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Twin-Helix Magnetic Channel for Parametric-resonance Ionization Cooling

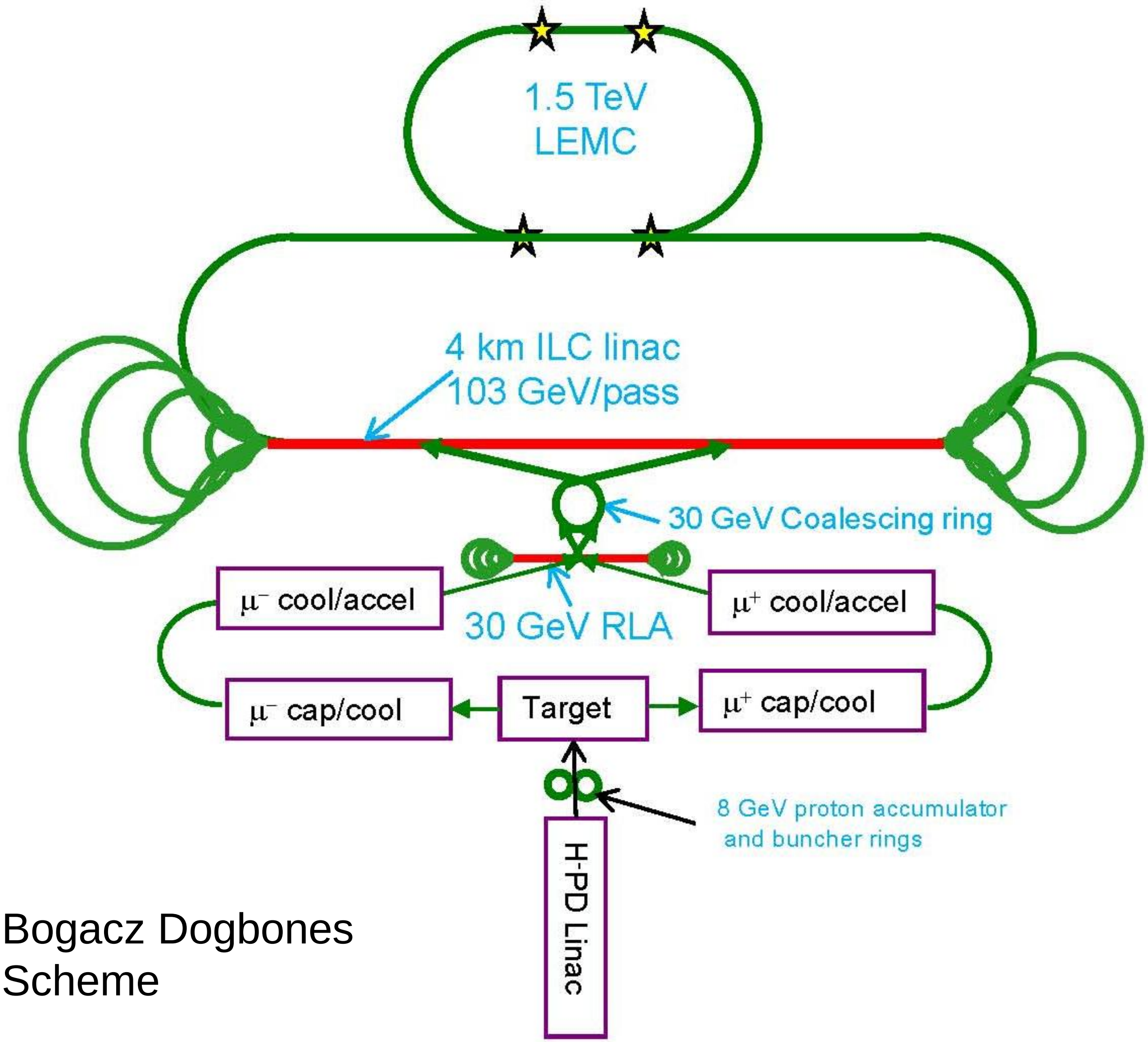
V.S. Morozov[1,2], A. Afanasev[2,3], Y.S. Derbenev[4], R.P. Johnson[2]

[1] *Old Domion University*
[2] *Muons, Inc.*
[3] *Hampton University*
[4] *Jefferson Lab*

Outline

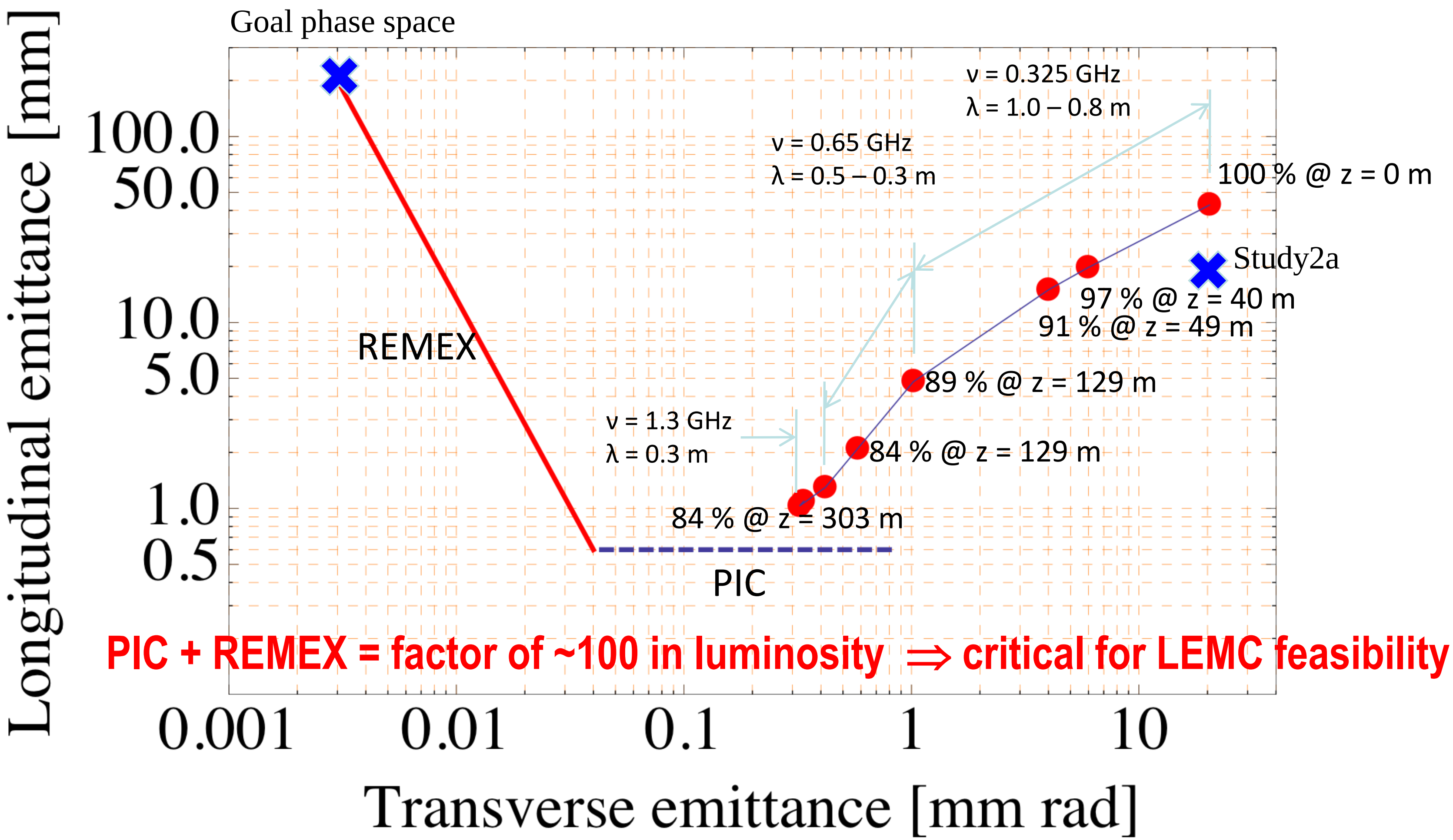
- Concept of Parametric-resonance Ionization Cooling (PIC)
- PIC linear optics requirements
- Epicyclic channel for PIC
- Twin-helix channel for PIC
  - Magnetic optics design
  - G4beamline simulations
  - Possible practical implementation
- Approach to compensating aberrations

LEMC Scenario



Bogacz Dogbones
Scheme

New Fernow-Neuffer plot

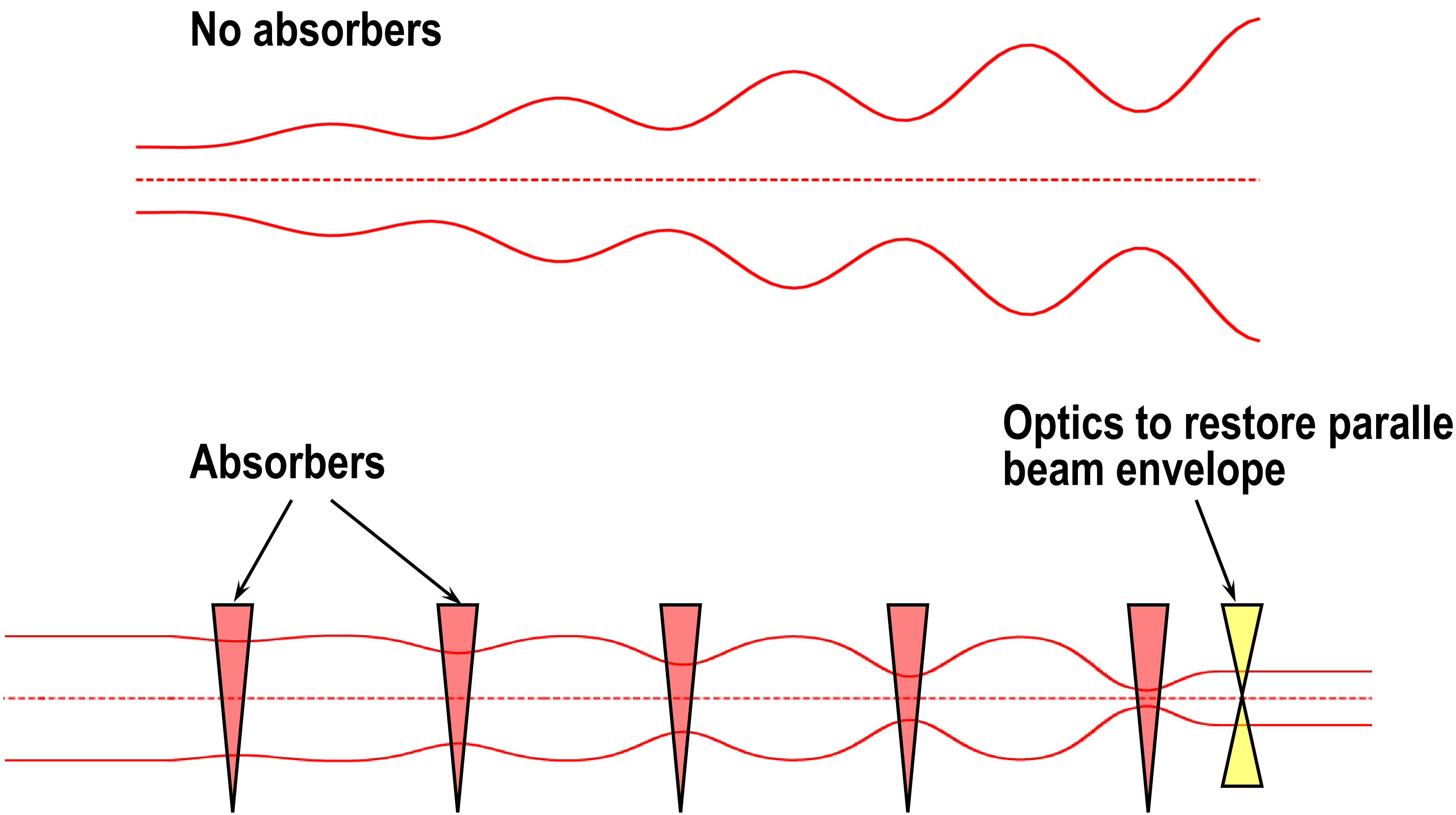


- GH2 pressure = 160 atm
- 60 μm Be RF window
- E ~ 27 MV/m
- Detailed parameter will be given in later slide (slide 15)

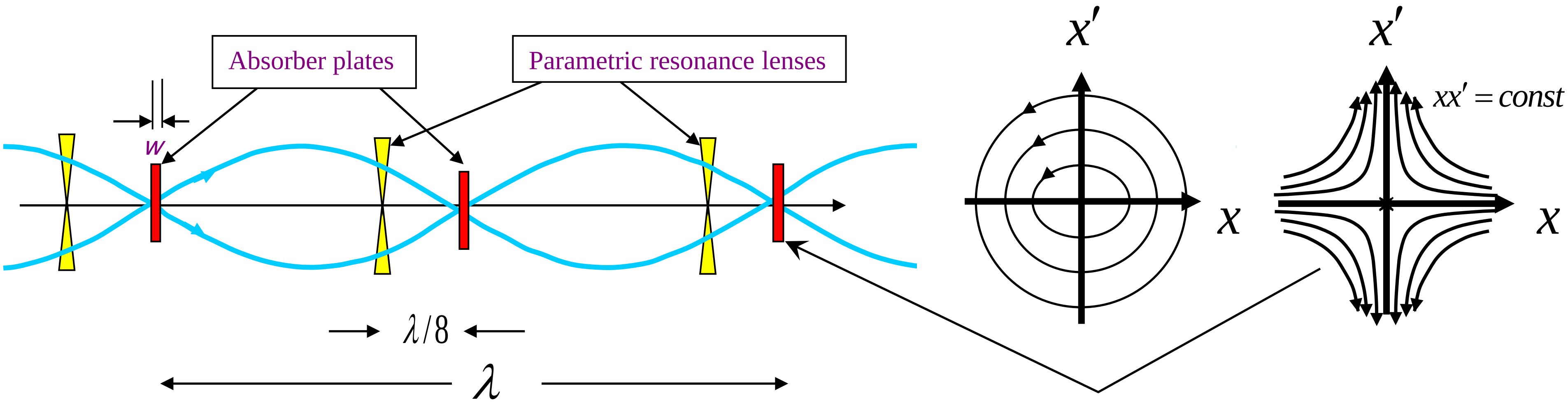
PIC Concept

- Parametric resonance induced in muon cooling channel
- Muon beam naturally focused with period of free oscillations
- Wedge-shaped absorber plates combined with energy-restoring RF cavities placed at focal points (assuming aberrations corrected)
  - Ionization cooling maintains constant angular spread
  - Parametric resonance causes strong beam size reduction
  - Emittance exchange at wedge absorbers produces longitudinal cooling
- Resulting equilibrium transverse emittances are an order of magnitude smaller than in conventional ionization cooling

Absorber Stabilizing Role



PIC Schematic



- **Equilibrium angular spread and beam size at absorber**

$$\theta_a^2 = \frac{3}{2}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu} \qquad \sigma_a = \frac{1}{2\sqrt{3}}\theta_a w$$

- **Equilibrium emittance**

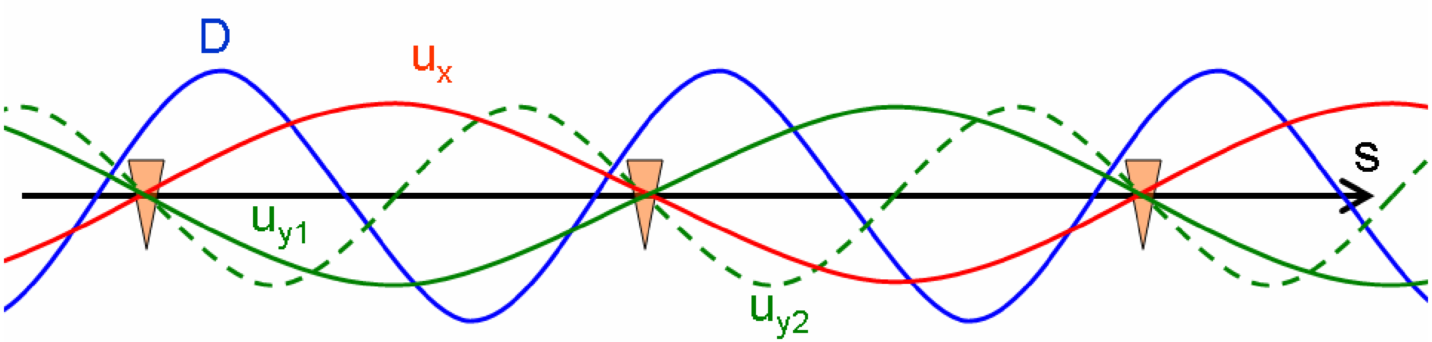
$$\varepsilon_n = \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w$$ **(a factor of** $\frac{\pi}{\sqrt{3}}\frac{w}{\lambda} = \frac{\pi}{2\sqrt{3}}\frac{\gamma'_{acc}}{\gamma'_{abs}}$ **improvement)**

PIC Optics Requirements

- Horizontal free oscillations' period λ_x equal to or low-integer multiple of vertical free oscillations' period λ_y
- Oscillating dispersion
  - small at absorbers to minimize energy straggling
  - non-zero at absorbers for emittance exchange
  - large between focal points for compensating chromatic and spherical aberrations

$$\Downarrow$$

- Correlated optics: correlated values of λ_x, λ_y and dispersion period λ_D
  - $\lambda_x = n\lambda_y = m\lambda_D$, *e.g.* $\lambda_x = 2\lambda_y = 4\lambda_D$ or $\lambda_x = 2\lambda_y = 2\lambda_D$



- Fringe-field-free design

Helical Harmonics

- **Practical fringe-field-free approach**
- **Periodic solutions of source-free Maxwell equations in vacuum**

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{B} = 0$$

- **Harmonic of order n given by**

$$B_\varphi^n(\varphi,\rho,z) = \left(\frac{2}{nk}\right)^{n-1} \left.\frac{\partial^{n-1} B_\varphi^n}{\partial \rho^{n-1}}\right|_{\substack{\varphi - kz + \varphi_0^n = 0 \\ \rho = 0}} [I_{n-1}(nk\rho) - I_{n+1}(nk\rho)]\cos(n[\varphi - kz + \varphi_0^n])$$

$$B_\rho^n(\varphi,\rho,z) = \left(\frac{2}{nk}\right)^{n-1} \left.\frac{\partial^{n-1} B_\varphi^n}{\partial \rho^{n-1}}\right|_{\substack{\varphi - kz + \varphi_0^n = 0 \\ \rho = 0}} [I_{n-1}(nk\rho) + I_{n+1}(nk\rho)]\sin(n[\varphi - kz + \varphi_0^n])$$

$$B_z^n(\varphi,\rho,z) = -2\left(\frac{2}{nk}\right)^{n-1} \left.\frac{\partial^{n-1} B_\varphi^n}{\partial \rho^{n-1}}\right|_{\substack{\varphi - kz + \varphi_0^n = 0 \\ \rho = 0}} I_n(nk\rho)\cos(n[\varphi - kz + \varphi_0^n])$$

- **Total field**

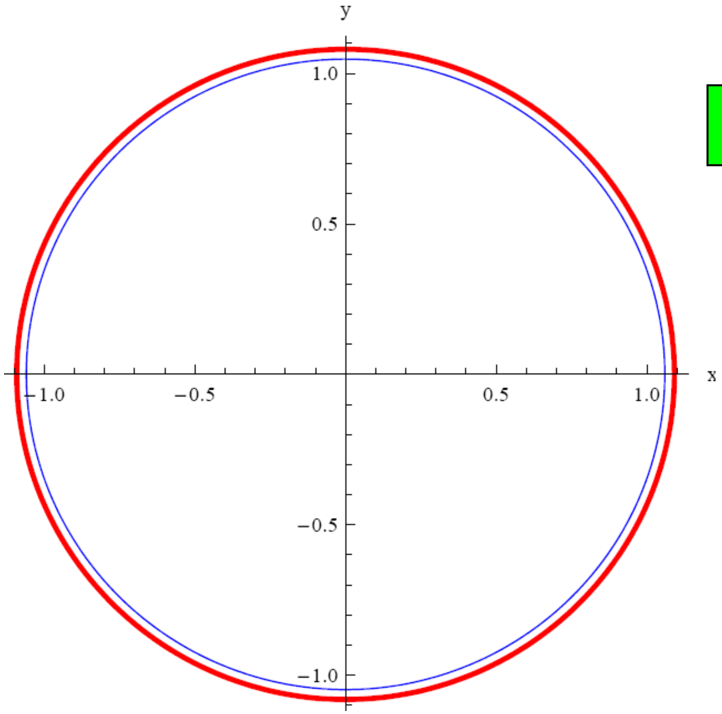
$$\vec{B} = \sum_n \vec{B}^n$$

Epicyclic Channel

- Two uniform (non-Maxwellian) transverse helical fields with wave numbers k_1 and k_2
$$b_1 + b_2 = B_1 e^{ik_1 z} + B_2 e^{ik_2 z}$$
- Equation of motion: $p_T' - ik_c p_T = i(b_1 + b_2), \qquad k_c = eB_z / p_z c$
- Analytic solution under approximation $k_c = \text{const} \;\; (p_z = \text{const})$
$$p_T = \frac{b_1}{k_1 - k_c} + \frac{b_2}{k_2 - k_c}, \qquad u = x + iy = -\frac{i}{p_z}\left(\frac{b_1/k_1}{k_1 - k_c} + \frac{b_2/k_2}{k_2 - k_c}\right)$$
- Dispersion function containing two oscillating terms
$$D = p\frac{\partial u}{\partial p} \propto \frac{b_1}{(k_1 - k_c)^2} + \frac{b_2}{(k_2 - k_c)^2}$$
- Condition for dispersion to periodically return to zero
$$\frac{B_2}{B_1} = -\left(\frac{k_2 - k_c}{k_1 - k_c}\right)^2, \quad k_1 \neq k_2$$
- $k_1 = -k_2 = -k_c / 2 \equiv k \quad \Rightarrow \quad B_2 = -B_1 / 9,\; u = u_0[2i\cos(kz) + \sin(kz)],\; D = u_0 \sin(kz)$

Oscillating Dispersion

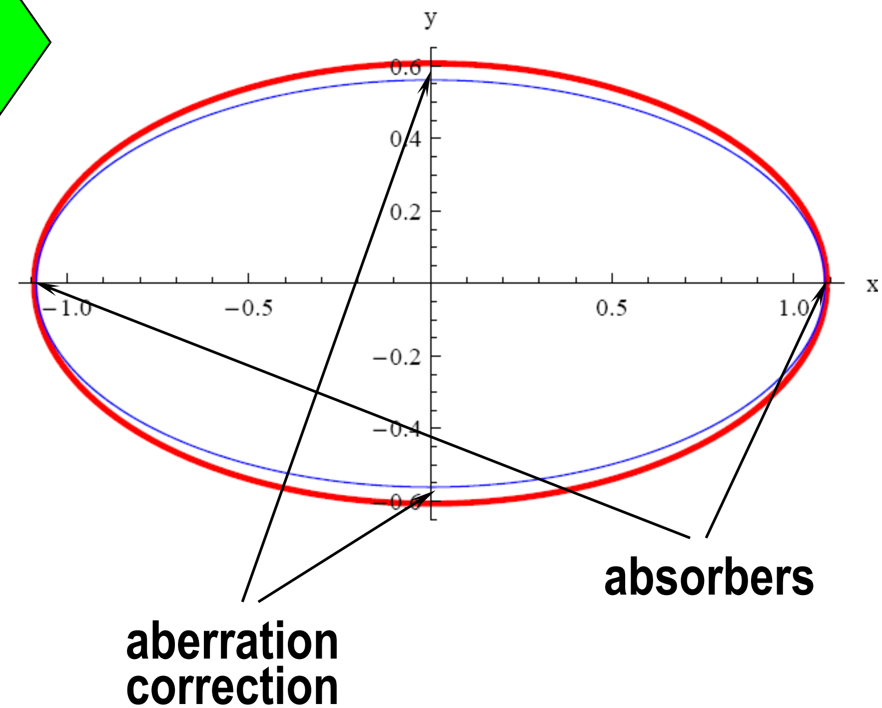
$$B_1 \neq 0, \;\; B_2 = 0 \;\; \text{(HCC)}$$



$$B_1 \neq 0, \;\; B_2 \neq 0 \;\; \text{(Epicyclic)}$$

$$k_1 = -k_2 = -k_c / 2$$

$p \rightarrow p + \Delta p$



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HCC-Based Approach to Designing Correlated Optics

- **Since there is no exact analytic solution for two Maxwellian helices, start with single helix considering second helix perturbation**
- **Using available analytic solution for dynamics in single Maxwellian helix, adjust desired free-oscillation period ratio** $\lambda_- / \lambda_+ = 1$ **or** 2 **for primary helix**
- **By choosing wave number** k_2 **of second helix, set dispersion oscillation period** $\lambda_D = |2\pi/(k_2 - k_1)|$ **such that** $\lambda_+ / \lambda_D = 2$
- **Adjust strength of second helix to create oscillating dispersion**
- **Iteratively adjust** λ_- / λ_+ **and** λ_+ / λ_D **by changing helices' parameters until correlated optics is achieved**

Single Helix

- **Equilibrium condition**

$$\frac{b}{B} = \frac{\kappa}{1+\kappa^2}\left(\frac{q}{q+1}\right), \qquad \kappa \equiv \frac{p_T}{p_z}, \qquad q \equiv \frac{k_c}{k} - 1$$

- **Orbit stability condition**

$$0 \quad < \quad G \equiv (q-g)\hat{D}^{-1} \quad < \quad R^2 \equiv \frac{1}{4}\left(1 + \frac{q^2}{1+\kappa^2}\right)^2$$

$$\hat{D}^{-1} = \frac{\kappa^2 + (1-\kappa^2)q}{1+\kappa^2} + g, \qquad g \equiv -\frac{(1+\kappa^2)^{3/2}}{pk^2}\frac{\partial b}{\partial a}$$

- **Betatron tunes**

$$Q_\pm^2 = R \pm \sqrt{R^2 - G}$$

- **For given** $r = Q_+/Q_-$ **, one can solve for** $\partial b/\partial a$ **if**

$$-(r^2-1)^2\frac{\kappa^4}{4} + [2(1+q^2)r^2 - (r^2+1)^2 q]\kappa^2 + (q^2 - r^2)(r^2q^2 - 1) < 0$$

Adjusting Betatron Tunes

- **No solution for** $Q_+ = Q_-$
- **Two solution regions for** $Q_+ = 2Q_-$
  - $|\kappa| << 1, \quad -2 < q < -1, \quad B_2/B_1 \sim 1$
  - $|\kappa| >> 1$

    Choose:

    $\kappa = -5.4$

    $q = -1.54$

    $B_{sol} = 2$ T

    $b_d = -0.154$ T

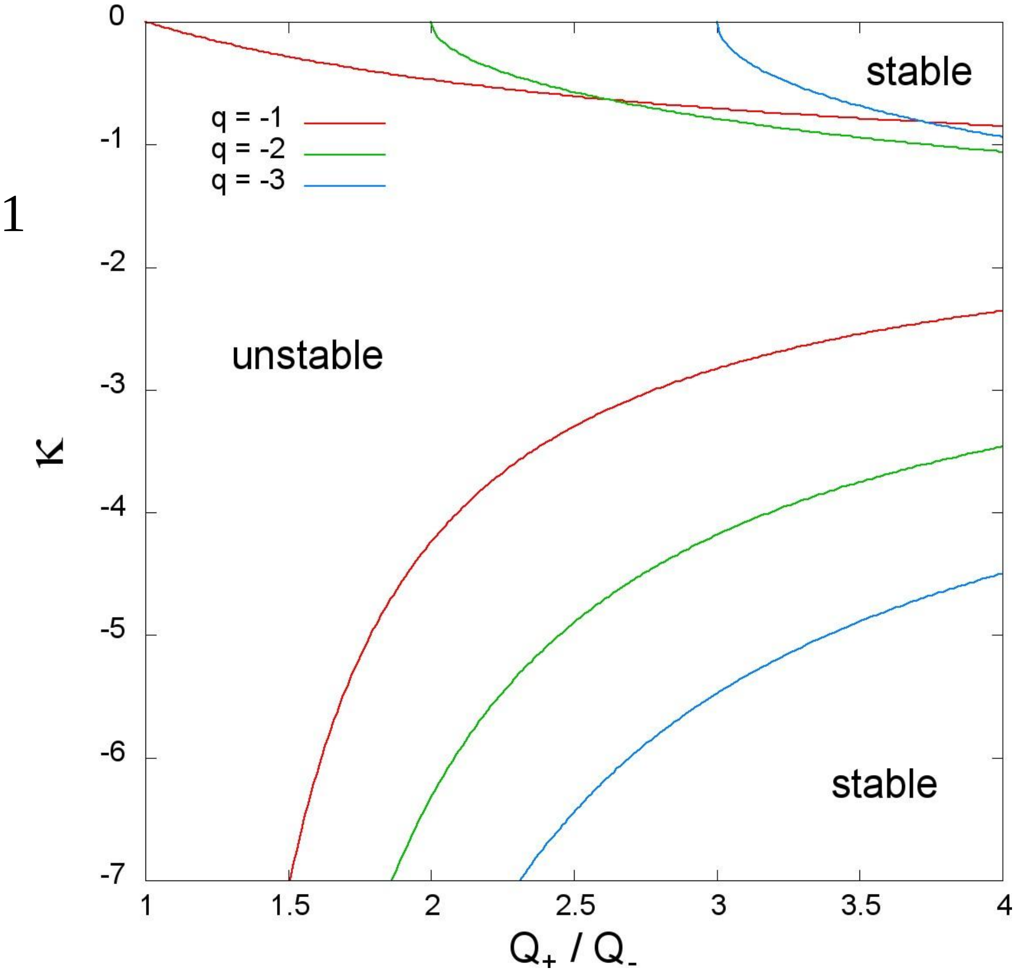
    $b_q = 0.065$ T/m

    $k_c = 32.9\ \mathrm{m}^{-1}$

    $k = -61.0\ \mathrm{m}^{-1}$

    $Q_- = 0.464, \quad Q_+ = 0.929$

    $B_2/B_1 \sim 0.04$



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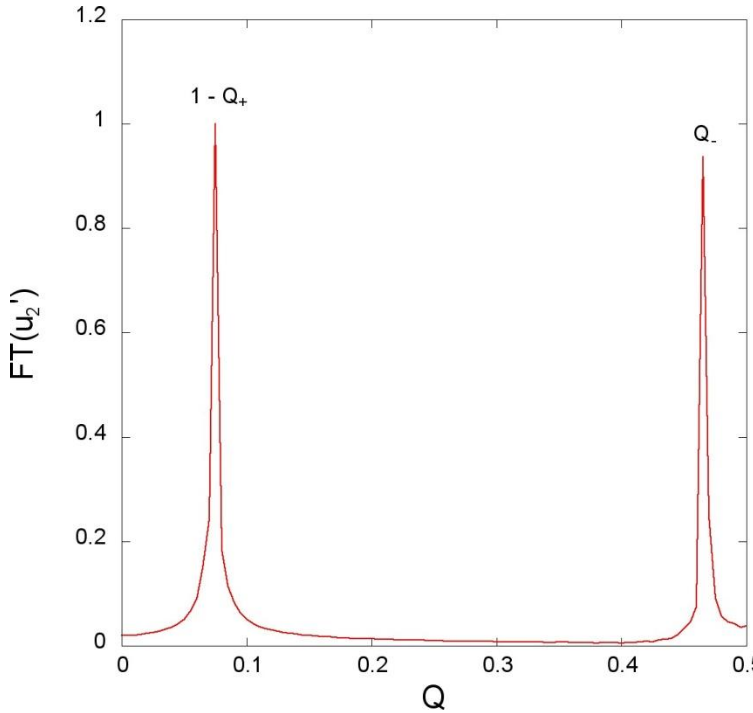
Determining Betatron Tunes

- **From one-period linear transfer matrix in terms of canonical coordinates**

$$\begin{pmatrix} u_1 \\ u_1' \\ u_2 \\ u_2' \end{pmatrix}_{n+1} = M \begin{pmatrix} u_1 \\ u_1' \\ u_2 \\ u_2' \end{pmatrix}_n , \quad M = \begin{pmatrix} A & E \\ F & B \end{pmatrix} = X \bar{M} X^{-1}, \quad \bar{M} = \begin{pmatrix} \bar{A} & 0 \\ 0 & \bar{B} \end{pmatrix}$$

$$\cos(2\pi Q_1) = \frac{1}{2}\mathrm{tr}\,\bar{A}, \quad \cos(2\pi Q_2) = \frac{1}{2}\mathrm{tr}\,\bar{B}$$

- **Track particle over many periods and take Fourier transform of coordinate vector component**



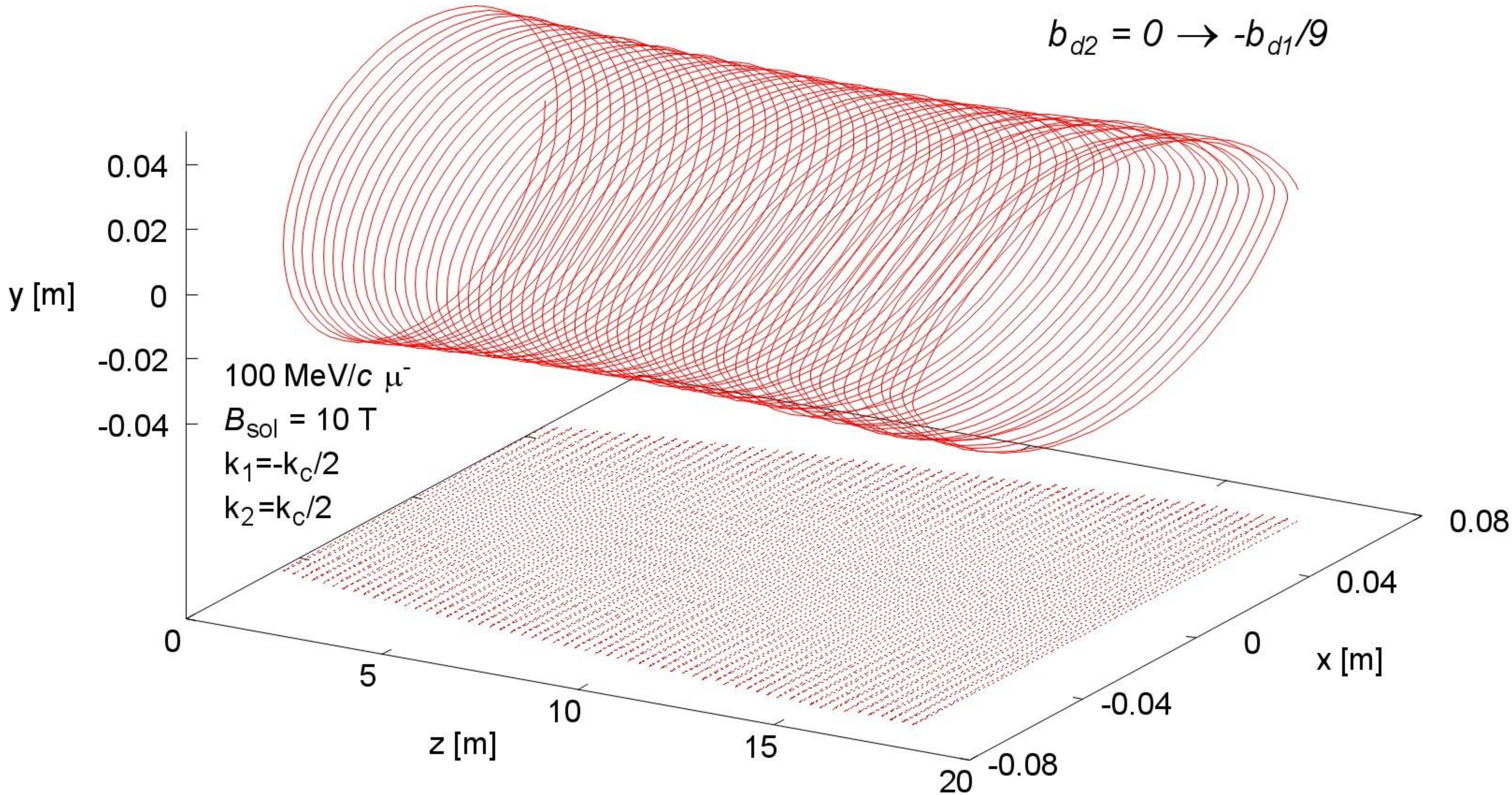
Finding Periodic Orbit

- No exact analytic solution in case of two helices
- Stable periodic orbit does not always exit
- Begin with single helix where stable periodic orbit is known to exist
- Use one or combination of the following to find periodic orbit when second helix is present
  - Adiabatically increase strength of second helix while tracking orbit
  - Use "friction" force making particle trajectory converge to periodic orbit

$$\dot{\vec{p}} = e[\vec{\upsilon} \times \vec{B}] - Q\frac{\vec{\upsilon}}{\upsilon} + Q\frac{\upsilon}{(\vec{\upsilon} \cdot \vec{e}_z)}\vec{e}_z$$

  - Increase second helix's strength from zero in small steps while iteratively determining periodic orbit on each step by locating fixed point in phase space

Adiabatic Turn On of Secondary Helix



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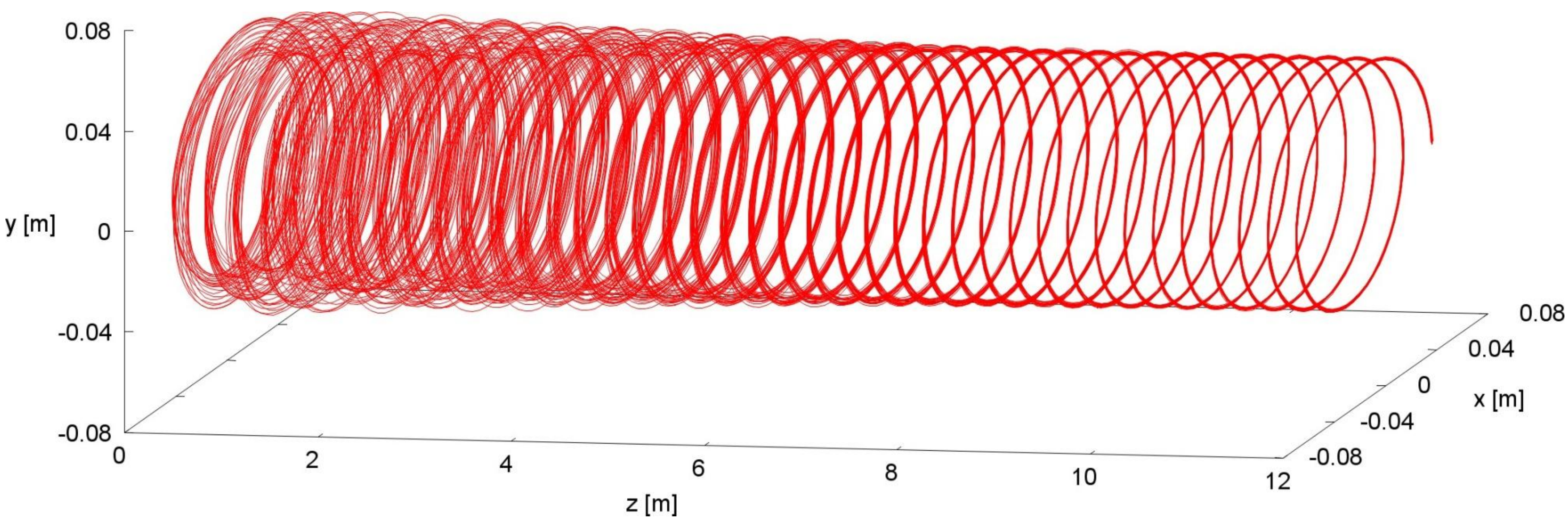
Friction Force Approach

- **Introduce effective "friction" force**

$$\dot{\vec{p}} = e[\vec{\upsilon}\times\vec{B}] - Q\frac{\vec{\upsilon}}{\upsilon} + Q\frac{\upsilon}{(\vec{\upsilon}\cdot\vec{e}_z)}\vec{e}_z$$

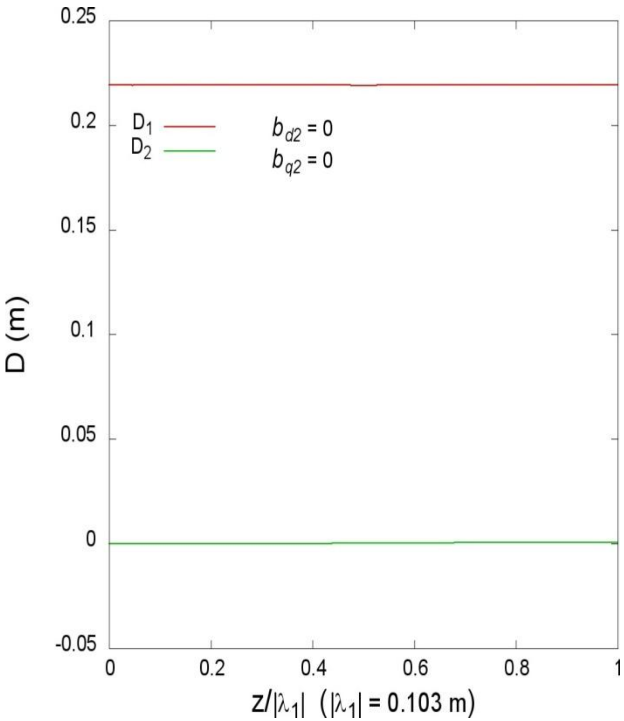
  - **total energy conserved while phase volume is not**
  - **analogous to cooling**
  - **all trajectories converge towards periodic orbit**
  - **aids in finding periodic orbit when analytic solution is not available**



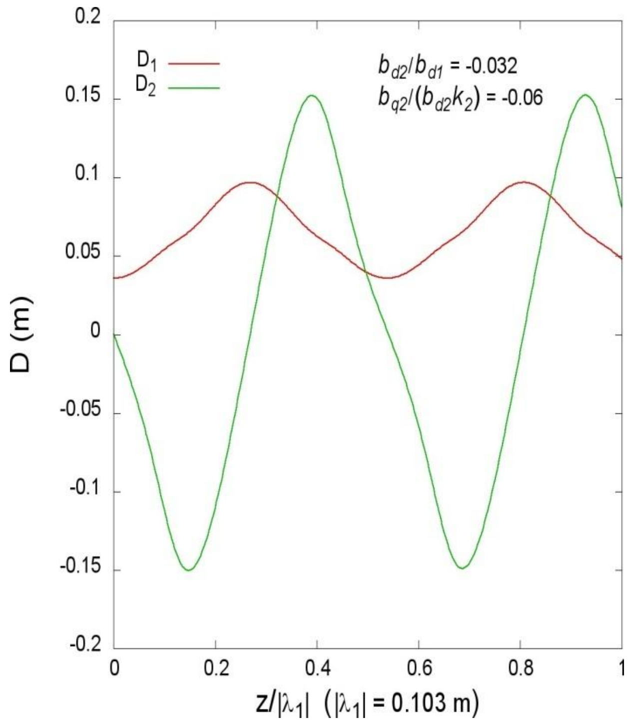
Dispersion in Epicyclic Channel

Second helix strength →

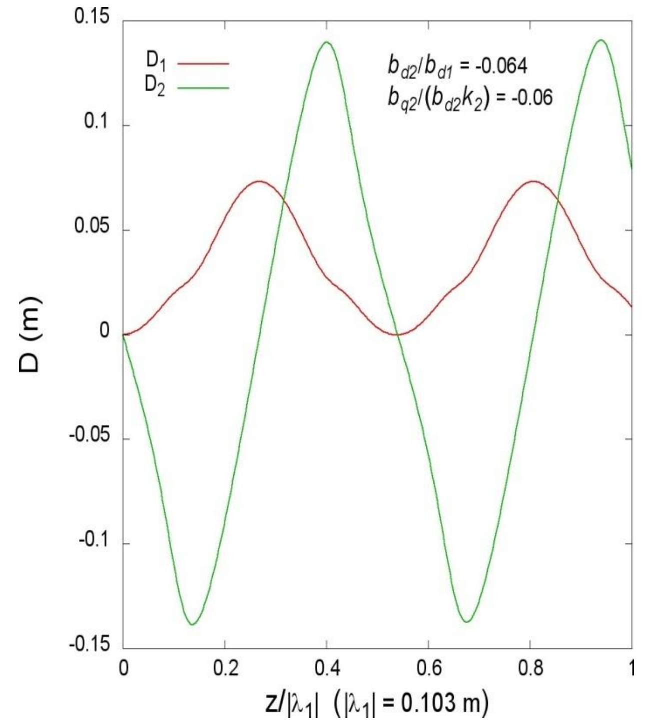
|*D*| = const



|*D*| oscillates not reaching 0

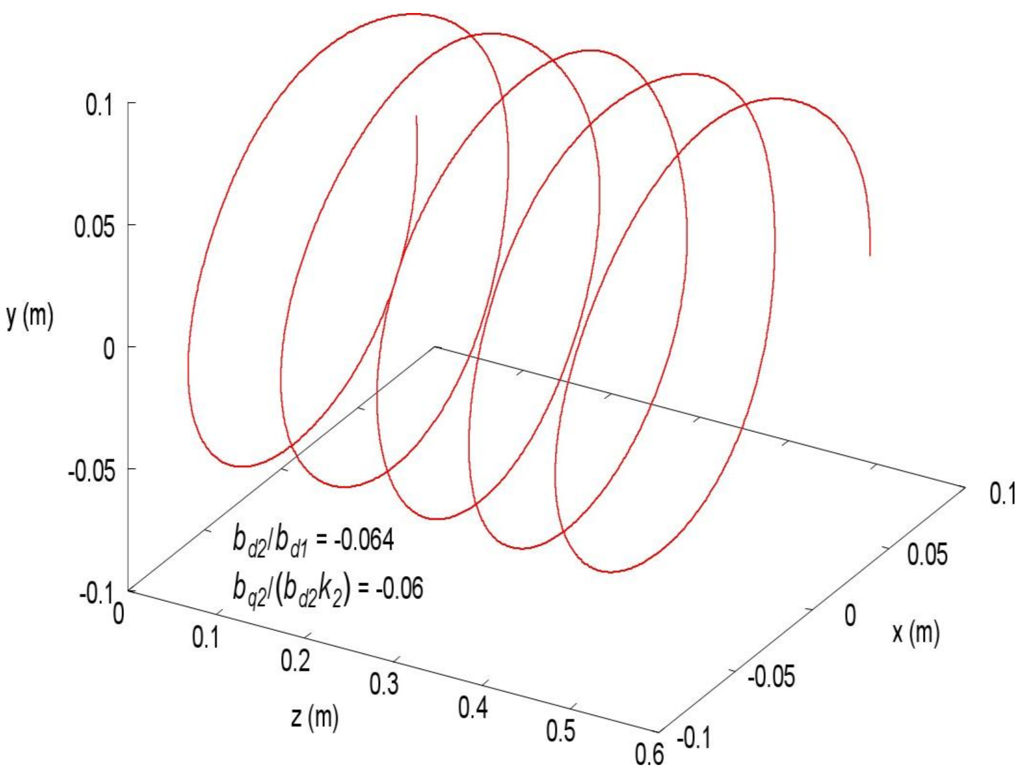


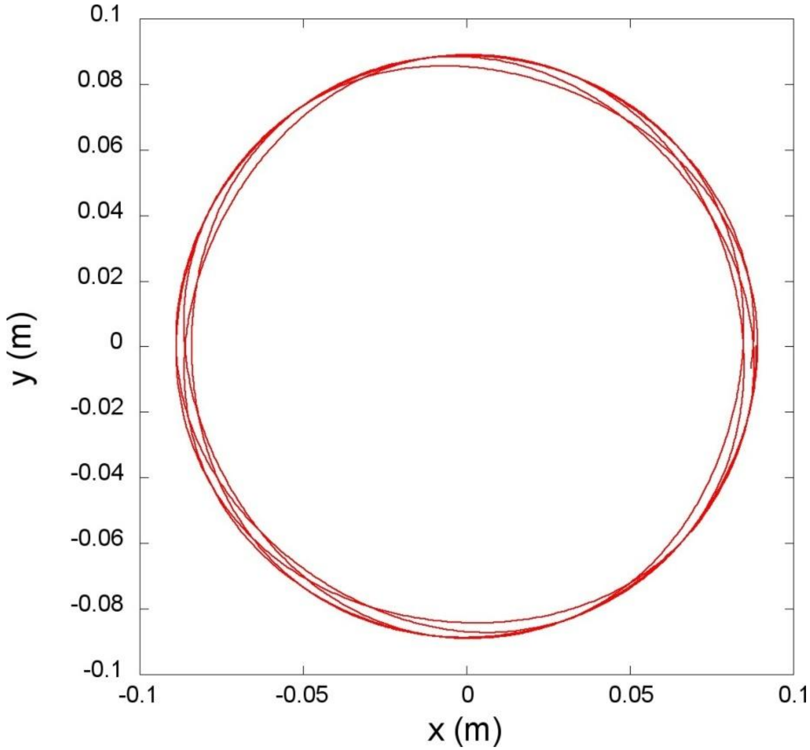
|*D*| oscillates reaching 0



Periodic Orbit in Epicyclic Channel

- **Problem with dynamic aperture**





Twin Helix

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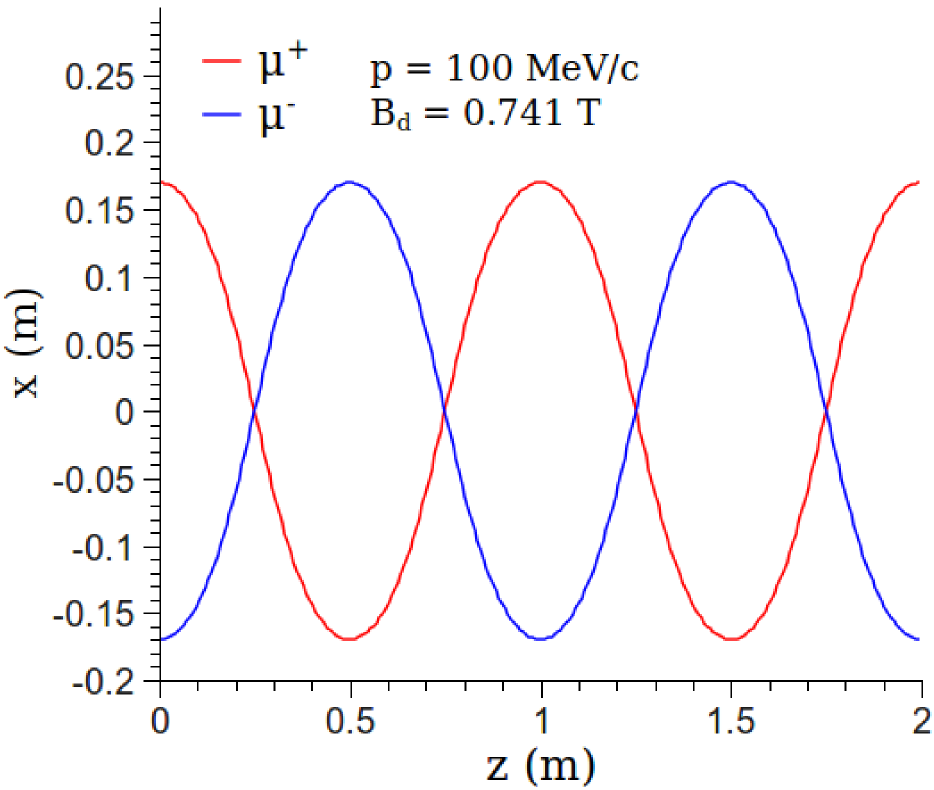
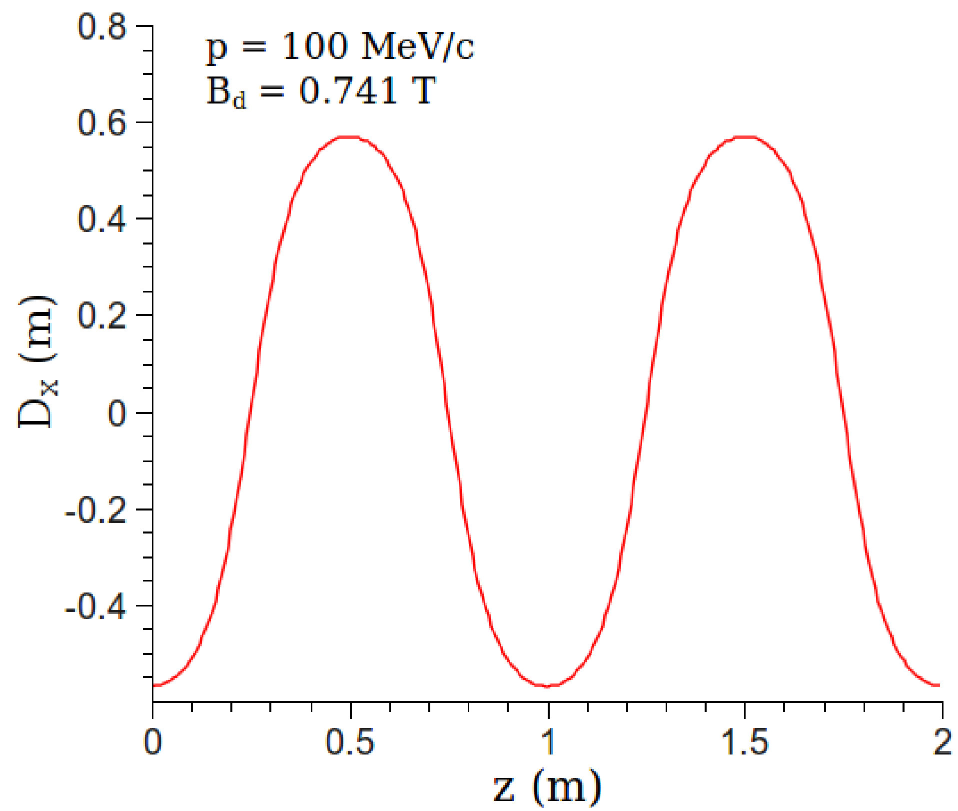
- Consider two dipole ($n = 1$) harmonics with $b_1 = b_2$ and $k_1 = -k_2 = 2\pi/\lambda$
- Vertical field only in horizontal plane $\Rightarrow$ Periodic orbit in horizontal plane
- Horizontal and vertical motion uncoupled
- Region of stable transverse motion in both planes
- $\lambda_D = \lambda \Rightarrow \lambda_x = 2\lambda_y = 4\lambda \Rightarrow \nu_x = 0.25, \nu_y = 0.5$





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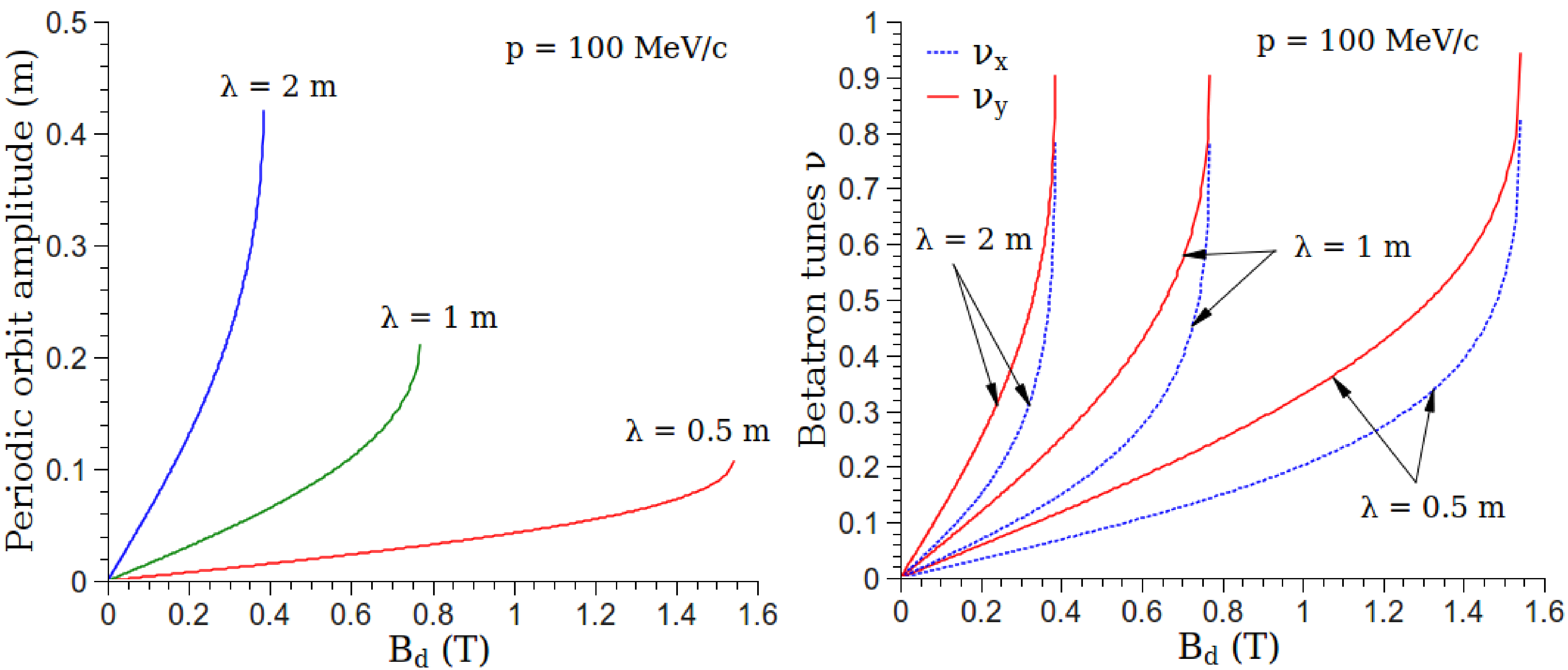
Periodic Orbit and Tunes

- Two dipole helical harmonics only
- Periodic orbit determined by locating fixed point in phase space
- Betatron tunes from linear transfer matrix for canonical coordinates
- $\nu_y = 1 \Rightarrow$ parametric resonance $\Rightarrow$ use $\nu_x = 0.25$, $\nu_y = 0.5$



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Adjusting Correlated Optics

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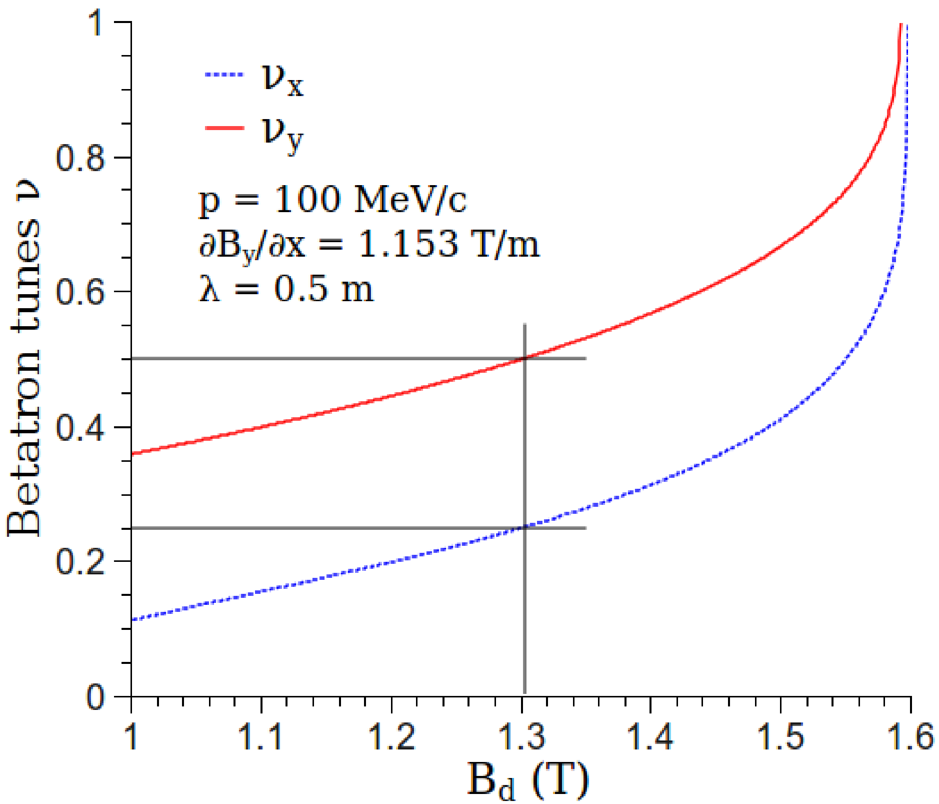
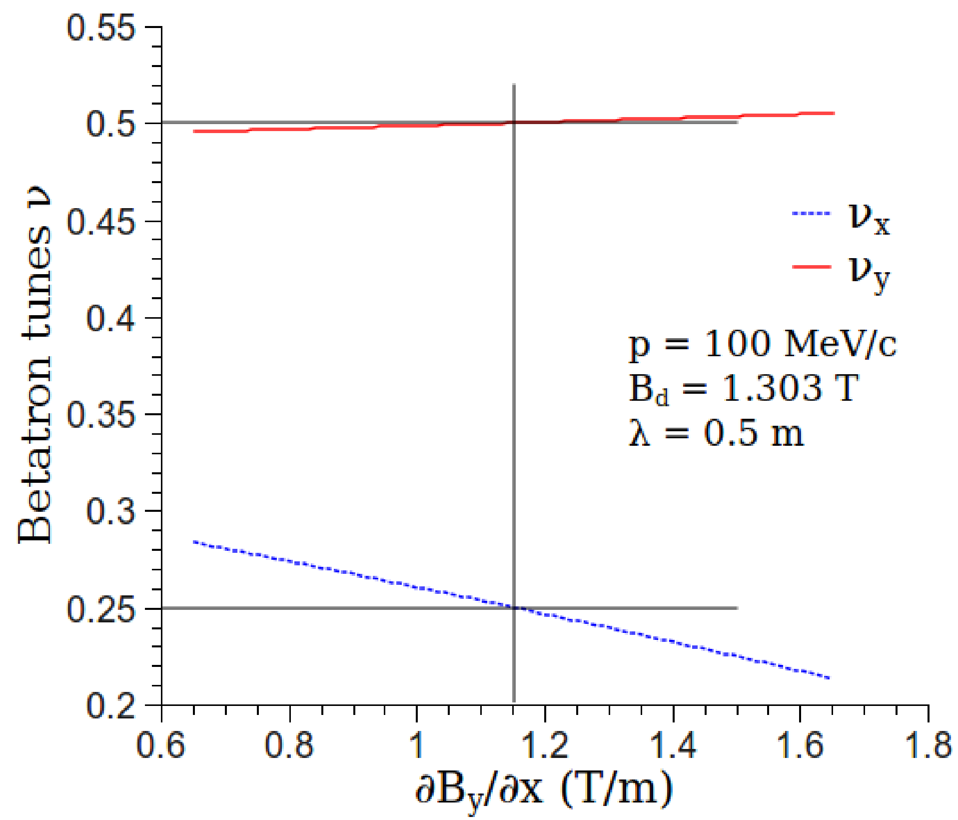
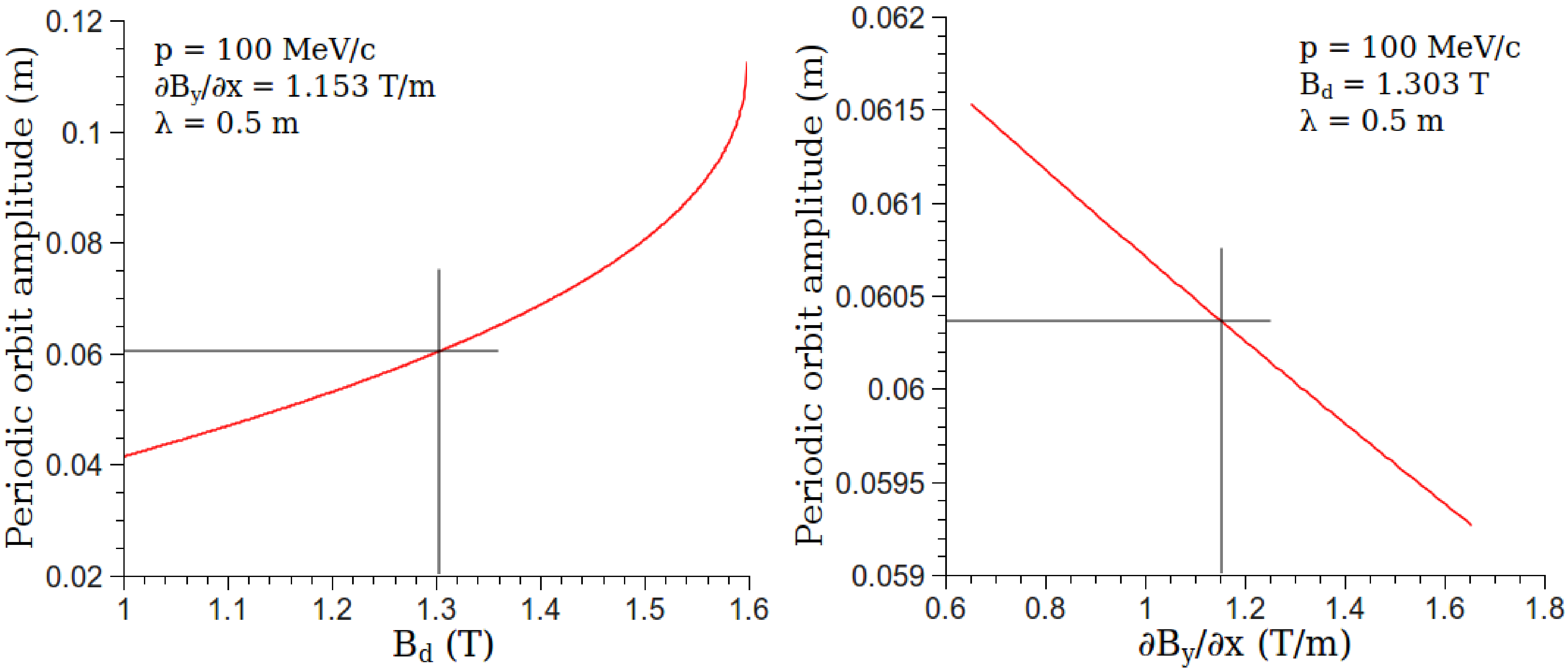
- Introduce straight quad to redistribute horizontal and vertical focusing
- Down side: cannot satisfy correlated optics conditions for both charges
- Iteratively adjust B_d and $\partial B_y/\partial x$ until correlated optics is reached
- No success applying same procedure with double helical quad



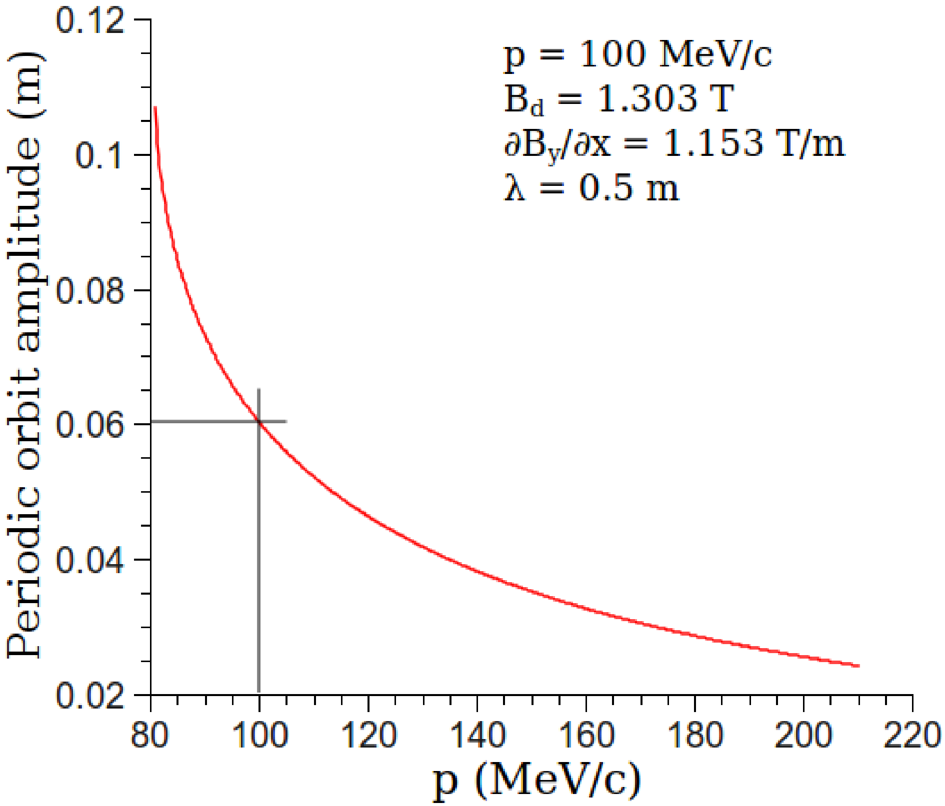


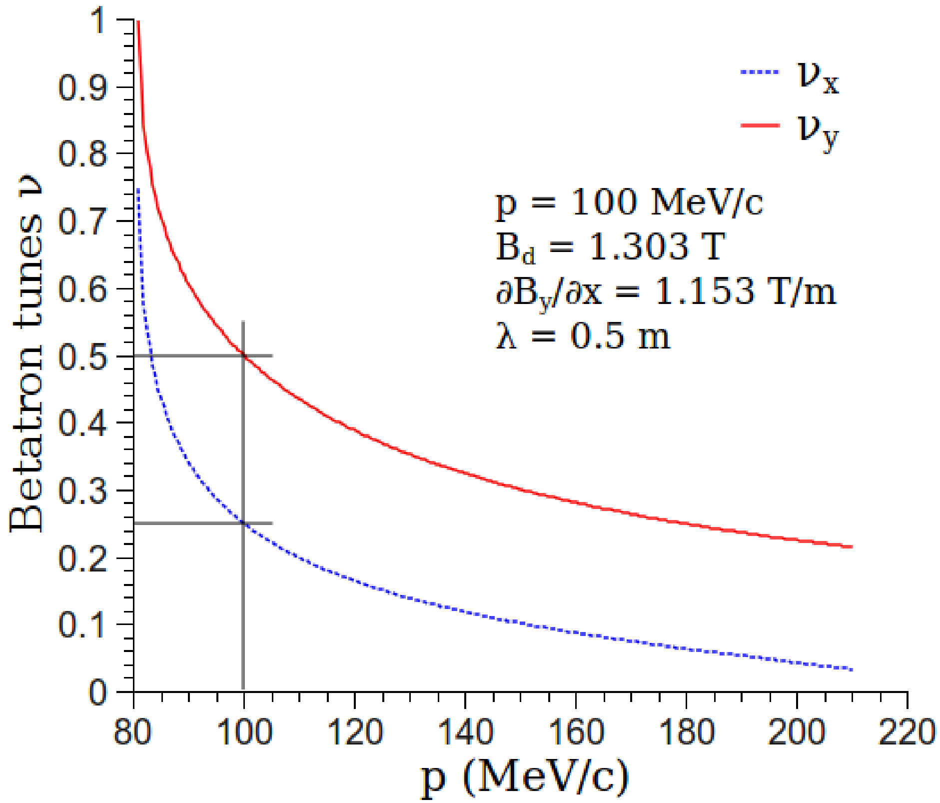
Periodic Orbit



Dispersion and Chromaticity

- **Dispersion:** $D_x^{\max} = p\,\partial x_a/\partial p = 0.098$ m
- **Chromaticity:** $\xi_x = p\,\partial \nu_x/\partial p = -0.646, \quad \xi_y = -0.798$
- **Scaling pattern:** $B_d \propto p/\lambda, \quad \partial B_y/\partial x \propto p/\lambda^2, \quad x_a, D_x \propto \lambda, \quad \xi_x,\ \xi_y \propto \text{const}$





G4BL Implementation

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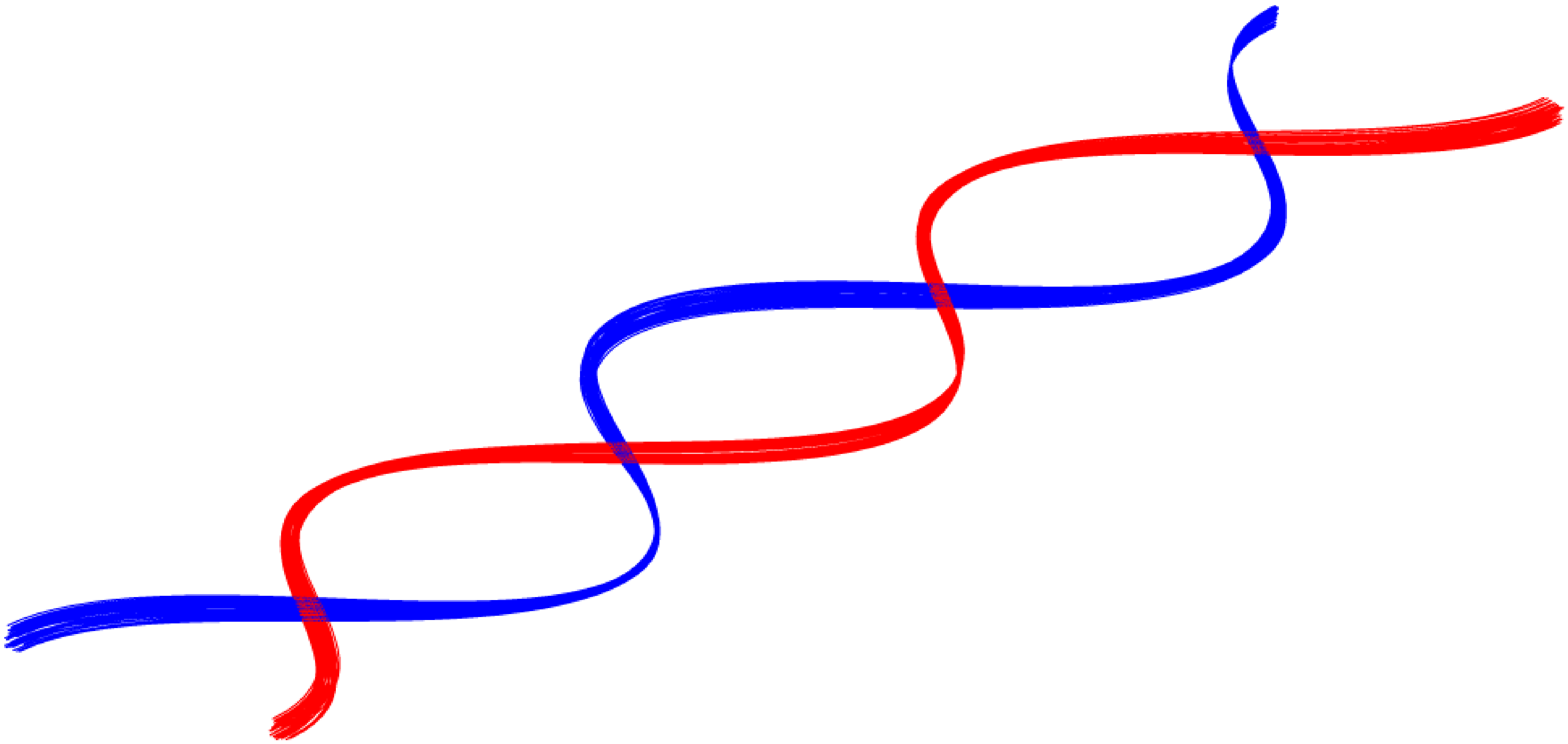
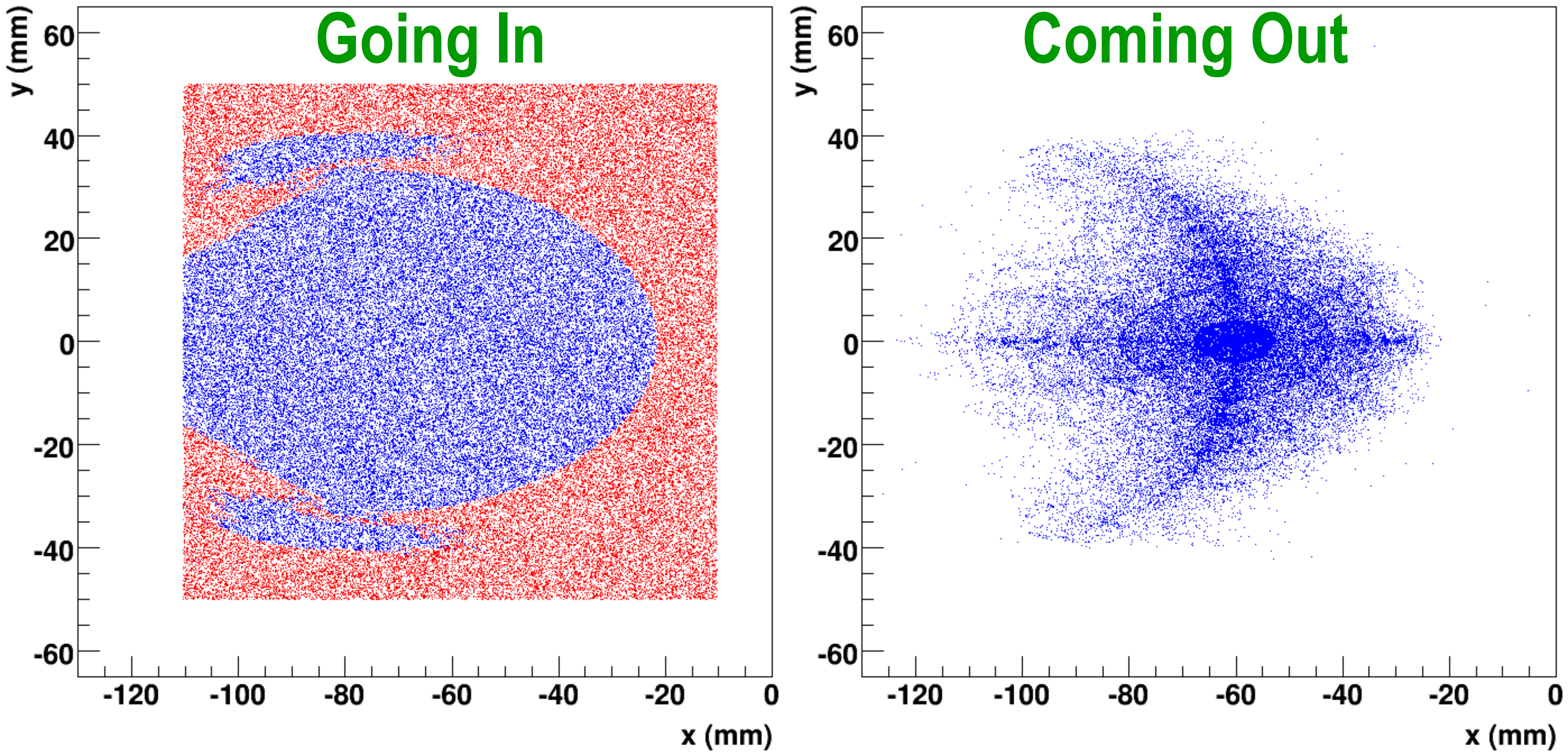
```
File  Edit  View  Terminal  Help
cmd: help helicalharmonic
helicalharmonic construct a helicalharmonic magnet.

             Creates a cylindrical region containing the field of a magnetic
             helical harmonic of given order [n]. The field is defined by the
             value of the (n-1) order derivative [b] of the vertical field
             component (when the initial phase is 0) with respect to the
             horizontal coordinate at the center of the helix:
               b=d^(n-1)B_phi/dr^(n-1) @ [r=0 & phi-k*z+phi0=0],
             where k=2*pi/lambda is the helix's wave number, [lambda] is the
             length of the helix's period, and phi0 is the initial phase. The
             field components in the cylindrical frame are given by:
               B_phi=(2/(n*k))^(n-1)*b*(I[n-1](n*k*r)-I[n+1](n*k*r))*
                     cos(n*(phi-k*z+phi0)),
               B_r  =(2/(n*k))^(n-1)*b*(I[n-1](n*k*r)+I[n+1](n*k*r))*
                     sin(n*(phi-k*z+phi0)),
               B_z  =-2*(2/(n*k))^(n-1)*b*I[n](n*k*r)*cos(n*(phi-k*z+phi0)),
             where I[n](x) is the modified Bessel function of the first kind
             of order [n].

             Note that this Element generates magnetic field only, and only
             within the cylinder defined by length and radius. So it has no
             solid associated with it, and is invisible.

             Named Arguments:
             radius        The radius of the field region (mm)
             length        The length of the field region (mm)
             n             Order of helical harmonic (i.e. n=1 for dipole)
             b             (n-1)-order derivative of the field at the center
                           (T/m^(n-1))
             lambda        Helix period along the Z axis (mm).
             phi0          The phase of the XY field at the entrance (rad).
cmd:
```

G4BL Simulation



G4BL Optics Test

- No absorber and no RF
- 10^5 100 MeV/c μ^- through 100 periods of “twin helix” with correlated optics
- Initially parallel beam uniformly distributed with 10×10 cm square



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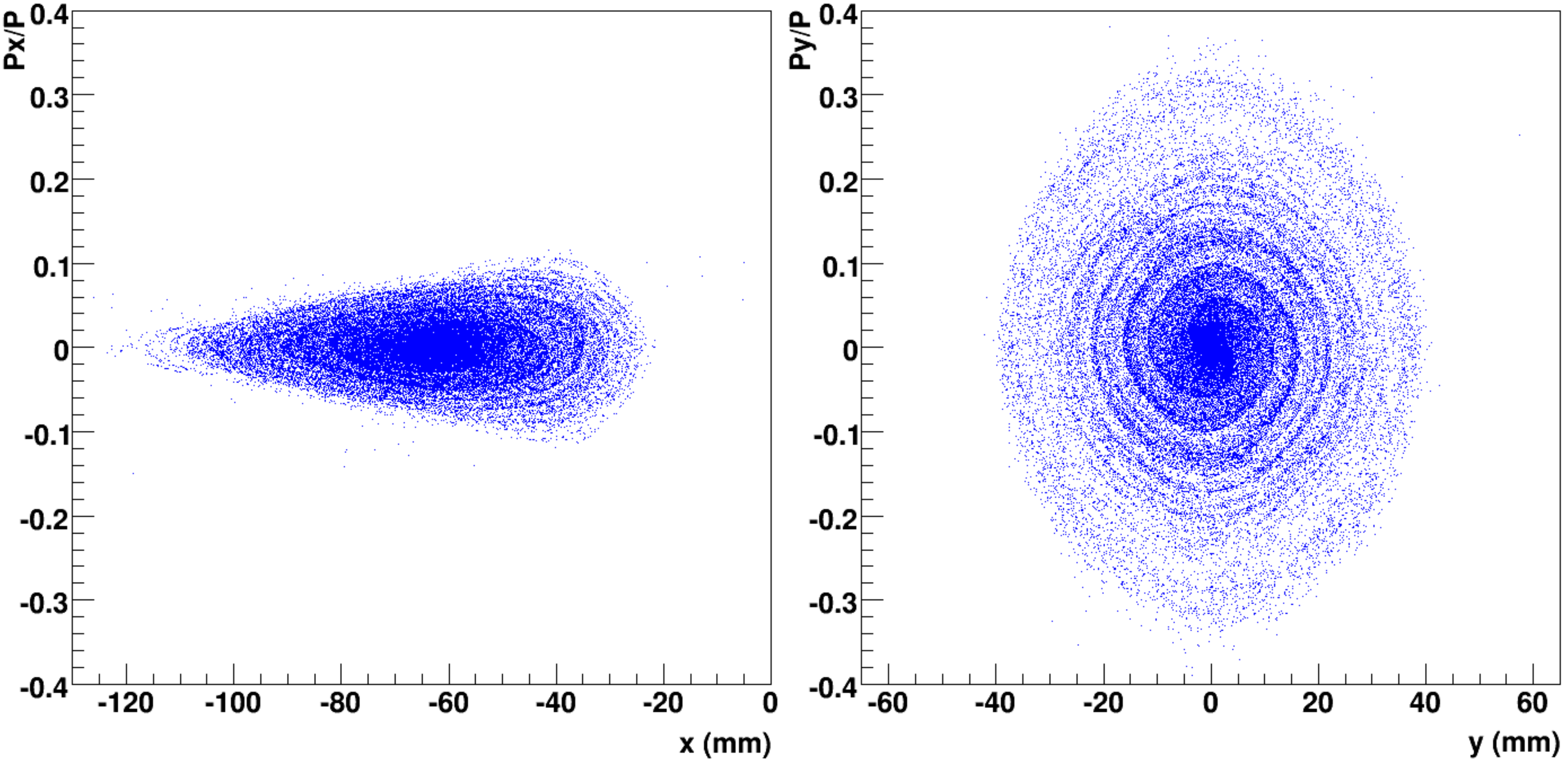
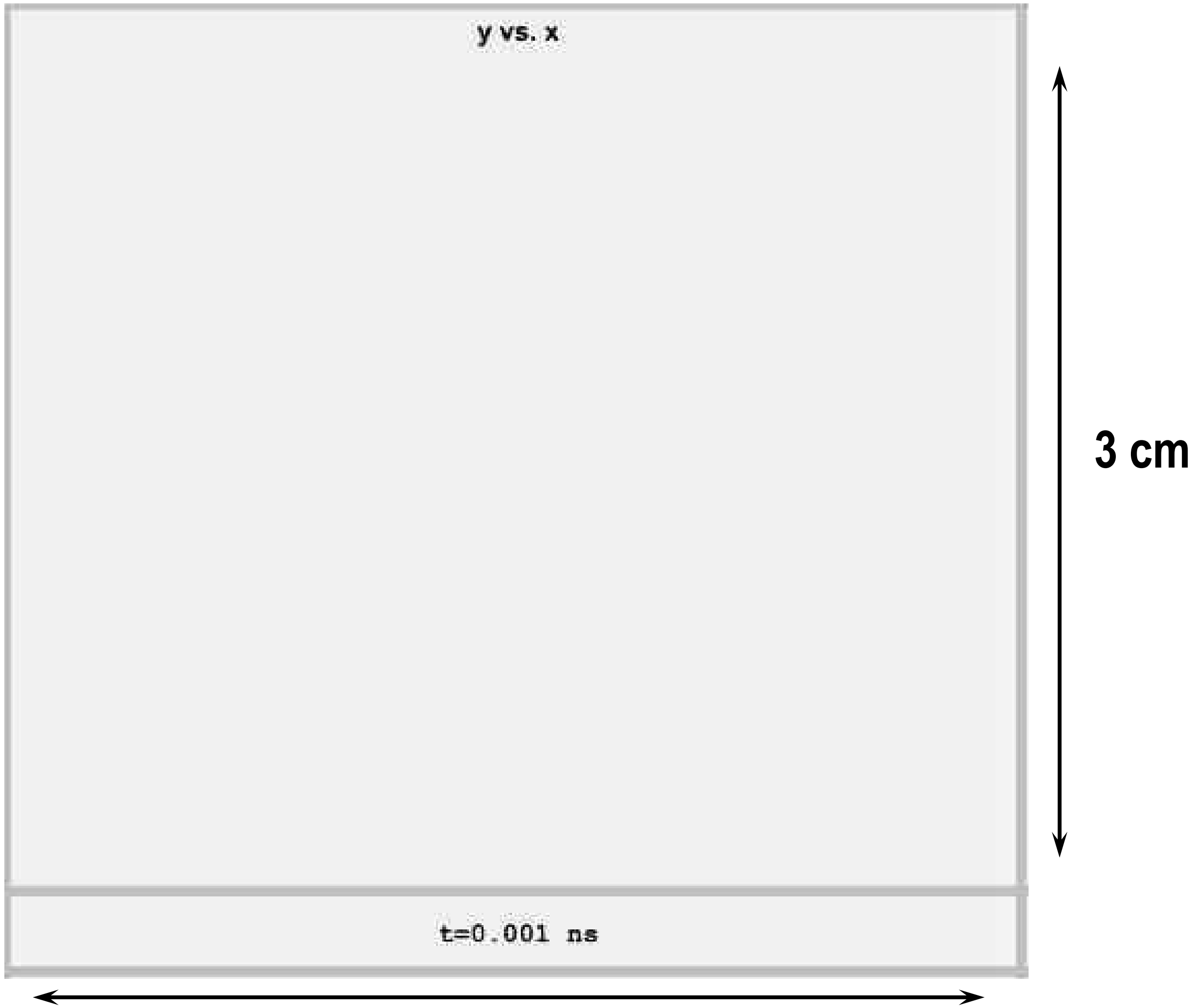
Final Phase Space

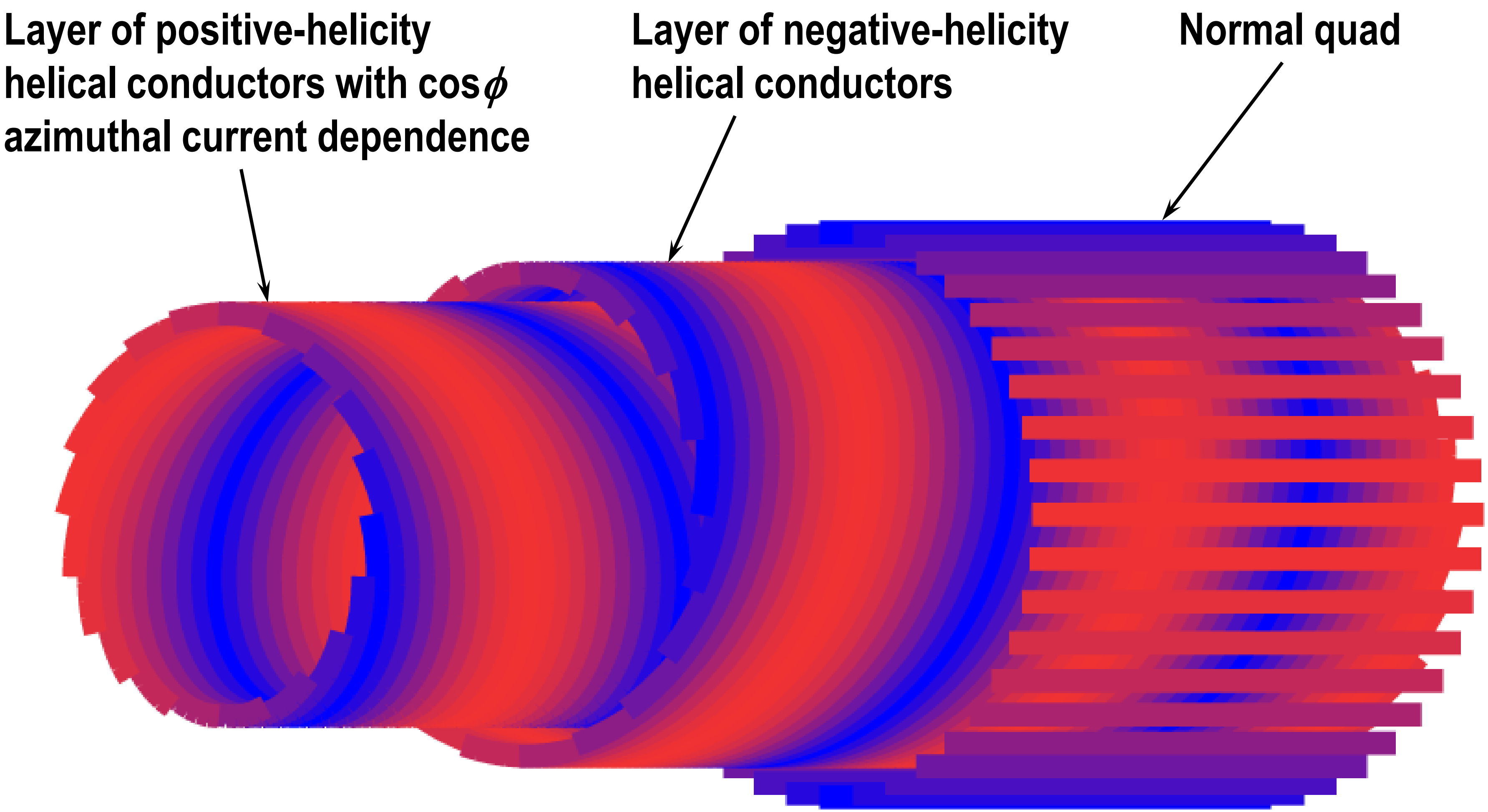


Transverse Motion
(relative to reference particle)

- 100 MeV/c μ^- beam from single point with uniform 0.1×0.1 rad angular spread



Possible Practical Implementation



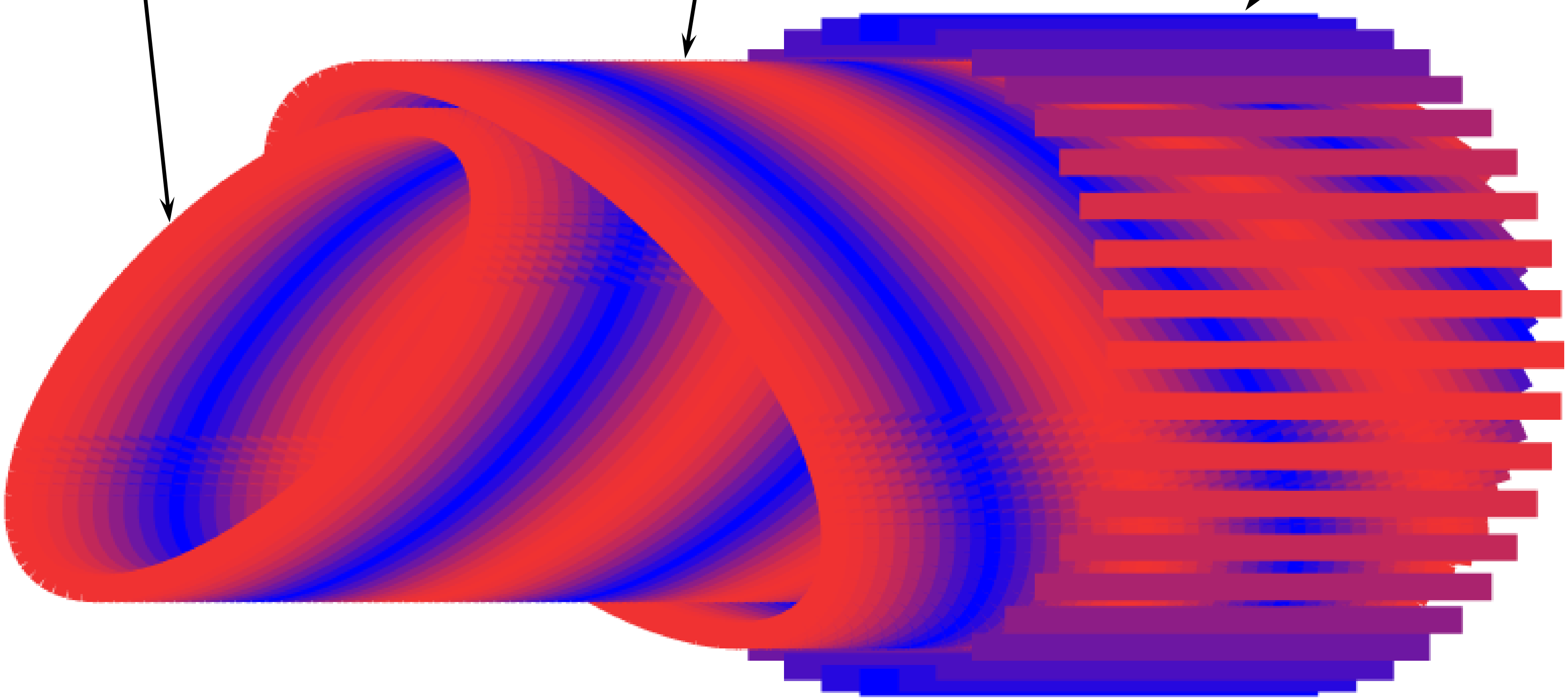
Possible Practical Implementation

- **Adopt existing technology?**

Layer of positive-tilted loops with cos z longitudinal current dependence

Layer of negatively-tilted loops

Normal quad



Reverse EMittance Exchange

- Another potential application of twin helix channel
- Longitudinal emittance after PIC smaller than needed for collider
- Reverse EMittance EXchange (REMEX)
  - Continue resonant regime
  - Reverse wedge gradient to maximize transverse cooling decrements at the cost of antidamping in longitudinal direction
  - Increase dispersion at absorber plates
  - Maintain reasonable relative momentum spread by bunch stretching in the first stage and by beam acceleration in the second stage
- Another order of magnitude reduction in transverse emittance

Aberration Compensation

- **Aberration compensation at beam focal points critical and most challenging for PIC**
- **Take full advantage of system's symmetry**
- **Compensation of 2nd-order terms with two sextupole harmonics**

$$n_s(s) = n_{s1}\sin(ks) + n_{s2}\sin(2ks)$$

- **Compensation of 3rd-order terms with three octupole harmonics**

$$n_o(s) = n_{o1} + n_{o2}\cos(ks) + n_{o3}\cos(2ks)$$

- **Conceptually very similar to problem of aberration compensation at collider interaction point**

Symmetry formulation of Achromatic IP
(Standard Model)

- **"Canonical" conditions (compensation for original chromatic terms)**

$$2\int Dn_s y_0^2 ds = \int_0^{*} n y_0^2 ds \equiv \int_0^{*} y_0'^2 ds \;; \qquad 2\int Dn_s x_0^2 ds = \int_0^{*} n x_0^2 ds \equiv -\int_0^{*} x_0'^2 ds$$

- **Conditions connected to the betatron and 2nd order dispersion beam sizes:**

$$\int n_s x_0 y_0^2 ds = 0\,; \qquad \int n_s x_0^3 ds = 0\,; \qquad \int (n_s D - n) D x_0 ds = 0$$

These 3 conditions on sextupoles can be satisfied "automatically", if to implement symmetry to the compensating block: symmetric x_0^2 and y_0^2 , while symmetry of D and n_s is opposite to symmetry of x_0.

<u>What is achieved with this compensation</u>:

Suppression of tune chromatic spread (usual)

Suppression of intrinsic chromatic and sextupole 3d smear of beam core at star point (new)

<u>What may not have been achieved</u>: maintaining the dynamical aperture

IR Design

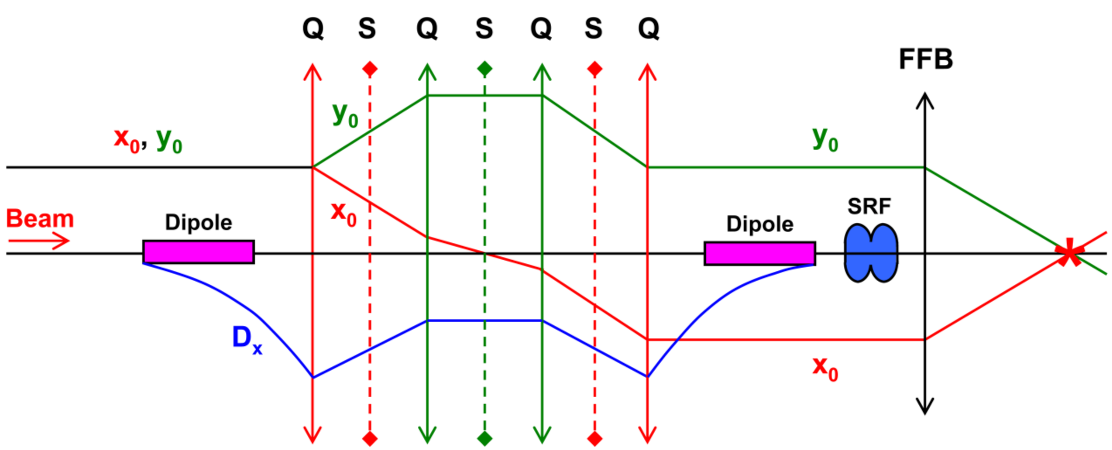
- **Modular approach**
- **Utilize COSY Infinity**
  - **calculates coefficients** $M(x|\alpha\beta\gamma\delta\lambda\mu)$ **of expansion of type**

$$x = \sum_{\substack{\alpha,\beta,\gamma,\\ \delta,\lambda,\mu}} M(x|\alpha\beta\gamma\delta\lambda\mu)\, x^{\alpha} x'^{\beta} y^{\gamma} y'^{\delta} t^{\lambda} q^{\mu}$$

to arbitrary order $(\alpha+\beta+\gamma+\delta+\lambda+\mu)$ **for each of coordinate components**

- **Design system such that**

$$x(s) = -x(-s)$$
$$y^2(s) = y^2(-s)$$
$$D(s) = D(-s)$$
$$n(s) = n(-s)$$
$$n_s(s) = n_s(-s)$$
$$n_o(s) = n_o(-s)$$



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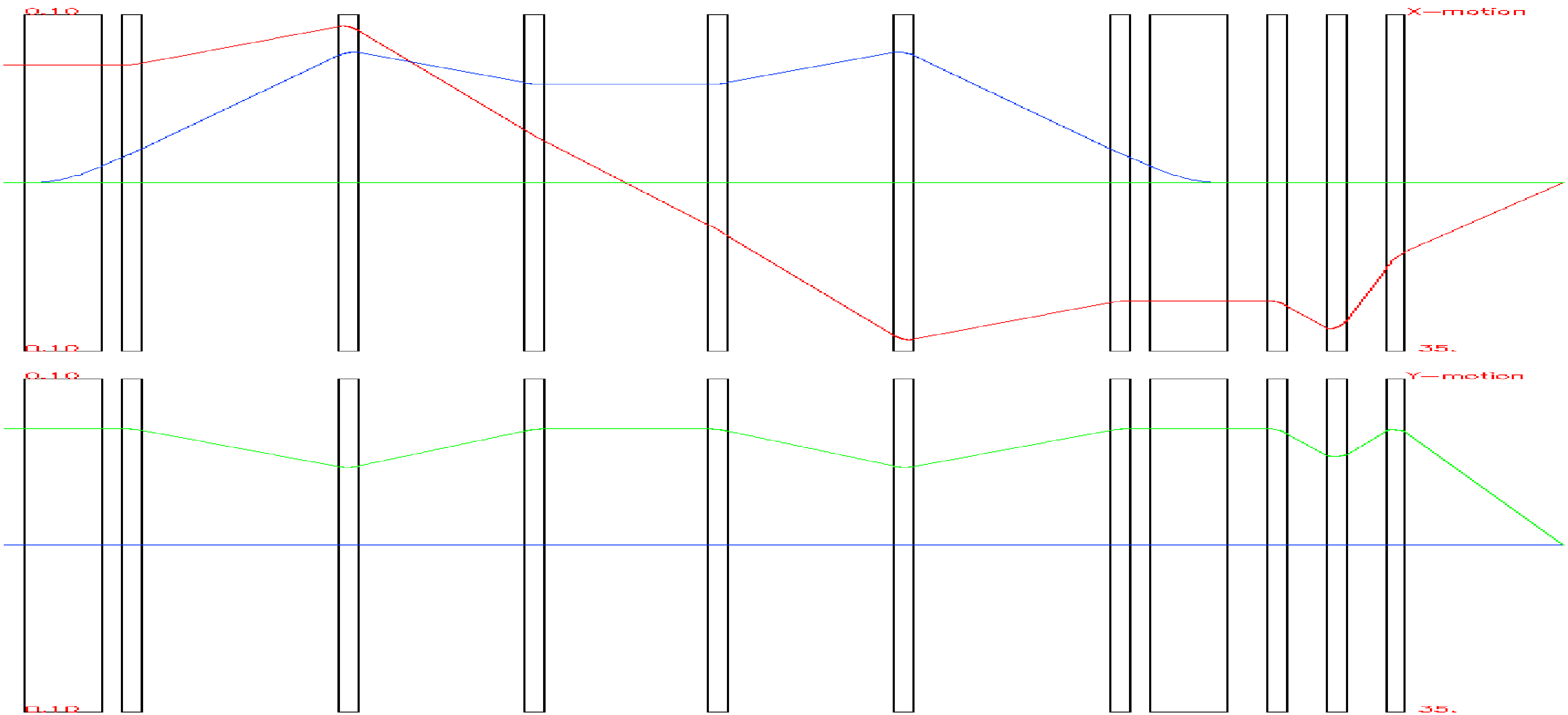
Interaction Region

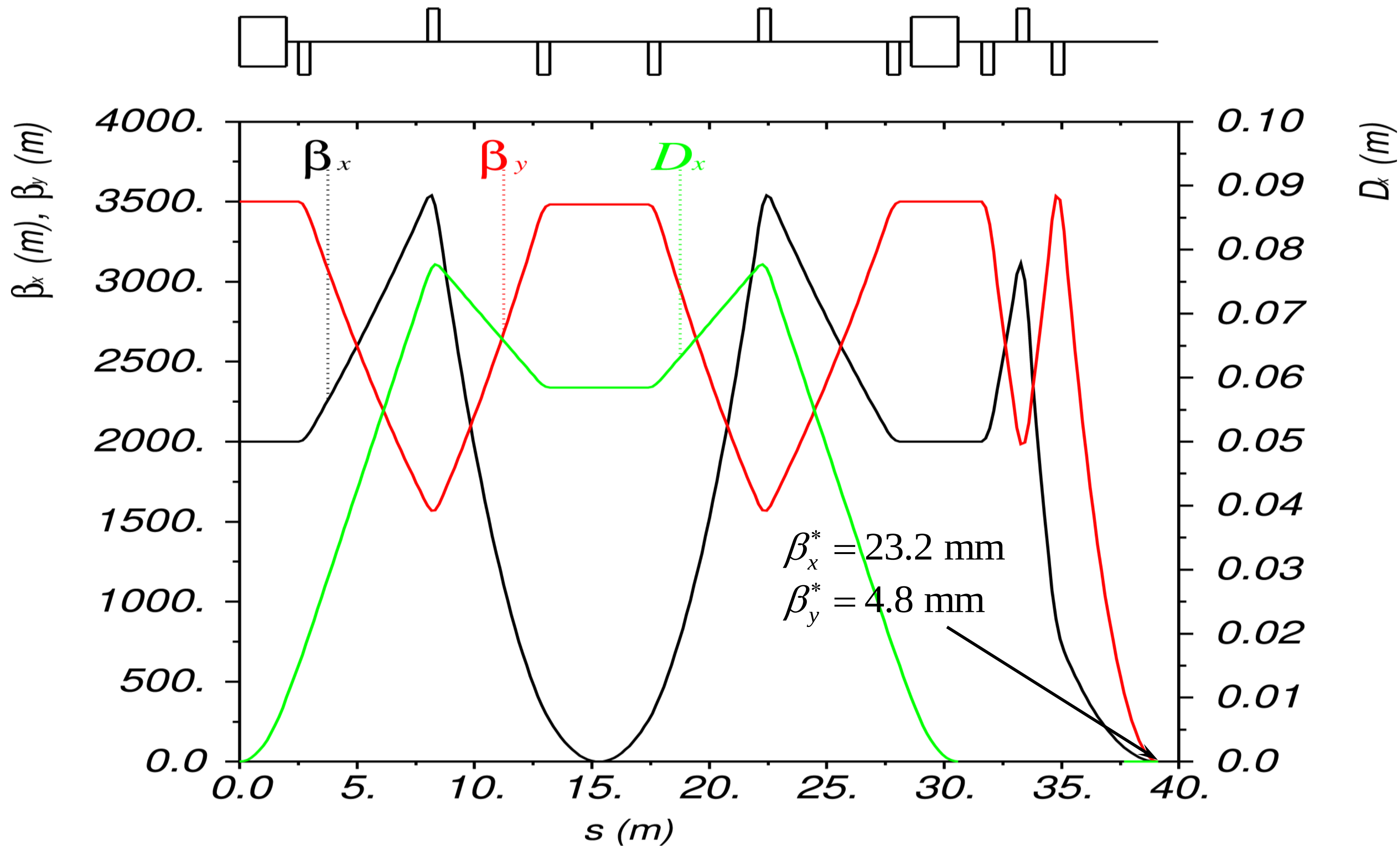
Old Dominion University

1st-order matrix:

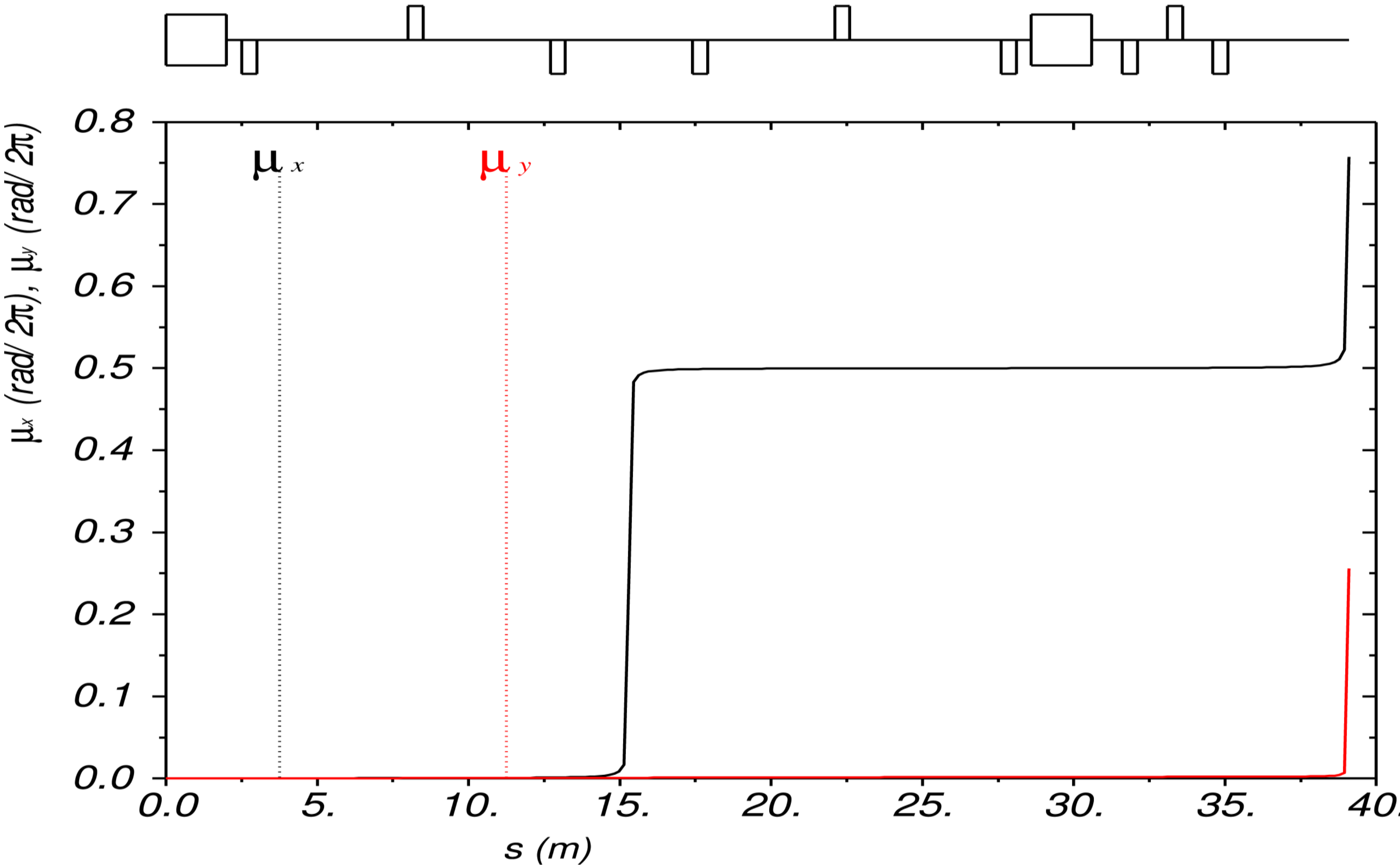
x	x'	y	y'	t		
-0.5789511E-09	0.1470106	0.000000	0.000000	0.000000	100000	$M(x\|x)$
-6.802231	-0.6030576	0.000000	0.000000	-0.1416656E-08	010000	
0.000000	0.000000	0.4386176E-08	0.2432977	0.000000	001000	$M(y\|y)$
0.000000	0.000000	4.110191	-10.45406	0.000000	000100	
0.000000	0.000000	0.000000	0.000000	1.000000	000010	
0.000000	-0.2082635E-09	0.000000	0.000000	-0.6655614E-04	000001	$M(x\|q)$



Interaction Region



IR Phase Advance



1st and 2nd-Order Aberrations

Assume: $10\ \mathrm{GeV/c}\ e^-,\quad \varepsilon_x^N = 85\ \mu m,\quad \varepsilon_y^N = 17\ \mu m,\quad \beta_x^i = 2\ \mathrm{km},$

$\beta_y^i = 3.5\ \mathrm{km},\quad \Delta E / E = 3\times10^{-4}$

1st-order aberrations:

Geometric beam size at IP due to emittance

-0.1706380E-11	0.4332949E-03	0.000000	0.000000	0.000000	100000	
-0.1002435E-04	0.8887174E-06	0.000000	0.000000	-0.2087706E-14	010000	$M(x\|x')$
0.000000	0.000000	0.7648125E-11	-0.4242355E-03	0.000000	001000	
0.000000	0.000000	0.2047684E-05	0.5208181E-05	0.000000	000100	$M(y\|y')$
0.000000	-0.6247905E-13	0.000000	0.000000	-0.1996684E-07	000001	

2nd-order aberrations:

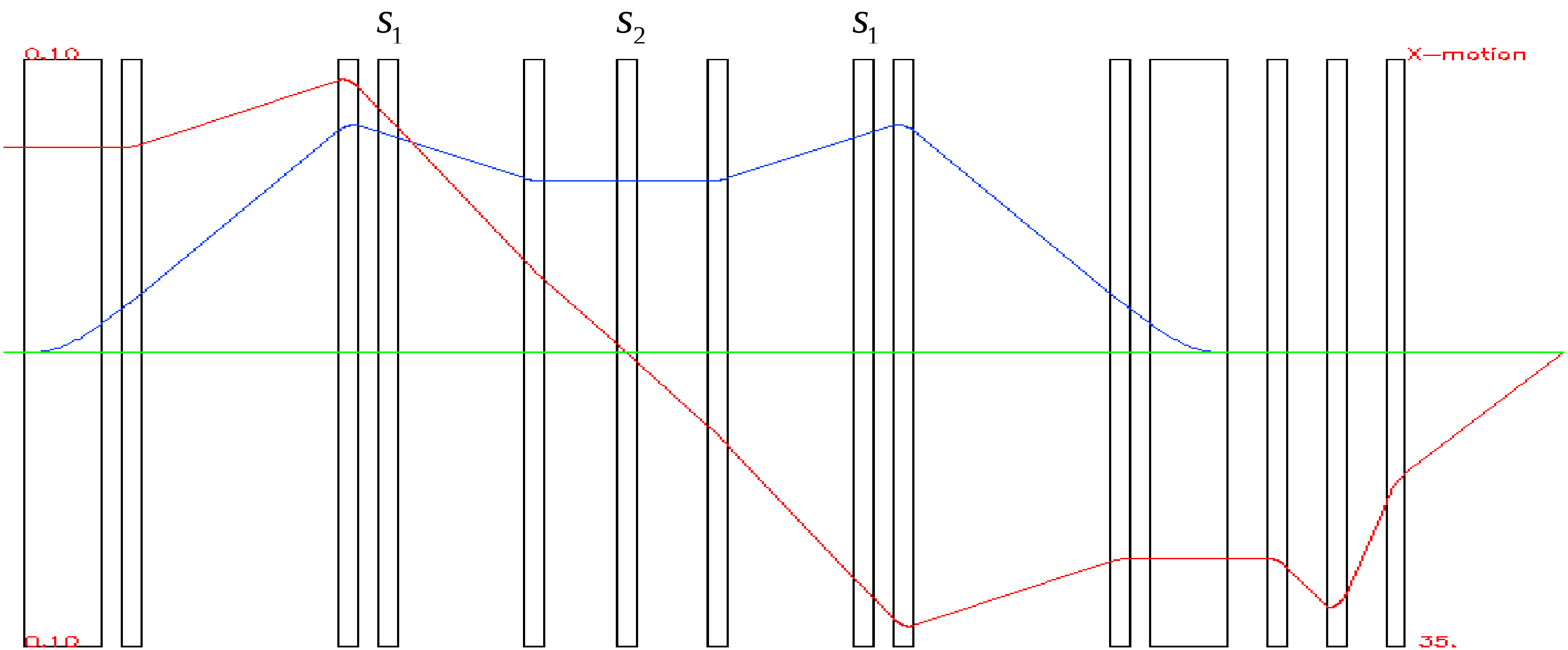
0.000000	0.000000	0.000000	0.000000	-0.4054288E-05	200000	
0.000000	0.000000	0.000000	0.000000	-0.6955882E-08	110000	
0.000000	0.000000	0.000000	0.000000	-0.3349380E-10	020000	
0.000000	0.000000	0.000000	0.6255384E-12	0.000000	101000	
0.000000	0.000000	0.000000	0.4691486E-15	0.000000	011000	
0.000000	0.000000	0.000000	0.000000	-0.6356532E-06	002000	
0.000000	0.000000	0.3019325E-14	-0.1480486E-15	0.000000	100100	
-0.2956197E-14	-0.3579176E-15	0.000000	0.000000	-0.1412937E-07	001100	
-0.5614128E-05	-0.7058949E-06	0.000000	0.000000	0.4812160E-15	100001	$M(x\|xq)$
-0.4816038E-08	-0.2431716E-08	0.000000	0.000000	-0.1618392E-09	010001	
0.000000	0.000000	0.8990100E-06	0.2165344E-06	0.000000	001001	$M(y\|yq)$
0.000000	0.000000	0.000000	0.000000	-0.9176189E-10	000200	
0.000000	0.000000	0.9991650E-08	0.1474259E-08	0.000000	000101	
0.5808878E-15	-0.2421690E-08	0.000000	0.000000	-0.2380987E-09	000002	

3rd-order aberrations: small

Sextupole Compensation

Make $M(x|xq) = 0$ **and** $M(y|yq) = 0$ **by adjusting**

$s_1 \rightarrow 0.40$ T @ 5 cm and $s_2 \rightarrow -1.15$ T @ 5 cm



1st and 2nd-Order Aberrations after Sextupole Compensation

Assume: $10\ \mathrm{GeV/c}\ \mathrm{e}^-,\ \ \varepsilon_x^N = 85\ \mu m,\ \ \varepsilon_y^N = 17\ \mu m,\ \ \beta_x^i = 2\ \mathrm{km},$
$\beta_y^i = 3.5\ \mathrm{km},\ \ \Delta E / E = 3\times 10^{-4}$

1st-order aberrations:

Geometric beam size at IP due to emittance

-0.1706382E-11	0.4332949E-03	0.000000	0.000000	0.000000	100000	
-0.1002435E-04	0.9970412E-06	0.000000	0.000000	-0.2087706E-14	010000	$M(x\|x')$
0.000000	0.000000	0.7648125E-11	-0.4242355E-03	0.000000	001000	
0.000000	0.000000	0.2047684E-05	0.5147575E-05	0.000000	000100	$M(y\|y')$
0.000000	-0.6247905E-13	0.000000	0.000000	-0.1996723E-07	000001	

2nd-order aberrations:

-0.8187338E-13	0.5960489E-04	0.000000	0.000000	0.000000	200000	
0.2757938E-05	0.3935176E-06	0.000000	0.000000	-0.8745170E-09	110000	$M(x\|xx')$
0.1378946E-08	0.3713441E-09	0.000000	0.000000	-0.1756283E-10	020000	
0.000000	0.000000	-0.2665783E-12	-0.5470744E-04	0.000000	101000	
0.000000	0.000000	-0.4416383E-06	0.5451167E-06	0.000000	011000	$M(y\|x'y)$
0.5765224E-12	0.9345175E-05	0.000000	0.000000	-0.1200334E-13	002000	
0.000000	0.000000	-0.2640598E-06	0.2076317E-07	0.000000	100100	$M(y\|xy')$
0.000000	0.000000	-0.2727589E-08	0.3752719E-08	0.000000	010100	
0.2585388E-06	0.1411477E-06	0.000000	0.000000	-0.6476140E-08	001100	
0.000000	-0.2617180E-07	0.000000	0.000000	0.4831748E-15	100001	$M(x\|xq)$
-0.6054890E-09	-0.1111433E-08	0.000000	0.000000	-0.1093650E-09	010001	
0.000000	0.000000	-0.1285563E-15	0.9487998E-06	0.000000	001001	$M(y\|yq)$
0.1519463E-08	0.4681611E-09	0.000000	0.000000	-0.7036362E-10	000200	
0.000000	0.000000	0.4579631E-08	0.9105008E-08	0.000000	000101	
0.1737153E-15	-0.1636490E-08	0.000000	0.000000	-0.1846897E-09	000002	

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3rd-Order Aberrations after Sextupole Compensation

Assume: $10\ \mathrm{GeV/c}\ e^-,\quad \varepsilon_x^N = 85\ \mu m,\quad \varepsilon_y^N = 17\ \mu m,\quad \beta_x^i = 2\ \mathrm{km},$

$\beta_y^i = 3.5\ \mathrm{km},\quad \Delta E / E = 3\times 10^{-4}$

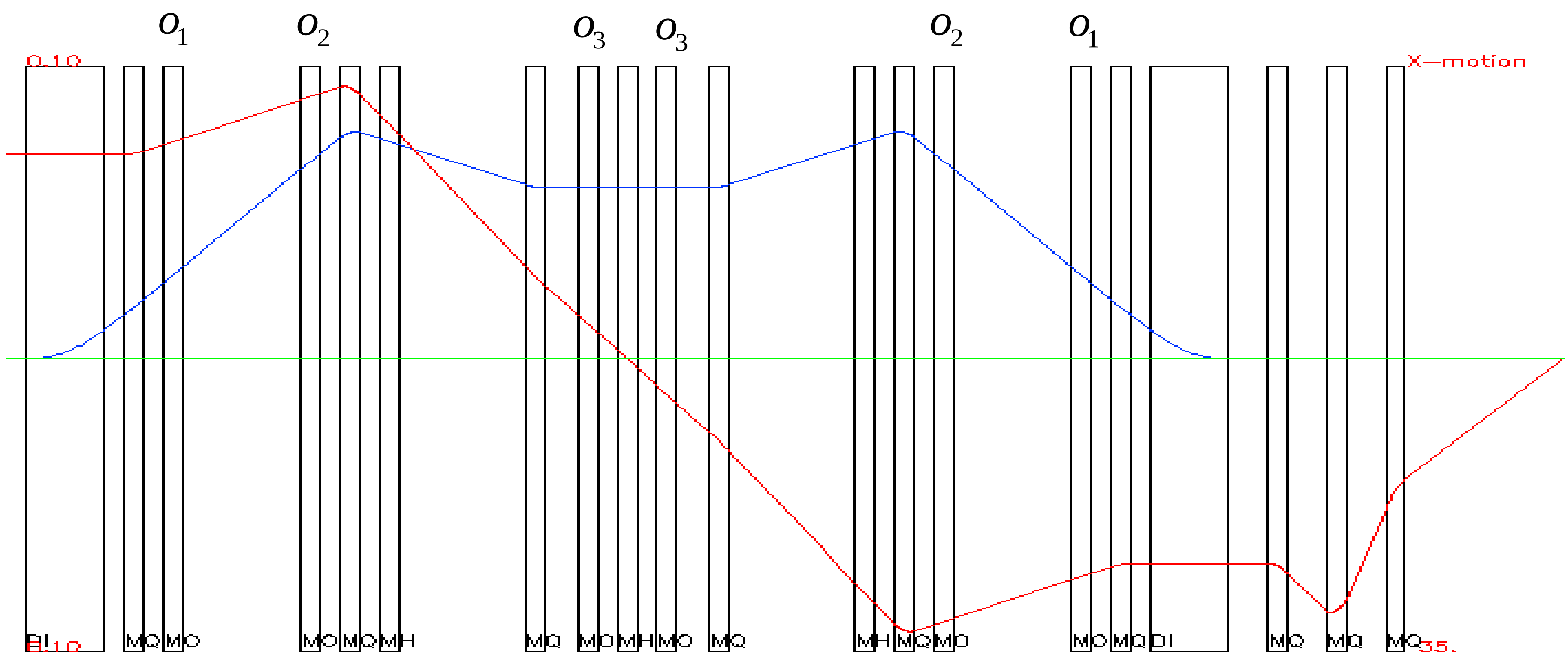
3rd-order aberrations:

-0.5899026E-04	0.1057326E-05	0.000000	0.000000	-0.2323975E-06	300000	$M(x\|x^3)$
-0.2781687E-06	0.2308882E-07	0.000000	0.000000	0.9147334E-09	210000	
0.7948680E-09	0.1207891E-09	0.000000	0.000000	0.7253813E-11	120000	
0.000000	0.000000	0.1435522E-05	-0.9151791E-05	0.000000	201000	$M(x\|x^2y)$
0.000000	0.000000	0.6218826E-07	-0.8854051E-07	0.000000	111000	
0.000000	0.000000	-0.7242334E-10	0.3902303E-10	0.000000	021000	
-0.1405508E-05	0.2320478E-05	0.000000	0.000000	-0.1015788E-06	102000	
0.1157951E-08	0.5751201E-08	0.000000	0.000000	0.1727379E-08	012000	
0.000000	0.000000	-0.3732884E-05	0.4248511E-05	0.000000	003000	$M(x\|y^3)$
0.000000	0.000000	0.7296707E-08	-0.5274028E-07	0.000000	200100	
0.000000	0.000000	0.3612654E-09	-0.5673026E-09	0.000000	110100	
-0.5207852E-07	0.2310078E-07	0.000000	0.000000	-0.1427363E-08	101100	
-0.1149810E-11	0.6099301E-10	0.000000	0.000000	0.3291855E-10	011100	
0.000000	0.000000	-0.6297188E-07	0.6769972E-07	0.000000	002100	
-0.4827146E-06	-0.2783686E-07	0.000000	0.000000	0.3514798E-08	200001	
0.1266666E-08	0.2709437E-09	0.000000	0.000000	0.2790265E-10	110001	
-0.1502093E-11	0.5038316E-12	0.000000	0.000000	0.7053000E-13	020001	
0.000000	0.000000	0.1436638E-06	-0.2637365E-07	0.000000	101001	
0.000000	0.000000	-0.1482003E-08	0.2651660E-09	0.000000	011001	
-0.7033002E-07	0.2899497E-08	0.000000	0.000000	0.3607059E-08	002001	
-0.2554838E-09	0.5553059E-10	0.000000	0.000000	0.2226419E-11	100200	
0.000000	0.000000	-0.3703736E-09	0.3750805E-09	0.000000	001200	
0.000000	0.000000	0.4347480E-09	-0.4858471E-09	0.000000	100101	
0.000000	0.000000	-0.7439565E-11	-0.2020415E-11	0.000000	010101	
-0.9882623E-09	-0.2147385E-10	0.000000	0.000000	0.7084583E-10	001101	
0.2433538E-08	0.2101202E-09	0.000000	0.000000	0.1914187E-10	100002	
-0.7751424E-12	0.1583433E-11	0.000000	0.000000	0.1503900E-12	010002	
0.000000	0.000000	-0.2550748E-08	0.3681286E-09	0.000000	001002	
0.000000	0.000000	-0.1893100E-10	-0.4231273E-11	0.000000	000102	
0.4417746E-11	0.2005542E-11	0.000000	0.000000	0.1217518E-12	000003	

Octupole Compensation

Make $M(x|x^3) = 0$, $M(y|x^2y) = 0$ **and** $M(y|y^3) = 0$ **by introducing 3 pairs of octupoles with**

$o_1 \rightarrow$ -0.80 T @ 5 cm, $o_2 \rightarrow$ 0.90 T @ 5 cm, and $o_3 \rightarrow$ -0.34 T @ 5 cm



Muons, Inc.

1st and 2nd-Order Aberrations after Octupole Compensation

Assume: $10\ \mathrm{GeV/c}\ e^-,\quad \varepsilon_x^N = 85\ \mu m,\quad \varepsilon_y^N = 17\ \mu m,\quad \beta_x^i = 2\ \mathrm{km},$
$\beta_y^i = 3.5\ \mathrm{km},\quad \Delta E / E = 3\times 10^{-4}$

1st-order aberrations:

Geometric beam size at IP due to emittance

-0.1706382E-11	0.4332949E-03	0.000000	0.000000	0.000000	100000	
-0.1002435E-04	-0.9970412E-06	0.000000	0.000000	-0.2087706E-14	010000	$M(x\|x')$
0.000000	0.000000	0.7648125E-11	-0.4242355E-03	0.000000	001000	
0.000000	0.000000	0.2047684E-05	-0.5147575E-05	0.000000	000100	$M(y\|y')$
0.000000	-0.6247759E-13	0.000000	0.000000	-0.1996723E-07	000001	

2nd-order aberrations:

-0.8187301E-13	0.5960489E-04	0.000000	0.000000	0.000000	200000	
0.2757938E-05	0.3935176E-06	0.000000	0.000000	-0.8745170E-09	110000	$M(x\|xx')$
0.1378946E-08	0.3713441E-09	0.000000	0.000000	-0.1756283E-10	020000	
0.000000	0.000000	-0.2665783E-12	-0.5470744E-04	0.000000	101000	
0.000000	0.000000	-0.4416383E-06	0.5451167E-06	0.000000	011000	$M(y\|x'y)$
0.5765224E-12	0.9345175E-05	0.000000	0.000000	-0.1200334E-13	002000	
0.000000	0.000000	-0.2640598E-06	0.2076317E-07	0.000000	100100	$M(y\|xy')$
0.000000	0.000000	-0.2727589E-08	0.3752719E-08	0.000000	010100	
0.2585388E-06	0.1411477E-06	0.000000	0.000000	-0.6476140E-08	001100	
0.000000	-0.2617180E-07	0.000000	0.000000	0.4831662E-15	100001	$M(x\|xq)$
-0.6054890E-09	-0.1111433E-08	0.000000	0.000000	-0.1093650E-09	010001	
0.000000	0.000000	-0.1285553E-15	0.9487998E-06	0.000000	001001	$M(y\|yq)$
0.1519463E-08	0.4681611E-09	0.000000	0.000000	-0.7036362E-10	000200	
0.000000	0.000000	0.4579631E-08	0.9105008E-08	0.000000	000101	
0.1737074E-15	-0.1636490E-08	0.000000	0.000000	-0.1846897E-09	000002	

Muons, Inc.

3^{rd}-Order Aberrations after Octupole Compensation

OLD DOMINION UNIVERSITY

Assume: $10\ \mathrm{GeV/c}\ e^-,\quad \varepsilon_x^N = 85\ \mu m,\quad \varepsilon_y^N = 17\ \mu m,\quad \beta_x^i = 2\ \mathrm{km},$

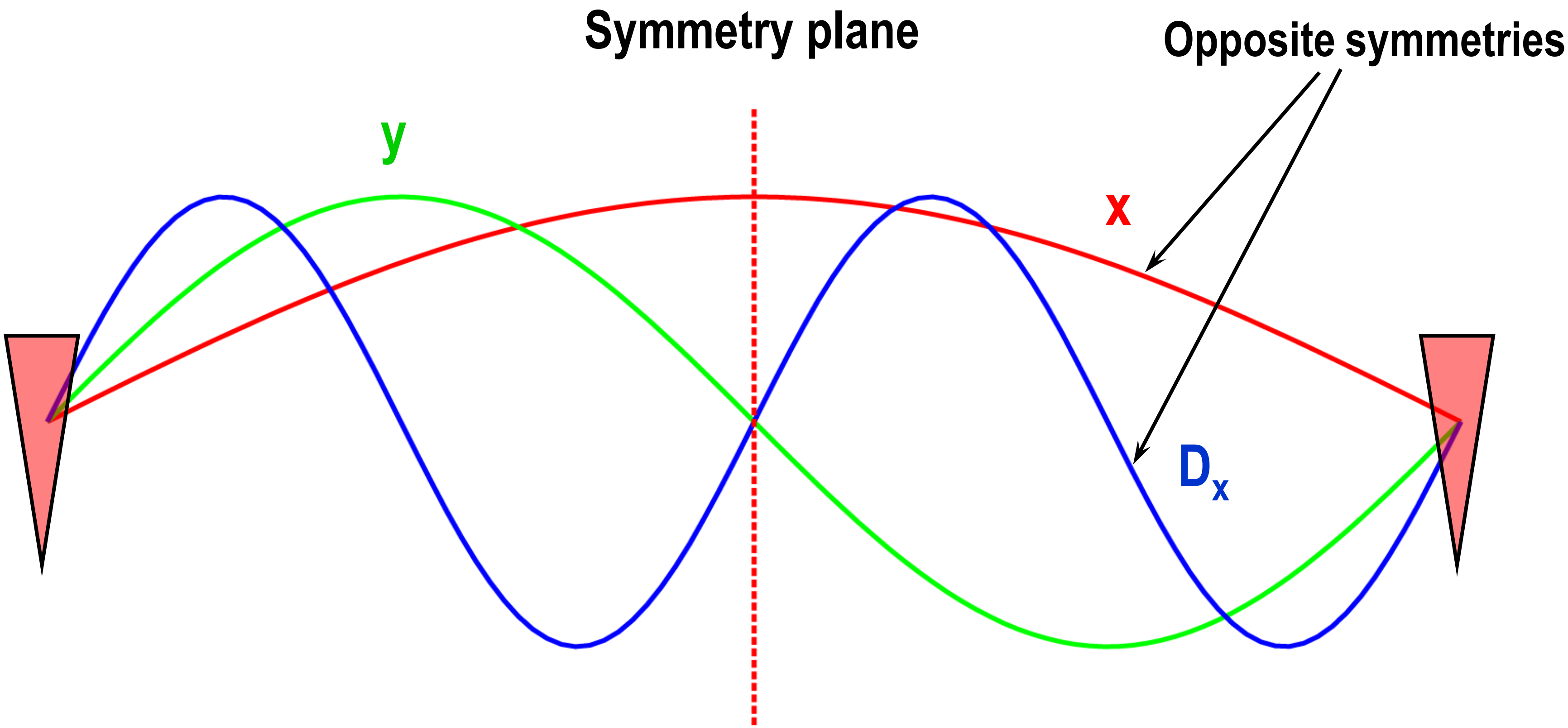
$\beta_y^i = 3.5\ \mathrm{km},\quad \Delta E / E = 3\times 10^{-4}$

3^{rd}-order aberrations:

0.000000	0.8199492E-05	0.000000	0.000000	-0.2323975E-06	300000	$M(x\|x^3)$
-0.1896848E-06	0.1146330E-06	0.000000	0.000000	0.6412818E-08	210000	
0.2709151E-08	0.4333882E-09	0.000000	0.000000	0.1275181E-10	120000	
0.000000	0.000000	0.000000	-0.7291008E-05	0.000000	201000	$M(x\|x^2y)$
0.000000	0.000000	0.6075276E-07	-0.1157797E-06	0.000000	111000	
0.000000	0.000000	-0.2828487E-10	-0.3274083E-10	0.000000	021000	
-0.4325833E-13	0.2490648E-05	0.000000	0.000000	-0.1015788E-06	102000	
0.1860693E-08	0.3953135E-08	0.000000	0.000000	0.1599289E-08	012000	
0.000000	0.000000	0.000000	-0.5902088E-06	0.000000	003000	$M(x\|y^3)$
0.000000	0.000000	-0.1140030E-08	-0.1321978E-06	0.000000	200100	
0.000000	0.000000	0.2123703E-09	-0.6357144E-09	0.000000	110100	
-0.3555784E-07	0.3104528E-07	0.000000	0.000000	-0.1023037E-08	101100	
0.1446322E-09	0.5948047E-10	0.000000	0.000000	0.3161510E-10	011100	
0.000000	0.000000	0.2843933E-08	0.1991208E-07	0.000000	002100	
-0.4827146E-06	0.1367050E-06	0.000000	0.000000	0.1470684E-07	200001	
0.8880075E-08	0.1357266E-08	0.000000	0.000000	0.3909450E-10	110001	
0.2304548E-11	0.1982152E-11	0.000000	0.000000	0.1397354E-12	020001	
0.000000	0.000000	0.1436638E-06	-0.8561024E-07	0.000000	101001	
0.000000	0.000000	-0.1300843E-08	0.7213526E-12	0.000000	011001	
-0.7033002E-07	-0.9338884E-09	0.000000	0.000000	0.3346314E-08	002001	
0.2202488E-09	0.1480645E-09	0.000000	0.000000	0.4602691E-11	100200	
0.000000	0.000000	0.1975608E-09	0.7998469E-10	0.000000	001200	
0.000000	0.000000	0.1488271E-09	-0.4633650E-09	0.000000	100101	
0.000000	0.000000	-0.6517825E-11	-0.2428764E-11	0.000000	010101	
-0.7083194E-09	-0.3263868E-10	0.000000	0.000000	0.6778098E-10	001101	
0.1018256E-07	0.1148323E-08	0.000000	0.000000	0.1914187E-10	100002	
0.3099303E-11	0.4039911E-11	0.000000	0.000000	0.2855707E-12	010002	
0.000000	0.000000	-0.2366361E-08	0.1291189E-09	0.000000	001002	
0.000000	0.000000	-0.1784734E-10	-0.4658336E-11	0.000000	000102	
0.4417746E-11	0.3354065E-11	0.000000	0.000000	0.2134776E-12	000003	

4^{th}-order aberrations: small

Twin Helix Symmetry



Conclusions and Future Plans

- Designed twin-helix magnetic structure satisfying PIC requirements of correlated linear optics
- Confirmed optics properties with GEANT4-based G4beamline simulations
- Large dynamic aperture suggested by simulations but more systematic study is needed
- Suggested straightforward possible practical implementations
- Next study aberration compensation following a well-defined approach (try adopting COSY Infinity?)
- Need to introduce transverse coupling for cooling decrement equalization i.e. by slightly offsetting spatial period of one helical harmonic