Coherent Synchrotron Radiation in the Cornell ERL

Chris Mayes – November 4, 2010

ERL Concept (1965)

A Possible Apparatus for Electron Clashing-Beam Experiments (*).

M. Tigner

Laboratory of Nuclear Studies, Cornell University - Ithaca, N. Y.

(ricevuto il 2 Febbraio 1965)

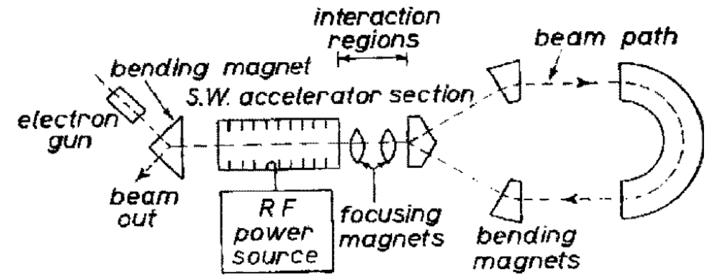
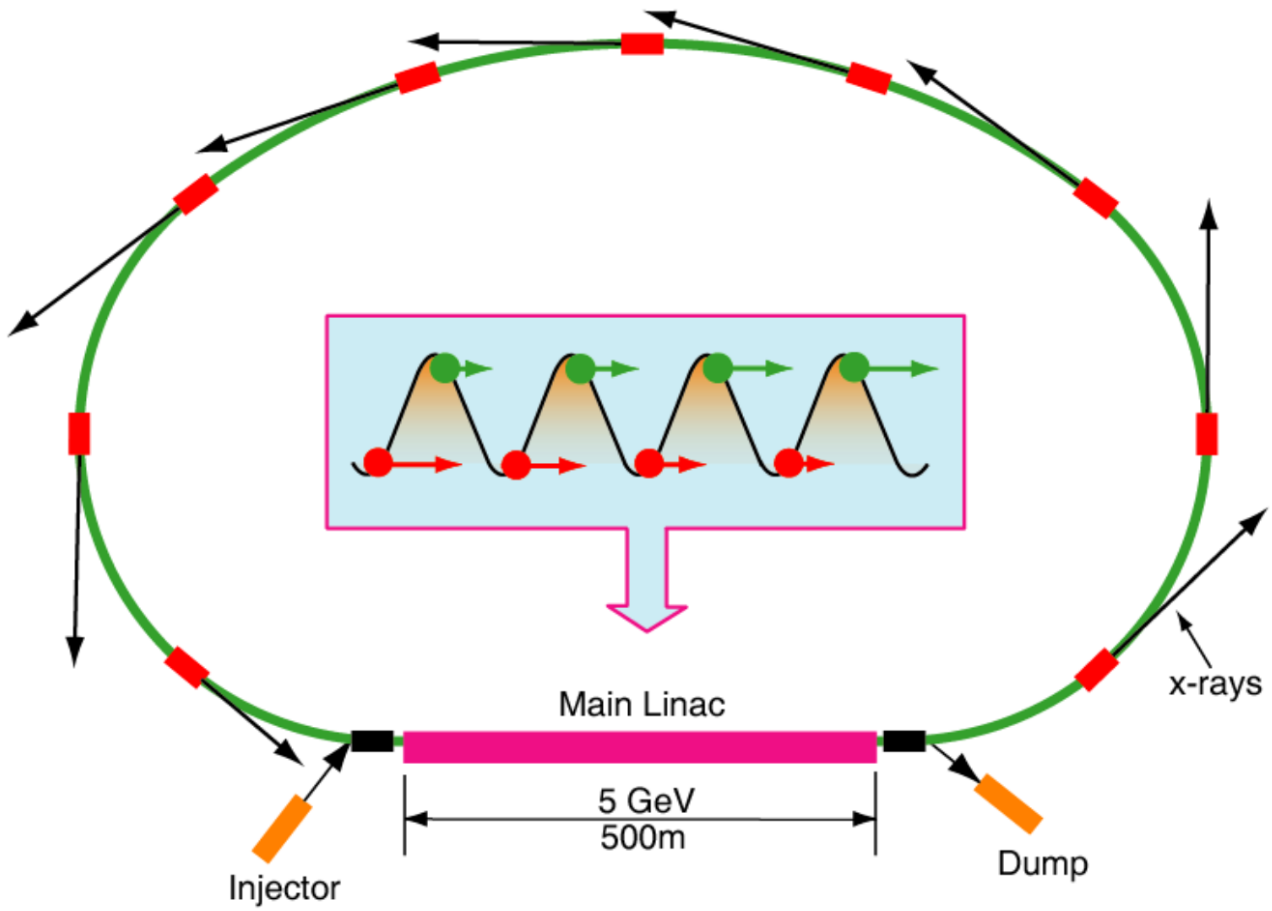


Fig. 3.

Cornell ERL Study (2001)



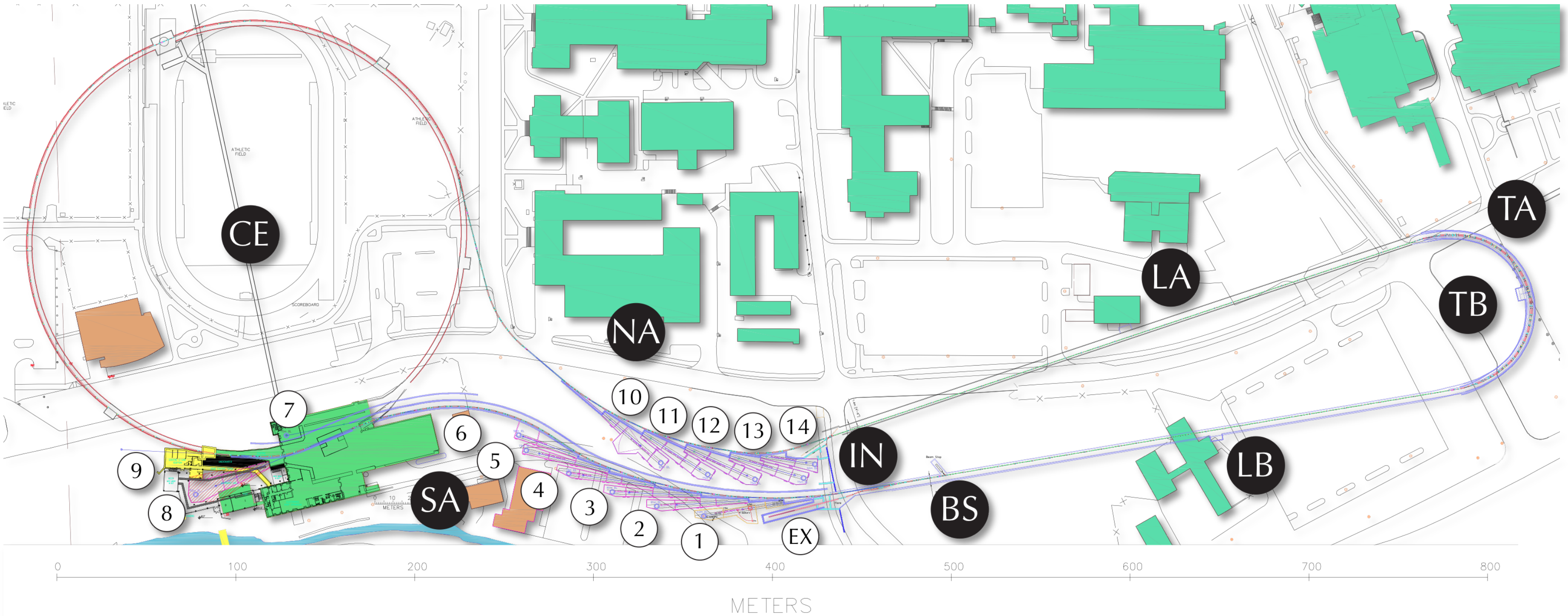
Without energy recovery . . .

$$P_{\text{linac}} = 1\,\text{MW}\left(\frac{I_{\text{av}}}{\text{mA}}\right)\left(\frac{\Delta\mathcal{E}}{\text{GeV}}\right)$$

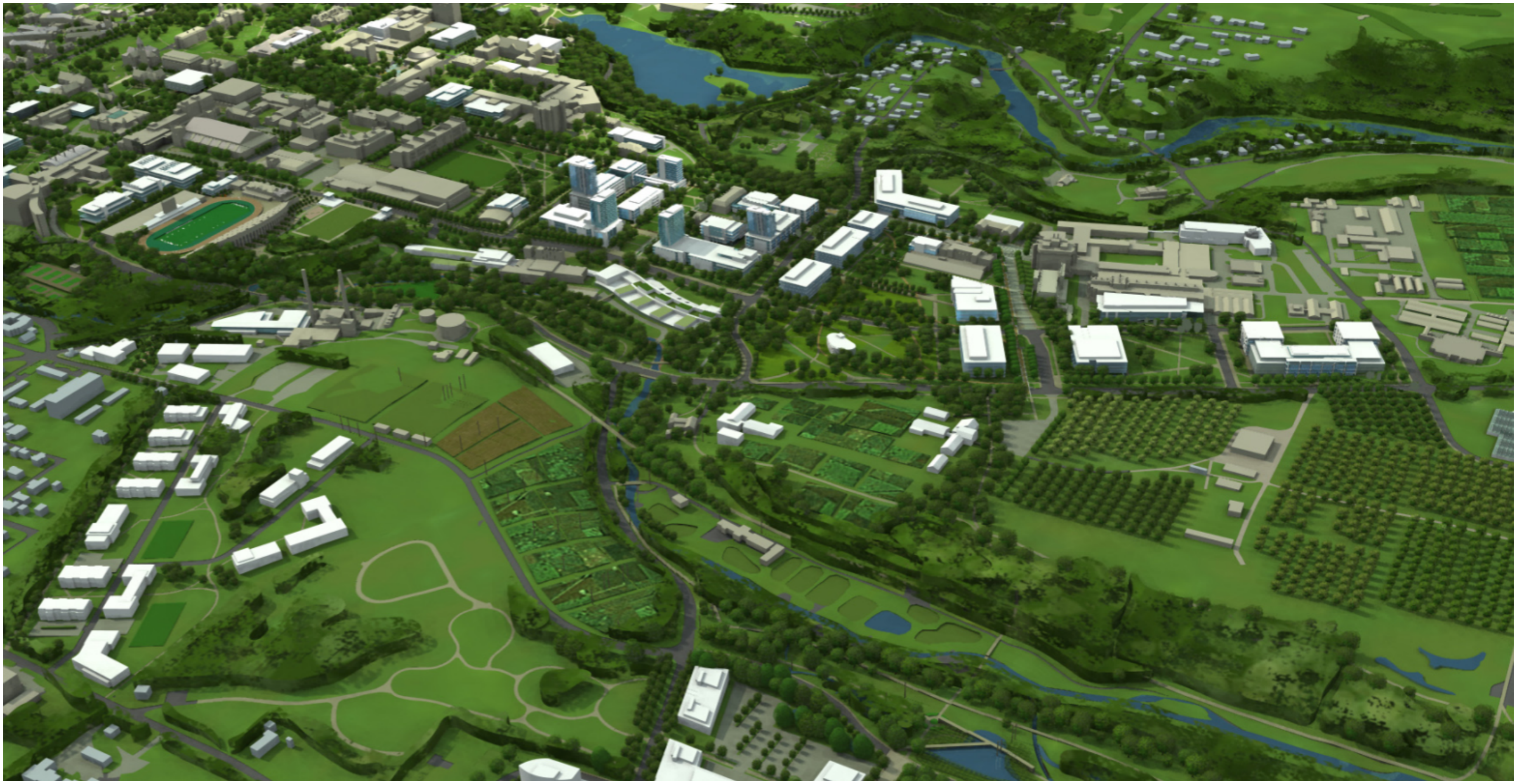
[Nine Mile Point Nuclear Power Plant, Oswego, NY]

Cornell ERL (2010)

Lightsource	Energy (GeV)	Current (mA)	Emittance (pm)	Bunch Length (ps)
ESRF	6	200	4000	20
APS	7	100	2514	20
SPring 8	8	100	3000	13
CERL mode A	5	100	31	2
mode B	5	25	8	2
mode C	5	25	70	0.1



Cornell Master Plan



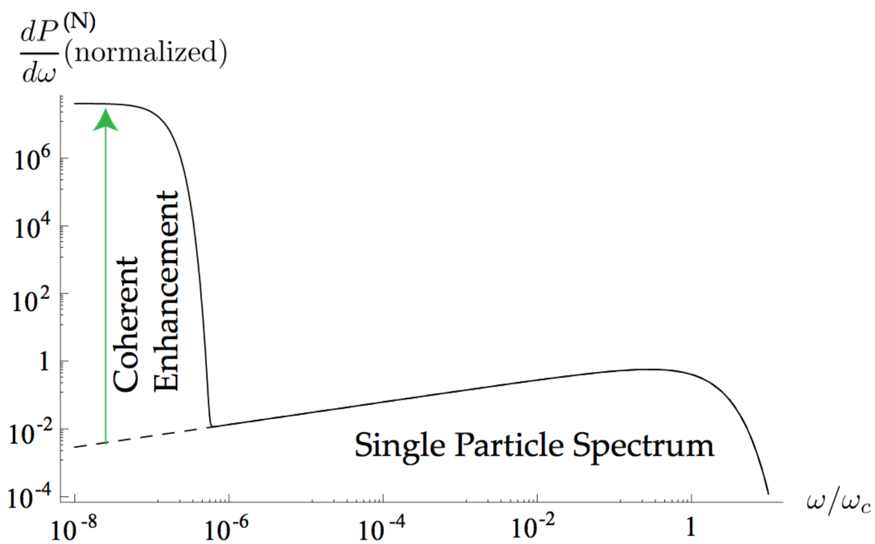
[www.masterplan.cornell.edu]

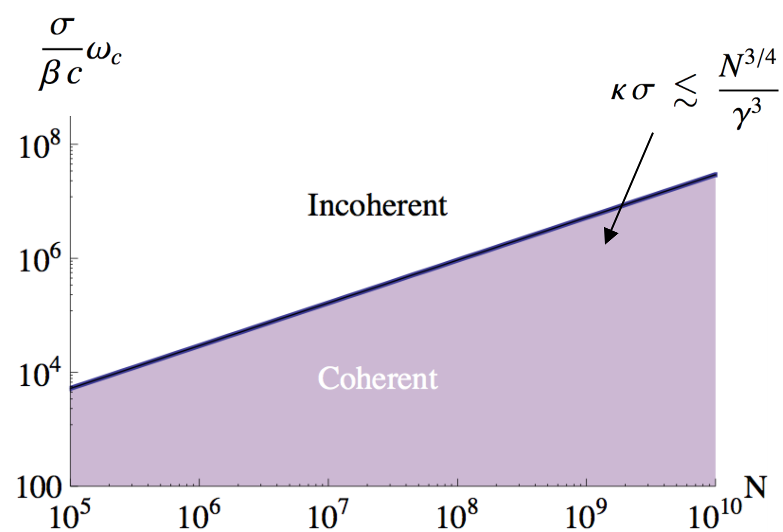
Coherent Synchrotron Radiation

Schwinger (1945)
(unpublished)

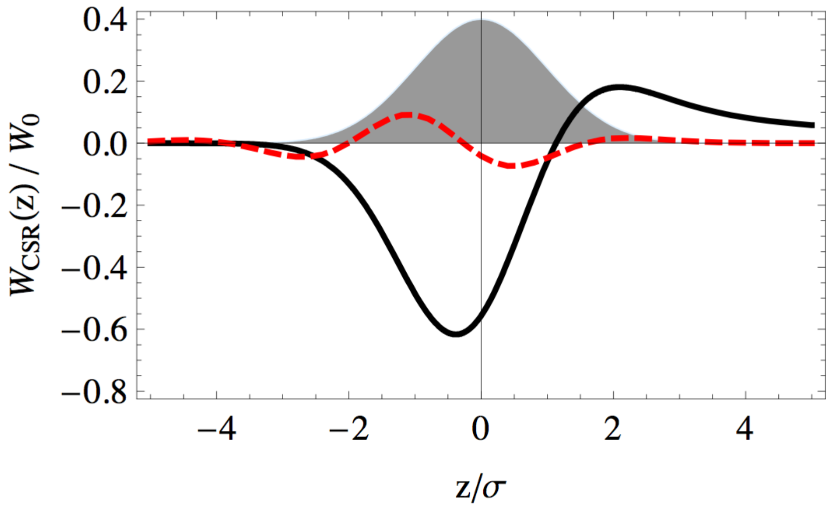

On Radiation by Electrons in a Betatron*
J. Schwinger
1945

$$\frac{dP^{(N)}}{d\omega}(\omega) \simeq \underbrace{N\frac{dP^{(1)}}{d\omega}}_{\text{incoherent}} + \underbrace{N(N-1)\left|\int \mathrm{d}s\,\lambda(s)\exp\left(i\,\frac{\omega s}{\beta c}\right)\right|^2 \frac{dP^{(1)}}{d\omega}}_{\text{coherent}}$$





CSR Wakefield and Shielding



$$W_{\mathrm{CSR}}(z) = -\frac{2\,N\,r_c\,mc^2}{3^{1/3}\,R^{2/3}} \left\{ \frac{\lambda(z-z_L)-\lambda(z-4z_L)}{z_L^{1/3}} + \int_{z-z_L}^{z} \frac{1}{(z-z')^{1/3}} \frac{d\lambda}{dz'}\,dz' \right\}$$

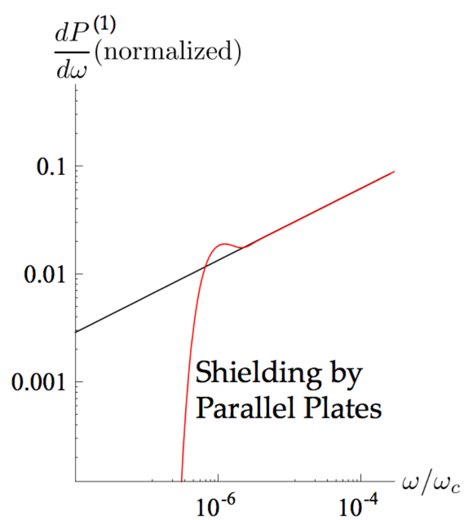
$$z_L \equiv \frac{R\,\phi^3}{24}\,,$$

[Saldin et al. (1997)]

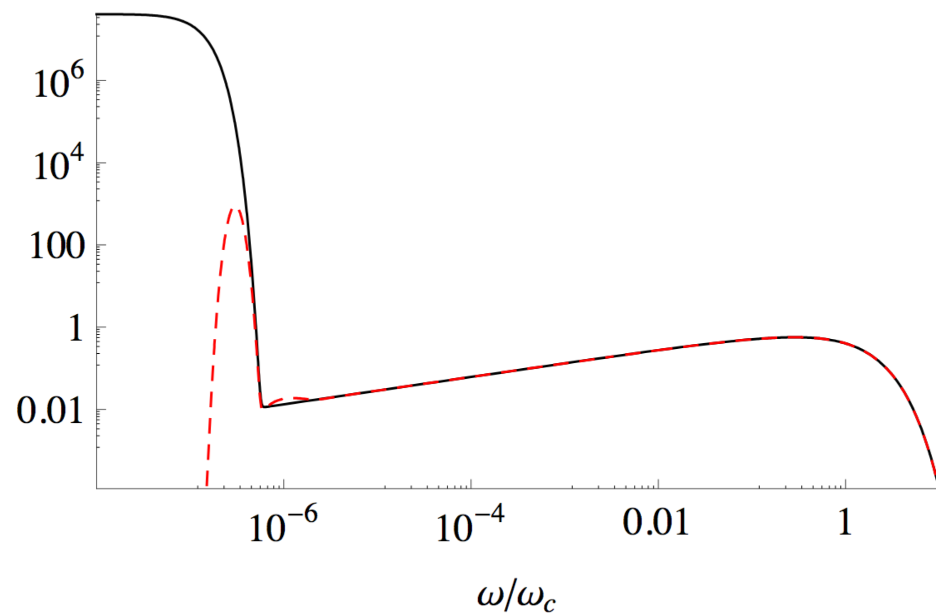
Free Space CSR Scales with

$$W_0 \equiv N r_c mc^2 \frac{\kappa^{2/3}}{\sigma^{4/3}}$$

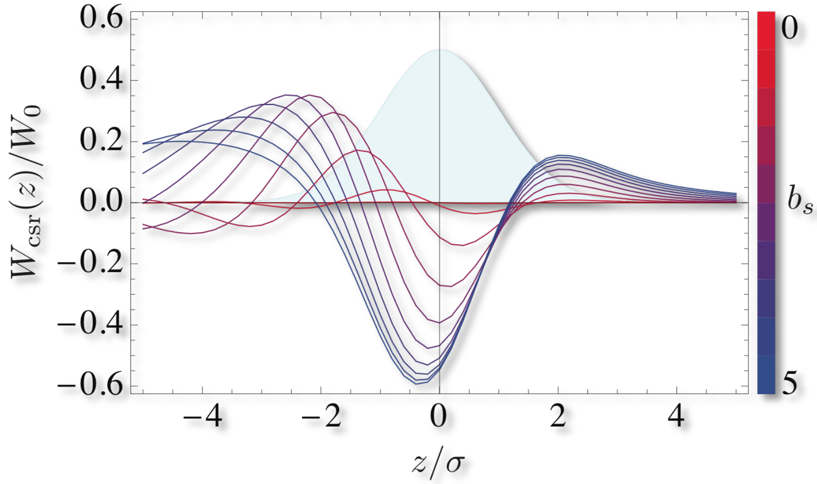
Single Particle Spectrum

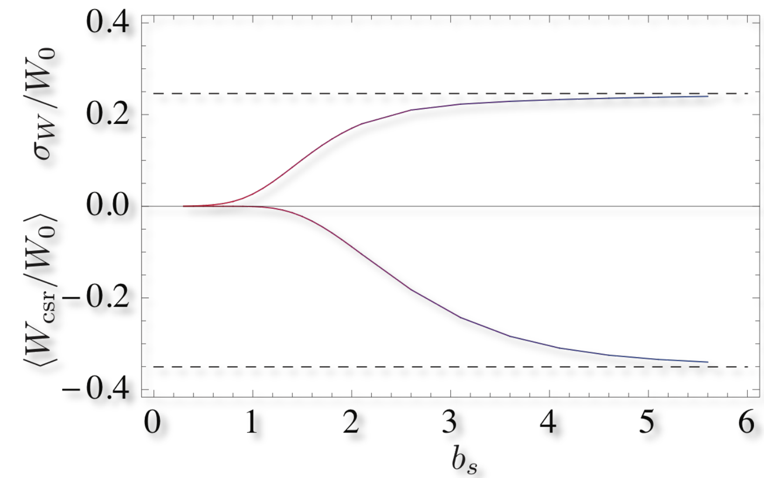


N Particle Spectrum



Shielding Factor $b_s \equiv h\left(\frac{\kappa}{\sigma^2}\right)^{1/3}$





$$W_{\substack{\text{p.p}\\ \gamma\to\infty}}(z) = -W_0\,\Re\left[\int_0^\infty I_b(\tilde{k})\,\hat{\lambda}\left(\frac{\tilde{k}}{\sigma}\right)\exp\left(i\tilde{k}\,\frac{z}{\sigma}\right)\,d\tilde{k}\right]$$

$$I_b\left(\tilde{k}\right) \equiv \frac{2^{10/3}\pi}{b_s\,\tilde{k}^{1/3}}\sum_{p=0}^{\infty} F_{AY}\left(\frac{(2p+1)^2\,\pi^2}{2^{2/3}\,b_s^2\,\tilde{k}^{4/3}}\right),$$

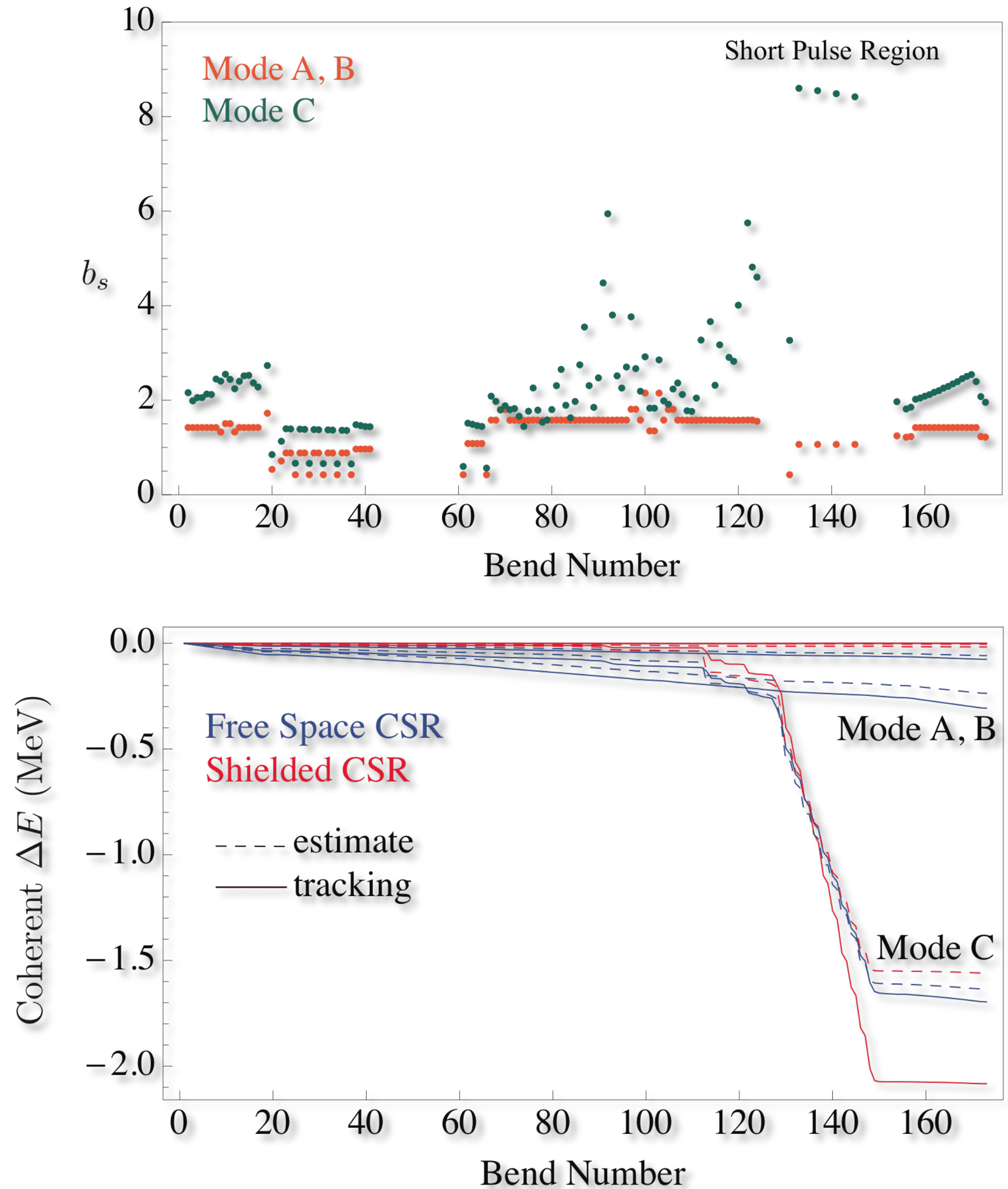
$$F_{AY}(x) \equiv \mathrm{Ai}'(x)\left[\mathrm{Ai}'(x) - i\,\mathrm{Bi}'(x)\right] + x\,\mathrm{Ai}(x)\left[\mathrm{Ai}(x) - i\,\mathrm{Bi}(x)\right].$$

[Simplified from Agoh & Yokoya (2004)]

$$\langle W_{\rm CSR}\rangle \equiv \int_{z_-}^{z_+} W_{\rm CSR}(z)\,\lambda(z)\,dz,$$

$$\sigma_W \equiv \left[\int_{z_-}^{z_+} W_{\rm CSR}^2(z)\,\lambda(z)\,dz - \langle W_{\rm CSR}\rangle^2\right]^{1/2}$$

Estimates for CSR in CERL Modes



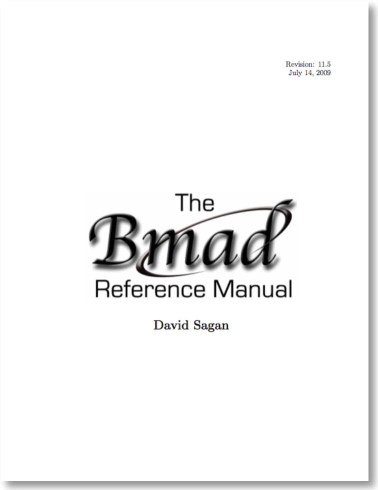
Simulation

Bmad and Tao

www.lns.cornell.edu/~dcs/bmad/

OPAL

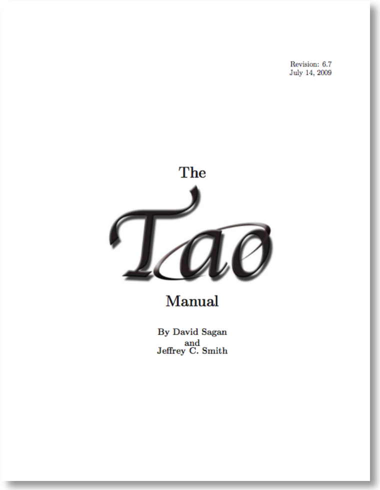
amas.web.psi.ch



Revision: 11.5
July 14, 2009

The Bmad Reference Manual

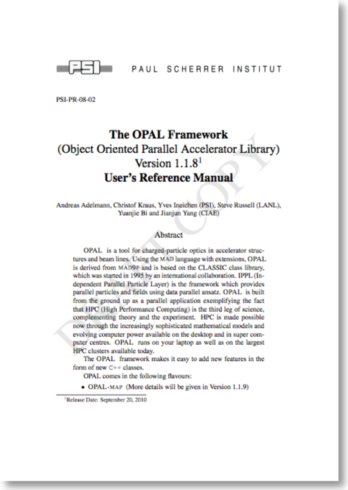
David Sagan



Revision: 6.7
July 14, 2009

The Tao Manual

By David Sagan and Jeffrey C. Smith



PAUL SCHERRER INSTITUT

PSI-PR-08-02

The OPAL Framework
(Object Oriented Parallel Accelerator Library)
Version 1.1.8[1]
User's Reference Manual

Andreas Adelmann, Christof Kraus, Yves Ineichen (PSI), Steve Russell (LANL), Yuanjie Bi and Jianjun Yang (CIAE)

Abstract

OPAL is a tool for charged-particle optics in accelerator structures and beam lines. Using the MAD language with extensions, OPAL is derived from MAD9P and is based on the CLASSIC class library, which was started in 1995 by an international collaboration. IPPL (Independent Parallel Particle Layer) is the framework which provides parallel particles and fields using data parallel ansatz. OPAL is built from the ground up as a parallel application exemplifying the fact that HPC (High Performance Computing) is the third leg of science, complementing theory and the experiment. HPC is made possible now through the increasingly sophisticated mathematical models and evolving computer power available on the desktop and in super computer centres. OPAL runs on your laptop as well as on the largest HPC clusters available today.

The OPAL framework makes it easy to add new features in the form of new C++ classes.

OPAL comes in the following flavours:

- OPAL-MAP (More details will be given in Version 1.1.9)

[1] Release Date: September 20, 2010

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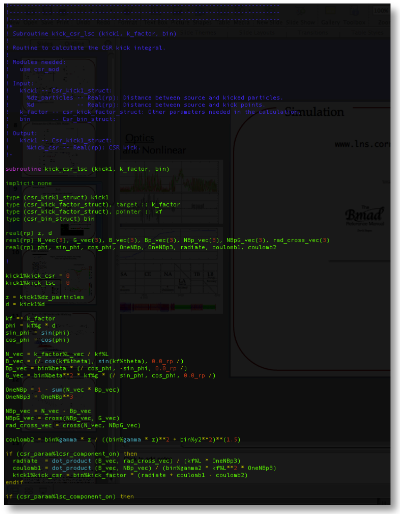
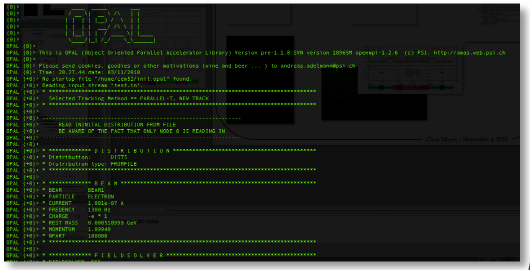
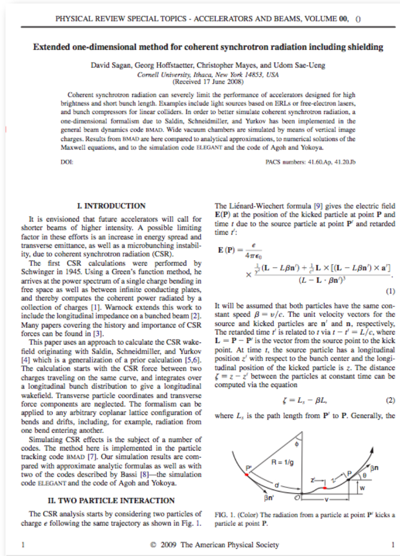




CSR Simulation


PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **00**, ()

Extended one-dimensional method for coherent synchrotron radiation including shielding

David Sagan, Georg Hoffstaetter, Christopher Mayes, and Udom Sae-Ueng
Cornell University, Ithaca, New York 14853, USA
(Received 17 June 2008)

Coherent synchrotron radiation can severely limit the performance of accelerators designed for high brightness and short bunch length. Examples include light sources based on ERLs or free-electron lasers, and bunch compressors for linear colliders. In order to better simulate coherent synchrotron radiation, a one-dimensional formalism due to Saldin, Schneidmiller, and Yurkov has been implemented in the general beam dynamics code BMAD. Wide vacuum chambers are simulated by means of vertical image charges. Results from BMAD are here compared to analytical approximations, to numerical solutions of the Maxwell equations, and to the simulation code ELEGANT and the code of Agoh and Yokoya.

DOI:
PACS numbers: 41.60.Ap, 41.20.Jb

I. INTRODUCTION

It is envisioned that future accelerators will call for shorter beams of higher intensity. A possible limiting factor in these efforts is an increase in energy spread and transverse emittance, as well as a microbunching instability, due to coherent synchrotron radiation (CSR).

The first CSR calculations were performed by Schwinger in 1945. Using a Green's function method, he arrives at the power spectrum of a single charge bending in free space as well as between infinite conducting plates, and thereby computes the coherent power radiated by a collection of charges [1]. Warnock extends this work to include the longitudinal impedance on a bunched beam [2]. Many papers covering the history and importance of CSR forces can be found in [3].

This paper uses an approach to calculate the CSR wakefield originating with Saldin, Schneidmiller, and Yurkov [4] which is a generalization of a prior calculation [5,6]. The calculation starts with the CSR force between two charges traveling on the same curve, and integrates over a longitudinal bunch distribution to give a longitudinal wakefield. Transverse particle coordinates and transverse force components are neglected. The formalism can be applied to any arbitrary coplanar lattice configuration of bends and drifts, including, for example, radiation from one bend entering another.

Simulating CSR effects is the subject of a number of codes. The method here is implemented in the particle tracking code BMAD [7]. Our simulation results are compared with approximate analytic formulas as well as with two of the codes described by Bassi [8]—the simulation code ELEGANT and the code of Agoh and Yokoya.

II. TWO PARTICLE INTERACTION

The CSR analysis starts by considering two particles of charge e following the same trajectory as shown in Fig. 1.

The Liénard-Wiechert formula [9] gives the electric field $\mathbf{E}(\mathbf{P})$ at the position of the kicked particle at point $\mathbf{P}$ and time t due to the source particle at point $\mathbf{P}'$ and retarded time t':

$$\mathbf{E}(\mathbf{P}) = \frac{e}{4\pi\epsilon_0} \times \frac{\frac{1}{\gamma^2}(\mathbf{L} - L\beta\mathbf{n}') + \frac{1}{c^2}\mathbf{L} \times [(\mathbf{L} - L\beta\mathbf{n}') \times \mathbf{a}']}{(L - \mathbf{L}\cdot\beta\mathbf{n}')^3}. \quad (1)$$

It will be assumed that both particles have the same constant speed $\beta = v/c$. The unit velocity vectors for the source and kicked particles are $\mathbf{n}'$ and $\mathbf{n}$, respectively. The retarded time t' is related to t via $t - t' = L/c$, where $\mathbf{L} = \mathbf{P} - \mathbf{P}'$ is the vector from the source point to the kick point. At time t, the source particle has a longitudinal position z' with respect to the bunch center and the longitudinal position of the kicked particle is z. The distance $\zeta = z - z'$ between the particles at constant time can be computed via the equation

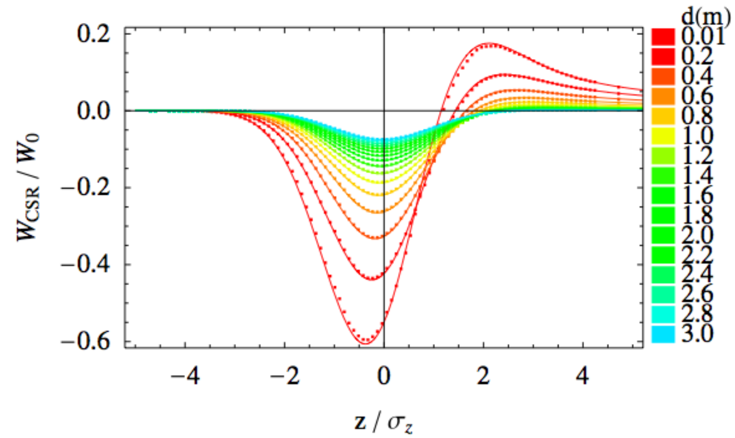
$$\zeta = L_s - \beta L, \quad (2)$$

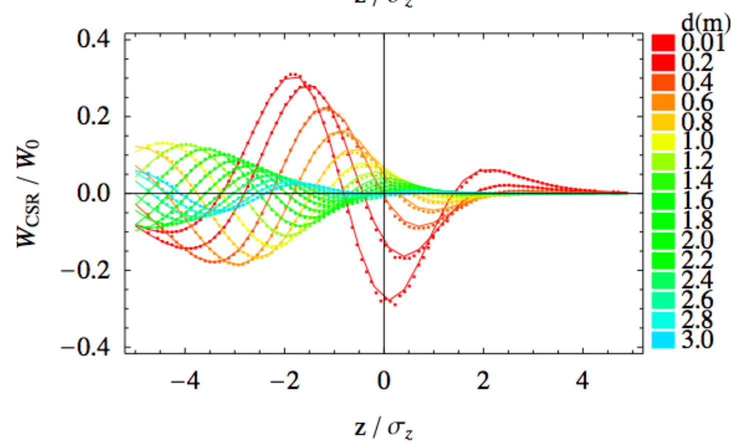
where L_s is the path length from $\mathbf{P}'$ to $\mathbf{P}$. Generally, the

FIG. 1. (Color) The radiation from a particle at point $\mathbf{P}'$ kicks a particle at point $\mathbf{P}$.

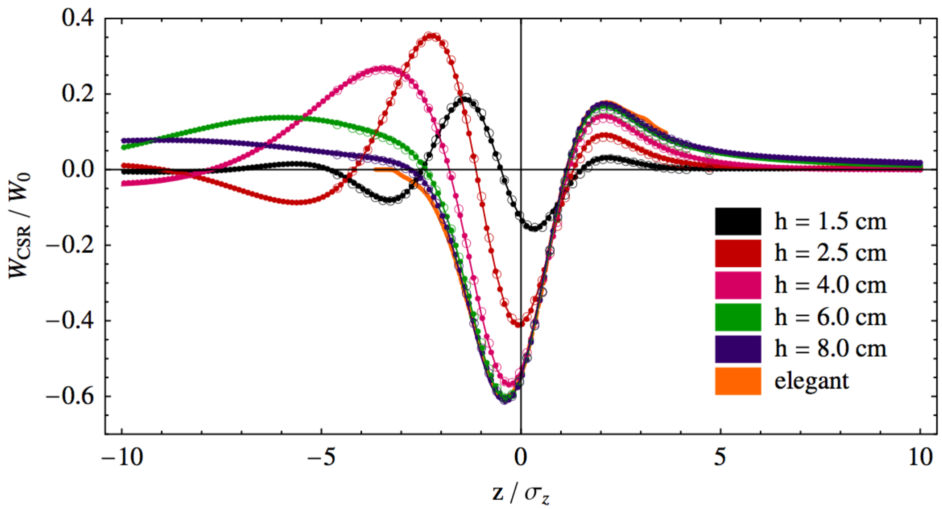
Sagan, Hoffstaetter, Mayes, Sae-Ueng
PRST-AB **12**, 040703 (2009)

Exit Transient



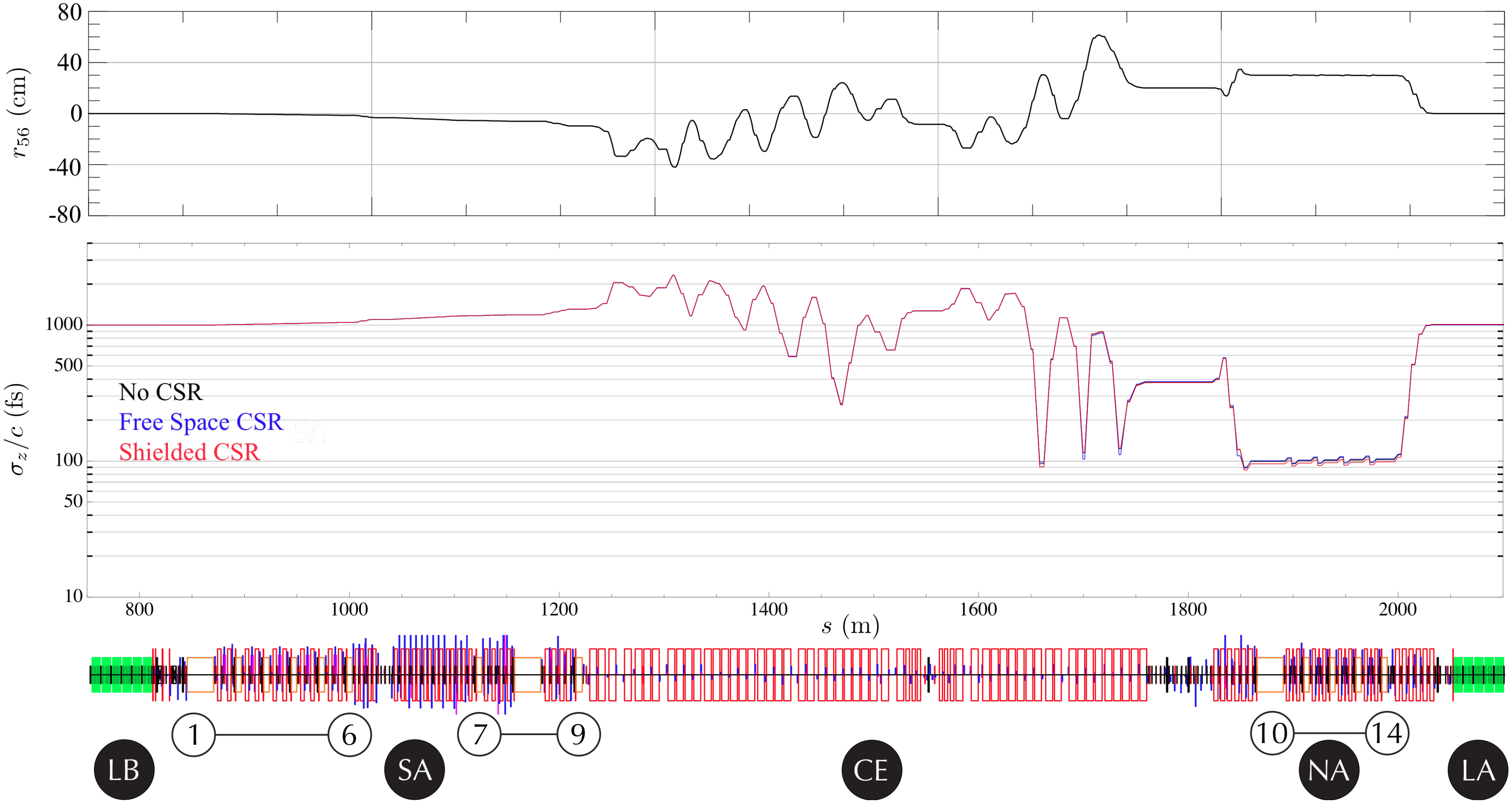


Compared with Agoh & Yokoya



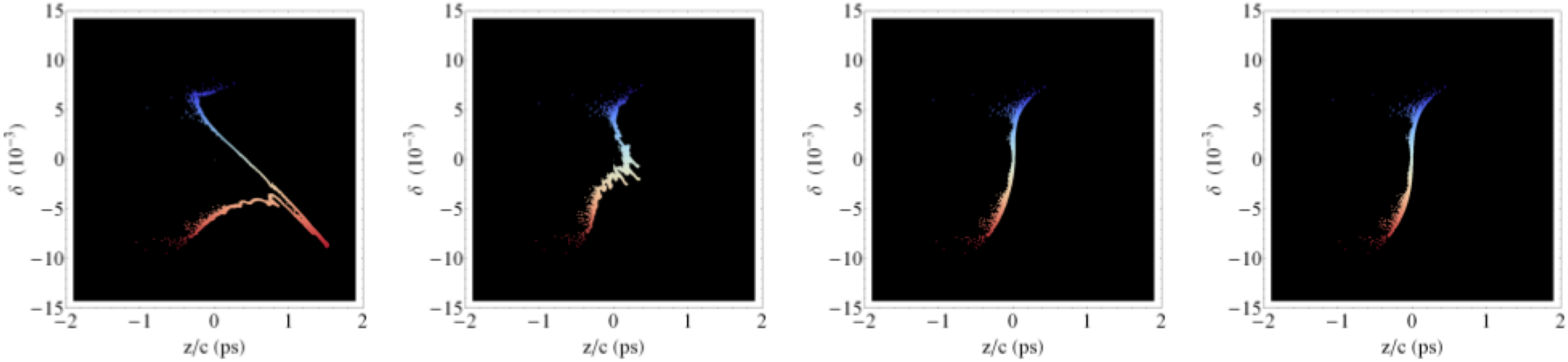
Bunch Lengths from Particle Tracking

Mode C: 19 pC, compressed to 100 fs

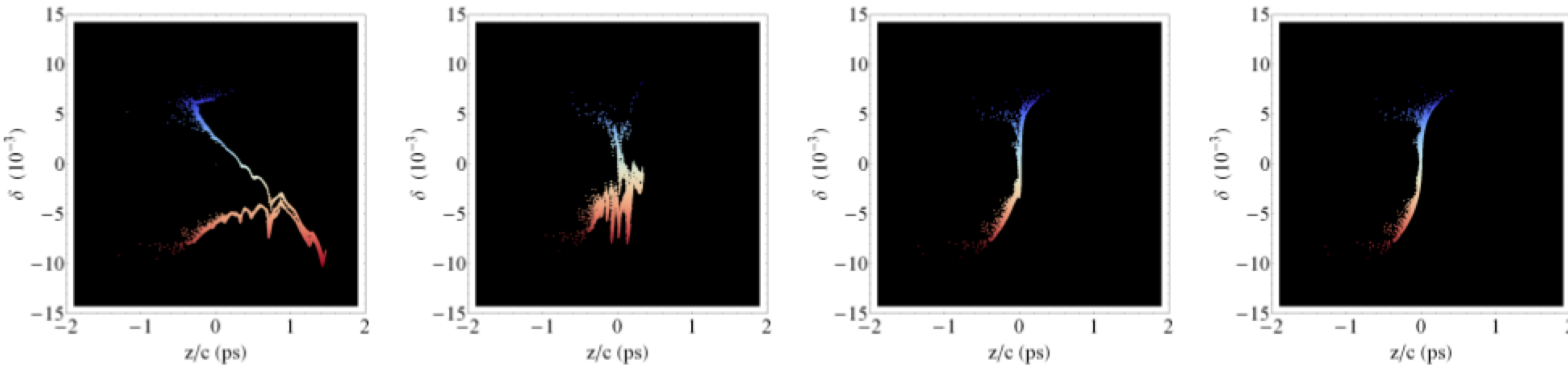


Longitudinal phase space for compressed bunches

In first short-pulse undulator (designed for 100 fs)



(a) 1 nC free space (b) 1 nC shielded (c) 77 pC shielded (d) No CSR



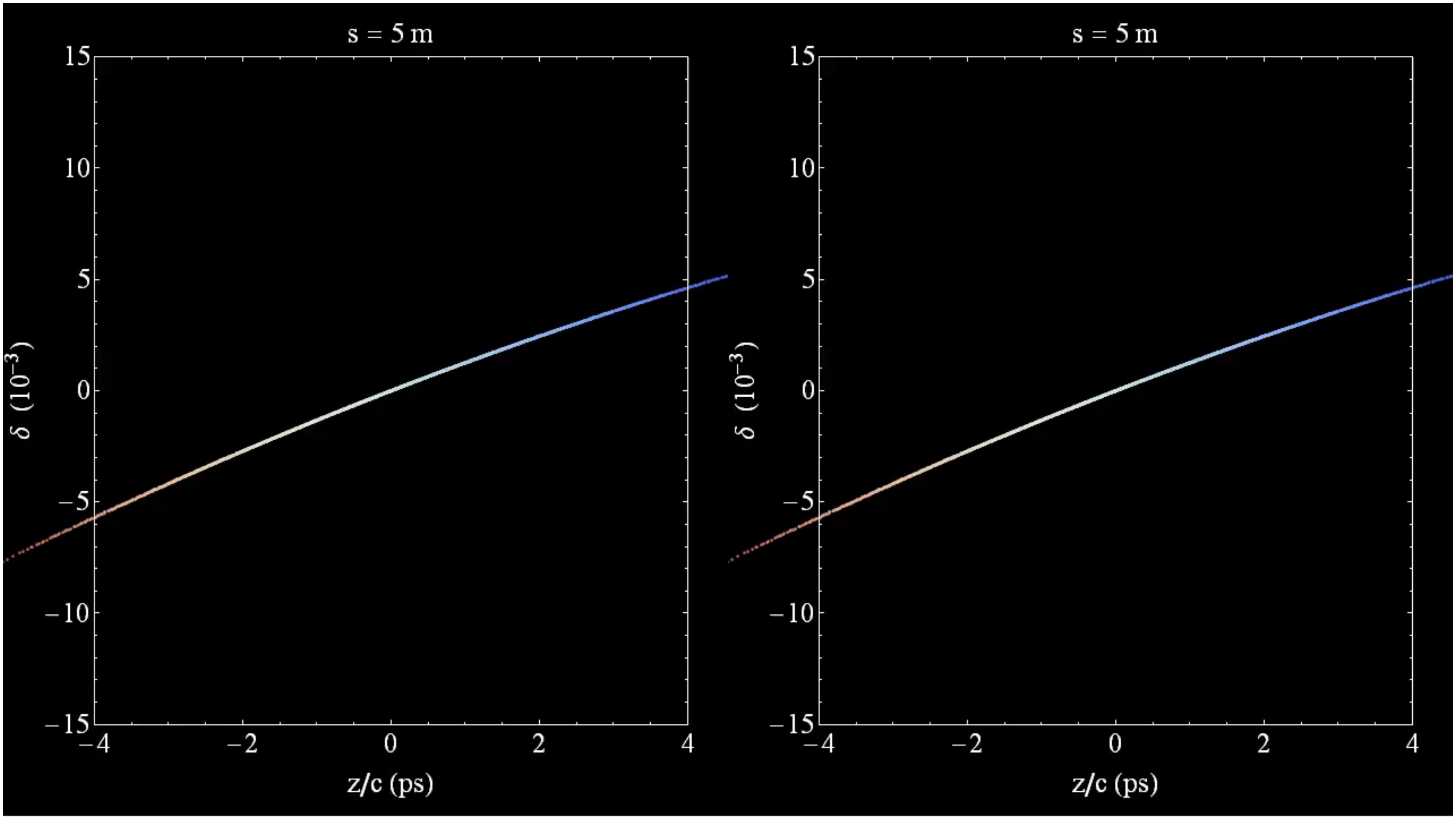
(e) 1 nC free space (f) 1 nC shielded (g) 77 pC shielded (h) No CSR

In last short-pulse undulator

CERL Bunch Compression

77 pC

1000 pC



Exact 1D CSR Model with Shielding

Mayes & Hoffstaetter
PRST-AB **12**, 024401 (2009)


PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 12, 024401 (2009)

Exact 1D model for coherent synchrotron radiation with shielding and bunch compression

Christopher Mayes and Georg Hoffstaetter

$$\mathbf{E}(\mathbf{x}, t) = \frac{1}{4\pi\epsilon_0} \int d^3x' \left[\frac{\mathbf{r}}{r^3} \rho(\mathbf{x}', t') + \frac{\mathbf{r}}{cr^2} \partial_{t'} \rho(\mathbf{x}', t') - \frac{1}{c^2 r} \partial_{t'} \mathbf{J}(\mathbf{x}', t') \right]_{t'=t-r/c},$$

$$W_{\underset{\text{s.s.}}{\text{CSR}}}(z) = N r_c m c^2 \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left[\frac{\beta^2 \cos(\alpha) - 1}{\sqrt{2 - 2\cos\alpha}} + \frac{\text{sgn}(\alpha)}{\gamma^2} \frac{1 - \frac{\beta \sin(\alpha)}{\sqrt{2 - 2\cos\alpha}}}{\alpha - \beta\sqrt{2 - 2\cos\alpha}} \right] \lambda'[z - \Delta(\alpha)],$$

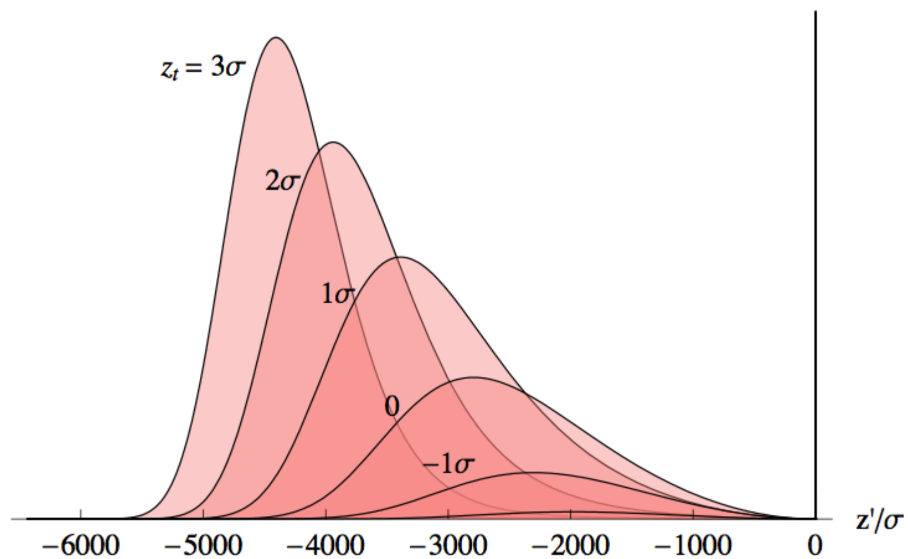


Figure 5.9: The steady-state retarded distribution $\lambda_{\text{ret,s.s.}}(z'; z_t)$ for various test particles z_t in Eq. (5.63) using a Gaussian bunch with standard deviation $\sigma = 0.3$ mm and energy 1 GeV, in a magnet of bending radius $\kappa^{-1} = 10.0$ m.

Exact 1D CSR Model with Shielding

Mayes & Hoffstaetter
PRST-AB **12**, 024401 (2009)

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 12, 024401 (2009)

Exact 1D model for coherent synchrotron radiation with shielding and bunch compression

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(Received 28 October 2008; published 19 February 2009)

Coherent synchrotron radiation has been studied effectively using a one-dimensional model for the charge distribution in the realm of small angle approximations and high energies. Here we use Jefimenko's form of Maxwell's equations, without such approximations, to calculate the exact wakefields due to this effect in multiple bends and drifts. It has been shown before that the influence of a drift can propagate well into a subsequent bend. We show, for reasonable parameters, that the influence of a previous bend can also propagate well into a subsequent bend, and that this is especially important at the beginning of a bend. Shielding by conducting parallel plates is simulated using the image charge method. We extend the formalism to situations with compressing and decompressing distributions, and conclude that simpler approximations to bunch compression usually overestimate the effect. Additionally, an exact formula for the coherent power radiated by a Gaussian bunch is derived by considering the coherent synchrotron radiation spectrum, and is used to check the accuracy of wakefield calculations.

DOI: 10.1103/PhysRevSTAB.12.024401 PACS numbers: 41.60.Ap, 41.75.−i, 41.85.Ew

I. INTRODUCTION

Coherent synchrotron radiation (CSR) is an important detrimental effect in modern particle accelerators with high bunch charges and short bunch lengths. It is a collective phenomenon where the energy radiated at wavelengths on the order of the bunch length is enhanced by the number of charges in the bunch. This CSR field can subsequently affect particle motion. A comprehensive collection of papers and references regarding this subject is found in Ref. [1].

CSR is difficult to model using discrete particles exactly because the problem scales with the number of particles N as N^2. To simplify this, the one-dimensional model projects the transverse particle density onto the longitudinal dimension. Formulas for the CSR field from this line charge are then used to calculate forces on each particle and then propagate the full bunch distribution. While this makes the calculation tractable, the electromagnetic fields on the world sheet of this charged line are singular. Pioneering efforts described in Refs. [2,3] circumvent this problem by examining the nonsingular terms only. In more detail, Ref. [4] regularizes the longitudinal force between two charges traveling on the arc of a circle by subtracting off the Coulomb force, calculated as if the charges traveled on a straight line, from the force calculated using Liénard-Wiechert fields. The result is an always finite CSR force.

This paper uses the less widely known Jefimenko forms of Maxwell's equations [5], which allow one to calculate electromagnetic fields by directly using the evolving charge and current densities, and which internally incorporate all retardation effects. These equations are related to forms used in Refs. [2,6]. This is in contrast to the usual Liénard-Wiechert approach, which gives fields due to charges at their retarded times t' and positions $\mathbf{x}(t')$, and one must invert equations of the form $t - t' = |\mathbf{x}(t') - \mathbf{x}_o|/c$ for the retarded time t', where $\mathbf{x}_o$ is an observation point at a later time t and c is the speed of light. While this latter method has proven useful in deriving equations for (incoherent) synchrotron radiation of single particles, the former is found to be useful for the coherent fields of particle distributions.

II. EXACT 1D APPROACHES TO STEADY-STATE CSR

In general, for given charge and current densities $\rho(\mathbf{x}, t)$ and $\mathbf{J}(\mathbf{x}, t)$ at position $\mathbf{x}$ and time t, the electric field $\mathbf{E}(\mathbf{x}, t)$ can be calculated using Jefimenko's form of Maxwell's equations [5],

$$\mathbf{E}(\mathbf{x}, t) = \frac{1}{4\pi\epsilon_0}\int d^3x'\left[\frac{\mathbf{r}}{r^3}\rho(\mathbf{x}', t') + \frac{\mathbf{r}}{cr^2}\partial_{t'}\rho(\mathbf{x}', t') - \frac{1}{c^2r}\partial_{t'}\mathbf{J}(\mathbf{x}', t')\right]_{t'=t-r/c}, \tag{1}$$

in which $\mathbf{r} \equiv \mathbf{x} - \mathbf{x}'$, $r \equiv \|\mathbf{r}\|$, ϵ_0 is the vacuum permittivity and t' is the retarded time. In this formulation, the retarded points $\mathbf{x}'$ and times t' are independent variables, so there are no functions that need to be inverted. Therefore, if one knows ρ, $\dot{\rho}$, and $\dot{\mathbf{J}}$ at all points in space $\mathbf{x}'$ and times $t' \le t$, with a dot denoting the time derivative, then this formula gives the electric field by direct integration.

Now consider a line charge distribution, which follows a path $\mathbf{X}(s)$ parametrized by distance s, has a unit tangent $\mathbf{u}(s) = d\mathbf{X}(s)/ds$, and moves with constant speed βc along this path. A bunch with total charge Q and normalized line density λ therefore has one-dimensional charge density and current

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$$\mathbf{E}(\mathbf{x}, t) = \frac{1}{4\pi\epsilon_0}\int d^3x'\left[\frac{\mathbf{r}}{r^3}\rho(\mathbf{x}', t') + \frac{\mathbf{r}}{cr^2}\partial_{t'}\rho(\mathbf{x}', t') - \frac{1}{c^2r}\partial_{t'}\mathbf{J}(\mathbf{x}', t')\right]_{t'=t-r/c},$$

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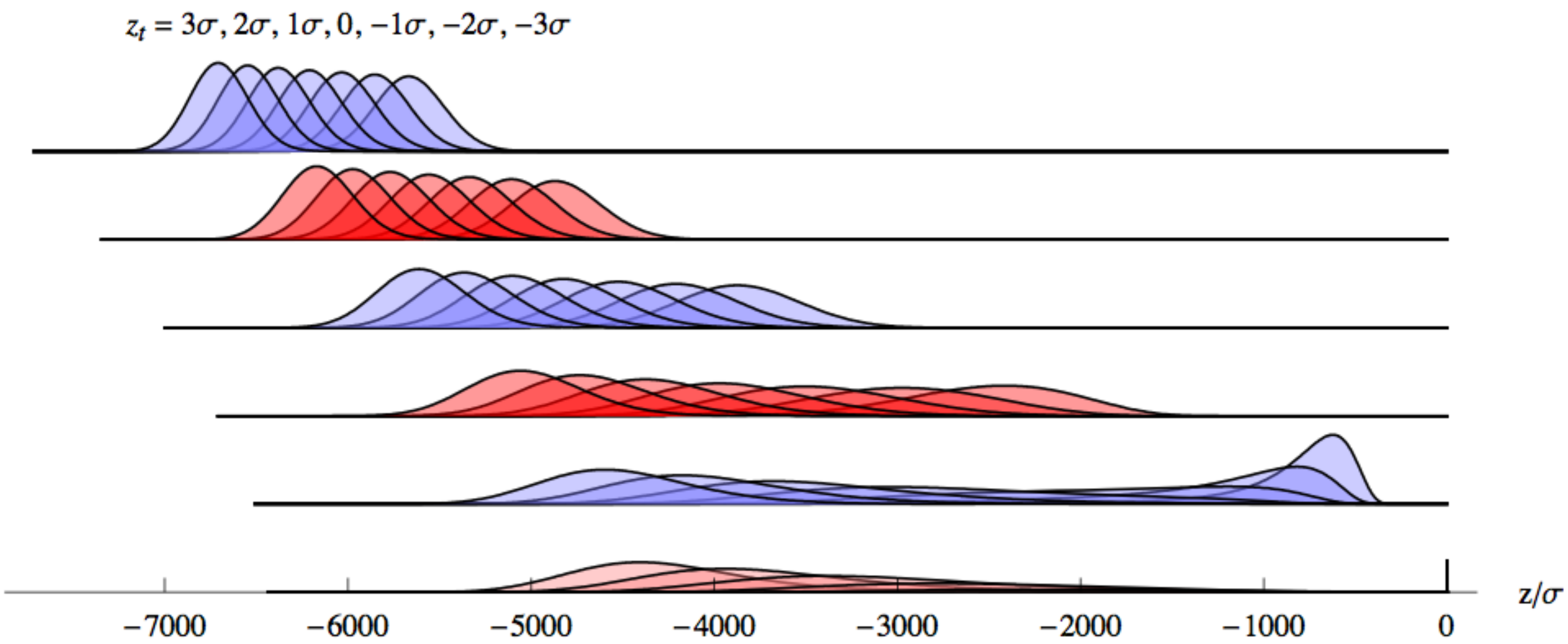


Exact 1D CSR Model with Shielding

Mayes & Hoffstaetter
PRST-AB **12**, 024401 (2009)

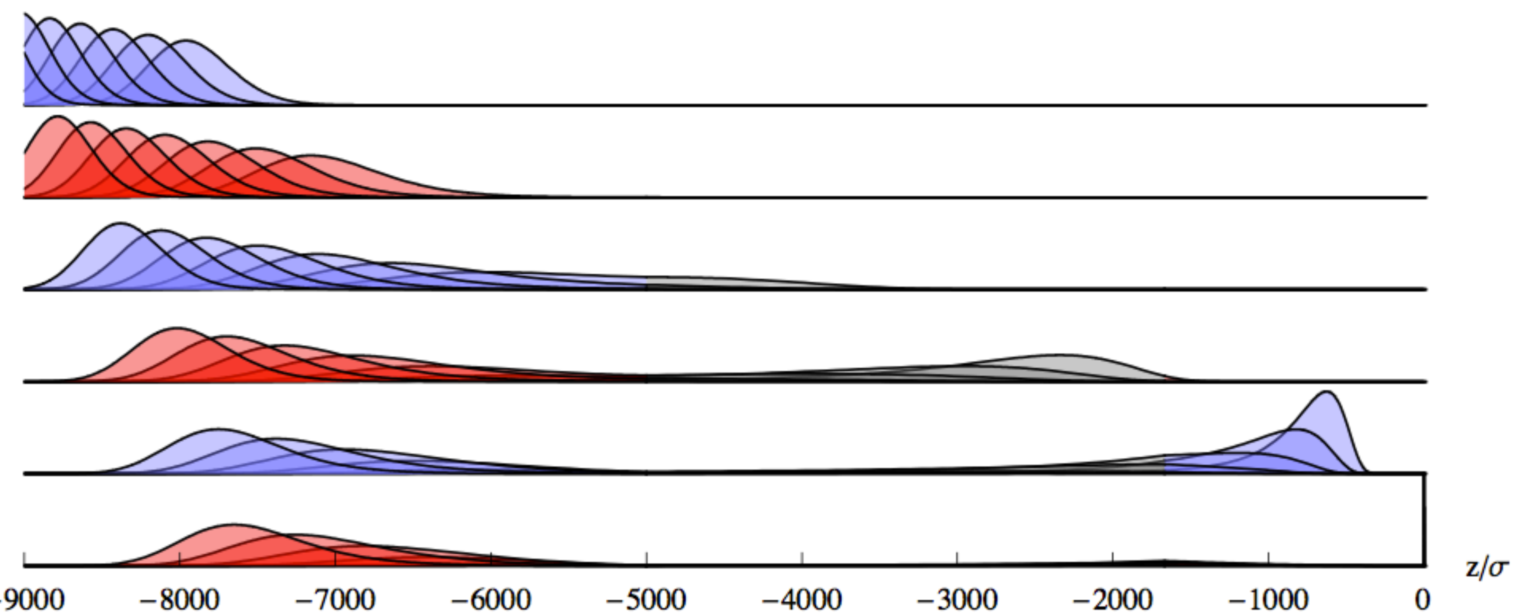

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 12, 024401 (2009)

Exact 1D model for coherent synchrotron radiation with shielding and bunch compression

Christopher Mayes and Georg Hoffstaetter

$$\mathbf{E}\left(\mathbf{x},t\right)=\frac{1}{4\pi\epsilon_0}\int d^3x'\left[\frac{\mathbf{r}}{r^3}\rho(\mathbf{x}',t')+\frac{\mathbf{r}}{cr^2}\partial_{t'}\rho(\mathbf{x}',t')-\frac{1}{c^2r}\partial_{t'}\mathbf{J}(\mathbf{x}',t')\right]_{t'=t-r/c},$$

$$W_{\underset{\text{s.s.}}{\text{CSR}}}(z)=Nr_cmc^2\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\left[\frac{\beta^2\cos(\alpha)-1}{\sqrt{2-2\cos\alpha}}+\frac{\text{sgn}(\alpha)}{\gamma^2}\,\frac{1-\frac{\beta\sin(\alpha)}{\sqrt{2-2\cos\alpha}}}{\alpha-\beta\sqrt{2-2\cos\alpha}}\right]\lambda'[z-\Delta(\alpha)],$$



Exact 1D CSR: Transient effects

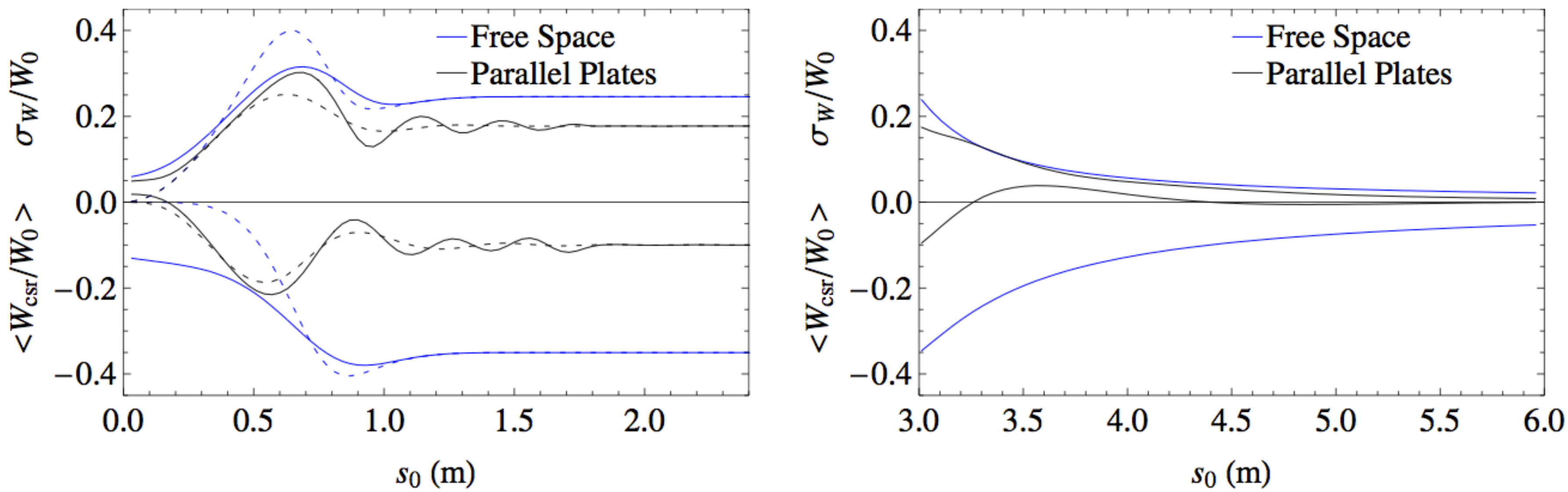


Figure 3: The average energy loss and energy spread induced, per unit length, of the CSR-wake for a Gaussian bunch through the length of a bend (left plot) and in the drift region (right plot) following that bend. Solid lines take into account a previous bend separated by 1 m from the current bend, while dashed lines have no previous bend. The bending radii are 10 m, and the bunch has a length $\sigma = 0.3$ mm with an energy of 1 GeV. In this example, shielding by parallel plates (separated by 2 cm) drastically reduces the energy loss, but only moderately reduces the energy spread, as compared to the free space calculations.

Exact 1D CSR: comparison with approximations

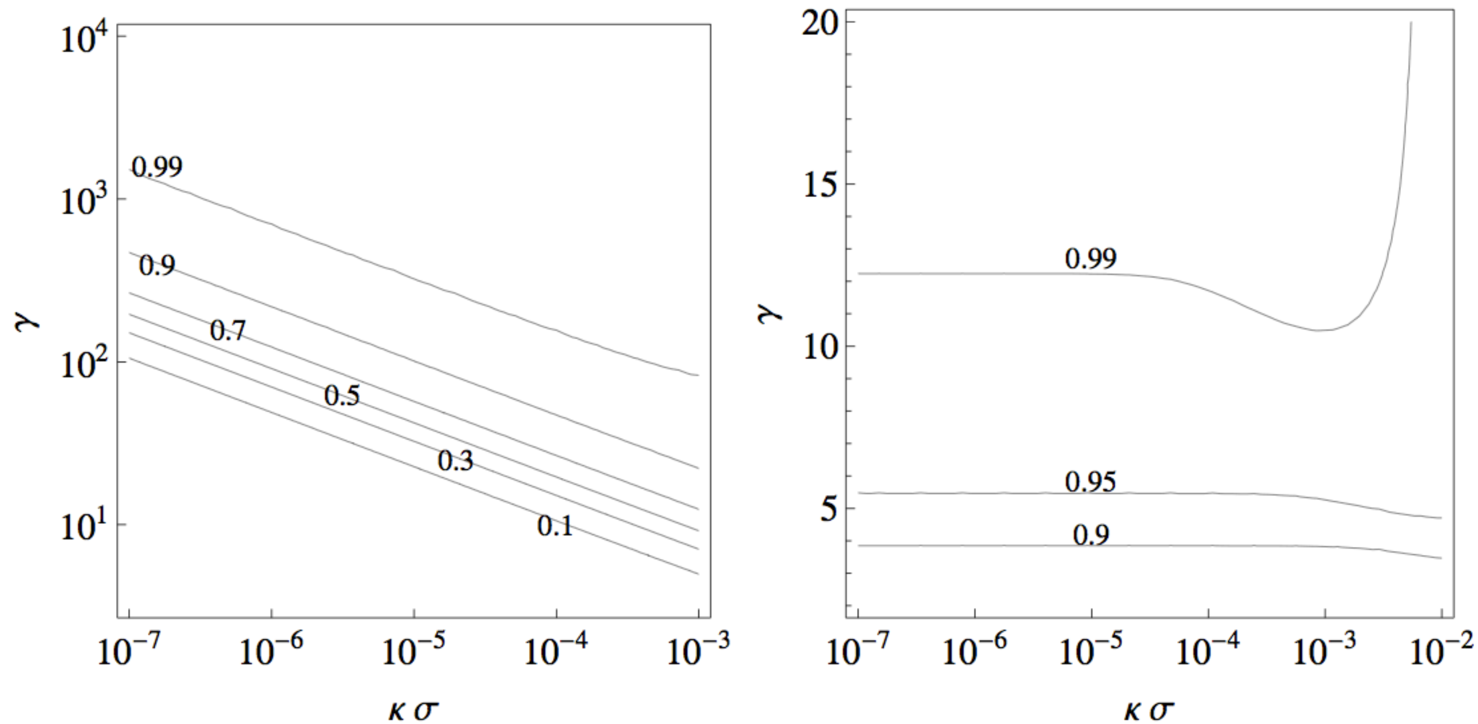


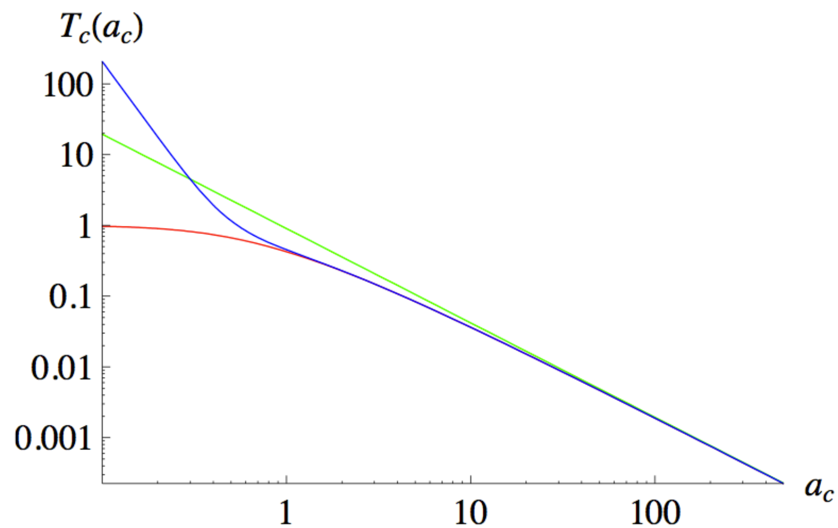
Figure 2: The ratio of the average energy lost by a Gaussian bunch using the exact CSR-wake to that using the ultra-relativistic, small angle approximation (left plot) described in Ref. [6] and to that using a significantly better approximation (right plot) described in Ref. [7]. Lower numbers therefore imply an overestimation of the CSR effect.

[6] M. Borland, Phys. Rev. ST-AB **4**, 070701 (2001)

[7] D.C. Sagan, G.H. Hoffstaetter, C.E. Mayes, and U. Sae-Ueng, Phys. Rev. ST-AB **12**, 040703 (2009)

Singularity?

$$W_0 \equiv N r_c m c^2 \frac{\kappa^{2/3}}{\sigma^{4/3}}$$



For a Gaussian distribution

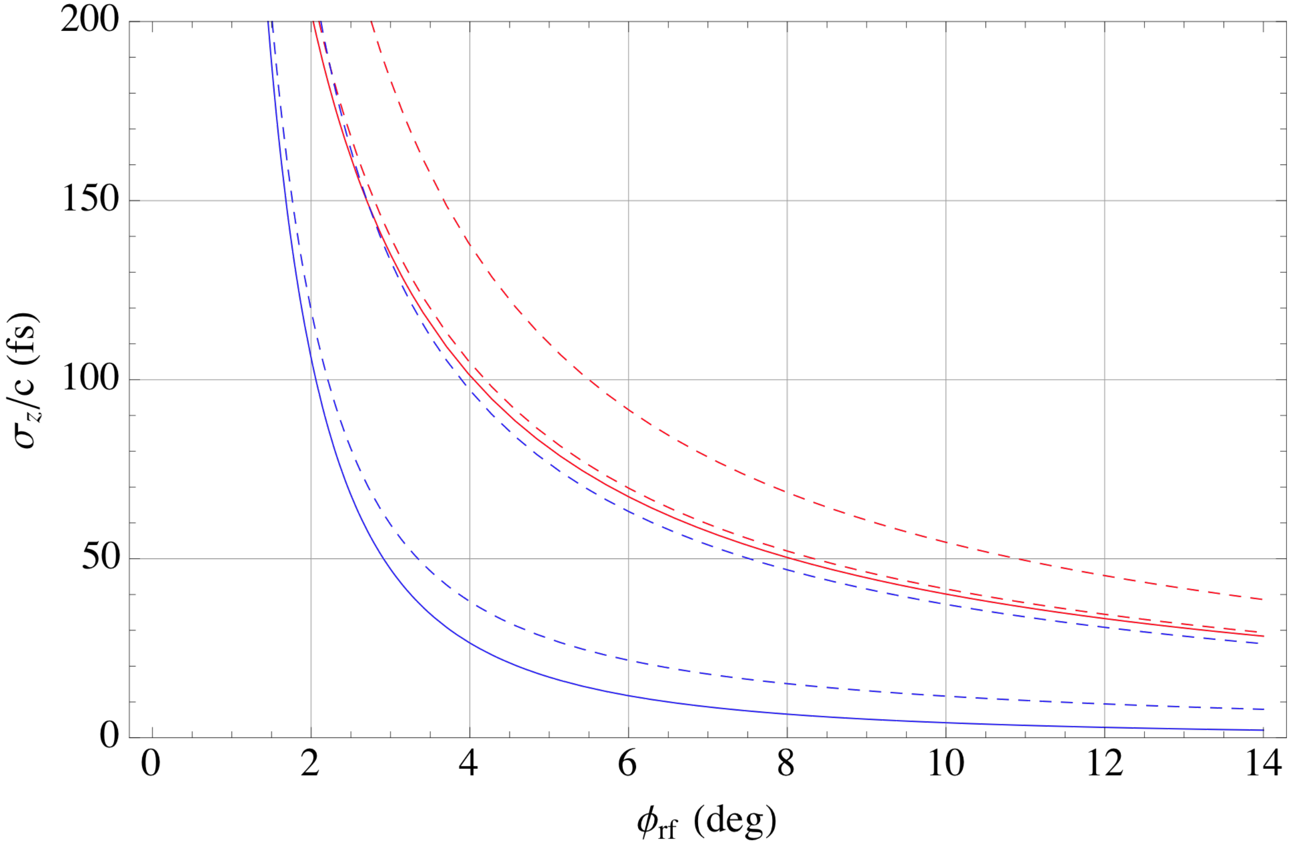
$$P^{(N)} = P^{(1)}N + P^{(1)}N(N-1)T\left(\frac{3\sigma_z\gamma^3}{2R\beta}\right),$$

$$T_c(a_c) \equiv \frac{9}{32\sqrt{\pi}\,a_c^3}\exp\left(\frac{1}{8a_c^2}\right)K_{5/6}\left(\frac{1}{8a_c^2}\right) - \frac{9}{16a_c^2}$$

$$T_c(a_c) \sim \frac{9\,\Gamma\left(\frac{5}{6}\right)}{16^{2/3}\sqrt{\pi}}\left(\frac{1}{a_c}\right)^{4/3} - \frac{9}{16}\left(\frac{1}{a_c}\right)^2 + \frac{9\,\Gamma\left(\frac{5}{6}\right)}{32\cdot 2^{2/3}\sqrt{\pi}}\left(\frac{1}{a_c}\right)^{10/3} + \ldots$$

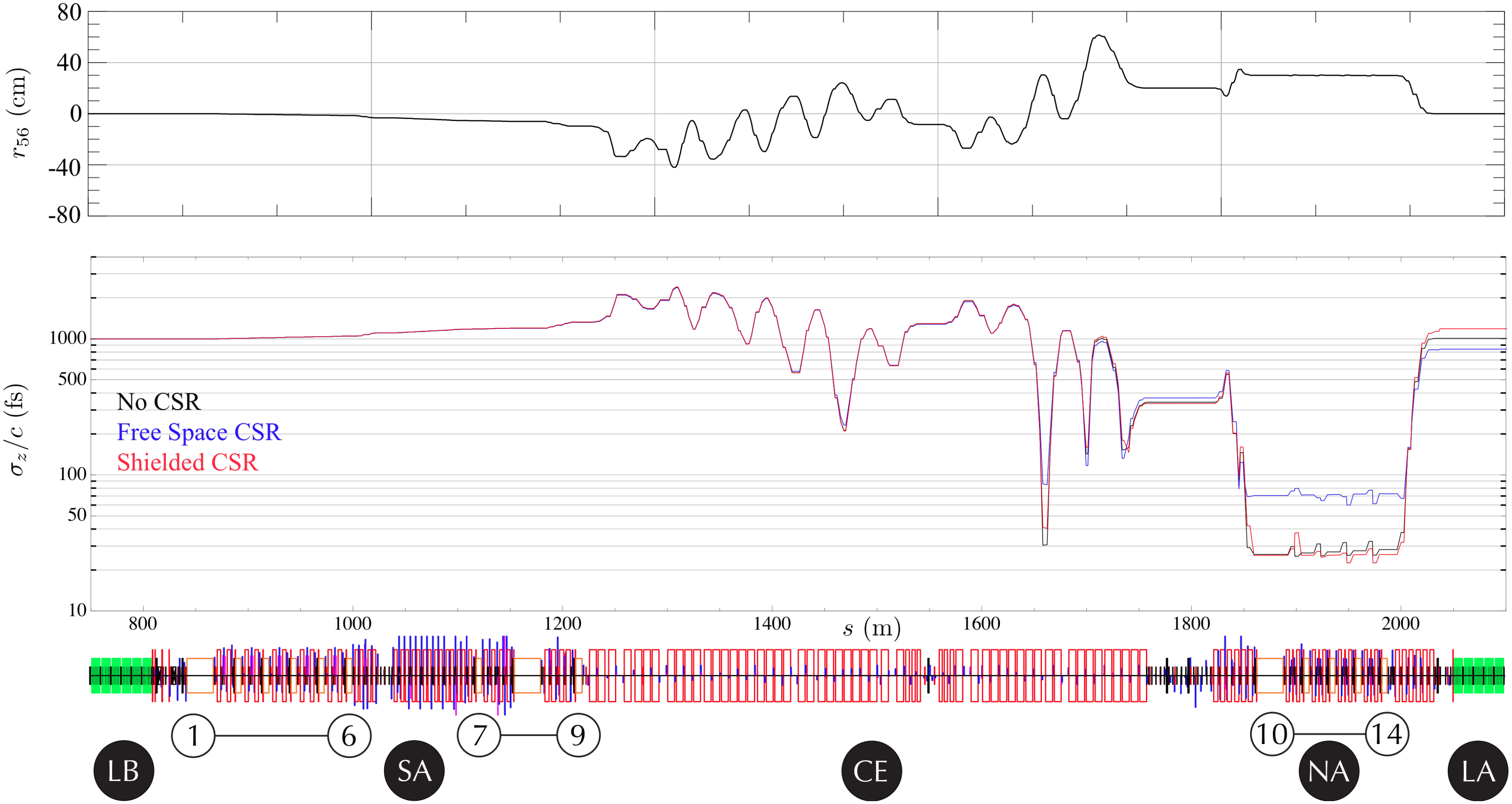
[First expansion term: Nodvick & Saxon (1954)]

Limits of Compression

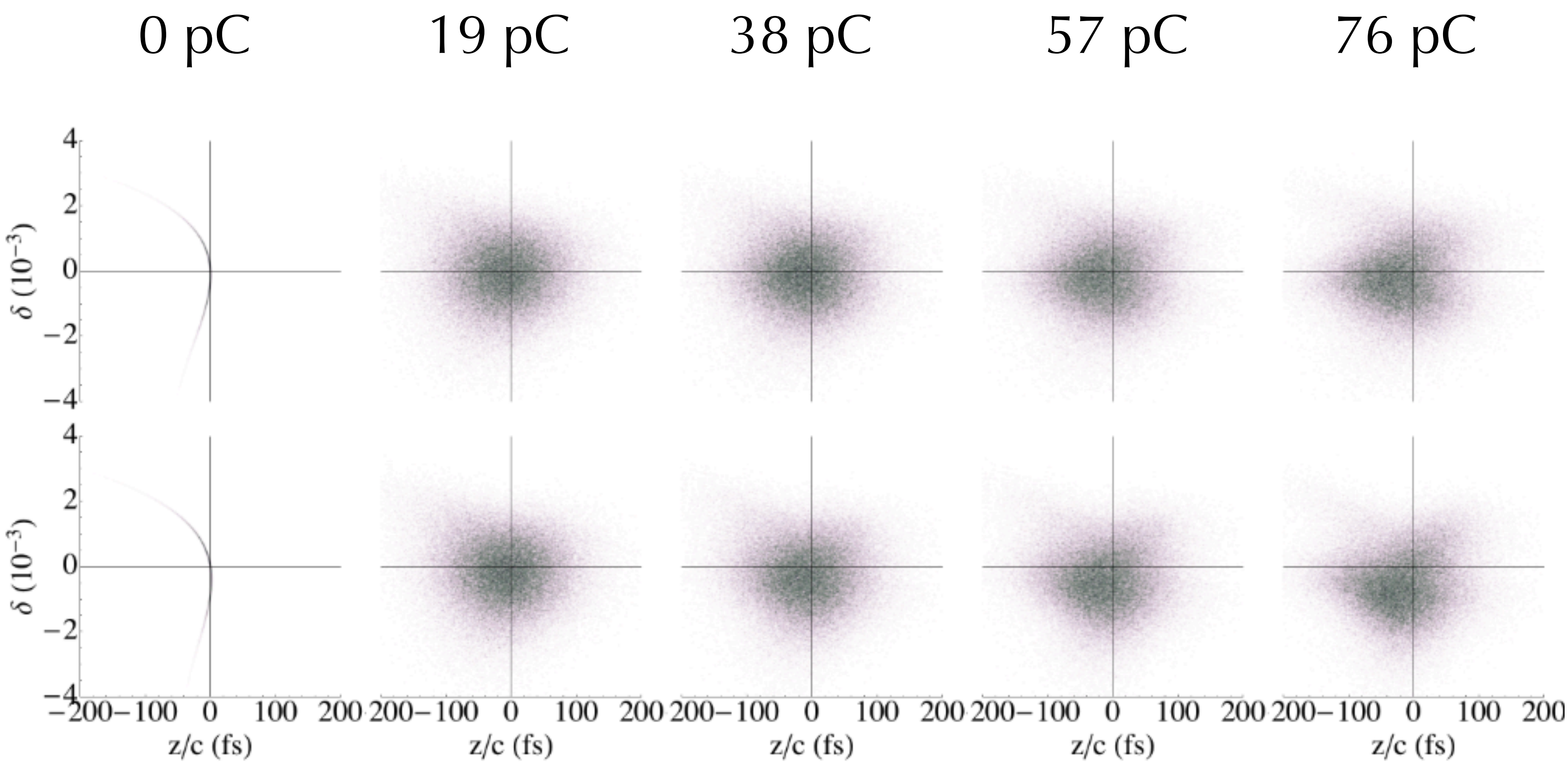


Bunch Lengths from Particle Tracking

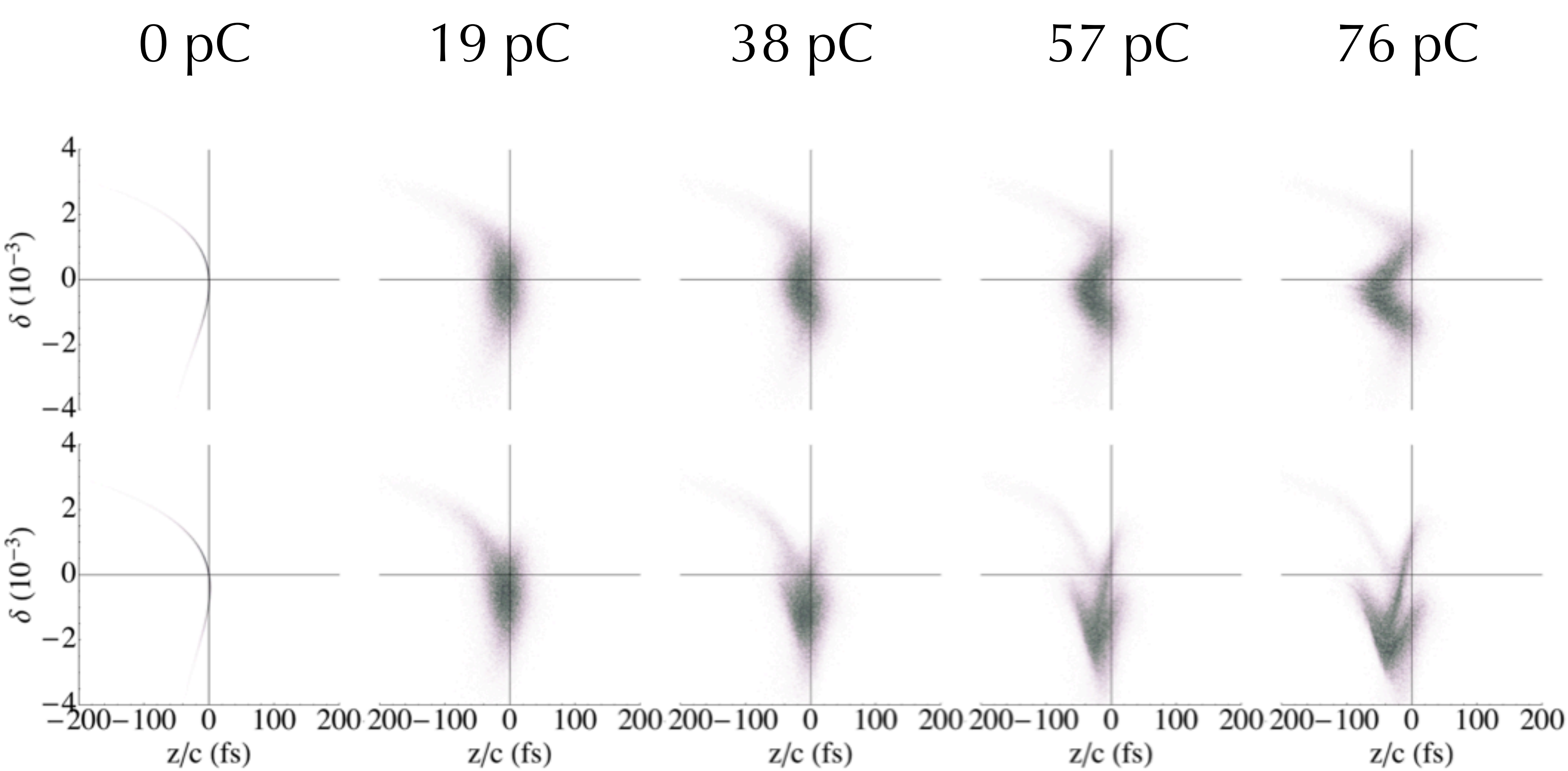
76 pC, compressed to 30 fs



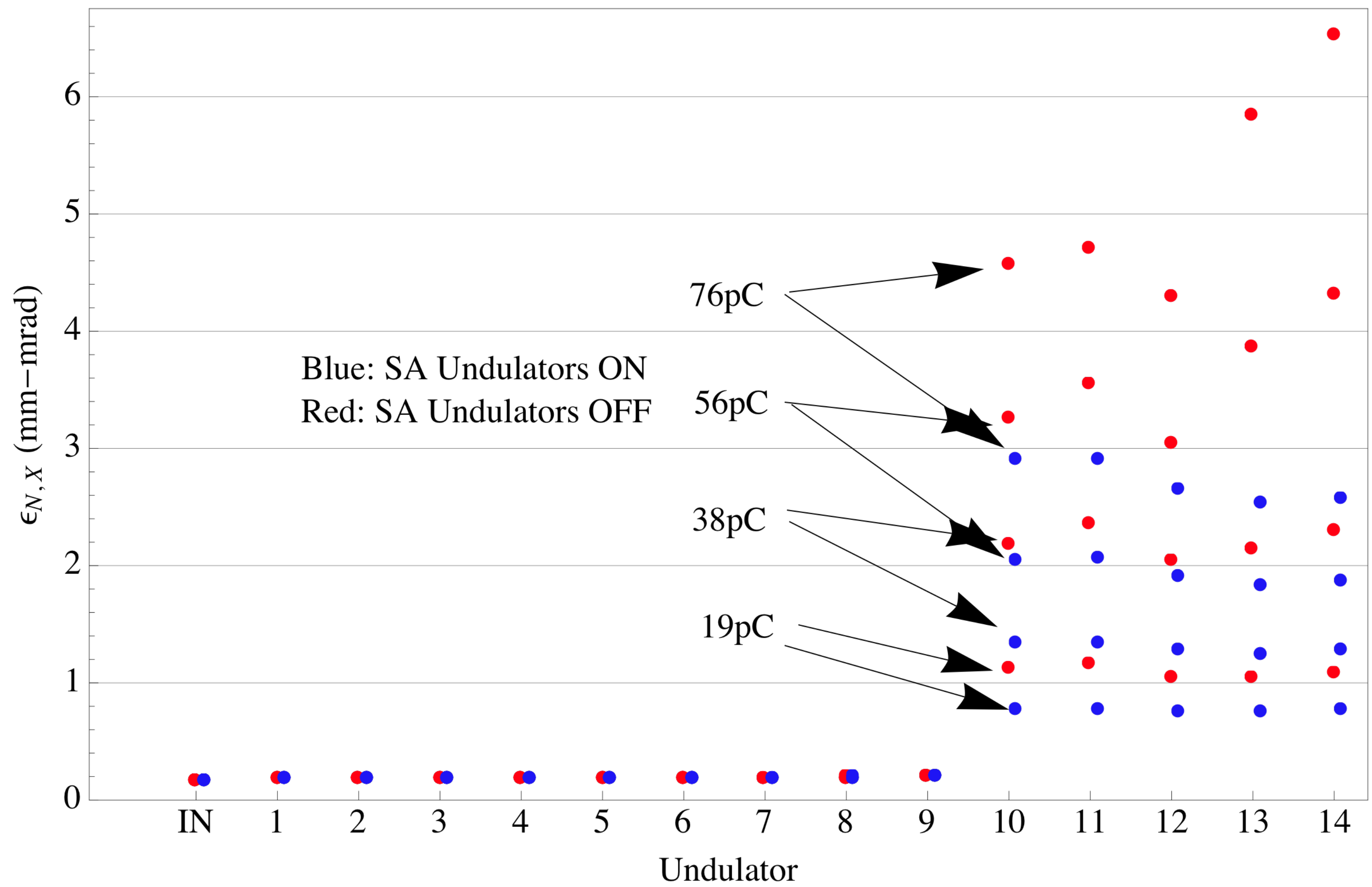
Compression with ISR energy spread



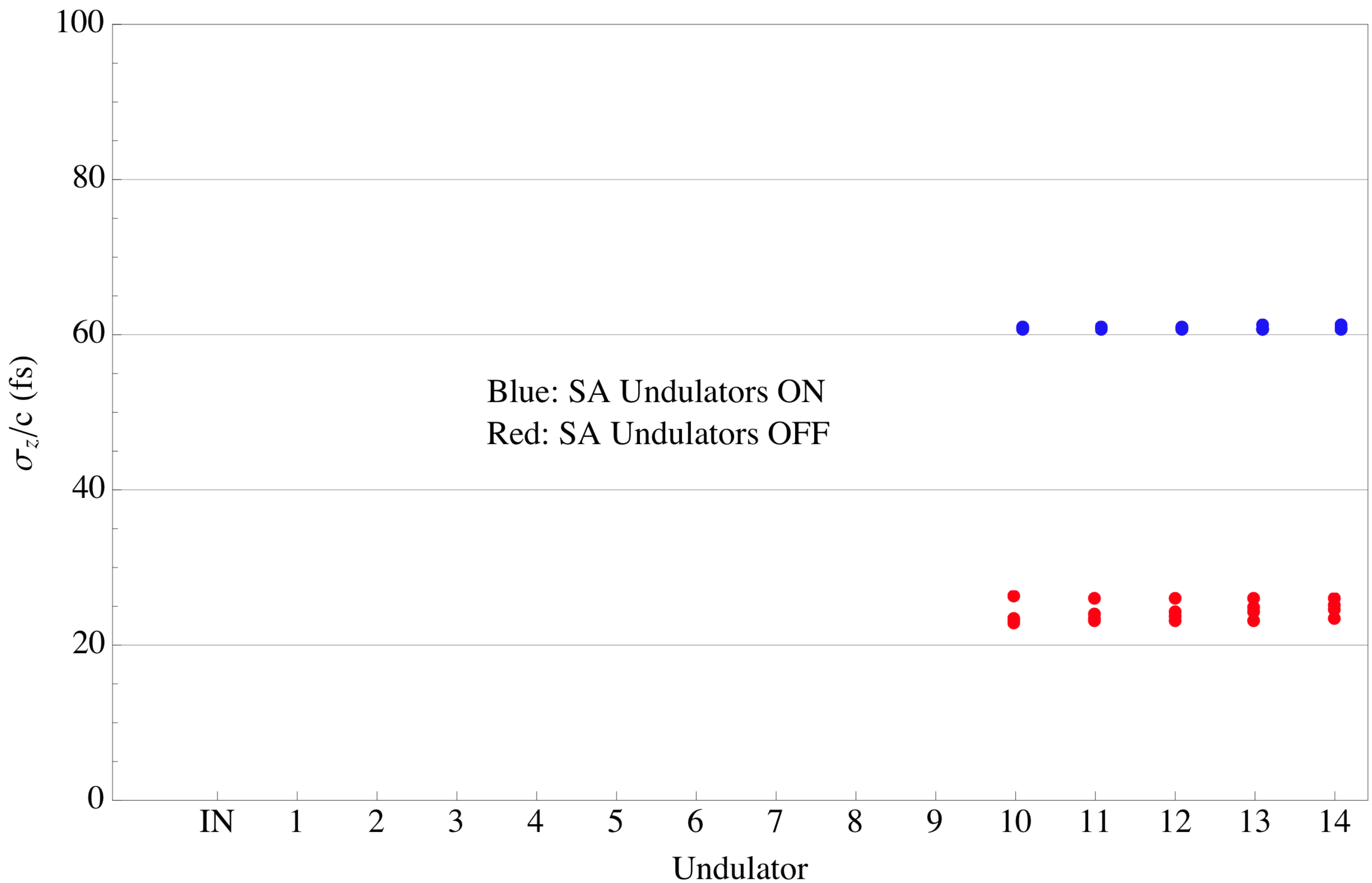
CSR Emittance Growth



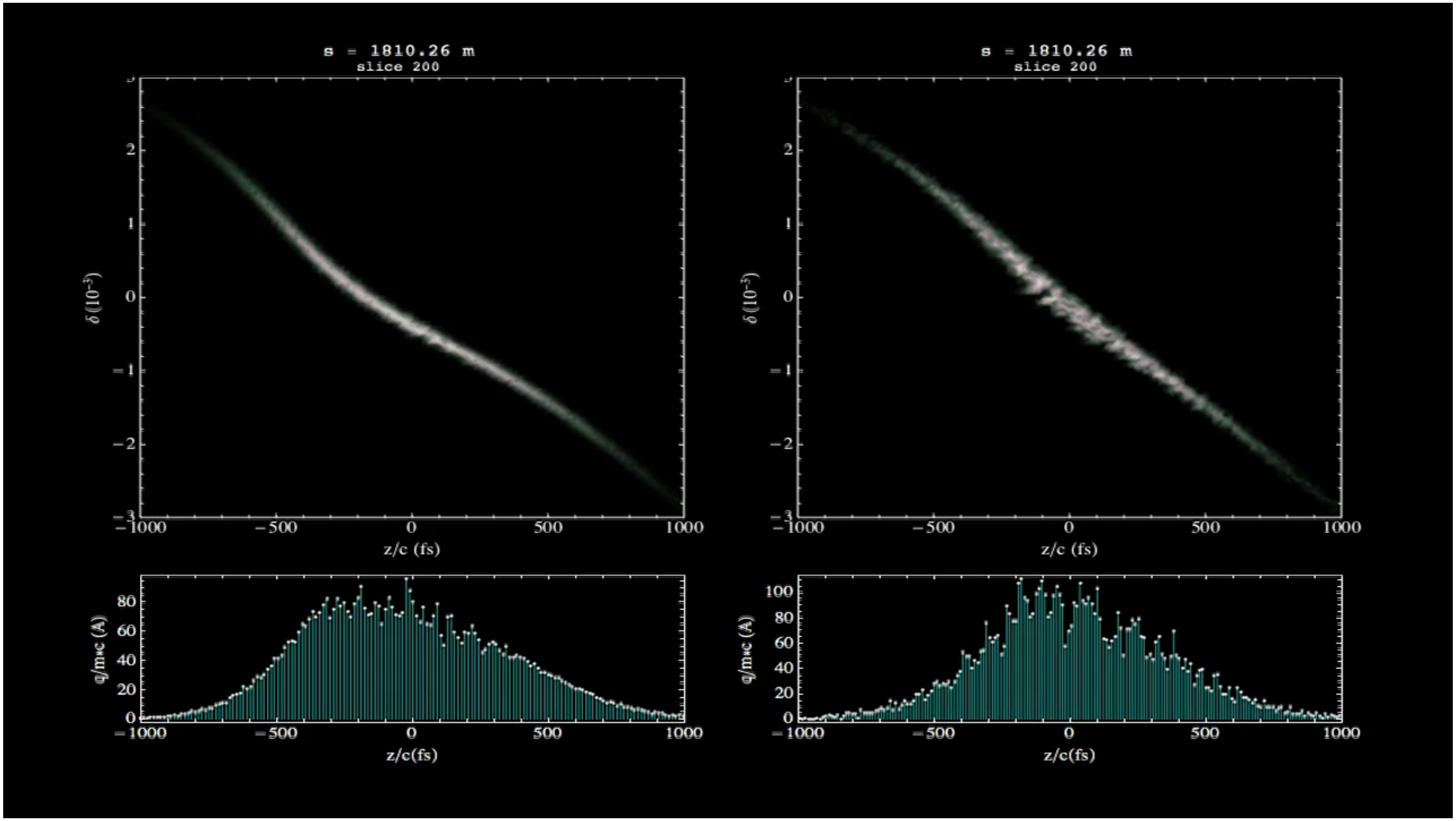
CSR Emittance Growth



CSR Emittance Growth



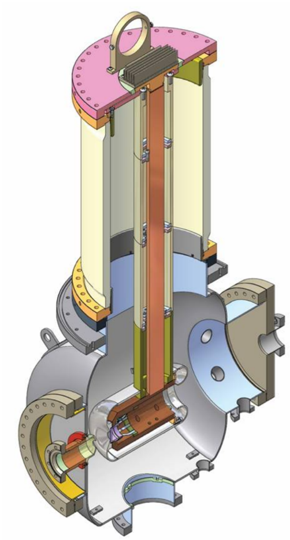
CERL Bunch Compression



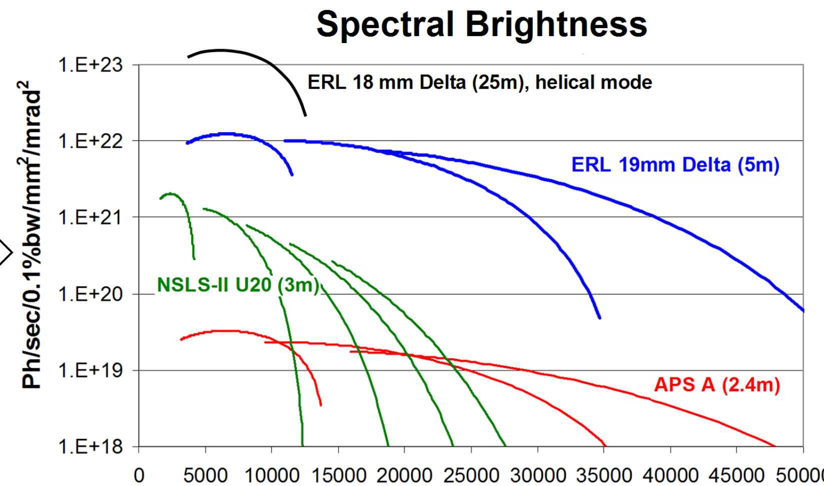
Free Space

1 inch beam pipe

Next Steps



OPAL & Bmad



I.V. Bazarov, S.A. Belomestnykh, D.H. Bilderback, M.G. Billing, J.D. Brock, E.P. Chojnacki, J.A. Crittenden, B.M. Dunham, M.P. Ehrlichman, M.J. Forester, S.M. Gruner, G.H. Hoffstaetter, C.J. Johnstone, Y.Li, M.U. Liepe, F. Loehl, C.E. Mayes, A.A. Mikhailichenko, H.S. Padamsee, S.B. Peck, D.C. Sagan, V.D. Shemelin, A. Temnykh, M. Tigner, V. Veshcherevich

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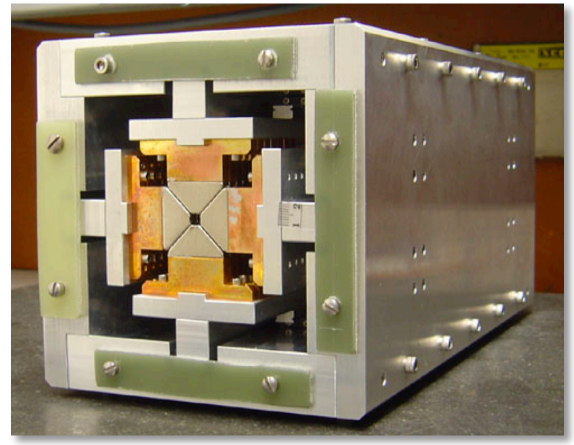
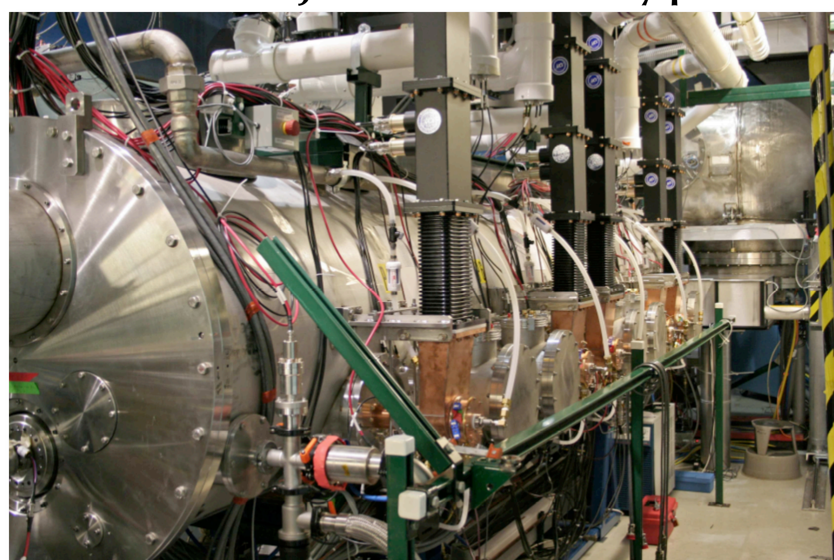
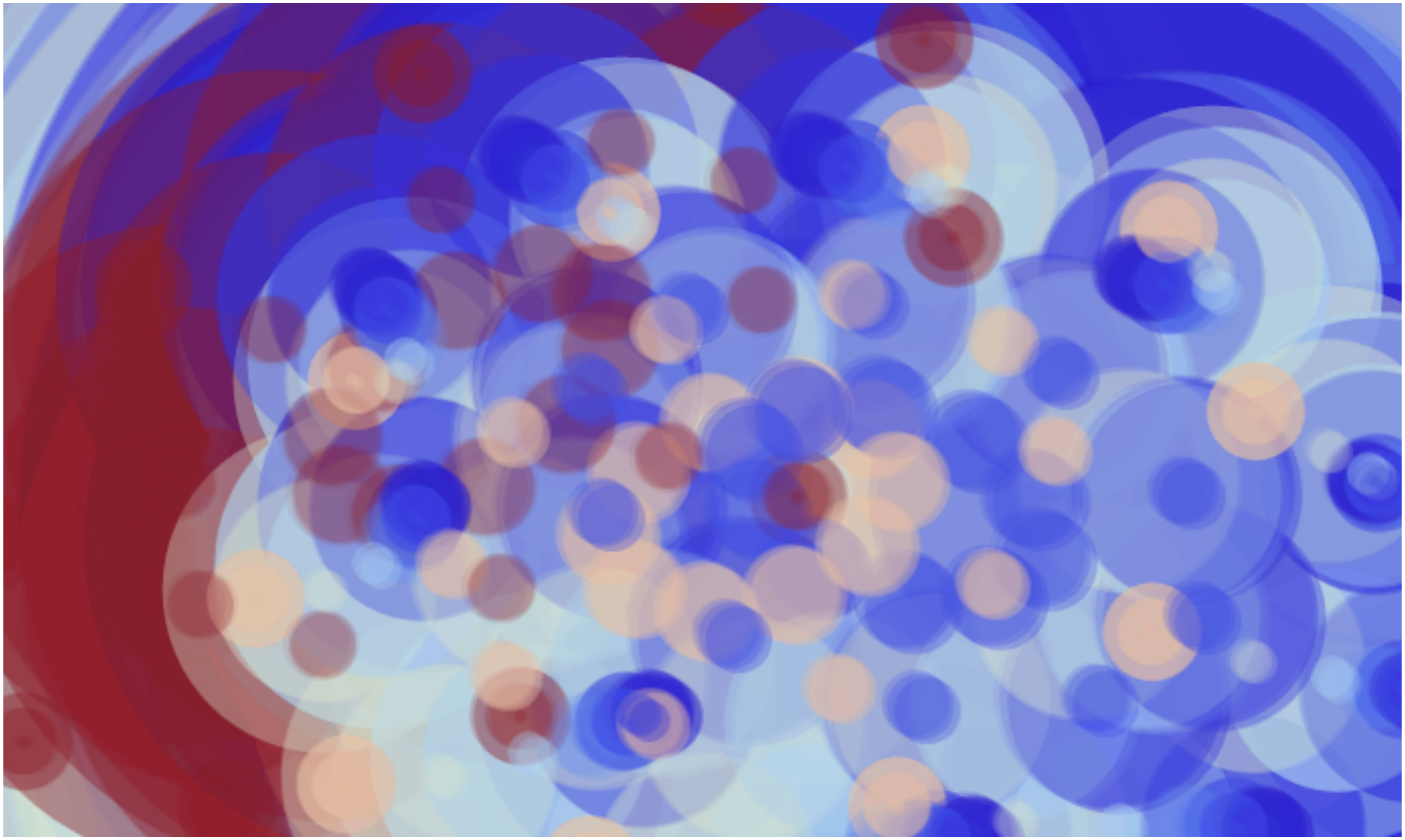
SRF Cavities

Delta Undulator



ERL Injector Prototype





End