Coherent electron cooling*

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Thanks to

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Content

- Why it is needed?
- Classical Coherent Electron Cooling (CeC)
- How to cool transversely
- Theory & Simulations
- Recent advances in CeC
  - Noise reduction in FEL
  - Spin cooling
- Proof of Principle test using R&D ERL
- Q&A

Why to cool hadron beams?

Measure of Collider Performance is the Luminosity

$$\dot{N}_{events} = \sigma_{A \to B} \cdot L$$

$$L = \frac{f_{coll} \cdot N_1 \cdot N_2}{4\pi\beta^* \varepsilon} \cdot g(\beta^*, h, \theta, \sigma_z)$$

Main sources of luminosity limitation

Large or growing emittance
Hour-glass effect
Crossing angle
Beam Intensity & Instabilities
Beam-Beam effects

eRHIC
(V. Litvinenko), Head
V. Ptitsyn, Deputy

Lattice & IR
(D. Trbojevic), GL
N. Tsoupas, Deputy
J. Beebe-Wang
(Y. Luo)
B. Parker, SMD
(S. Tepikian)

R&D ERL
D. Kayran, GL
B. Sheehy, Deputy
D. Pate
T. Rao, ID

CeC
(V. Litvinenko), GL
Y. Hao
G. Wang
S. Webb+

Polarized Sources
(I. Ben-Zvi), GL
X. Chang, Deputy
J. Skaritka
E. Tsentalovich++, MIT
(A. Zelenski)

Engineering
(J. Tuozzolo), GL
A. Jain, SMD
(G. Mahler)
(W. Meng)

Why to coherent electron cooling?

- Traditional stochastic cooling does not have enough bandwidth to cool modern-day proton beams
- Efficiency of traditional electron cooling falls as a high power of hadron's energy
- Synchrotron radiation is too fable – event at LHC energy cooling time is more than 10 hours
- Optical stochastic cooling is not suitable for cooling hadrons with large range of energies and has a couple of weak points:
    - Hadron do not like to radiate or absorb photons, the process which OSC uses twice
    - Tunabiity and power of laser amplifiers are limited

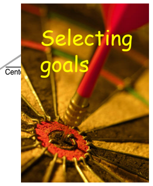
Examples of hadron beams cooling

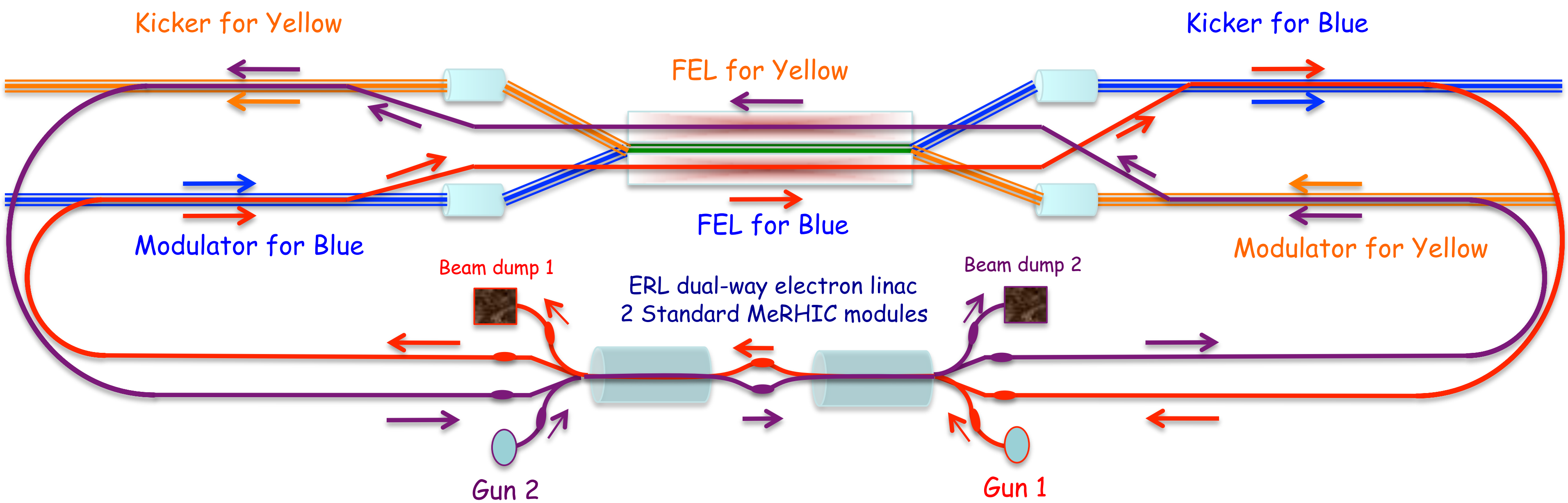
Machine	Species	Energy GeV/n	Trad. Stochastic Cooling, hrs	Synchrotron radiation, hrs	Trad. Electron cooling hrs	**Coherent Electron Cooling, hrs 1D/3D**
RHIC PoP	***Au***	***40***	-	**-**	**~ 1**	***0.02/0.06***
eRHIC	Au	130	~1	20,961 ∞	~ 1	0.015/0.05
eRHIC	**p**	**325**	**~100**	**40,246 ∞**	**> 30**	**0.1/0.3**
LHC	p	7,000	~ 1,000	13/26	∞ ∞	0.3/<1

Potential increases in luminosities:

RHIC polarized pp ~ 6 fold, eRHIC ~ 5-10 fold, LHC ~ 2 fold

One possible layout in RHIC IP of CeC driven by a single linac





E_p, GeV	γ	E_e, MeV
100	106.58	54.46
250	**266.45**	**136.15**
325	346.38	177.00

Gains from coherent e-cooling:

Coherent Electron Cooling vs. IBS

$$X = \frac{\varepsilon_x}{\varepsilon_{xo}};\ S = \left(\frac{\sigma_s}{\sigma_{so}}\right)^2 = \left(\frac{\sigma_E}{\sigma_{sE}}\right)^2;$$

$$\frac{dX}{dt} = \frac{1}{\tau_{IBS\perp}}\frac{1}{X^{3/2}S^{1/2}} - \frac{\xi_\perp}{\tau_{CeC}}\frac{1}{S};$$

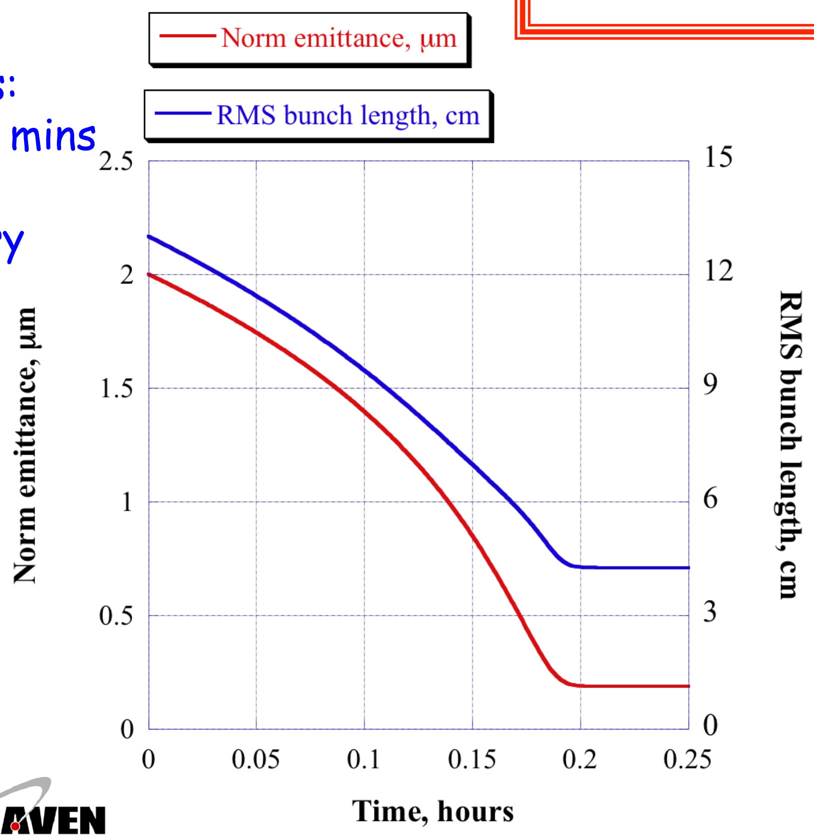
$$\frac{dS}{dt} = \frac{1}{\tau_{IBS//}}\frac{1}{X^{3/2}Y} - \frac{1-2\xi_\perp}{\tau_{CeC}}\frac{1}{X};$$

$$X = \frac{\tau_{CeC}}{\sqrt{\tau_{IBS//}\tau_{IBS\perp}}}\frac{1}{\sqrt{\xi_\perp(1-2\xi_\perp)}};\quad S = \frac{\tau_{CeC}}{\tau_{IBS//}}\cdot\sqrt{\frac{\tau_{IBS\perp}}{\tau_{IBS//}}}\cdot\sqrt{\frac{\xi_\perp}{(1-2\xi_\perp)^3}}$$

$$\varepsilon_{xn0} = 2\,\mu m;\ \sigma_{s0} = 13\ cm;\ \sigma_{\delta 0} = 4\cdot 10^{-4}$$
$$\tau_{IBS\perp} = 4.6\ \ hrs;\ \tau_{IBS//} = 1.6\ \ hrs$$

IBS in RHIC for eRHIC, 250 GeV, $N_p = 2\cdot 10^{11}$
Beta-cool, ©A.Fedotov

Dynamics:
Takes 12 mins to reach stationary point



$$\varepsilon_{xn} = 0.2\,\mu m;\ \sigma_s = 4.9\ \ \mathrm{cm}$$

This allows

a) keep the luminosity as it is
b) reduce polarized beam current down to 50 mA (10 mA for e-I)
c) increase electron beam energy to 20 GeV (30 GeV for e-I)
d) increase luminosity by reducing β* from 25 cm down to 5 cm

List of potential p-p luminosity increases from CeC

	Average IP Luminosity	Average Vertex Luminosity
Leveling luminosity at L_{peak}	2	2
Long stores with L=const	2	2
Short bunches	1.4	6 for ±10 cm 2 for ±30 cm
Total	5.6	24 for ±10 cm 8 for ±30 cm

Luminosity in eRHIC

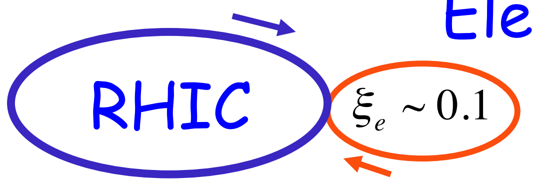
	eRHIC IR1		eRHIC IR2	
	p /A	e	p /A	e
Energy (max), GeV	325/130	20	325/130	20
Number of bunches	166	74 nsec	166	74 nsec
Bunch intensity (u) , 10^{11}	2.0	0.24	2.0	0.24
Bunch charge, nC	32	4	32	4
Beam current, mA	420	50	420	50
Normalized emittance, 1e-6 m, 95% for p / rms for e	1.2	25	1.2	25
Polarization, %	70	80	70	80
rms bunch length, cm	4.9	0.2	4.9	0.2
β^*, cm	25	25	5	5
Luminosity, $cm^{-2}s^{-1}$	2.8×10^{33}		1.4×10^{34}	

Luminosity for 30 GeV e-beam operation will be at 20% level

2007 Choosing the focus: ERL or ring for electrons
CeC is the key ingredient

- Two main design options for eRHIC:
  - Ring-ring:

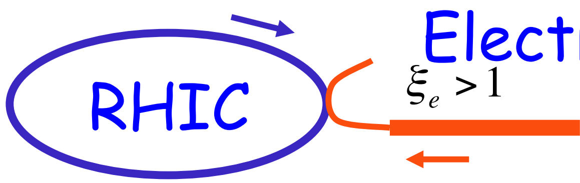
$$L = \left(\frac{4\pi\gamma_h\gamma_e}{r_h r_e}\right)(\xi_h\xi_e)(\sigma'_h\sigma'_e)f$$



Electron storage ring

  - Linac-ring:

$$L = \gamma_h f N_h \frac{\xi_h Z_h}{\beta^*_h r_h}$$



Electron linear accelerator

L x 10

Natural staging strategy

Evolution of beam in LHC at 7 TeV

(assuming nominal LHC bunch intensity 1.15e11 p/bunch and 40% of CeC cooling capability)

$$\frac{\sigma_\varepsilon^2}{\tau_{IBS//}} = \frac{N r_c^2 c}{2^5 \pi \gamma^3 \varepsilon_x^{3/2} \sigma_s} \left\langle \frac{f(\chi_m)}{\beta_y v} \right\rangle; \quad \frac{\varepsilon_x}{\tau_{IBS\perp}} = \frac{N r_c^2 c}{2^5 \pi \gamma^3 \varepsilon_x^{3/2} \sigma_s} \left\langle \frac{H}{\beta_y^{1/2}} f(\chi_m) \right\rangle; \kappa = 1$$

$$f(\chi_m) = \int_{\chi_m}^{\infty} \frac{d\chi}{\chi} \ln\left(\frac{\chi}{\chi_m}\right) e^{-\chi}; \ \chi_m = \frac{r_c m^2 c^4}{b_{\max} \sigma_E^2}; b_{\max} \cong n^{-1/3}; \ r_c = \frac{e^2}{mc^2}; \quad (e->Ze; m->Am)$$

J.LeDuff, "Single and Multiple Touschek effects", Proceedings of CERN Accelerator School, Rhodes, Greece, 20 September - 1 October, 1993, Editor: S.Turner, CERN 95-06, 22 November 1995, Vol. II, p. 573

$$X = \frac{\varepsilon_x}{\varepsilon_{xo}}; S = \left(\frac{\sigma_s}{\sigma_{so}}\right)^2 = \left(\frac{\sigma_E}{\sigma_{sE}}\right)^2;$$

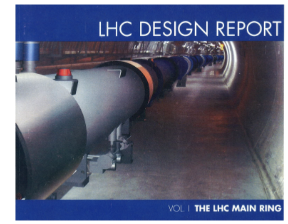
$$\frac{dX}{dt} = \frac{1}{\tau_{IBS\perp}} \frac{1}{X^{3/2} S^{1/2}} - \frac{\xi_\perp}{\tau_{CeC}} \frac{1}{S};$$

$$\frac{dS}{dt} = \frac{1}{\tau_{IBS//}} \frac{1}{X^{3/2} S^{1/2}} - \frac{1 - 2\xi_\perp}{\tau_{CeC}} \frac{1}{X};$$

$$\varepsilon_{xn0} = 3.75\,\mu m; \ \sigma_{s0} = 7.55\,cm$$
$$\tau_{IBS\perp} = 80 \ \ hrs; \ \tau_{IBS//} = 61 \ \ hrs$$

IBS rates in LHC from

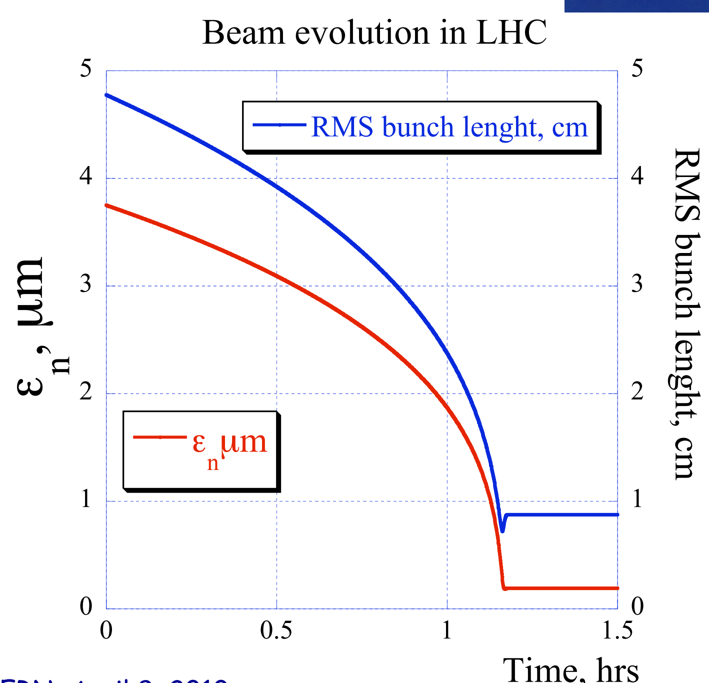
Table 2.2



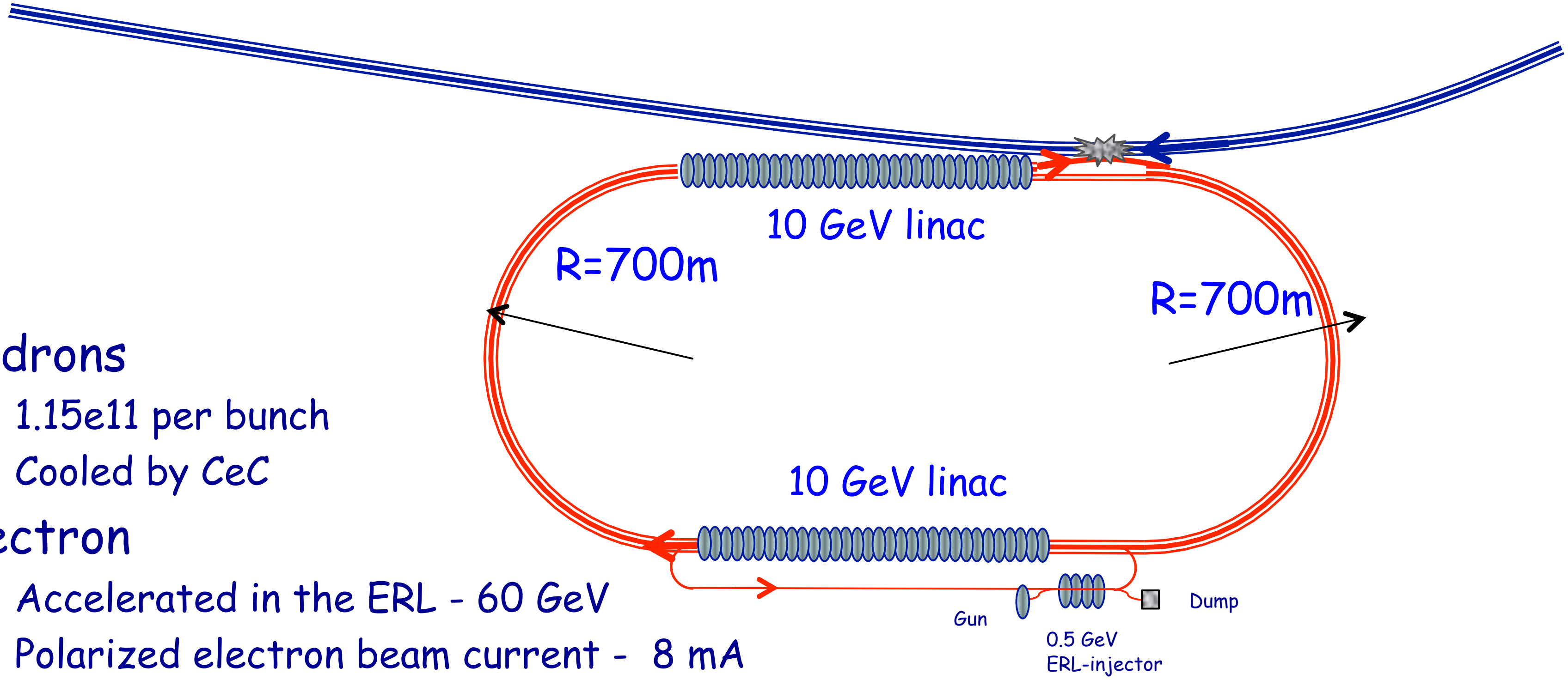
Stationary solution for τ_{CeC} = 0.8 hrs

$$X = \frac{\tau_{CeC}}{\sqrt{\tau_{IBS//}\tau_{IBS\perp}}} \frac{1}{\sqrt{\xi_\perp (1 - 2\xi_\perp)}}; \quad S = \frac{\tau_{CeC}}{\tau_{IBS//}} \cdot \sqrt{\frac{\tau_{IBS\perp}}{\tau_{IBS//}}} \cdot \sqrt{\frac{\xi_\perp}{(1 - 2\xi_\perp)^3}}$$

$$\varepsilon_{xn} = 0.19\,\mu m; \ \sigma_s = 0.87 \ \ \text{cm}$$



Layout for ERL based LHC

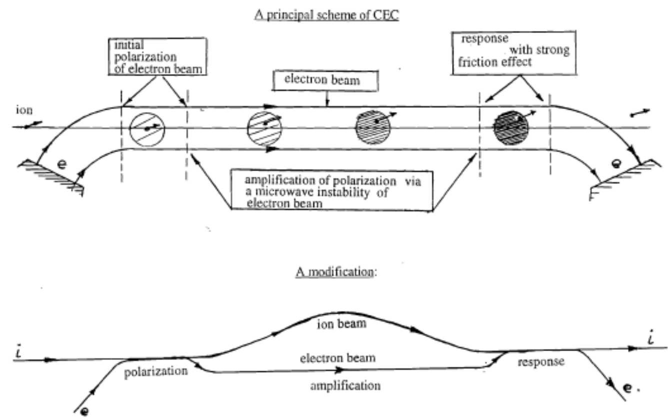


- Hadrons
  - 1.15e11 per bunch
  - Cooled by CeC
- Electron
  - Accelerated in the ERL - 60 GeV
  - Polarized electron beam current - 8 mA
- Number of passes – 3
- AC power consumption – 100 MW
- Crab-crossing
- β^{*}=12 cm
- L = $2 \cdot 10^{34}$ cm^{-2} sec^{-1}

History

possibility of coherent electron cooling was discussed qualitatively by Yaroslav Derbenev about 30 years ago

- Y.S. Derbenev, Proceedings of the 7th National Accelerator Conference, V. 1, p. 269, (Dubna, Oct. 1980)
- Coherent electron cooling, Ya. S. Derbenev, Randall Laboratory of Physics, University of Michigan, MI, USA, UM HE 91-28, August 7, 1991
- Ya.S.Derbenev, Electron-stochastic cooling, DESY , Hamburg, Germany, 1995 ……….



UM HE 91-28

August 7, 1991

COHERENT ELECTRON COOLING

1. Physics of the method in general

Ya. S. Derbenev
Randall Laboratory of Physics, University of Michigan
Ann Arbor, Michigan 48109-1120 USA

CONCLUSION

The method considered above combines principles of electron and stochastic cooling and microwave amplification. Such an unification promises to frequently increase the cooling rate and stacking of high-temperature, intensive heavy particle beams. Certainly, for the whole understanding of new possibilities thorough theoretical study is required of all principle properties and other factors of the method.

What's new in today's presentation?

- It is a new CeC is the scheme and the first with complete analytical and quantitative evaluation
- The spirit of amplifying the interaction remains the same as in 80's. but the underlying physics of interaction is different and also specific
- Now we can analytically estimate and numerically calculate CeC cooling decrements for a wide variety of cases
- FEL theory relevant for CeC is expanded significantly
- Practical scheme for very high gain FEL amplifiers with noise suppression was developed
- Recently we suggested spin cooling
- There are two practical schemes to test CeC at RHIC

Start from longitudinal cooling , ultra-relativistic case (γ>>1)

At a half of plasma oscillation

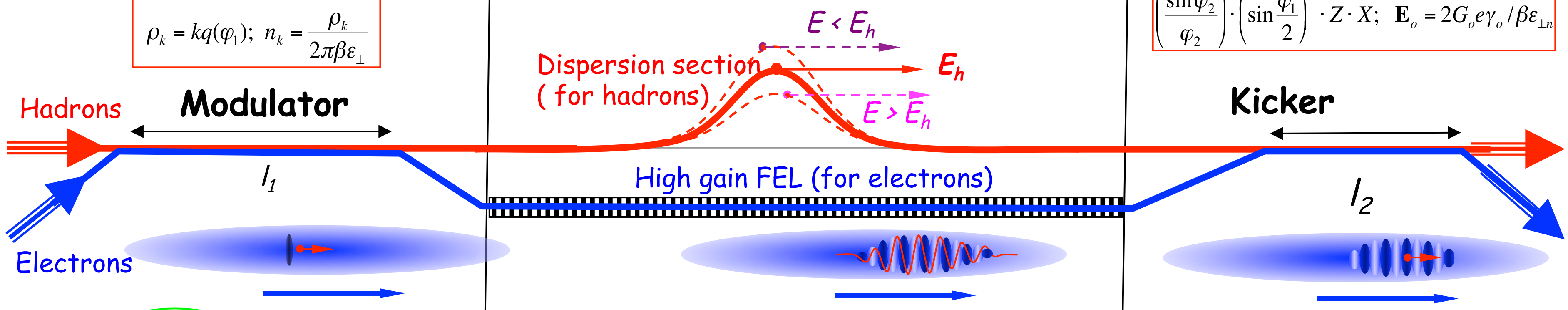
$$q_{\lambda_{FEL}} \approx \int_0^{\lambda_{FEL}} \rho(z)\cos(k_{FEL}z)dz$$

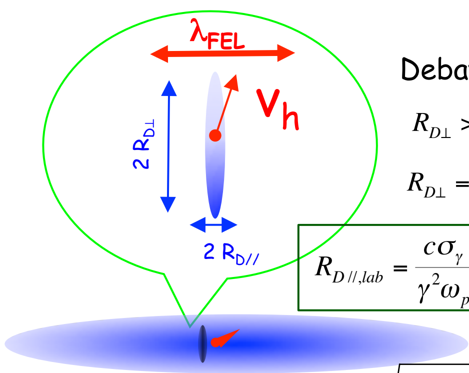
$$\rho_k = kq(\varphi_1);\; n_k = \frac{\rho_k}{2\pi\beta\varepsilon_\perp}$$

Dispersion

$$c\Delta t = -D\cdot\frac{\gamma-\gamma_o}{\gamma_o};\; D_{free} = \frac{L}{\gamma^2};\; D_{chicane} = l_{chicane}\cdot\theta^2 ....$$

$$\Delta E_h = -e\cdot\mathbf{E}_o\cdot l_2\cdot\sin\left(k_{FEL}D\frac{E-E_o}{E_o}\right)\cdot\left(\frac{\sin\varphi_2}{\varphi_2}\right)\cdot\left(\sin\frac{\varphi_1}{2}\right)^2\cdot Z\cdot X;\; \mathbf{E}_o = 2G_o e\gamma_o/\beta\varepsilon_{\perp n}$$





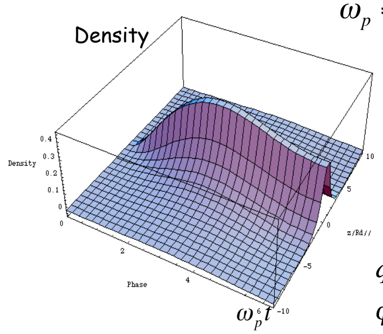
Debay radii

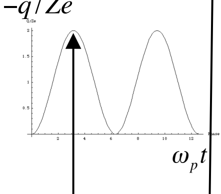
$$R_{D\perp} >> R_{D//}$$

$$R_{D\perp} = \frac{c\gamma\sigma_{\theta e}}{\omega_p}$$

$$\boxed{R_{D//,lab} = \frac{c\sigma_\gamma}{\gamma^2\omega_p} << \lambda_{FEL}}$$

$$\omega_p = \sqrt{4\pi n_e e^2/\gamma_o m_e}$$



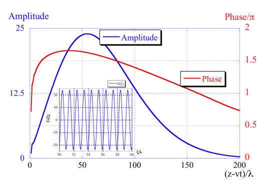


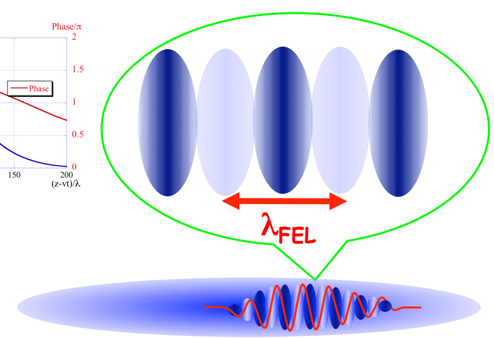
$$q = -Ze\cdot(1-\cos\varphi_1)$$

$$\varphi_1 = \omega_p l_1/c\gamma$$

$$q_{peak} = -2Ze$$

Amplifier of the e-beam modulation in an FEL with gain G_{FEL}~10^2-10^3





$$\lambda_{fel} = \lambda_w\left(1+\langle\vec a_w^2\rangle\right)/2\gamma_o^2$$

$$\vec a_w = e\vec A_w/mc^2$$

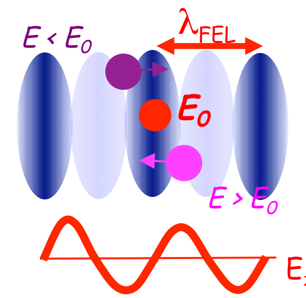
$$L_{Go} = \frac{\lambda_w}{4\pi\rho\sqrt3}$$

$$L_G = L_{Go}(1+\Lambda)$$

$$G_{FEL} = e^{L_{FEL}/L_G}$$

$$\Delta\varphi = \frac{L_{FEL}}{\sqrt3 L_G}$$

$$A_\perp = 2\pi\beta_\perp\varepsilon_n/\gamma_o$$



$$k_{FEL} = 2\pi/\lambda_{FEL};\; k_{cm} = k_{FEL}/2\gamma_o$$

$$\boxed{n_{amp} = G_o\cdot n_k\cos(k_{cm}z)}$$

$$\Delta\varphi = 4\pi en \Rightarrow \varphi = -\varphi_0\cdot\cos(k_{cm}z)$$

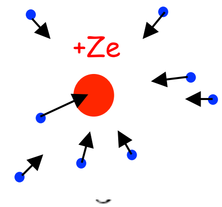
$$\vec{\mathbf{E}} = -\vec\nabla\varphi = -\hat z\mathbf{E}_o\cdot X\sin(k_{cm}z)$$

$$\mathbf{E}_o = 2G_o\gamma_o\frac{e}{\beta\varepsilon_{\perp n}}$$

$$X = q/e \cong Z(1-\cos\varphi_1) \sim Z$$

e-Density modulation caused by a hadron (co-moving frame)

Induces charge: $q = -Ze \cdot (1 - \cos\omega_p t)$



Analytical: for kappa-2 anisotropic electron plasma,

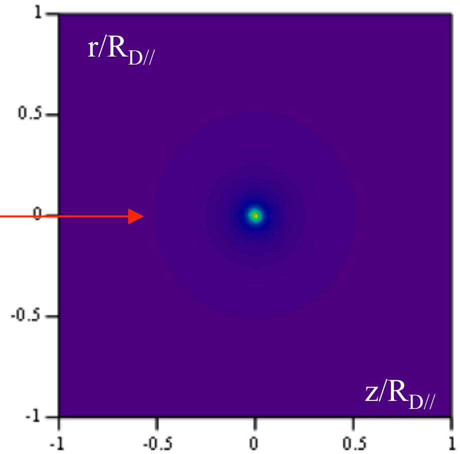
G. Wang and M. Blaskiewicz, Phys Rev E 78, 026413 (2008)

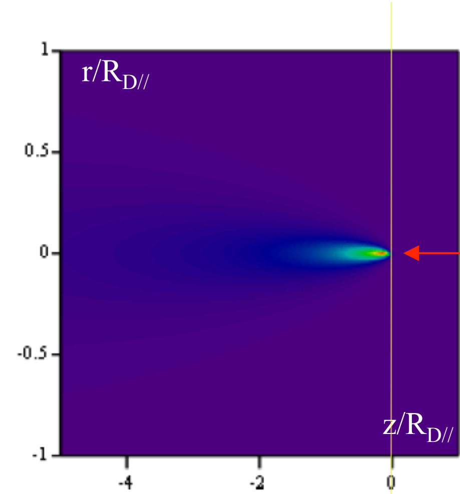
$$\tilde{n}(\vec{r},t) = \frac{Z n_o \omega_p^3}{\pi^2 \sigma_{vx}\sigma_{vy}\sigma_{vz}} \int_0^{\omega_p t} \tau \sin\tau \left(\tau^2 + \left(\frac{x - v_{hx}\tau/\omega_p}{r_{Dx}}\right)^2 + \left(\frac{y - v_{hy}\tau/\omega_p}{r_{Dy}}\right)^2 + \left(\frac{z - v_{hz}\tau/\omega_p}{r_{Dz}}\right)^2 \right)^{-2} d\tau$$

Density plots for a quarter of plasma oscillation

Ion rests in c.m.

(0,0) is the location of the ion





Ion moves in c.m. with

$$v_{hz} = 10\sigma_{vze}$$

(0,0) is the location of the ion

Numerical: VORPAL @ TechX)

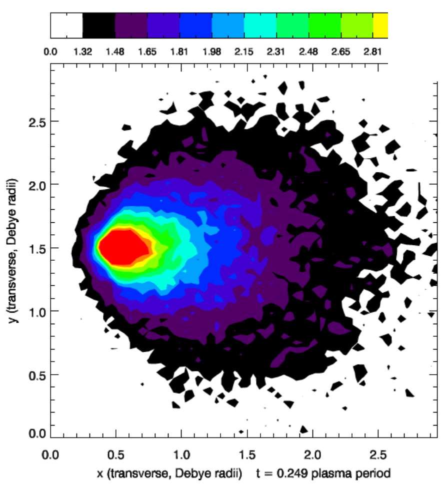


Figure 3: A transverse cross section of the wake behind a gold ion, with the color denoting density enhancement.

Parameters of the problem

$$R_{D_\alpha} \propto \left(|v_\alpha| + \sigma_{v_\alpha}\right)/\omega_p; \quad \alpha = x,y,z$$

$$t = \tau/\omega_p; \ \vec{v} = \vec{\tilde{v}}\sigma_{v_z}; \ \vec{r} = \vec{\rho}\sigma_{v_z}/\omega_p; \ \omega_p = \sqrt{\frac{4\pi e^2 n_e}{m}} \qquad s = r_{D_z} = \frac{\sigma_{v_z}}{\omega_p}$$

$$R = \frac{\sigma_{v_\perp}}{\sigma_{v_z}}; \ T = \frac{v_{hx}}{\sigma_{v_z}}; \ L = \frac{v_{hz}}{\sigma_{v_z}}; \ \xi = \frac{Z}{4\pi n_e R^2 s^3};$$

$$A = \frac{a}{s}; \ X = \frac{x_{ho}}{a}; Y = \frac{y_{ho}}{a}.$$

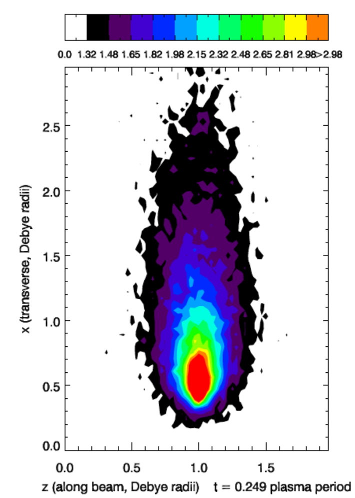
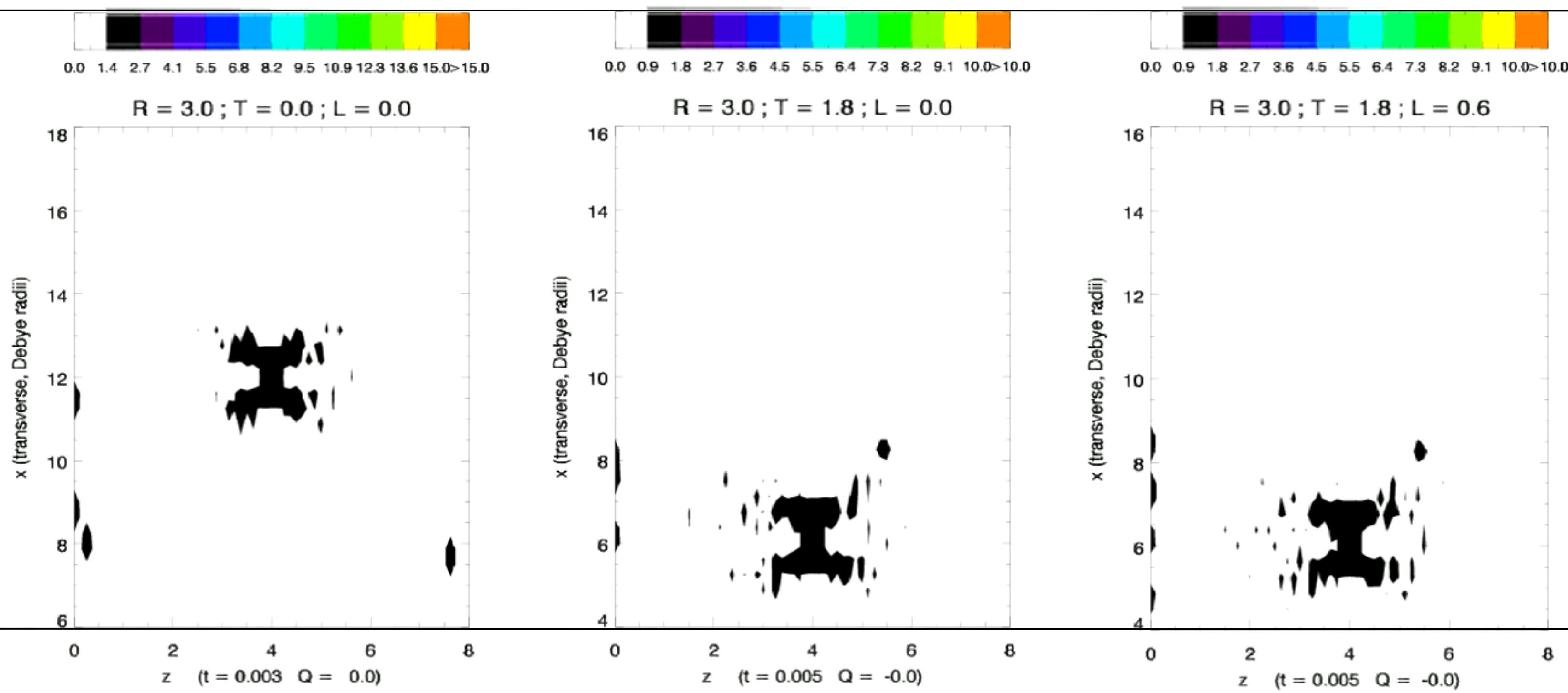


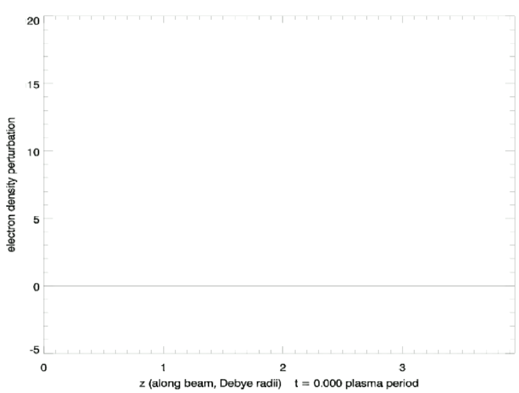
Figure 4: A longitudinal cross section of the wake behind a gold ion, with the color denoting density enhancement.

V.N. Litvinenko, Jlab/CASA April 29, 2010

Numerical simulations (VORPAL @ TechX) Provides for simulation with arbitrary distributions and finite electron beam size

VORPAL Simulations Relevant to Coherent Electron Cooling, G.I. Bell et al., EPAC'08, (2008)



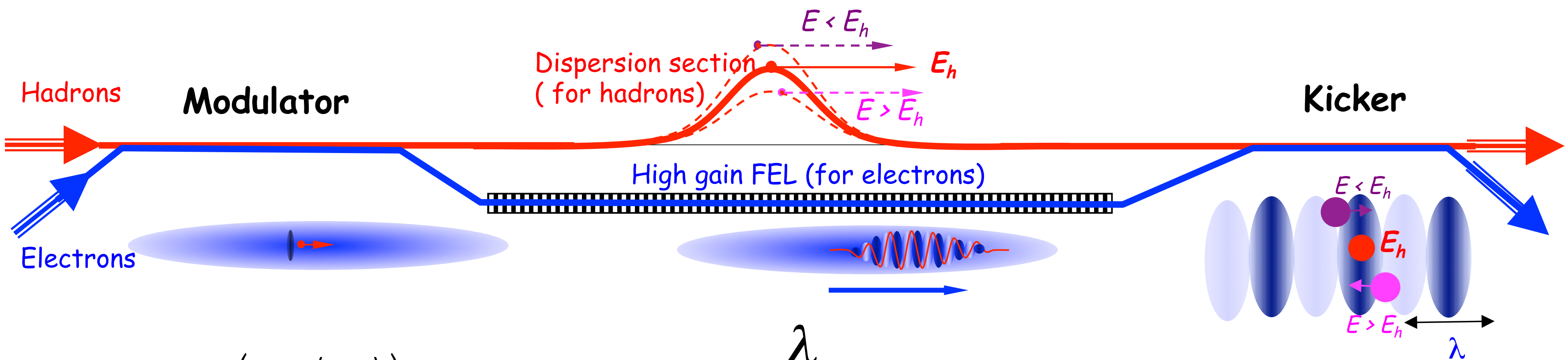


$$R = \frac{\sigma_{v_\perp}}{\sigma_{v_z}};\ T = \frac{v_{hx}}{\sigma_{v_z}};\ L = \frac{v_{hz}}{\sigma_{v_z}}$$

$$q = -Ze \cdot (1 - \cos\omega_p t)$$

Central Section of CeC

$$D = D_{free} + D_{chicane};\; D_{free} = \frac{L}{\gamma^2};\quad D_{chicane} = l_{chicane} \cdot \theta^2$$



$$\lambda_{fel} = \lambda_w \left(1 + \left\langle \vec{a}_w^{\,2} \right\rangle\right)/2\gamma_o^{\;2} \qquad L_{Go} = \frac{\lambda_w}{4\pi\rho\sqrt{3}} \qquad L_G = L_{Go}(1+\Lambda)$$

Electron density modulation is amplified in the FEL and made into a train with duration of $N_c \sim L_{gain}/\lambda_w$ alternating hills (high density) and valleys (low density) with period of FEL wavelength λ. Maximum gain for the electron density of High Gain FEL is ~ 10^3.

$$v_{group} = (c + 2v_{//})/3 = c\left(1 - \frac{1 + a_w^2}{3\gamma^2}\right) = c\left(1 - \frac{1}{2\gamma^2}\right) + \frac{c}{3\gamma^2}\left(1 - 2a_w^2\right) = v_{hadrons} + \frac{c}{3\gamma^2}\left(1 - 2a_w^2\right)$$

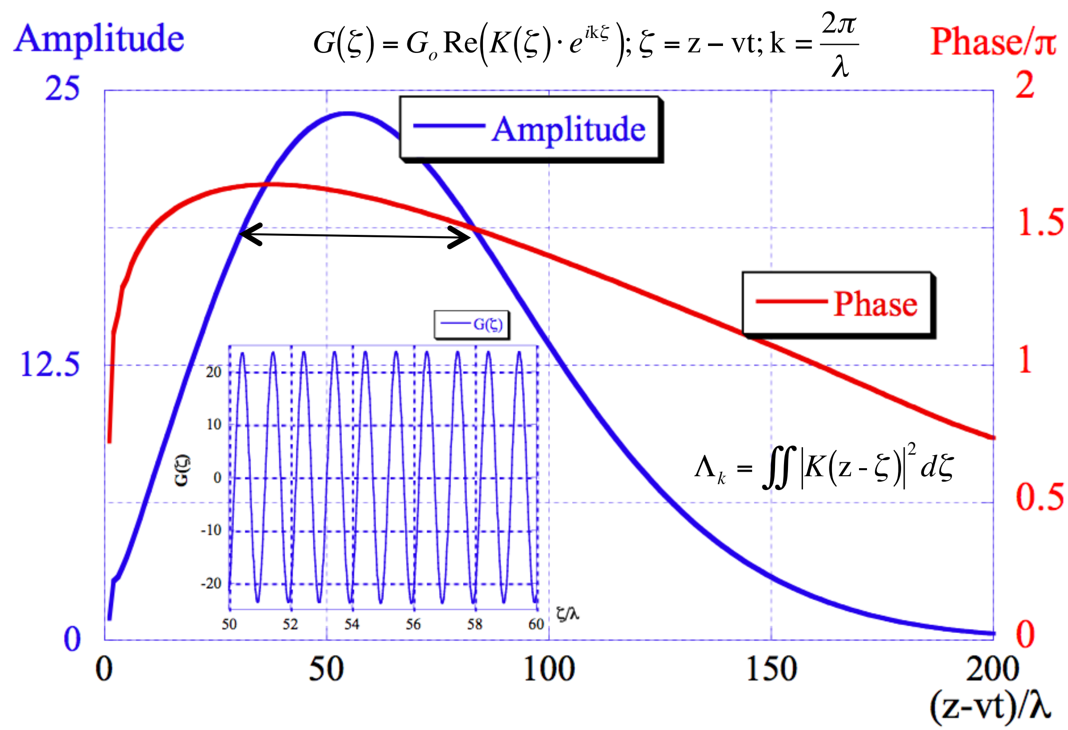
Economic option requires: $2a_w^2 < 1$!!!

3D FEL response
calculated Genesis 1.3, confirmed by RON

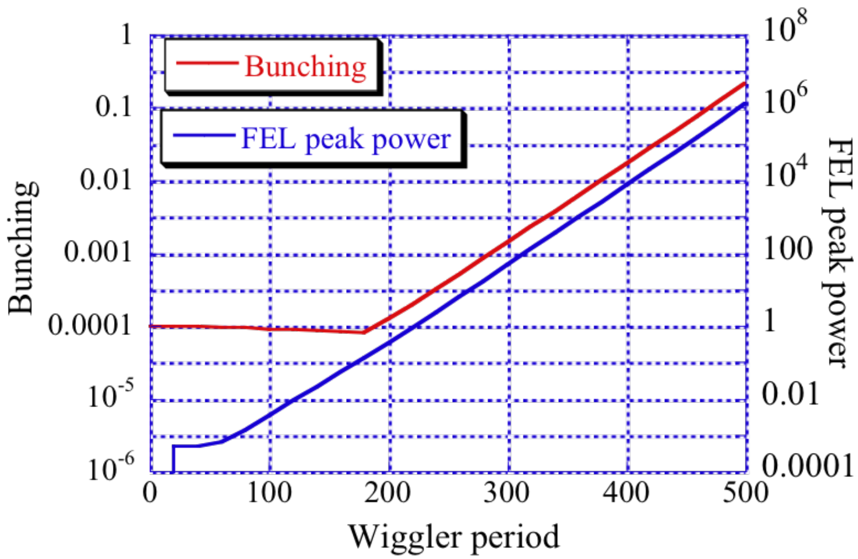
Main FEL parameters for eRHIC with 250 GeV protons

Energy, MeV	136.2	γ	266.45
Peak current, A	100	λ_o, nm	700
Bunchlength, psec	50	λ_w, cm	5
Emittance, norm	5 mm mrad	a_w	0.994
Energy spread	0.03%	Wiggler	Helical

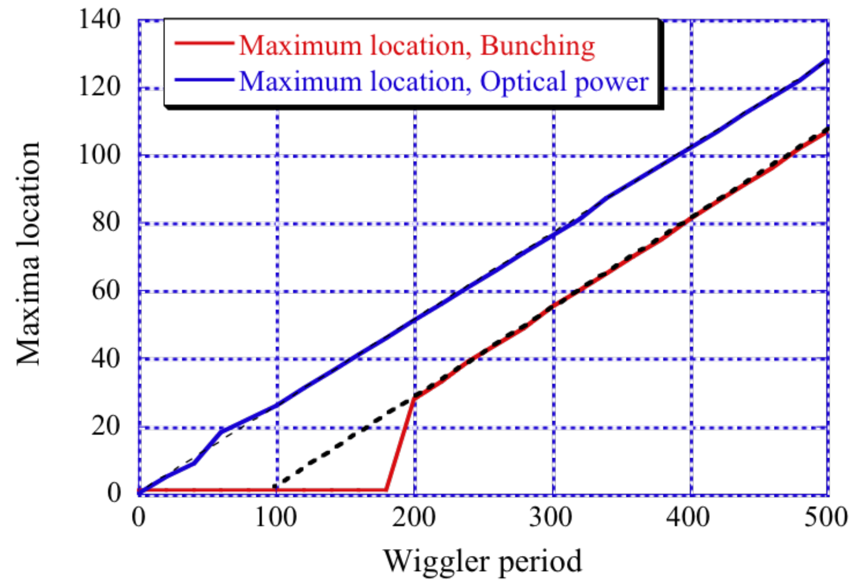
The amplitude (**blue line**) and the phase (**red line**, in the units of π) of the FEL gain envelope after 7.5 gain-lengths (300 period). Total slippage in the FEL is 300λ, λ=0.5 μm. A clip shows the central part of the full gain function for the range of ζ= {50λ, 60λ}.



Genesis: 3D FEL



Evolution of the maximum bunching in the e-beam and the FEL power simulated by Genesis.
The location of the maxima, both for the optical power and the bunching progresses with a lower speed compared with prediction by 1D theory,
i.e. electrons carry ~75% for the "information"



Evolution of the maxima locations in the e-beam bunching and the FEL power simulated by Genesis.
Gain length for the optical power is 1 m (20 periods) and for the amplitude/modulation is 2m (40 periods)

$$\boxed{\mathrm{v}_g \cong \frac{c + 3\langle \mathrm{v}_z \rangle}{4} = c\left(1 - \frac{3}{8}\frac{1 + a_w^2}{\gamma_o^2}\right)}$$

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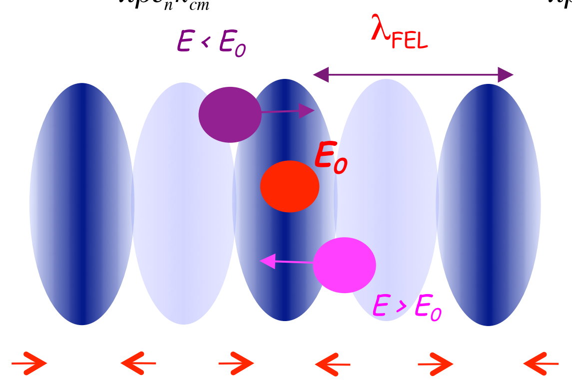
The Kicker

A hadron with central energy (E_o) phased with the hill where longitudinal electric field is zero, a hadron with higher energy ($E > E_o$) arrives earlier and is decelerated, while hadron with lower energy ($E < E_o$) arrives later and is accelerated by the collective field of electrons

Analytical estimation

$$\Delta\varphi = 4\pi\rho \Rightarrow \varphi = -\frac{8G\cdot Ze}{\pi\beta\varepsilon_n k_{cm}}\cdot\cos(k_{cm}z);\quad \vec{\mathbf{E}} = -\vec{\nabla}\varphi = -\hat{z}\frac{8G\cdot Ze}{\pi\beta\varepsilon_n}\cdot\sin(k_{cm}z)$$



Periodical longitudinal electric field

$$\frac{d\mathbf{E}}{dz} = -eE_{peak}\cdot\sin\left\{kD\frac{\mathbf{E}-\mathbf{E}_o}{\mathbf{E}_o}\right\};$$

$$kD\sigma_\delta \sim 1$$

$$\sigma_\delta = \frac{\sigma_{\mathbf{E}}}{\mathbf{E}_o}$$

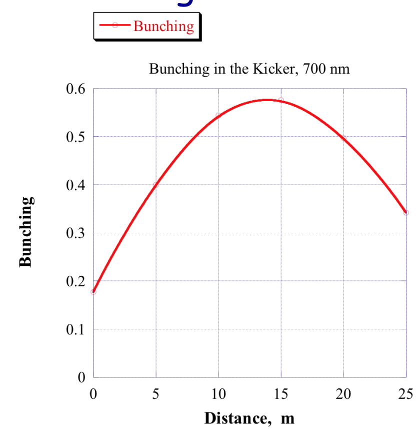
$$\zeta_{CEC} = -\frac{\Delta\mathbf{E}}{\mathbf{E}-\mathbf{E}_o} \approx \frac{e\cdot E_o\cdot l_2}{\gamma_o m_p c^2\cdot\sigma_\varepsilon}\cdot\frac{Z^2}{A}$$

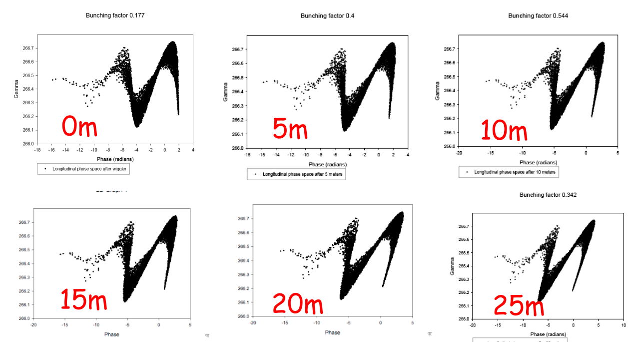
Simulations: only started

Step 1: use 3D FEL code out output + tracking
First simulation indicate that <u>equations on the left significantly underestimate the kick</u>, i.e. the density modulation continues to grow after beam leaves the FEL

Output from Genesis propagated for 25 m

©I.Ben Zvi





Step 2:
use VORPAL with input from Genesis, in preparation

Analytical formula for damping decrement

- 1/2 of plasma oscillation in the modulator creates a pancake of electrons with the charge -2Ze
- electron clamp is well within $\Delta z \sim \lambda_{FEL} / 2\pi$
- gain in SASE FEL is $G \sim 10^2$-10^3
- electron beam is wider than $2\gamma_o \lambda_{FEL}$ - it is 1D field
- Length of the kicker is ~ β-function

$$\zeta = -\frac{\Delta E_i}{E - E_o} = A \cdot \frac{L_2}{\beta} \cdot \chi \cdot \frac{\sin\varphi_3}{\varphi_3} \cdot \frac{\sin\varphi_2}{\varphi_2} \cdot \left(\sin\frac{\varphi_1}{2}\right)^2$$

$$A = 2G_o \frac{Z^2}{A} \cdot \frac{r_p}{\varepsilon_{\perp n}\sigma_\delta}; \quad \chi = k_{FEL} D \cdot \sigma_\delta;$$

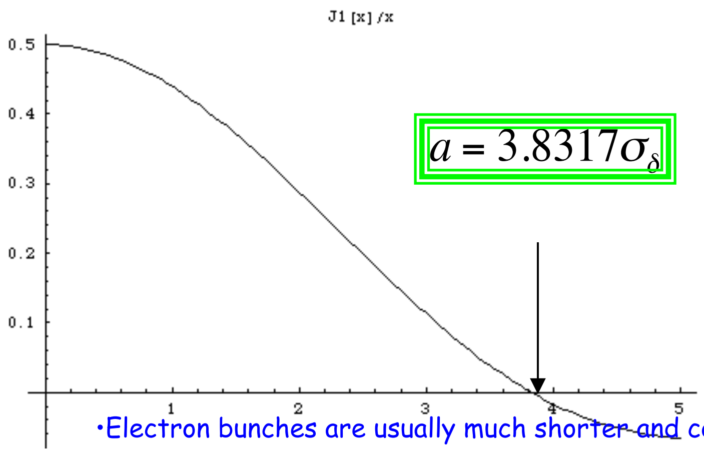
$$\varphi_3 = k_{FEL} D\delta; \quad \delta = \frac{E - E_o}{E_o}$$

$$\delta = a \cdot \sin\Omega_s t$$

$$\left\langle \delta^2 \right\rangle' = -\left\langle 2A \cdot a^2 \cdot \cos^2\Omega_s t \cdot \sin\left(\frac{a}{\sigma_\delta} \cdot \chi \cdot \sin\Omega_s t\right)\right\rangle$$

$$= -2A \cdot \left\langle \delta^2 \right\rangle \cdot J_1\left(\chi \cdot \frac{a}{\sigma_\delta}\right)$$

$$\frac{L_2}{\beta} \cdot \chi \cdot \mathrm{sinc}(\varphi_3) \cdot \mathrm{sinc}\varphi_2 \cdot \left(\sin\frac{\varphi_1}{2}\right)^2 \sim 1$$



$$\boxed{a = 3.8317\sigma_\delta}$$

Beam-Average decrement

$$\int \frac{2J_1(x)}{x} e^{-x^2/2} dx = 0.889$$

- Electron bunches are usually much shorter and cooling time for the entire bunch is proportional to the bunch-lengths ratios

$$\boxed{\chi = 1}$$

$$\left\langle \zeta_{CeC} \right\rangle = \zeta \frac{\sigma_{z,e}}{\sigma_{z,h}} = 2 \frac{G_o \sigma_{z,e}}{\sigma_\delta \sigma_{z,h}} \frac{Z^2}{A} \frac{r_p}{\varepsilon_{\perp n}} \cdot \kappa; \qquad \kappa \sim 1$$

ε_1

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Analytical formula for damping decrement

$$\left\langle \zeta_{CeC} \right\rangle = \zeta \frac{\sigma_{\tau,e}}{\sigma_{\tau,h}} = \kappa \cdot 2G_o \cdot \frac{Z^2}{A} \cdot \frac{r_p \cdot \sigma_{\tau,e}}{\varepsilon_{\perp n}\left(\sigma_\delta \cdot \sigma_{\tau,h}\right)}; \ \kappa \sim 1$$

$$\left\langle \zeta_{CeC} \right\rangle \sim \frac{1}{\varepsilon_{long,h}\varepsilon_{trans,h}}$$

Note that damping decrement

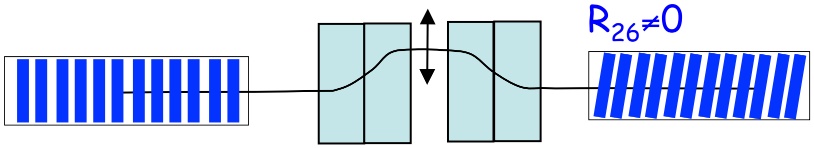
a) **Does not depend on the energy of particles !**
b) **Improves as cooling goes on**

It makes it realistic to think about cooling intense proton beam in RHIC & LHC at 100s of GeV and 7 TeV energies
Even though LHC needs one more trick *(back up slides)*

Transverse cooling

- Transverse cooling can be obtained by using coupling with longitudinal motion via transverse dispersion
- Sharing of cooling decrements is similar to sum of decrements theorem for synchrotron radiation damping, i.e. decrement of longitudinal cooling can be split into appropriate portions to cool both transversely and longitudinally: $J_s+J_h+J_v=1$
- Vertical (better to say the second eigen mode) cooling is coming from transverse coupling

Non-achromatic chicane installed at the exit of the FEL before the kicker section turns the wave-fronts of the charged planes in electron beam



$$\delta(ct) = -R_{26} \cdot x$$

$$\Delta\mathbf{E} = -eZ^2 \cdot E_o \cdot l_2 \cdot \sin\left\{k\left(D\frac{\mathbf{E}-\mathbf{E}_o}{\mathbf{E}_o} + R_{16}x' - R_{26}x + R_{36}y' + R_{46}y\right)\right\};$$

$$\Delta x = -D_x \cdot eZ^2 \cdot E_o \cdot L_2 \cdot kR_{26}x + \ldots.$$

$$\zeta_\perp = J_\perp \zeta_{CeC}; \quad \zeta_{//} = (1-2J_\perp)\zeta_{CeC};$$

$$\frac{d\varepsilon_x}{dt} = -\frac{\varepsilon_x}{\tau_{CeC\perp}}; \frac{d\sigma_\varepsilon^2}{dt} = -\frac{\sigma_\varepsilon^2}{\tau_{CeC//}}$$

$$\tau_{CeC\perp} = \frac{1}{2J_\perp\zeta_{CeC}}; \quad \tau_{CeC\perp} = \frac{1}{2(1-2J_\perp)\zeta_{CeC}};$$

Example: CeC vs. IBS at RHIC

J.LeDuff, "Single and Multiple Touschek effects",
Proceedings of CERN Accelerator School,
Rhodes, Greece, 20 September - 1 October, 1993,
Editor: S.Turner, CERN 95-06, 22 November 1995, Vol. II, p. 573

$$\frac{\sigma_\varepsilon^2}{\tau_{IBS//}} = \frac{N r_c^2 c}{2^5 \pi \gamma^3 \varepsilon_x^{3/2} \sigma_s} \left\langle \frac{f(\chi_m)}{\beta_y v} \right\rangle; \quad \frac{\varepsilon_x}{\tau_{IBS\perp}} = \frac{N r_c^2 c}{2^5 \pi \gamma^3 \varepsilon_x^{3/2} \sigma_s} \left\langle \frac{H}{\beta_y^{1/2}} f(\chi_m) \right\rangle; \kappa = 1$$

$$f(\chi_m) = \int_{\chi_m}^{\infty} \frac{d\chi}{\chi} \ln\left(\frac{\chi}{\chi_m}\right) e^{-\chi}; \ \chi_m = \frac{r_c m^2 c^4}{b_{\max} \sigma_E^2}; b_{\max} \cong n^{-1/3}; \ r_c = \frac{e^2}{mc^2}; \quad (e -> Ze; m -> Am)$$

IBS in RHIC for 250 GeV, N_p=2·10[11] were scaled from the data below
Reference value was provided by A.Fedotov using Beta-cool code © Dubna

$$X = \frac{\varepsilon_x}{\varepsilon_{xo}}; \ S = \left(\frac{\sigma_s}{\sigma_{so}}\right)^2 = \left(\frac{\sigma_E}{\sigma_{sE}}\right)^2;$$

$$\frac{dX}{dt} = \frac{1}{\tau_{IBS\perp}} \frac{1}{X^{3/2} S^{1/2}} - \frac{\xi_\perp}{\tau_{CeC}} \frac{1}{S};$$

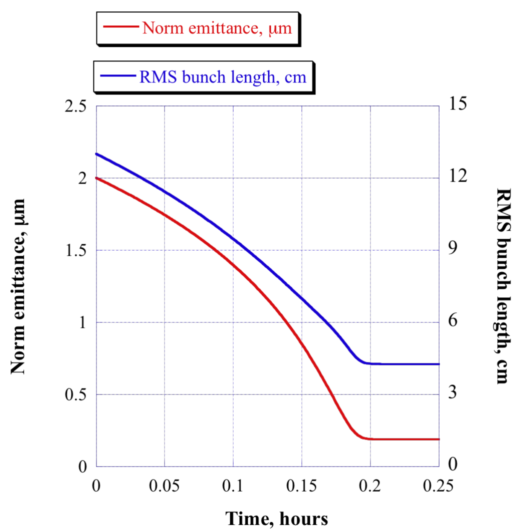
$$\frac{dS}{dt} = \frac{1}{\tau_{IBS//}} \frac{1}{X^{3/2} S^{1/2}} - \frac{1 - 2\xi_\perp}{\tau_{CeC}} \frac{1}{X};$$

$$\varepsilon_{xn0} = 2\,\mu m; \ \sigma_{s0} = 13\ cm; \ \sigma_{\delta 0} = 4 \cdot 10^{-4}$$
$$\tau_{IBS\perp} = 4.6\ \ hrs; \ \tau_{IBS//} = 1.6\ \ hrs;$$

Stationary solution:

$$X = \frac{\tau_{CeC}}{\sqrt{\tau_{IBS//} \tau_{IBS\perp}}} \frac{1}{\sqrt{\xi_\perp (1 - 2\xi_\perp)}}; \quad S = \frac{\tau_{CeC}}{\tau_{IBS//}} \cdot \sqrt{\frac{\tau_{IBS\perp}}{\tau_{IBS//}}} \cdot \sqrt{\frac{\xi_\perp}{(1 - 2\xi_\perp)^3}}$$

$$\boxed{\varepsilon_{xn} = 0.2\,\mu m; \ \sigma_s = 4.9\ \ \mathrm{cm}}$$



This may allow

a) RHIC pp - keep the luminosity at beam-beam limit all the time
b) RHIC pp – reduce bunch length to few cm (from present 1 m)
   1. to reduce hourglass effect
   2. To concentrate event in short vertexes of the detectors
c) eRHIC - reduce polarized beam current down to 50 mA while keeping the same luminosity
d) eRHIC - increase electron beam energy to 20 GeV
e) Both - increase luminosity by reducing β* to 5-10 cm from present 0.5m

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Effects of the surrounding particles

Each charged particle causes generation of an electric field wave-packet proportional to its charge and synchronized with its initial position in the bunch

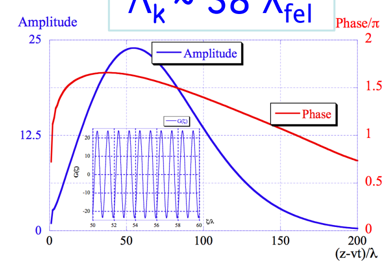
$$\mathbf{E}_{total}(\zeta) = \mathrm{E}_o \cdot \mathrm{Im}\left(X \cdot \sum_{i,hadrons} K(\zeta - \zeta_i) e^{ik(\zeta - \zeta_i)} - \sum_{j,electrons} K(\zeta - \zeta_j) e^{ik(\zeta - \zeta_j)} \right)$$

$$\mathbf{E}_o = 2G_o \cdot \gamma_o \cdot \frac{e}{\beta \varepsilon_{\perp n}}$$

$$X = q/e \cong Z(1 - \cos\varphi_1) \sim Z$$

Evolution of the RMS value resembles stochastic cooling!
Best cooling rate achievable is ~ $1/N_{eff}$, N_{eff} is effective number of hadrons in coherent sample ($\Lambda_k = N_c \lambda$)

$\Lambda_k \sim 38\ \lambda_{fel}$



$$\boxed{\left\langle \delta^2 \right\rangle' = -2\xi \left\langle \delta^2 \right\rangle + D}$$

$$\Lambda_k = \iint \left| K(z - \zeta) \right|^2 d\zeta$$

$$N_{eff} \cong N_h \frac{\Lambda_k}{\sqrt{4\pi}\sigma_{z,h}} + \frac{N_e}{X^2} \frac{\Lambda_k}{\sqrt{4\pi}\sigma_{z,e}}$$

$$\xi = -g \left\langle \delta_i \mathrm{Im}\left(K(\Delta\zeta_i) e^{ik\Delta\zeta_i} \right) \right\rangle / \left\langle \delta^2 \right\rangle; \ D = g^2 N_{eff}/2;$$

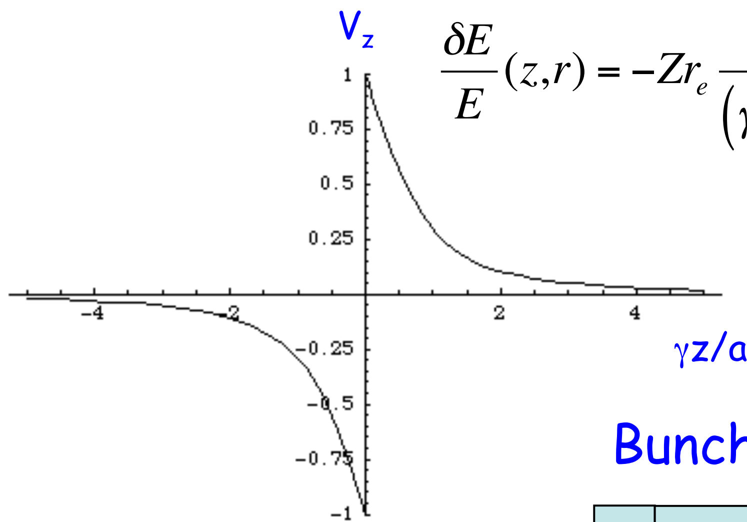
$$g = G_o \frac{Z^2}{A} \frac{r_p}{\varepsilon_{\perp n}} \left\{ 2 f(\varphi_2)(1 - \cos\varphi_1) \frac{l_2}{\beta} \cdot \right\},$$

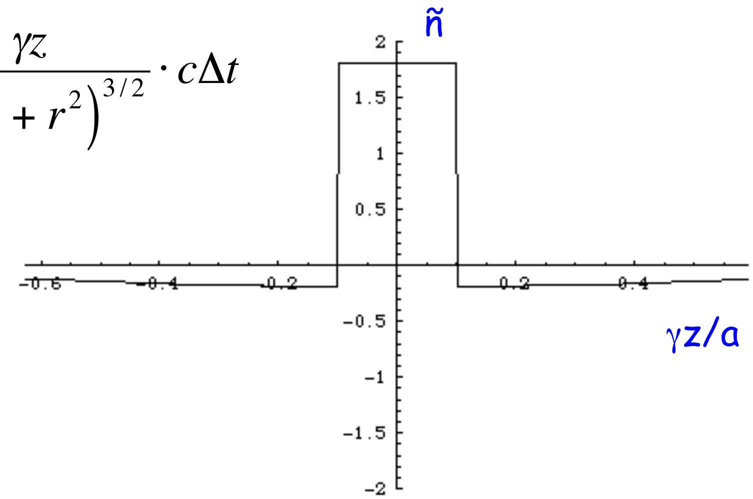
$$\boxed{\xi_{CeC}(\max) = \frac{\Delta}{2\sigma_\gamma} = \frac{2}{N_{eff}} \left(kD\sigma_\varepsilon \right) \propto \frac{1}{N_{eff}}}$$

Fortunately, the bandwidth of FELs $\Delta f \sim 10^{13}$-10^{15} Hz is so large that this limitation does not play any practical role in most HE cases

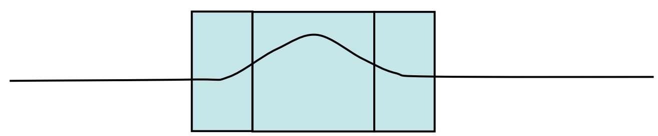
Velocity map & buncher (γ>1000)

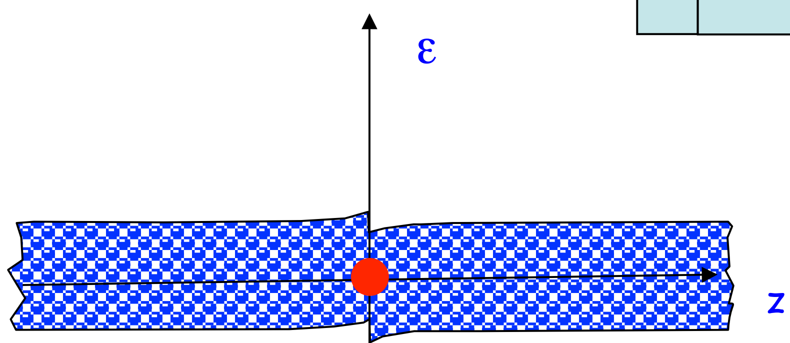
$$\frac{\delta E}{E}(z,r) = -Zr_e \frac{\gamma z}{\left(\gamma^2 z^2 + r^2\right)^{3/2}} \cdot c\Delta t$$

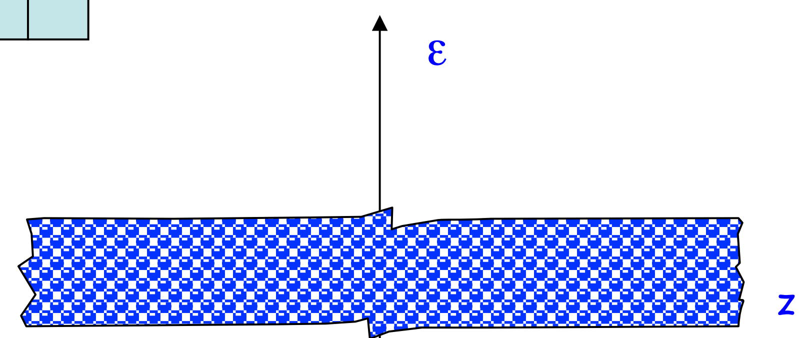




Buncher







$$\left\langle \frac{\delta E}{E} \right\rangle \cong -2Z \frac{r_e}{a^2} \cdot \frac{L_{pol}}{\gamma} \cdot \left(\frac{z}{|z|} - \frac{z}{\sqrt{a^2/\gamma^2 + z^2}} \right)$$

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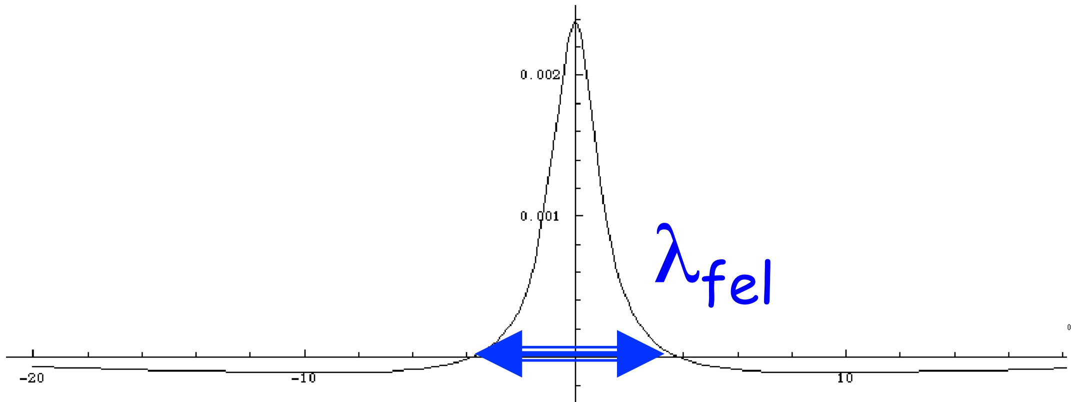
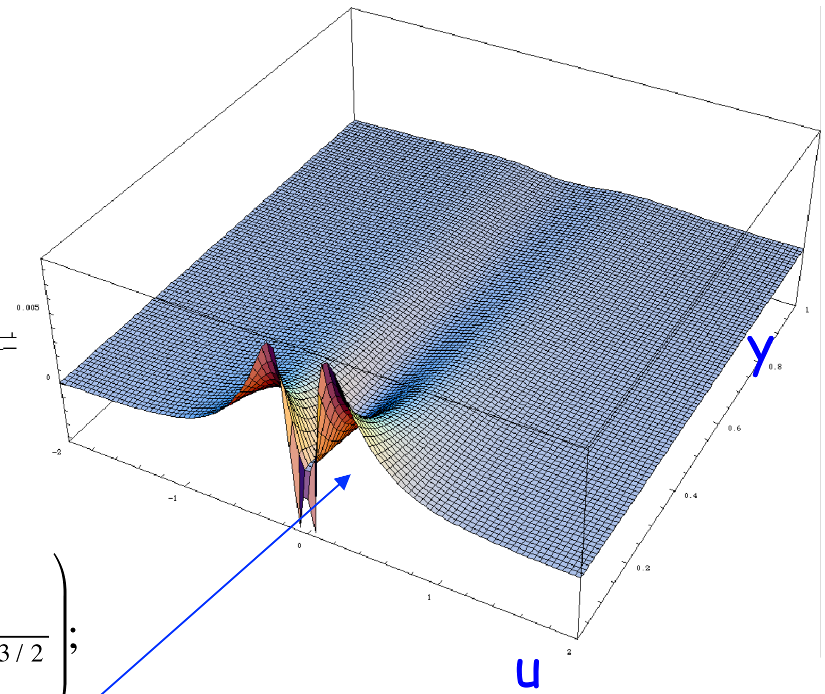
Exact calculations: solving Vlasov equation





$$f_o(r,\vec{p}_\perp,z,\gamma)=\frac{\theta(r-a)}{a^2/2}\cdot\frac{\theta(z-l_z)}{l_z}\cdot\frac{1}{\sqrt{2\pi}\sigma_\gamma}e^{-\frac{(\gamma-\gamma_o)^2}{2\sigma_\gamma^2}}\cdot g(\vec{p}_\perp)$$

$$\frac{\delta\gamma}{\gamma_o}=\frac{\delta\gamma_i}{\gamma_o}-A\frac{\gamma_o z_i}{\left(r_i^2+\gamma_o^2 z_i^2\right)^{3/2}};\quad z=z_i+D\left(\frac{\delta\gamma_i}{\gamma_o}-A\frac{\gamma_o z_i}{\left(r_i^2+\gamma_o^2 z_i^2\right)^{3/2}}\right);$$

$$l_z\rho(z)=\Phi(s)=\frac{1}{\kappa^2\sqrt{2\pi}}\int_0^{\kappa^2}dy\int_{-L/2}^{L/2}\left[\exp\left\{-\frac{1}{2}\left(s-u\left(1-\frac{G}{\left(y+u^2\right)^{3/2}}\right)\right)^2\right\}-\exp\left\{-\frac{(s-u)^2}{2}\right\}\right]du;$$

$$G=Z\frac{r_e L_{\text{mod}}|D|}{\left(\gamma_o\sigma_{p_1}|D|\right)^3};\ \kappa=\frac{a}{\gamma_o\sigma_{p_1}|D|};\ L=\frac{l_z}{\sigma_{p_1}|D|}$$

$$u=\frac{x_1}{\sigma_{p_1}|D|};\ s=\frac{z}{\sigma_{p_1}|D|};\ y=\frac{r^2}{\left(\gamma_o\sigma_{p_1}|D|\right)^2}$$

For 7 TeV p in LHC CeC case: simple “gut-feeling” estimate gave 22.9 boost in the induced charge by a buncher, while exact calculations gave 21.7.

Beyond the basics toward the complete theory and simulation package

Analytical, Numerical and Computer Tools to:

1. find reaction *(distortion of the distribution function of electrons)* on a presence of moving hadron inside an electron beam

$$\frac{\partial f_e}{\partial t} + \frac{\partial f_e}{\partial \vec{\mathrm{v}}} \cdot \frac{e\vec{E}}{m} + \frac{\partial f_e}{\partial \vec{r}} \cdot \vec{\mathrm{v}} = 0; \quad \vec{r}_h(t) \cong \vec{r}_o + \vec{\mathrm{v}}_{\mathrm{h}} t;$$

$$\left(\vec{\nabla} \cdot \vec{E}\right) = 4\pi e n_e \left(\frac{Z}{n_e} \delta\left(\vec{r} - \vec{r}_h(t)\right) - \int f_e d\vec{\mathrm{v}}^3 \right).$$

$$\boxed{f \Rightarrow f_o + \delta f}$$

2a. Find how an arbitrary δf is amplified in high-gain FEL

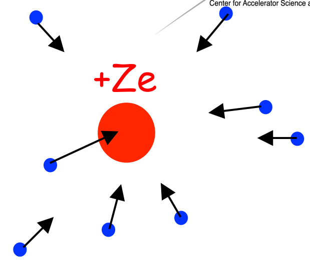
$$f_{exit}(\vec{r}_\perp, \vec{\mathrm{p}}, \mathrm{t}) = f_{o\ exit}(\vec{r}_\perp, \vec{\mathrm{p}}) + \int K\left(\vec{r}_\perp, \vec{\mathrm{p}}, \vec{r}_{\perp 1}, \vec{\mathrm{p}}_1, t - t_1\right) \cdot \delta f(\vec{r}_1, \vec{\mathrm{p}}_1, t_1) \cdot d\vec{r}_{\perp 1} d\vec{\mathrm{p}}_1 dt_1$$

2b. Design cost effective lattice for hadrons + coupling

3. Find how the amplified reaction of the e-beam acts on the hadron (including coupling to transverse motion)

Modulator

Dimensionless equations of motion



$$\frac{\partial f_e}{\partial t} + \frac{\partial f_e}{\partial \vec{\mathrm{v}}} \cdot \frac{e\vec{E}}{m} + \frac{\partial f_e}{\partial \vec{r}} \cdot \vec{\mathrm{v}} = 0; \quad \vec{r}_h(t) \cong \vec{r}_o + \vec{\mathrm{v}}_{\mathrm{h}} t;$$

$$\left(\vec{\nabla} \cdot \vec{E}\right) = 4\pi e n_e \left(\frac{Z}{n_e} \delta\left(\vec{r} - \vec{r}_h(t)\right) - \int f_e d\vec{\mathrm{v}}^3 \right).$$

$$\Rightarrow$$

$$\frac{\partial f_e}{\partial \tau} + \frac{\partial f_e}{\partial \vec{v}} \cdot \vec{g} + \frac{\partial f_e}{\partial \vec{\rho}} \cdot \vec{v} = 0; \quad \vec{g} = \frac{e\vec{E}}{m \omega_p^{\,2} s};$$

$$\left(\vec{\nabla}_n \cdot \vec{g}\right) = \frac{Z}{s^3 n_e} \delta\left(\vec{\rho} - \vec{\rho}_i(t)\right) - \int f_e d\vec{v}^3; \quad \vec{\nabla}_n \equiv \partial_{\vec{\rho}}.$$

$$t = \tau/\omega_p; \quad \vec{\mathrm{v}} = \vec{v}\sigma_{\mathrm{v}_z}; \quad \vec{r} = \vec{\rho}\sigma_{\mathrm{v}_z}/\omega_p; \; \omega_p^2 = \frac{4\pi e^2 n_e}{m} \qquad s = r_{D_z} = \sigma_{\mathrm{v}_z}/\omega_p$$

Parameters of the problem

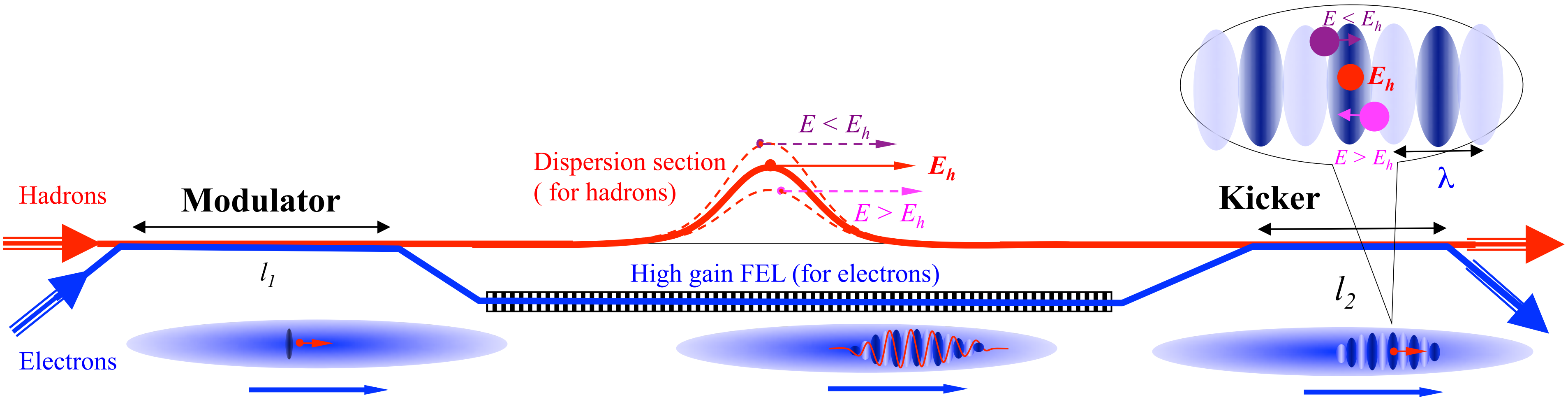
$$\mathrm{R} = \frac{\sigma_{v_\perp}}{\sigma_{v_z}}; \; \mathrm{T} = \frac{\mathrm{v}_{\mathrm{hx}}}{\sigma_{v_z}}; \; \mathrm{L} = \frac{\mathrm{v}_{\mathrm{hz}}}{\sigma_{v_z}}; \; \xi = \frac{Z}{4\pi n_e R^2 s^3};$$

$$\mathrm{A} = \frac{a}{s}; \; \mathrm{X} = \frac{\mathrm{x}_{\mathrm{ho}}}{a}; \mathrm{Y} = \frac{\mathrm{y}_{\mathrm{ho}}}{a}.$$

CeC: FEL response

$$f_{input}(\vec{r}_\perp,\vec{p},t) = f_{o\ input}(\vec{r}_\perp,\vec{p}) + \delta f(\vec{r}_\perp,\vec{p},t)$$

$$f_{exit}(\vec{r}_\perp,\vec{p},t) = f_{o\ exit}(\vec{r}_\perp,\vec{p}) + \int K\left(\vec{r}_\perp,\vec{p},\vec{r}_{\perp 1},\vec{p}_1,t-t_1\right)\cdot \delta f(\vec{r}_1,\vec{p}_1,t_1)\cdot d\vec{r}_{\perp 1} d\vec{p}_1 dt_1$$



1D FEL response

$$\boxed{\rho_{exit}(t;z) = \rho_o + \int G(\tau;z)\cdot \delta\rho(t-\tau;0)\cdot d\tau}$$

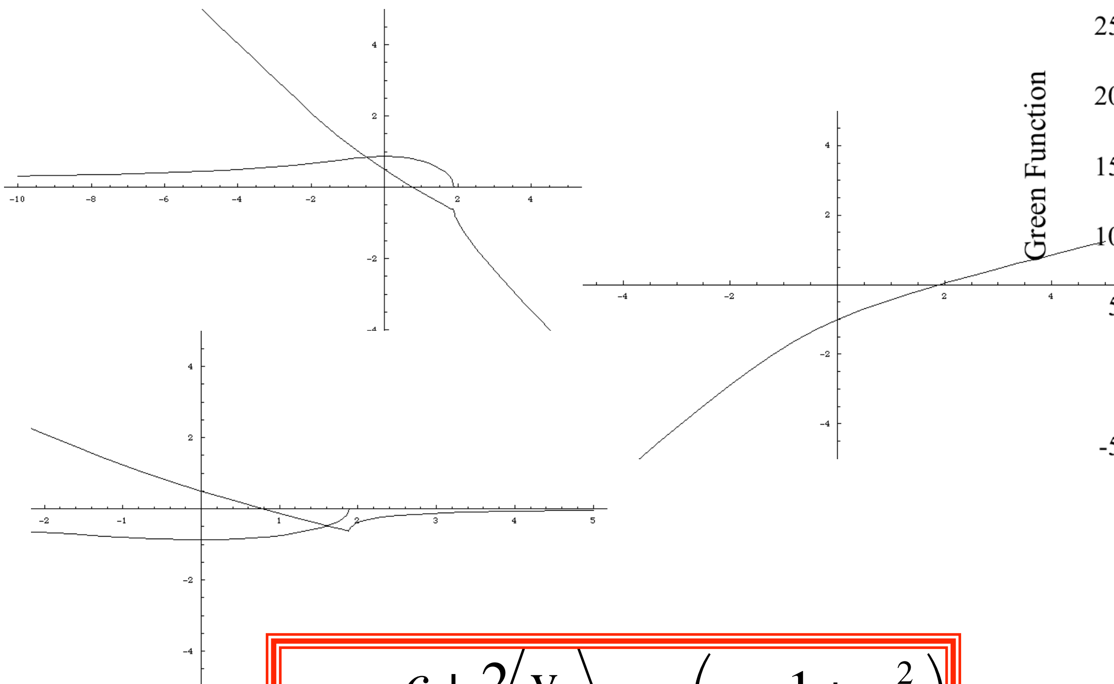
$$\omega_0 = \frac{2\pi c}{\lambda_o};$$

Response - 1D FEL after 10 gain lengths

$$\left\{\left\{x\to\frac{1}{3}\left(-2ic-\frac{2^{1/3}c^2}{\left(27i-2ic^3+3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}+\frac{\left(27i-2ic^3+3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}{2^{1/3}}\right)\right\},\right.$$

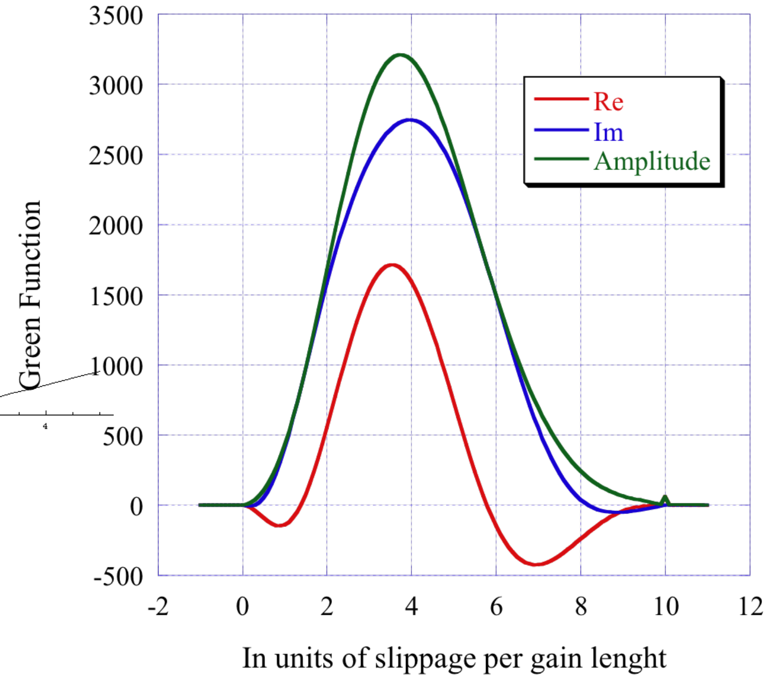
$$\left\{x\to-\frac{2ic}{3}+\frac{\left(1+i\sqrt{3}\right)c^2}{3\,2^{2/3}\left(27i-2ic^3+3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}-\frac{\left(1-i\sqrt{3}\right)\left(27i-2ic^3+3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}{6\,2^{1/3}}\right\},$$

$$\left.\left\{x\to-\frac{2ic}{3}+\frac{\left(1-i\sqrt{3}\right)c^2}{3\,2^{2/3}\left(27i-2ic^3+3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}-\frac{\left(1+i\sqrt{3}\right)\left(27i-2ic^3+3\sqrt{3}\sqrt{-27+4c^3}\right)^{1/3}}{6\,2^{1/3}}\right\}\right\}$$



$$\boxed{v_g=\frac{c+2\langle v_z\rangle}{3}=c\left(1-\frac{1+a_w^2}{3\gamma_o^2}\right)}$$

Green-function envelope (Abs, Re and Im)



Maximum located at 3.744 slippage units,
(i.e. just a bit further that expected 3 and 1/3)
The Green function (with oscillations) had effective RMS length of 1.48 slippage units.

New theoretical developments beyond 1D

Inserting (8) into (5) generates

$$\frac{\partial}{\partial \hat{z}}\tilde{R}\left(\hat{z},\hat{k}_\perp,\hat{C}\right)=-i\int_0^{\hat{z}} d\hat{z}'\left(\hat{z}'-\hat{z}\right)\exp\left[i\left(\hat{C}-\hat{k}_\perp^2\right)\left(\hat{z}'-\hat{z}\right)-\left|\hat{q}\left(\hat{z}'-\hat{z}\right)\right|\right]\left\{\tilde{R}\left(\hat{z}',\hat{k}_\perp,\hat{C}\right)+i\Lambda_p^2\frac{\partial}{\partial \hat{z}'}\tilde{R}\left(\hat{z}',\hat{k}_\perp,\hat{C}\right)\right\} \quad (9)$$

, which is similar to 1D FEL theory (equation (6.68)~ (6.69) of Gang's thesis) of except that $\hat{C}$ there is replaced by $\hat{C}_{3d}=\hat{C}-\hat{k}_\perp^2$. Thus equation (9) can be reduced to a third order ODE the same way as what is done in 1D FEL theory, i.e.

$$\frac{d^3}{d\hat{z}^3}\tilde{R}(\hat{z})+2\left(i\hat{C}_{3d}+\hat{q}\right)\frac{d^2}{d\hat{z}^2}\tilde{R}(\hat{z})+\left[\hat{\Lambda}_p^2+\left(i\hat{C}_{3d}+\hat{q}\right)^2\right]\frac{d}{d\hat{z}}\tilde{R}(\hat{z})-i\tilde{R}(\hat{z})=0\,. \qquad (10)$$

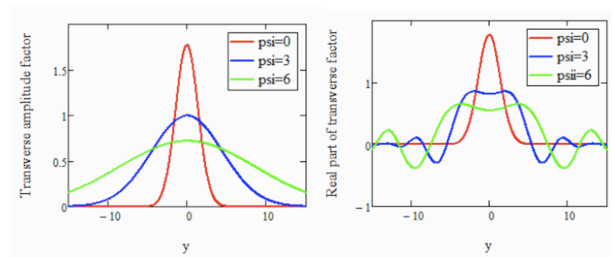


Figure 3. Transverse profile for an initial Gaussian perturbation as calculated from equation (30). The left plot shows the amplitude and the right plot shows the real part.

$$\tilde{j}_1(x,y,z,t)=\frac{c^2\Gamma^4\gamma_z^4}{4\pi^3\theta_s\rho}\frac{1}{\xi(z,t)}e^{-i\frac{\hat{x}^2+\hat{y}^2}{4\xi(z,t)}}e^{\pm i(1\pm1)\frac{\pi}{4}}e^{-i\hat{k}_w\xi(z,t)}$$
$$\times\int_{-\infty}^{\infty}\sum_{i=1}^{3}A_i\left(\hat{C}_{3d}\right)\lambda_i\left(\hat{C}_{3d}\right)e^{\lambda_i\left(\hat{C}_{3d}\right)\hat{z}}e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}}d\hat{C}_{3d}$$

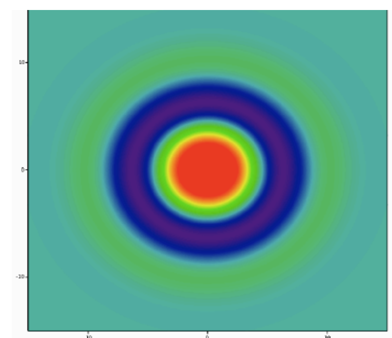

(c)

Figure 2. Amplitude and phase of the growth mode radial distribution factor in eq. (28) for (a) $\hat{C}_0=0$, $\hat{q}=0$, $\hat{\Lambda}_p=0$ and $\hat{z}=6$; (b) $\hat{z}=15$ [1]; (c) plots the real part of the radial distribution for $\hat{z}=15$.

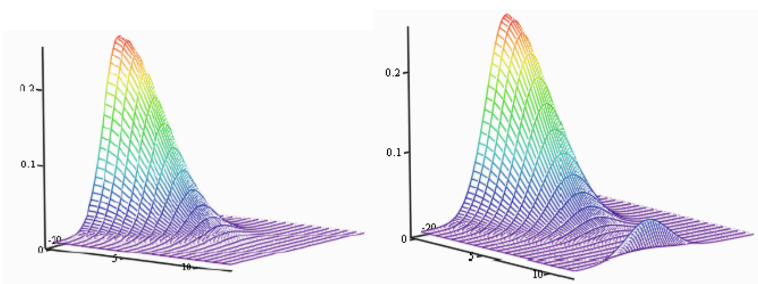

(a) (b)

Figure 8. Comparison of equation (46) with 20 terms expansion and equation (37) by direct integration. (a) calculation done by (37); (b) calculation by (46). For both plots, $\hat{\sigma}_x=2$, $\hat{\sigma}_t=0.1$ and $\xi=0$ are used.

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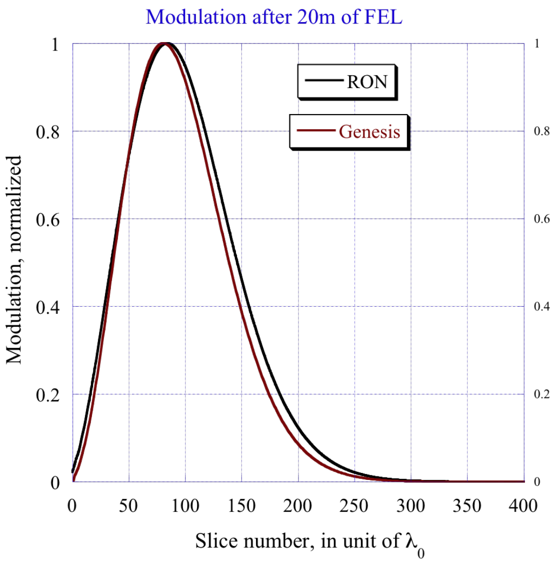
FEL's Green Function_
1D - analytical approach $G(\tau;z) = \mathrm{Re}\left(\tilde{G}_z(\tau)e^{i\omega_o\tau}\right)$
3D - 3D FEL codes RON and Genesis 1.3

FEL parameters for Genesis 1.3 and RON simulations

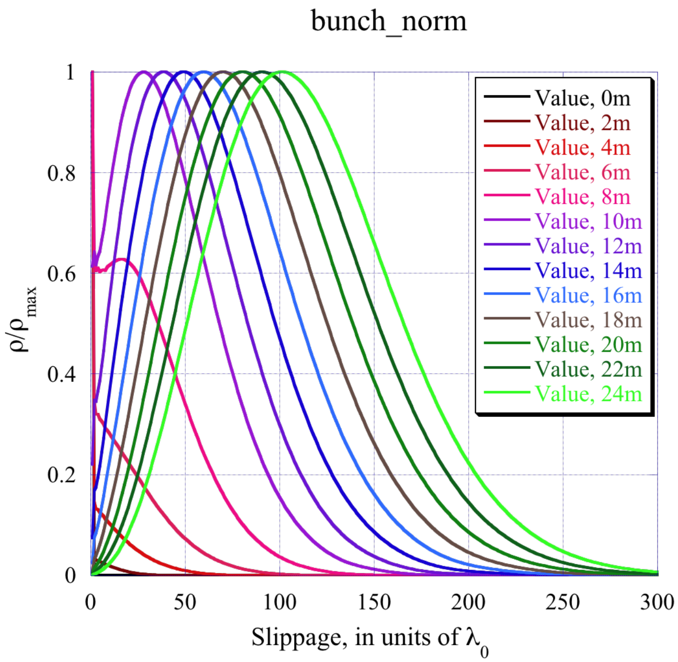
FEL gain length: 1 m (power), 2m (amplitude)

Main FEL parameters for eRHIC with 250 GeV protons

Energy, MeV	136.2	γ	266.45
Peak current, A	100	λ_o, nm	700
Bunchlength, psec	50	λ_w, cm	5
Emittance, norm	5 mm mrad	a_w	0.994
Energy spread	0.03%	Wiggler	Helical



Genesis: 3D FEL



Evolution of the normalized bunching envelope

The Green function (with oscillations) after 10 gain-lengths had also smaller effective RMS length [1] of 0.96 slippage units (i.e. about 38 optical wavelengths, or 27 microns

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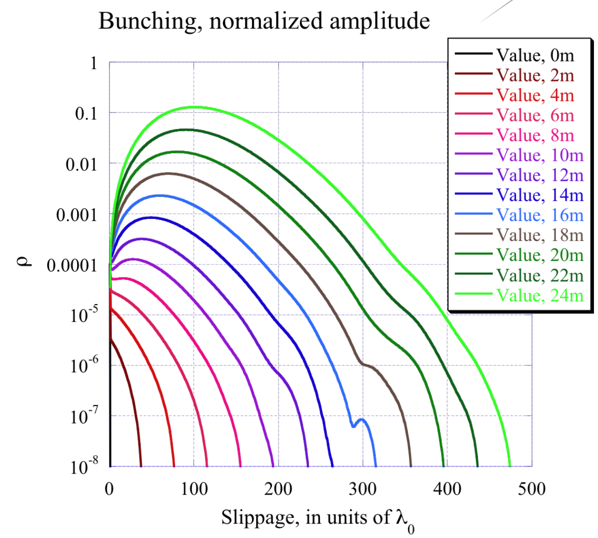

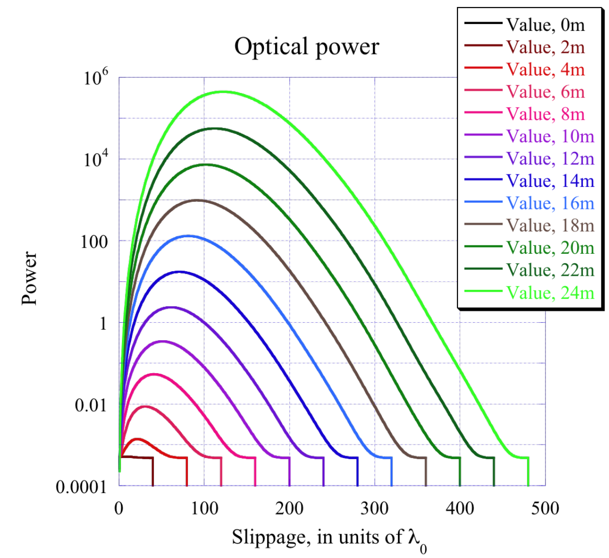

Evolution of the bunching and optical power envelopes (vertical scale is logarithmic)

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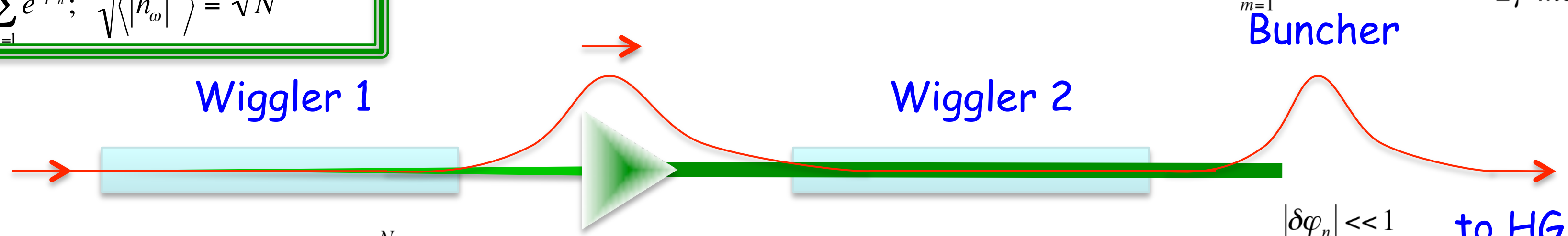
Shot-Noise Suppressor (in time)

$$\varphi = k_l \cdot c\tau$$

$$\boxed{h_\omega^o = \sum_{n=1}^{N} e^{i\varphi_n}; \quad \sqrt{\left\langle \left|h_\omega^o\right|^2 \right\rangle} = \sqrt{N}}$$

$$\delta\varphi_n = g_o \cdot \mathrm{Im}\sum_{m=1}^{N} e^{i(\varphi_n - \varphi_m)}; \quad g_o = \frac{kD \cdot ea_o L_w \cdot K_w}{2\gamma^2 mc^2}$$



$$|\delta\varphi_n| << 1$$

$$E = \mathrm{Re}\left(ae^{i\omega t}\right) \quad a = a_o \sum_{n=1}^{N} e^{i\varphi_n}$$

$$\delta E_n = -ea_o L_w \cdot \frac{K_w}{2\gamma} \cdot \mathrm{Im}\sum_{m=1}^{N} e^{i(\varphi_n - \varphi_m)}$$

$$h_\omega = \sum_{n=1}^{N} e^{i(\varphi_n + \delta\varphi_n)} \cong \sum_{n=1}^{N} e^{i\varphi_n}\left(1 + i\delta\varphi_n\right)$$

$$h_\omega = \sum_{n=1}^{N} e^{i\varphi_n} + \frac{g_o}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} e^{i(2\varphi_n - \varphi_m)} - \frac{g_o}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} e^{i\varphi_m} =$$

$$\left(1 - \frac{g_o N}{2}\right) \cdot h_\omega^o + \frac{g_o}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} e^{i(2\varphi_n - \varphi_m)}.$$

$$g_o = 2/N$$

$$\boxed{h_\omega = \frac{1}{N}\sum_{n=1}^{N}\sum_{m=1}^{N} e^{i(2\varphi_n - \varphi_m)}. \qquad \sqrt{\left\langle \left|h_\omega\right|^2 \right\rangle} = 1}$$

Such system will reduce the amplitude of shot-noise by the factor N $^{1/2}$, and the power of short-noise (spontaneous radiation, SASE) by a factor of N.

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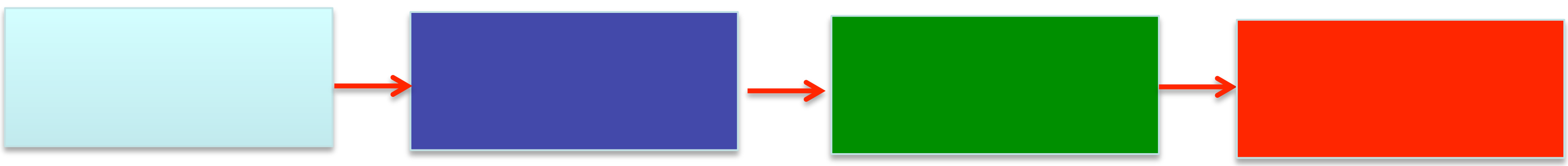
CASE Center for Accelerator Science and Education

Bunch compression and extension to shorter wavelength

- Bunch compression needed for X-ray FELs is also useful for removing the noise at X-ray wavelength using optical (V/IR) noise canceller
- There are many possible schemes – here is the simplest one



Noise killer at $\lambda_l = 2\pi / k_l$

Exchanger

$$M_e = \begin{pmatrix} 1 & R_{56} \\ -1/R_{56} & 0 \end{pmatrix}$$

Compressor

$$M_c = \begin{pmatrix} 0 & r_{56} \\ -1/r_{56} & 1 \end{pmatrix}$$

FEL amplifier

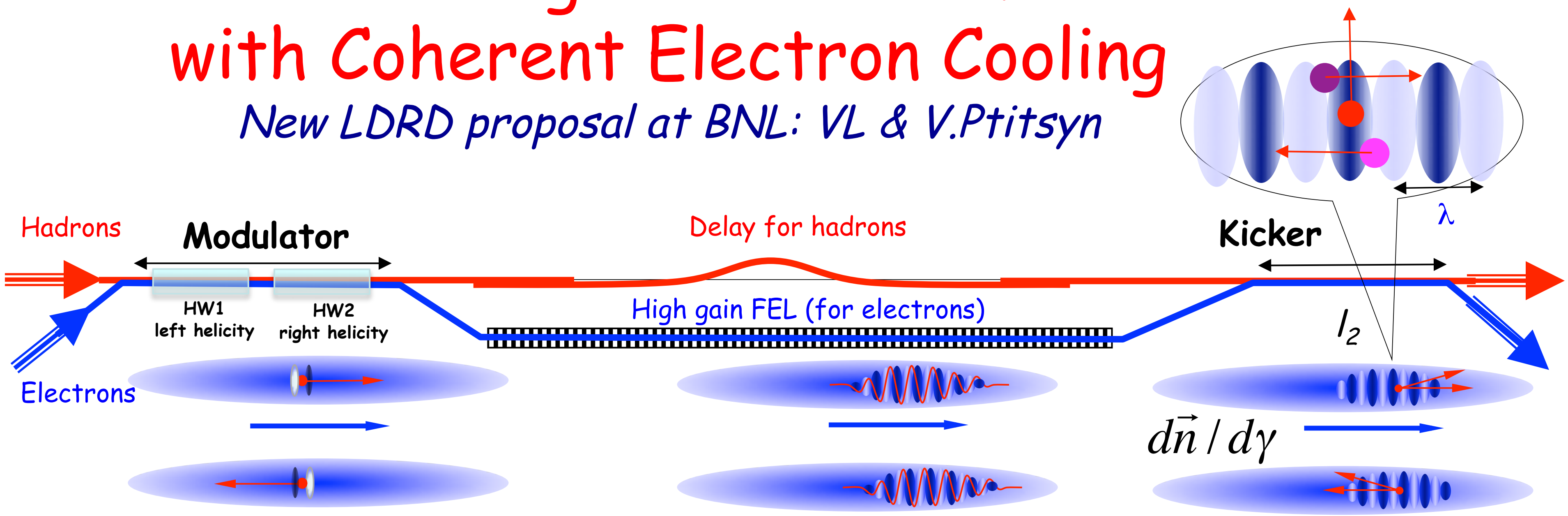
$$M_t = \begin{pmatrix} \dfrac{r_{56}}{R_{56}} & 0 \\ -\dfrac{1}{R_{56}} - \dfrac{1}{r_{56}} & \dfrac{R_{56}}{r_{56}} \end{pmatrix}$$

$$\varphi_x = k_X (c\tau); \quad h_X = \sum_{n=1}^{N} e^{ik_X (c\tau_n)_3} = \sum_{n=1}^{N} e^{ik_X \frac{r_{56}}{R_{56}} (c\tau_n)_1} \rightarrow 0 \quad \forall k_l = k_X \frac{r_{56}}{R_{56}}$$

Thus it is possible using visible/IR noise killer to suppress noise in X-ray FEL

Polarizing Hadron Beams with Coherent Electron Cooling

New LDRD proposal at BNL: VL & V.Ptitsyn



Modulation of the electron beam density around a hadron is caused by value of spin component along the longitudinal axis, z. Hardons with spin projection onto z-axis will attract electrons while traveling through helical wiggler with left helicity and repel them in the helical wiggler with right helicity.

The high gain FEL amplify the imprinted modulation.

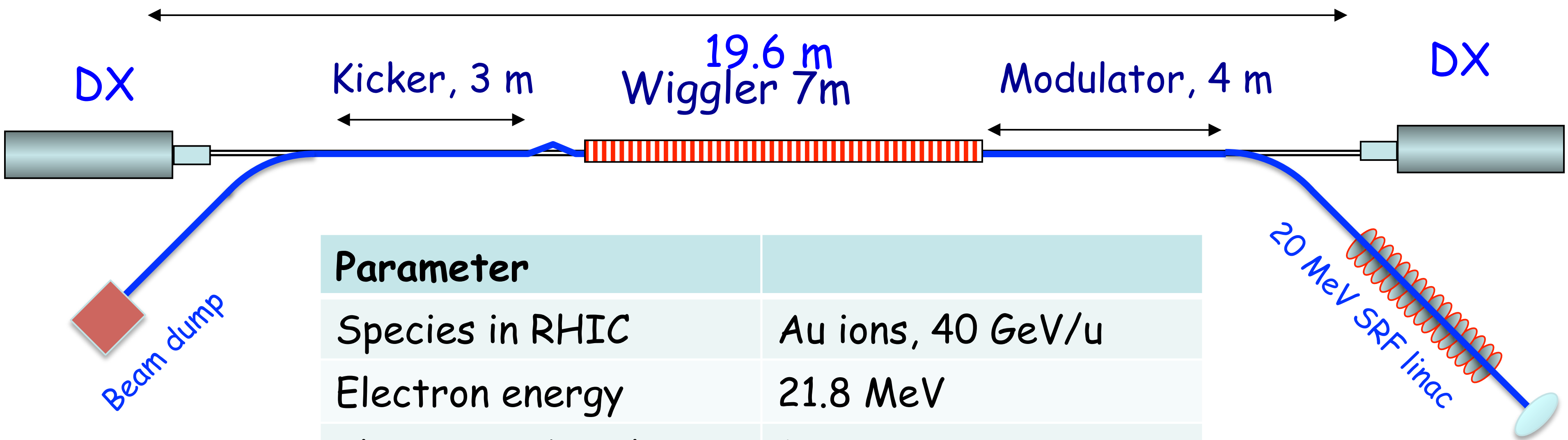
Hadrons with z-component of spin will have an energy kick proportional to the value and the sigh if the projection. Placing the kicker in spin dispersion will result in reduction of z-component of the spin and in the increase of the vertical one.

This process will polarize the hadron beam

Polarizing Hadron Beams with Coherent Electron Cooling

- It is very provocative proposal and requires very high gain FELs for attaining reasonable spin-cooling times – no time to go into many details
- For LHC this technique could an unique opportunity to operate with polarized hadrons
- in RHIC, bringing polarization of proton beams close to 100% would provide for a nearly eight-fold boost of the observables and could be of critical for solving long-standing proton spin crisis
- The methods can be used for polarizing other hadrons, such as deuterons – polarization of which is considered to be impossible in RHIC, He3 and other ions
- The technique can open a unique possibility of getting polarized antiprotons

Layout for Coherent Electron Cooling proof-of-principle experiment in RHIC IR
Join BNL-Jlab option
Collaboration is opened for all interested labs/people



Parameter	
Species in RHIC	Au ions, 40 GeV/u
Electron energy	21.8 MeV
Charge per bunch	1 nC
Train	5 bunches
Rep-rate	78.3 kHz
e-beam current	0.39 mA
e-beam power	8.5 kW

Conclusions

- Coherent electron cooling has potential of cooling high intensity TeV scale proton and ion beams with reasonable (under an hour) cooling time
- Electron accelerator of choice for such cooler is energy recovery linac (ERL)
- ERL seems to be capable of providing required beam quality for such coolers
- Majority of the technical limitation and requirements on the beam and magnets stability are well within limit of current technology, even though satisfying all of them in nontrivial fit
- We plan a proof of principle experiment of coherent electron cooling with Au ions in RHIC at ~ 40 GeV/n