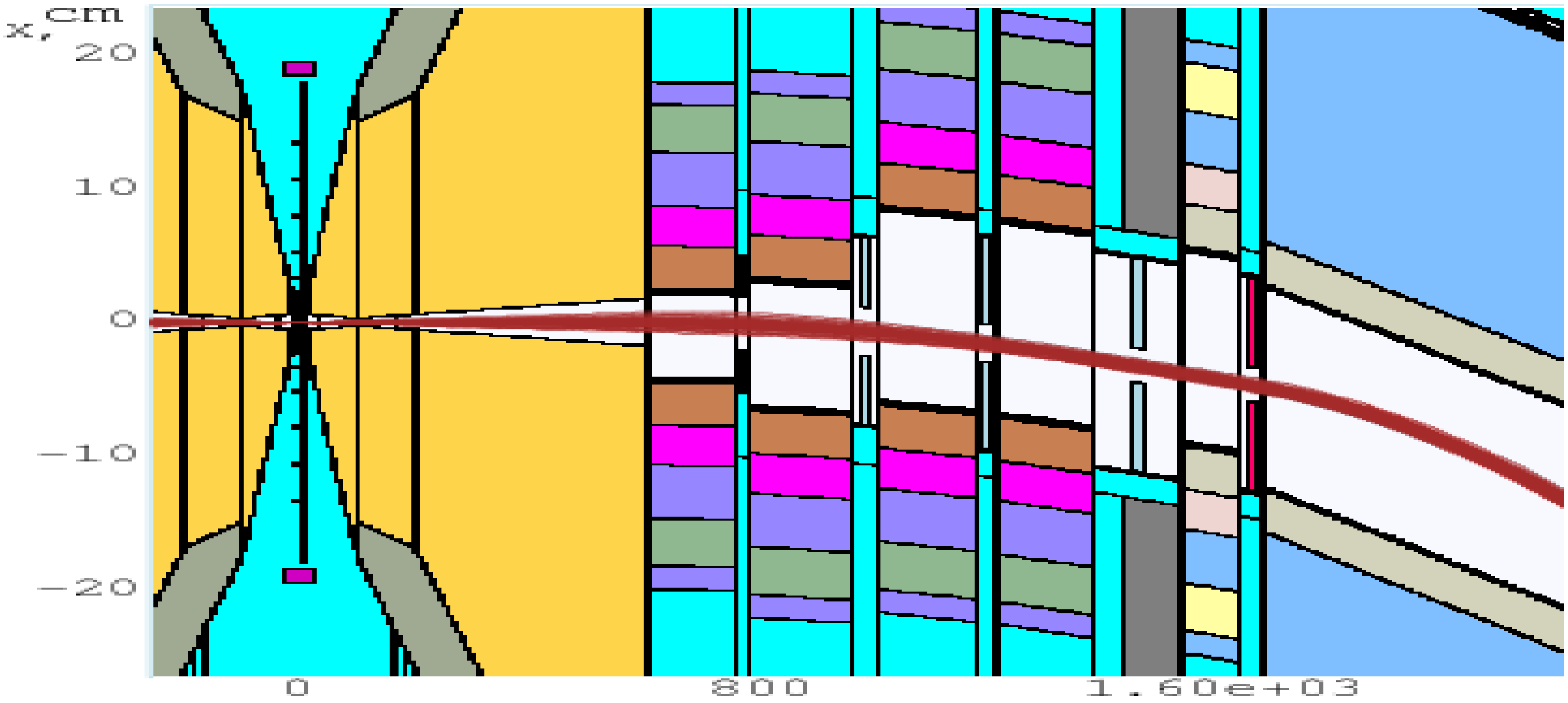
- Dipole field component in the FF quads will deflect the decay electrons more than muons so that they may hit the detector instead of passing through.
- There will be more X-radiation from the decay electrons.

Three cases presented here:

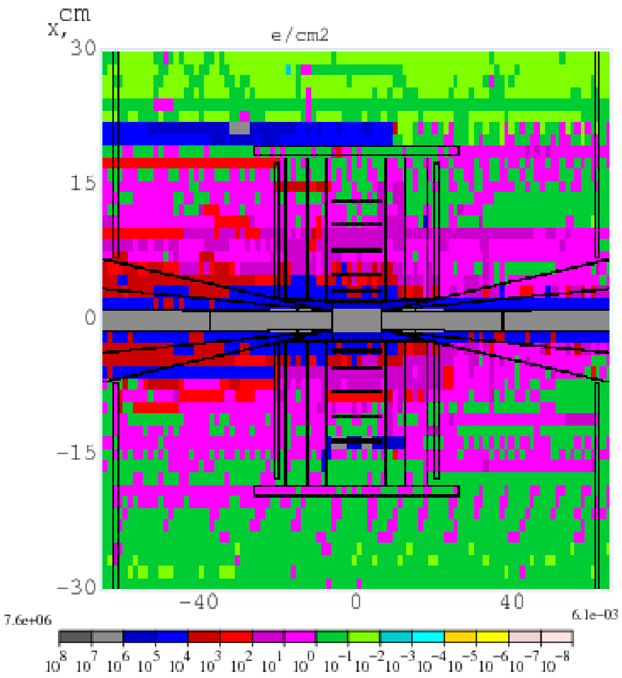
- Initial cone configuration (6°, 5σ inner radius up to 2m from IP), no masks between FF quads - reported at the November workshop at FNAL,
- Cone angle increased to 10 °, 5σ inner radius up to 1m from IP, 5σ masks inserted between FF quads
- The same as above + FF quads displacement by 1/10 of the aperture

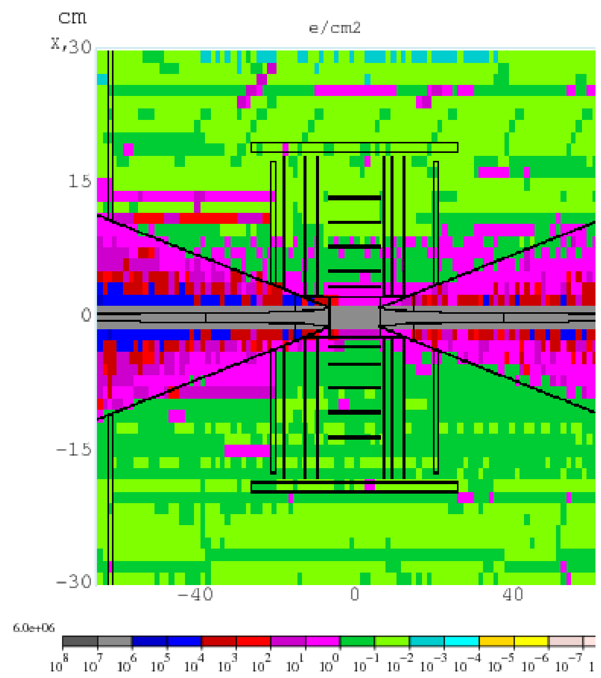
To speed up calculations processes involving neutrons were excluded

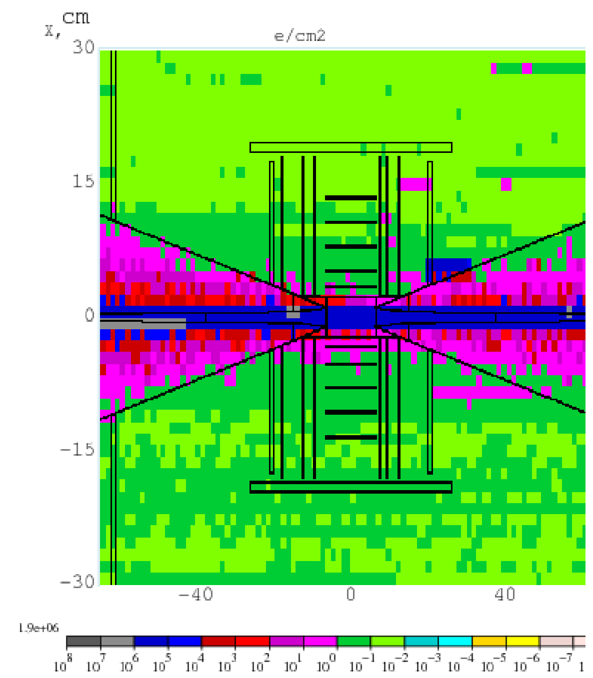
IR Layout (a view from MARS)



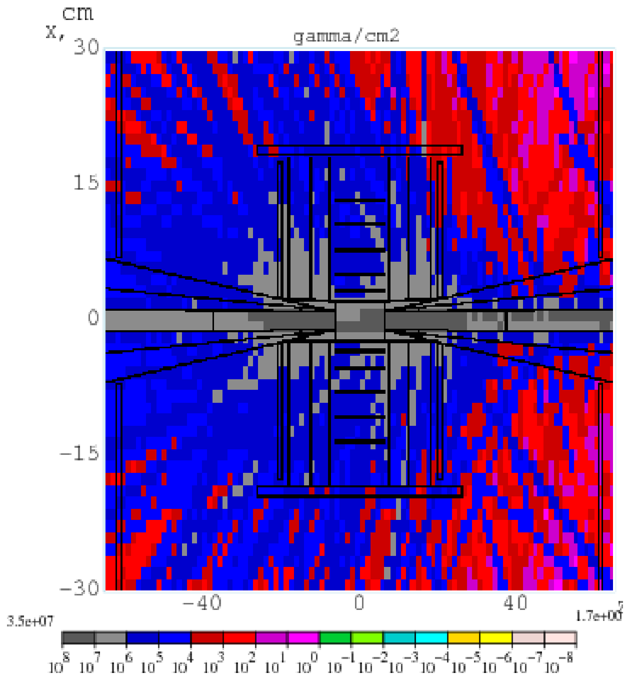
Electron Fluxes

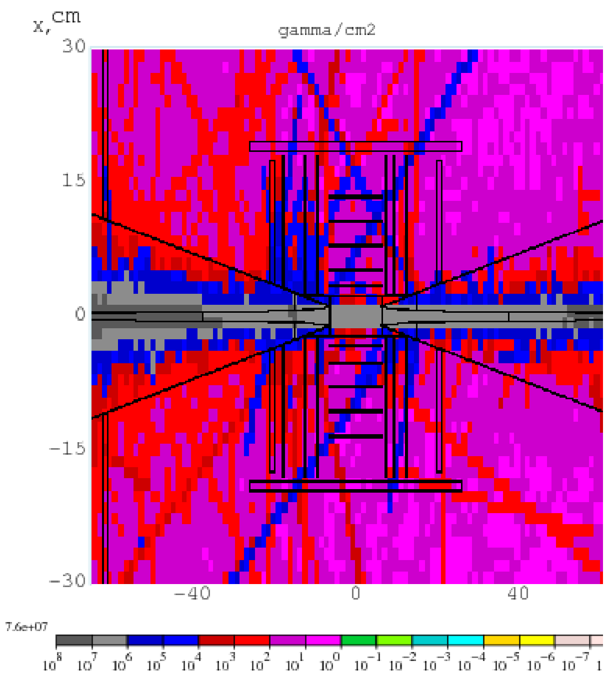


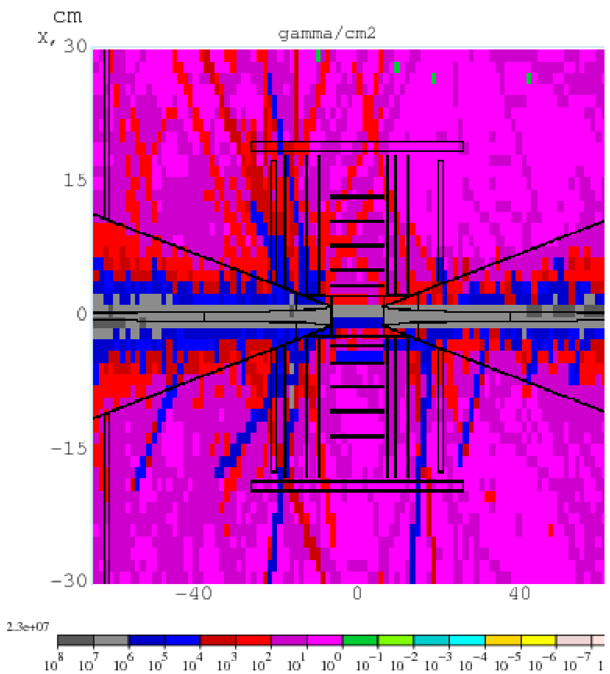




Gamma Fluxes



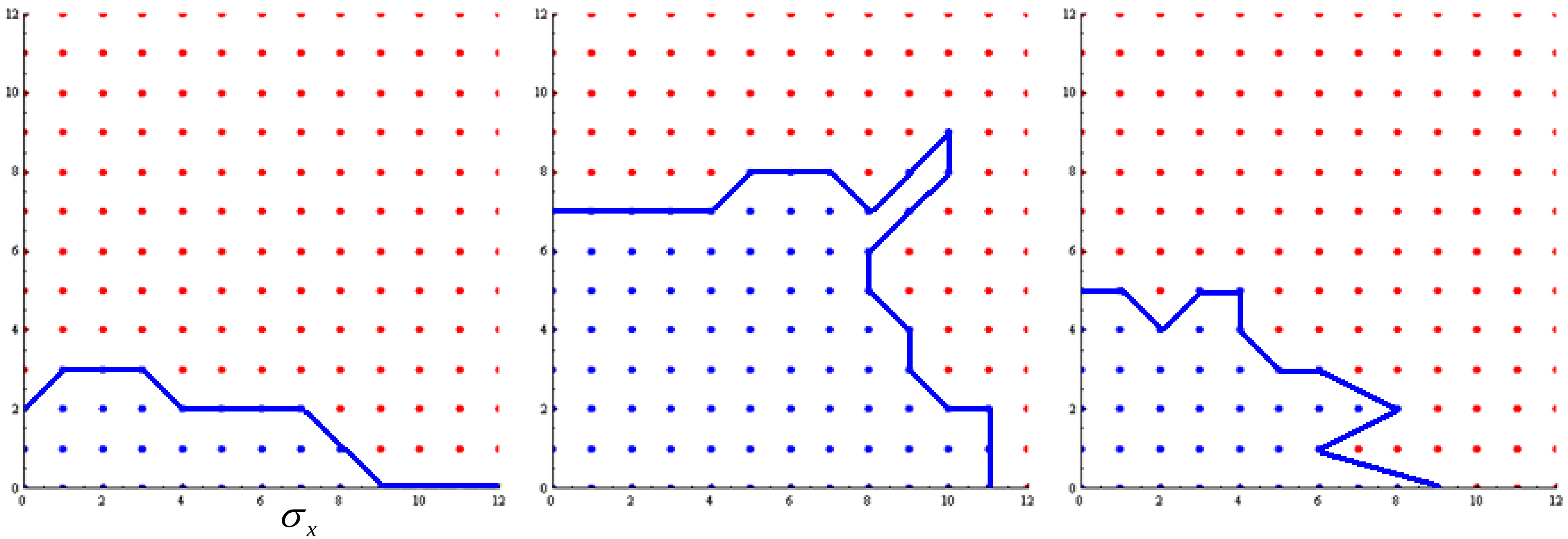




Dynamic Aperture Studies

Dynamic Aperture vs Constant Momentum Deviation

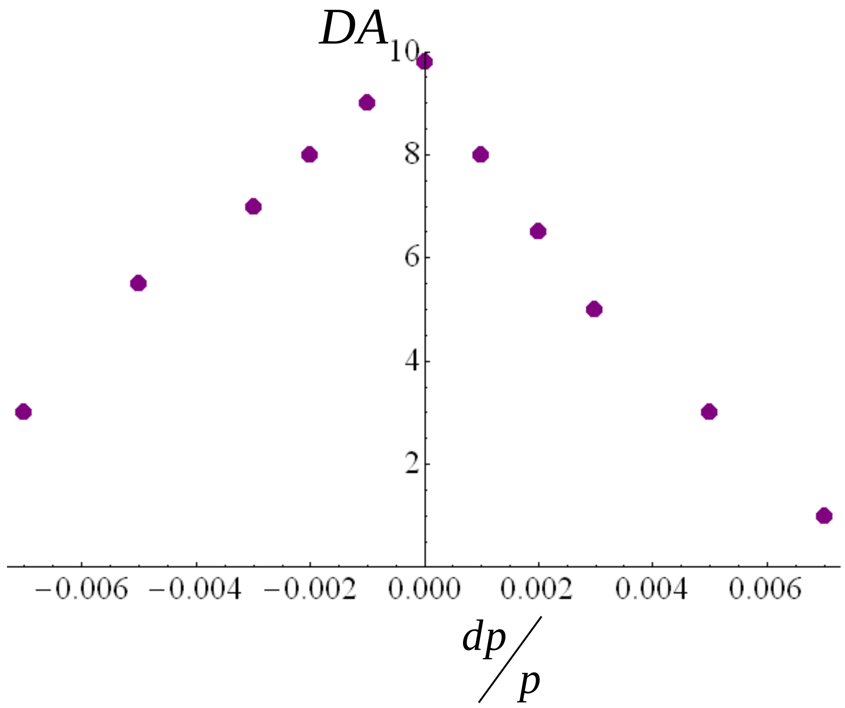
DELTAP = 0.005 DELTAP = 0 DELTAP = -0.005



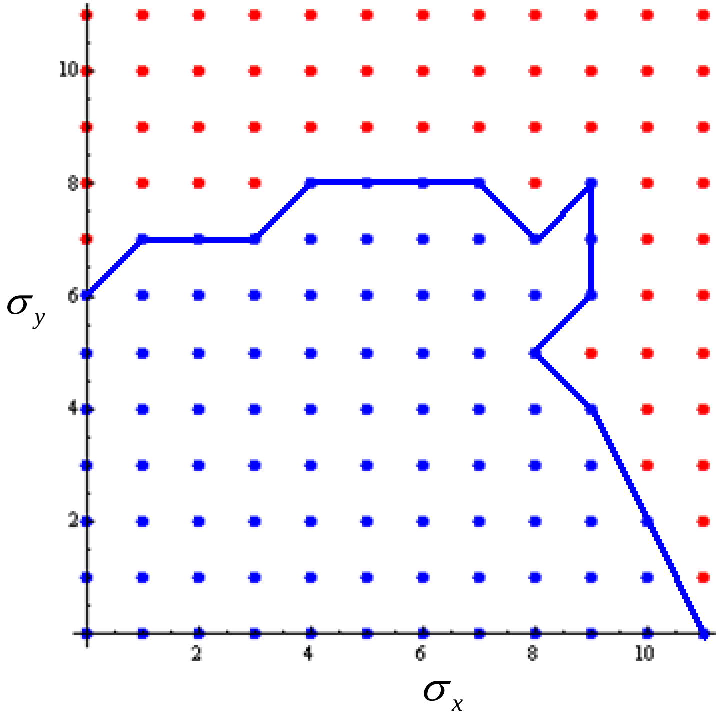
(Calculated using MAD-8 with lie4 method, BeamBeam included, 1024 turns)

$$E_{\perp N} = 10\pi \cdot mm \cdot mrad \qquad \xi_{BB} = 0.095 \qquad \sigma_z = 0.01m \qquad \sigma_{x,z} = 5.7\mu m$$

Dynamic Aperture Radius vs Momentum Deviation



DA diagonal, MAD-8 calculation (4D tracking) for different constant dp/p, BeamBeam included, 1024 turns

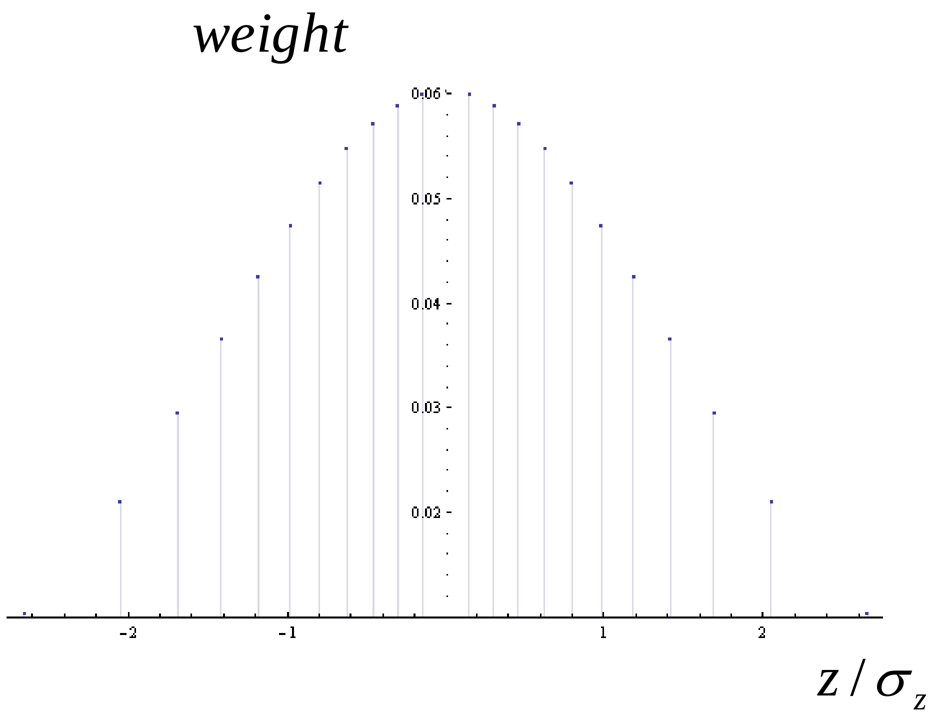


MAD-X calculation 6D tracking with synchrotron oscillations, no BeamBeam, 1024 turns

$f_{RF} = 800\,MHz \qquad V_{RF} = 4 \cdot 16\,MV$

$Q_s \cong 10^{-3}$

Beam-Beam Element Simulation (Mathematica)



Number of slices = 23

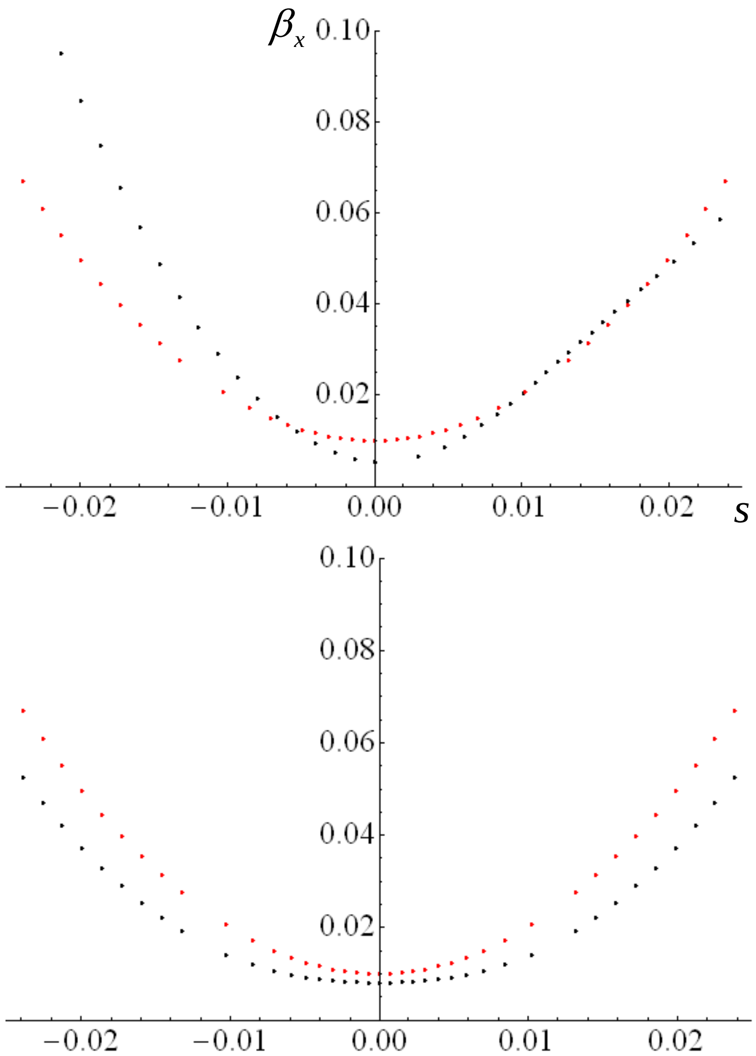
$$\sigma_z = 0.01m$$
$$N_b = 2\cdot 10^{12}$$

Thin lens model
strength of slice:

$$\frac{1}{f_i} = \frac{-2N\cdot weight_i \cdot r_0}{\gamma\sigma_{xi}(\sigma_{xi}+\sigma_{y_i})}$$

In quasi Strong-Strong approach we iteratively calculate new self consistent beta-functions.

Iteratively Calculated new Beta-Functions at IP



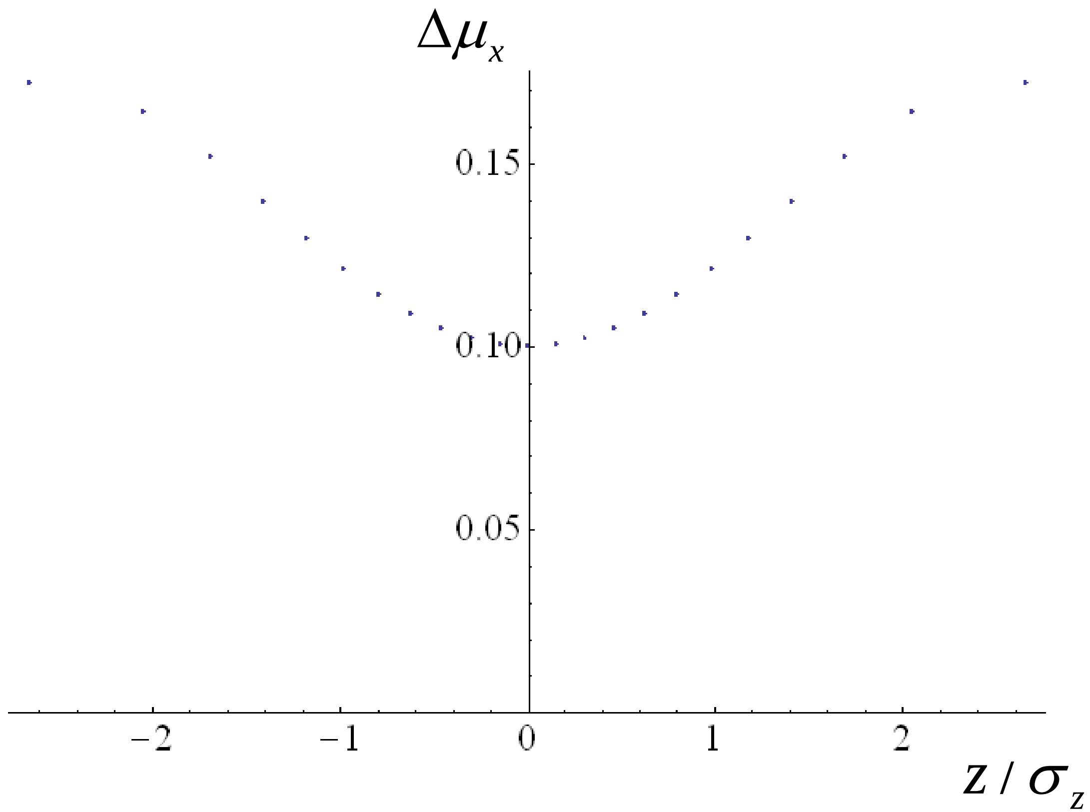
←First slice of the bunch (black dots)

(red dots represent initial beta-function)

←Middle slice of the bunch (black dots)

For each slice we introduce a thin lenses at points where it meats the slices of opposite bunch, find new beta-functions, assign them for both bunches and repeat till converged.

Calculated Tuneshifts for Different Slices in the Bunch

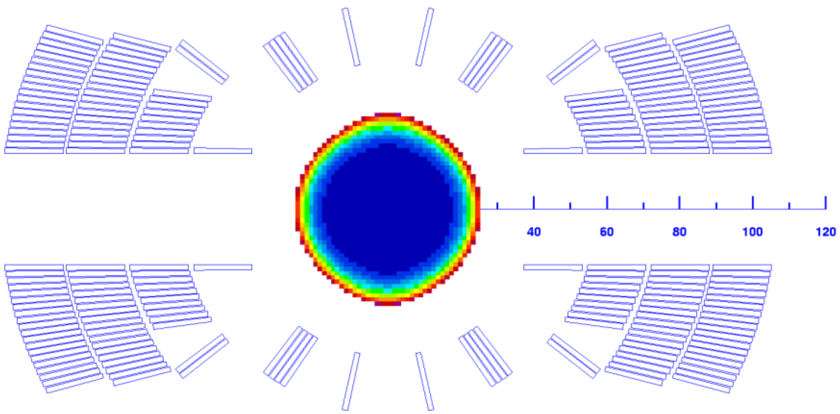


$$\Delta\mu_{x,centrtal} = 0.1$$

(as far as we have round beams and almost the same fractional parts of tunes, vertical tune sifts are the same)

Dynamics of Beam-Beam interaction in Mathematica file

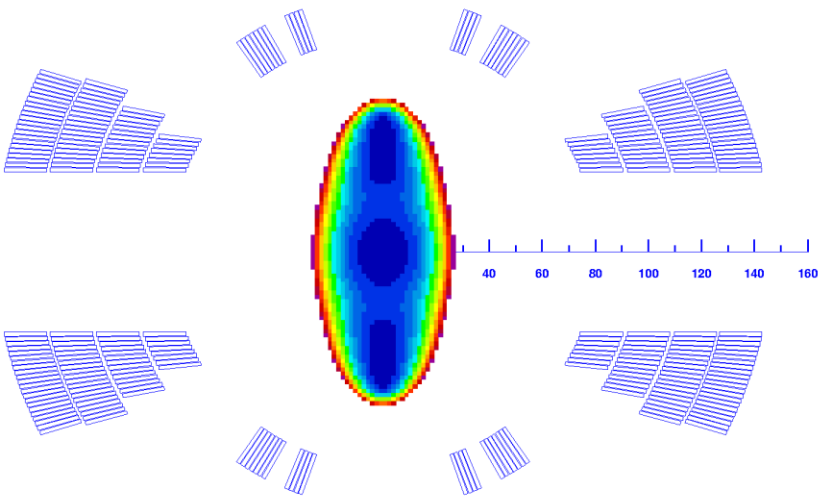
Dipole Magnet Field Imperfections



Magnetic field multipole expansion:

$$B_\theta(r,\theta) = B_{ref} \times 10^{-4} \sum_{n=1}^{\infty} \left(\frac{r}{r_{ref}} \right)^{n-1} \left[b_n \cos(n\theta) - a_n \sin(n\theta) \right]$$

$$B_r(r,\theta) = B_{ref} \times 10^{-4} \sum_{n=1}^{\infty} \left(\frac{r}{r_{ref}} \right)^{n-1} \left[b_n \sin(n\theta) + a_n \cos(n\theta) \right]$$



IR dipole:	Ring dipole:
Rref=40mm	Rref=20mm
b1=10000	b1=10000
b3=-5.875	b3=0.003
b5=-18.320	b5=-0.012
b7=-17.105	b7=0.154
b9=-4.609	b9=-1.185
b11=0.390	b11=-0.118
b13=0.103	b13=0.053

(V.V.Kashikhin)

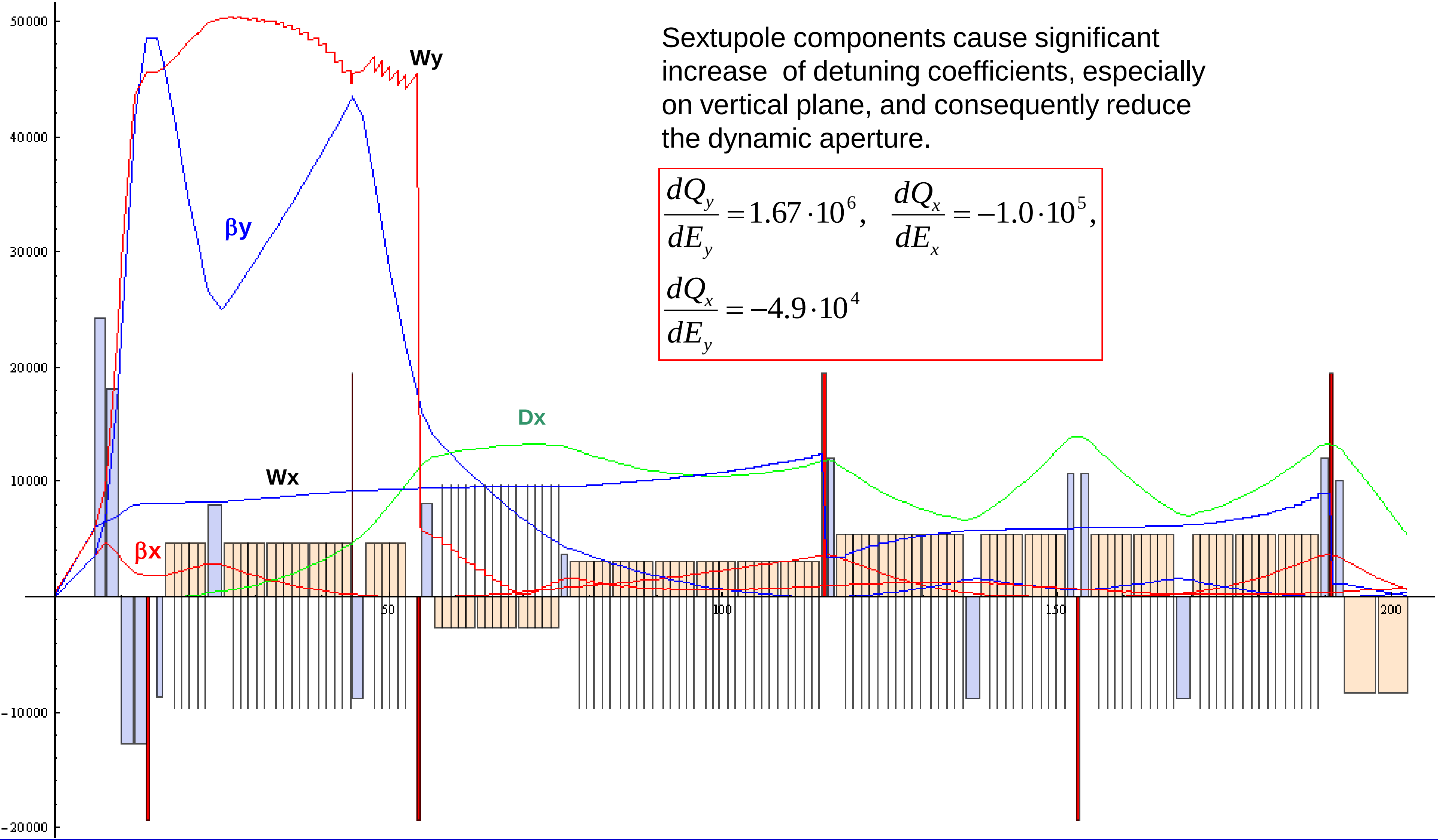
Field Imperfection Impact and Correction

Magnets sliced in 5 pieces and thin multipoles introduced between them.

Sextupole components cause significant increase of detuning coefficients, especially on vertical plane, and consequently reduce the dynamic aperture.

$$\frac{dQ_y}{dE_y} = 1.67 \cdot 10^6, \quad \frac{dQ_x}{dE_x} = -1.0 \cdot 10^5,$$

$$\frac{dQ_x}{dE_y} = -4.9 \cdot 10^4$$



Plans

Lattice update including:

- FF quads displacement to produce dipole field ~2T
- Momentum compaction factor correction (making it slightly negative)
- Arc magnet length reduction to get 10T bending field $\rightarrow$ even smaller circumference!

Next steps:

- Study the effect of magnet imperfections with Vadim Kashikhin's magnet design
- Possibility to change β^* in a wide range w/o changing the layout
- Collimation system design (are special sections necessary?)
- Study effects of fringe fields, add nonlinear correctors if necessary
- Design orbit correction and tuning circuits
- Study the effect of random misalignments and magnet imperfections

Muon Collider Parameters

$\sqrt{s}$ (TeV)	1.5	3
Av. Luminosity / IP (10^{34}/cm^2/s)	1.2*	5
Max. bending field (T)	9.2**	14
Av. bending field in arcs (T)	7.7	12
Circumference (km)	2.6	4
No. of IPs	2	2
Repetition Rate (Hz)	15	12
Beam-beam parameter / IP	0.087	0.087
β^* (cm)	1	0.5
Bunch length (cm)	1	0.5
No. bunches / beam	1	1
No. muons/bunch (10^{12})	2	2
Norm. Trans. Emit. (μm)	25	25
Energy spread (%)	0.1	0.1
Norm. long. Emit. (m)	0.07	0.07
Total RF voltage (MV) at 800MHz	60	700
μ+ in collision / 8GeV proton	0.008	0.007
8 GeV proton beam power (MW)	4.8	4.3

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*) With **increase** by the beam-beam effect
**) Not 10T just by mistake

$$\langle \mathcal{L} \rangle = f_0 \frac{n_b N_\mu^2}{4\pi \varepsilon_\perp \beta^*} h \times \frac{1}{2} \tau f_{rep} \sim \frac{P_\mu \xi}{C \beta^*} h \tau$$

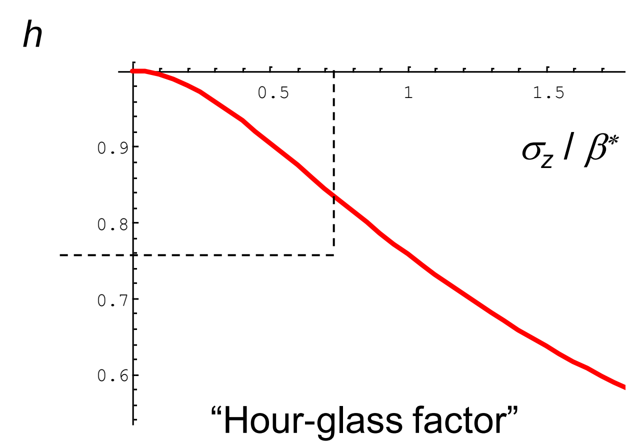
P_μ – average muon beam power ($\sim \gamma$)

$\xi = \dfrac{r_\mu N_\mu}{4\pi \gamma \varepsilon_\perp}$ – beam-beam parameter

C – collider circumference ($\sim \gamma$ if B=const)

τ – muon lifetime ($\sim \gamma$)

β^* – beta-function at IP



"Hour-glass factor"