Novel Features of Computational EM and Particle-in-Cell Simulations

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Outline

Part-I

- EM Structure
- Motivation
- Method
- Modes , Radiation Leakage and HOM Damper
- Conclusions

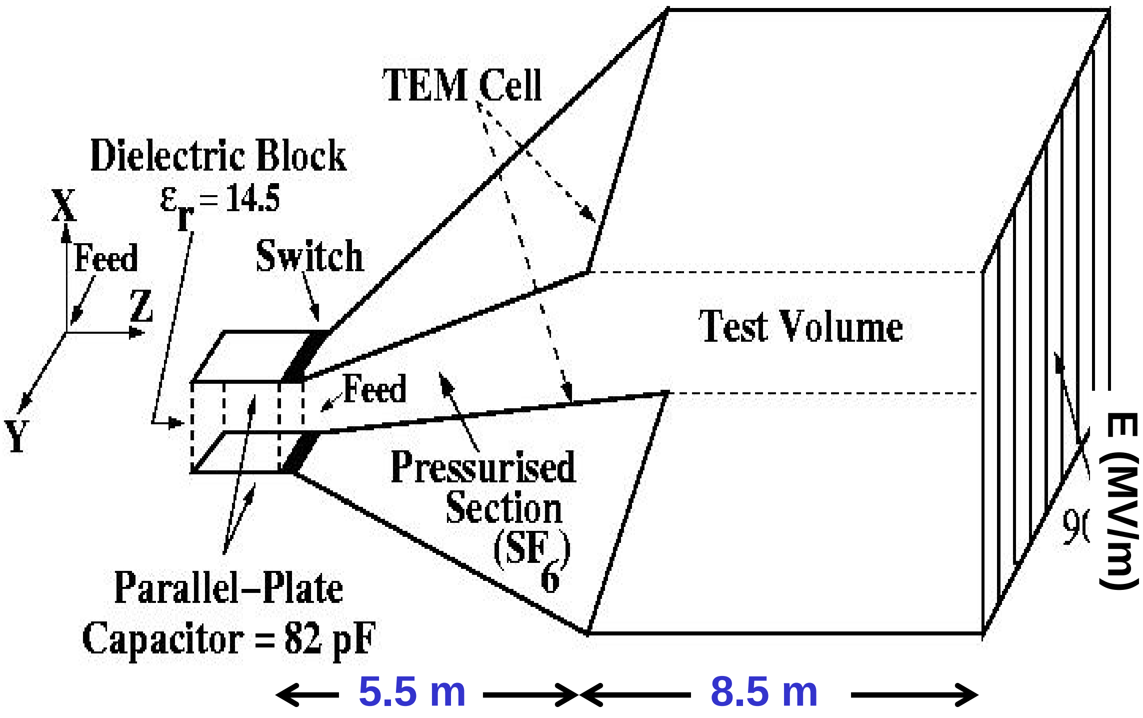
Part-II

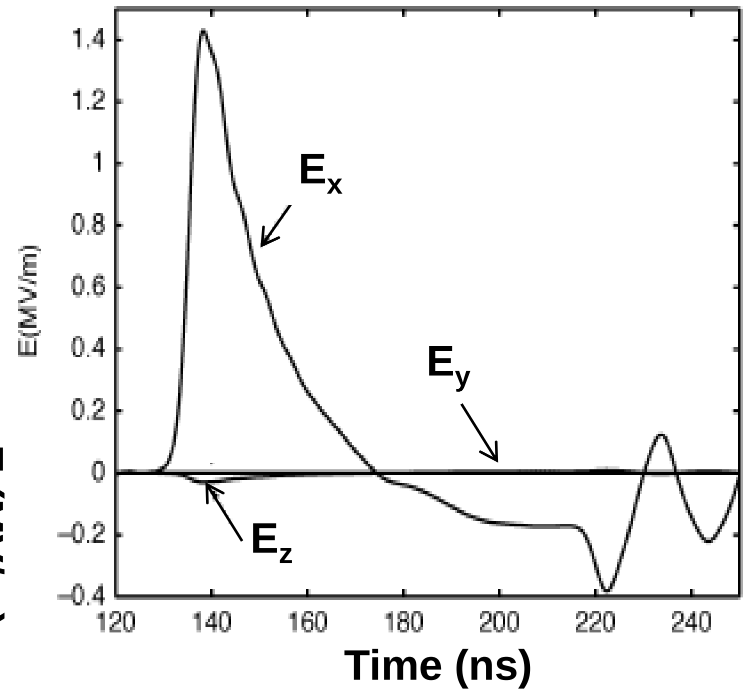
- Beam Matter Interaction
- Motivation
- Simulation studies
- Space charge physics
- Conclusions

Pulsed Electromagnetic (EM) Wave Generator

- Study of lightning / ESD interaction with test objects.
- w / h = Constant = Characteristic Impedance (Z_0) , pulse waveform preserved.

w_a = 2.32 m , h_a = 1. 49 m , Z_0 ~ 90 Ω

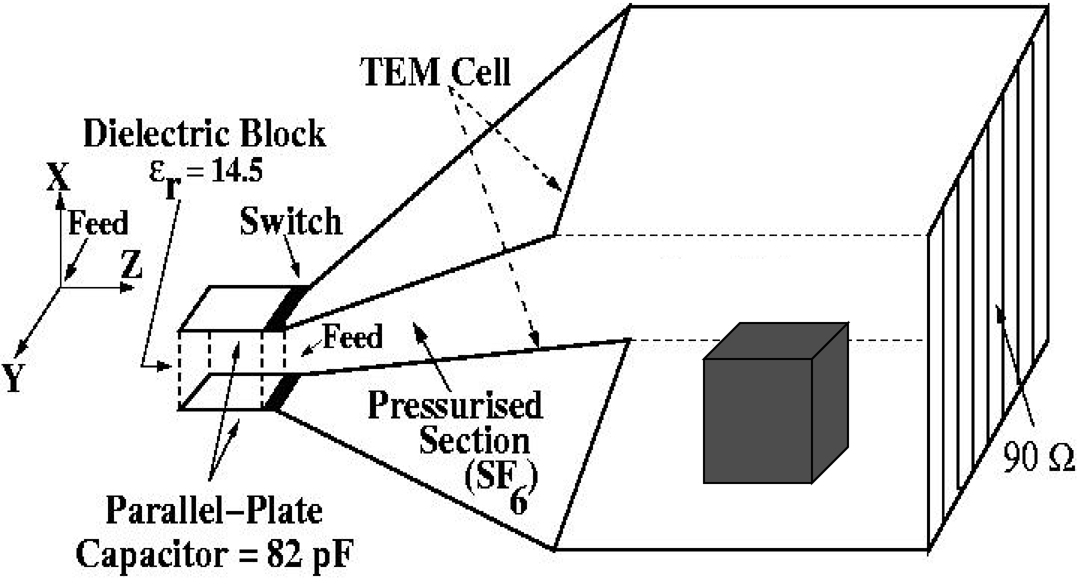




Typical EM Pulse

Motivation

- Natural mode (free space): TEM.
- Experiment: Scattering / reflections excite Higher Order Modes (HOM).
- Results deviate from real effect observed in free-space.
- Quantitative measurement of TEM mode polluted by higher-order modes.
- Given wide frequency spectrum, complex structures with varying material properties require numerical techniques for modal analysis.



Possible modes

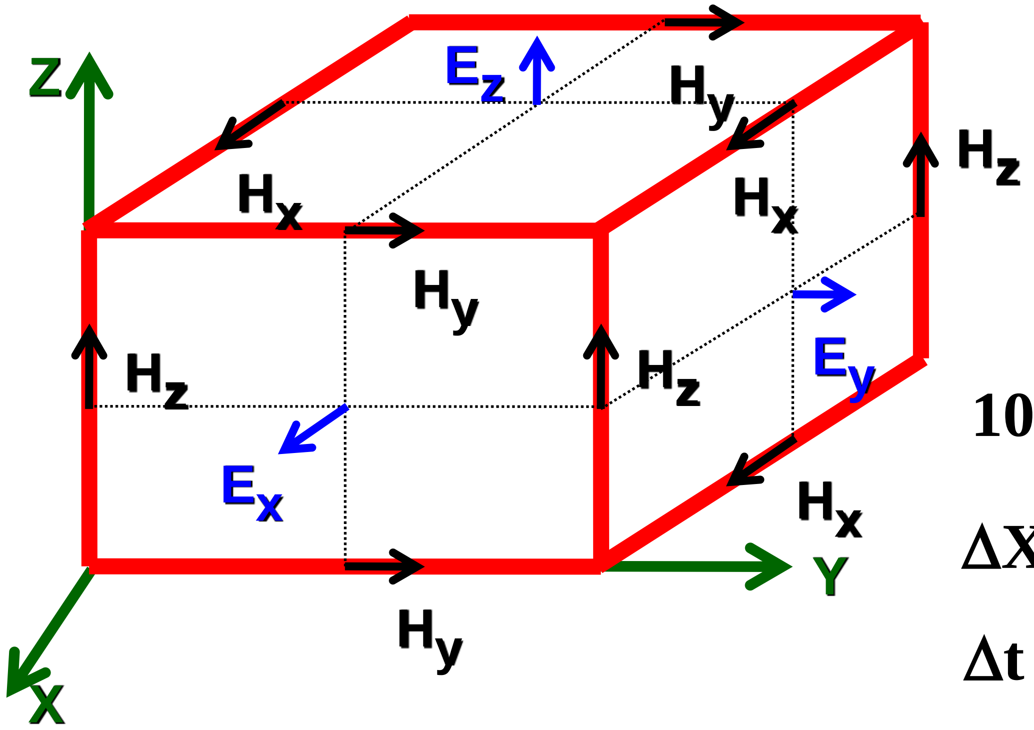
$E_y \sim 0$, TE modes insignificant

$E_z \neq 0$, TM modes exist

E_x is common to TEM & TM
Best choice for comparison

FDTD Model

- Well established in computational electromagnetics.
- EM data from self-consistent simulations are reliable.



$$\nabla \times E = -\frac{\partial B}{\partial t}$$

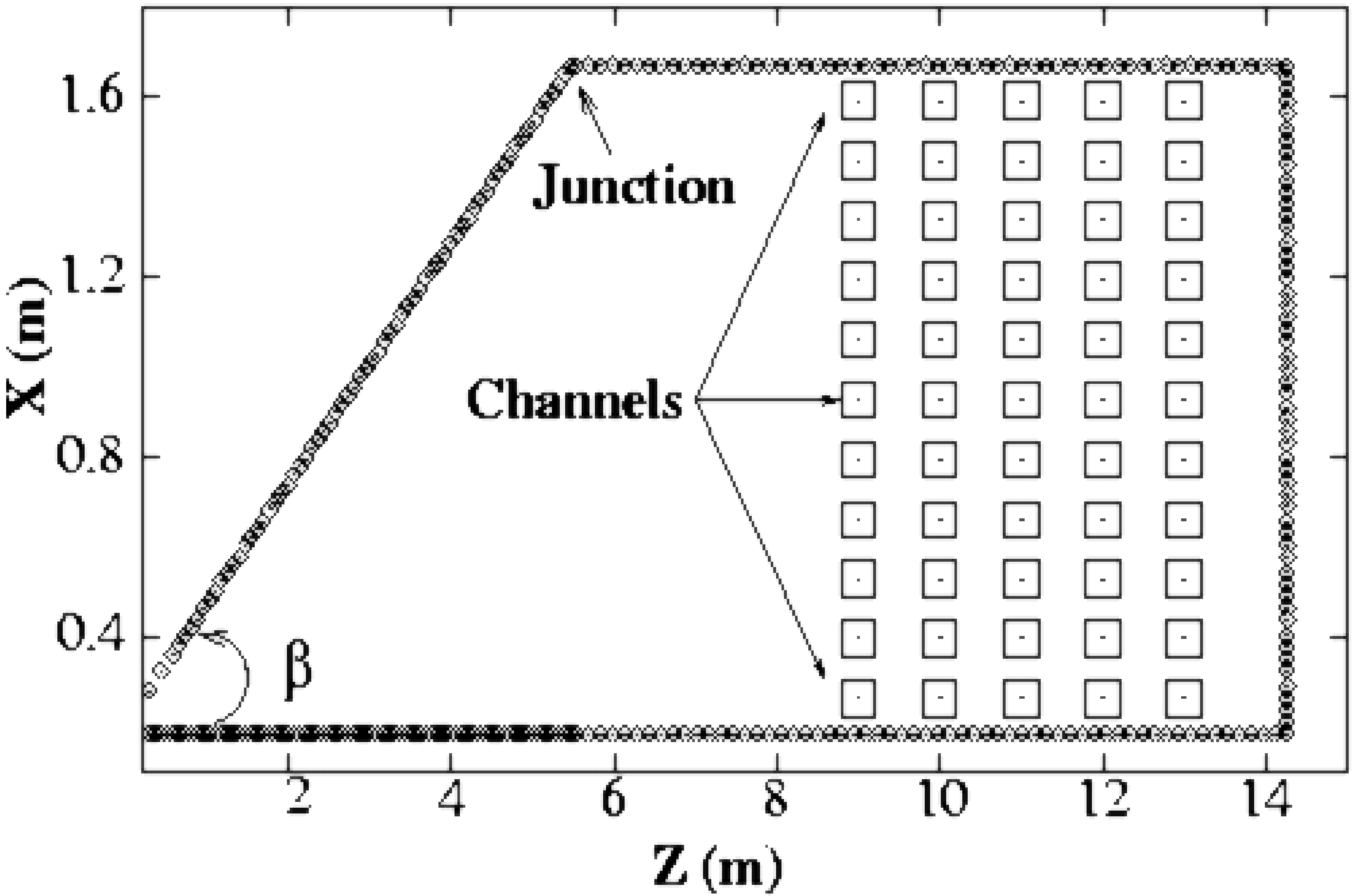
$$\nabla \times B = \frac{\partial D}{\partial t} + J$$

100 x 156 x 600

$\Delta X = \Delta Y = \Delta Z = 1.85$ cm

$\Delta t = 35.6$ pS

Channels: Monitor Fields at FDTD Mesh Points



Modal Analysis Using SVD

Singular Value Decomposition (SVD)

- **Diagonalize rectangular matrices.**
- **Extract all possible modes in a single analysis.**
- **Filter out noise from real data.**

Physical quantity '*E*' : E_{ij}

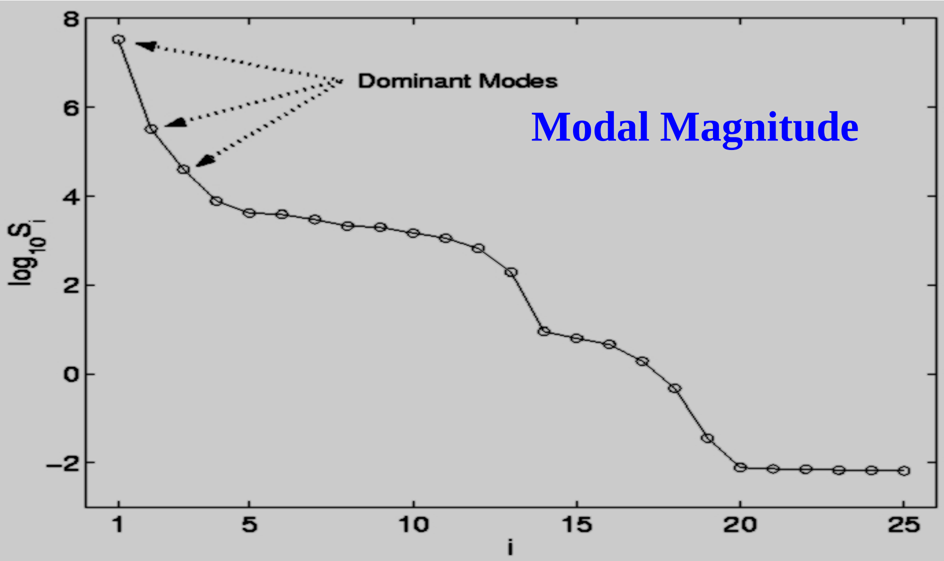
$i \rightarrow$ **Channels** , $j \rightarrow$ **time**

$$\text{SVD } [E] = [U][S][V^T]$$

- [U] → **Temporal Vectors**
- [S] → **Singular Values**
- [V^T] → **Spatial Vectors**

SVD Analysis of FDTD Data

SVD [E_x (25,11000)]



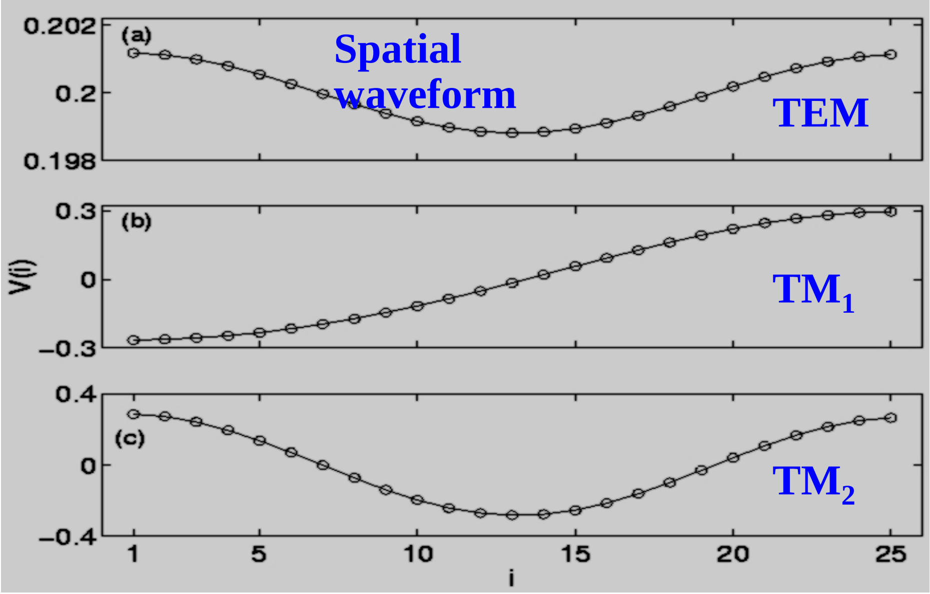
TM-Mode waveform:

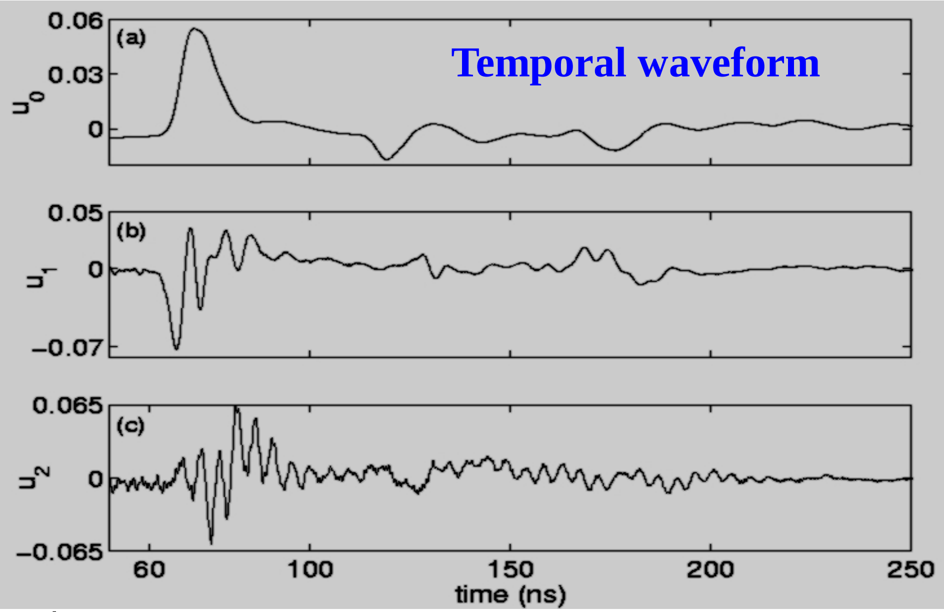
$E_x = E_{x0}$ Cos[m π x/ h]

f_c = mc/2h ; m = mode number (0 , 1 , 2)

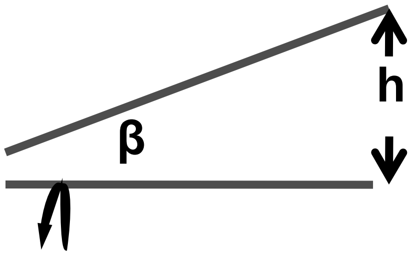
[$S_0 / S_i \sim 10$] ; i = 1 , 2

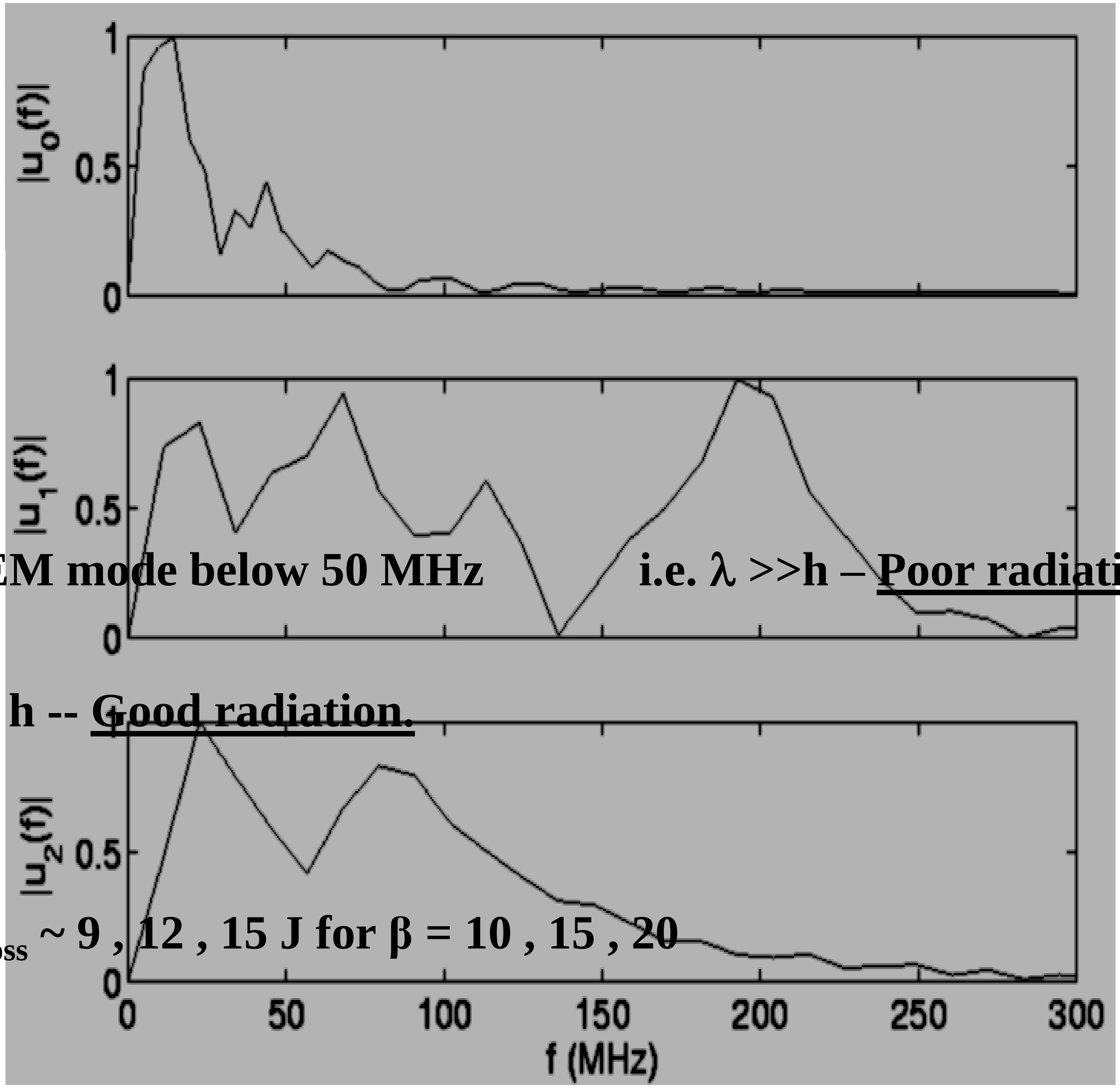
Dominant modes reconstruct original signal , rest are noise.





HOMs and Radiation Leakage





For EM Energy Confinement:
$kh << 1$; $k = 2\pi / \lambda$

- **Significant energy content of TEM mode below 50 MHz i.e. $\lambda >> h$ – <u>Poor radiati</u>**
- **Fair portion of TM_1 and TM_2 $\lambda \leq h$ -- <u>Good radiation.</u>**

Radiation loss increase with angle β

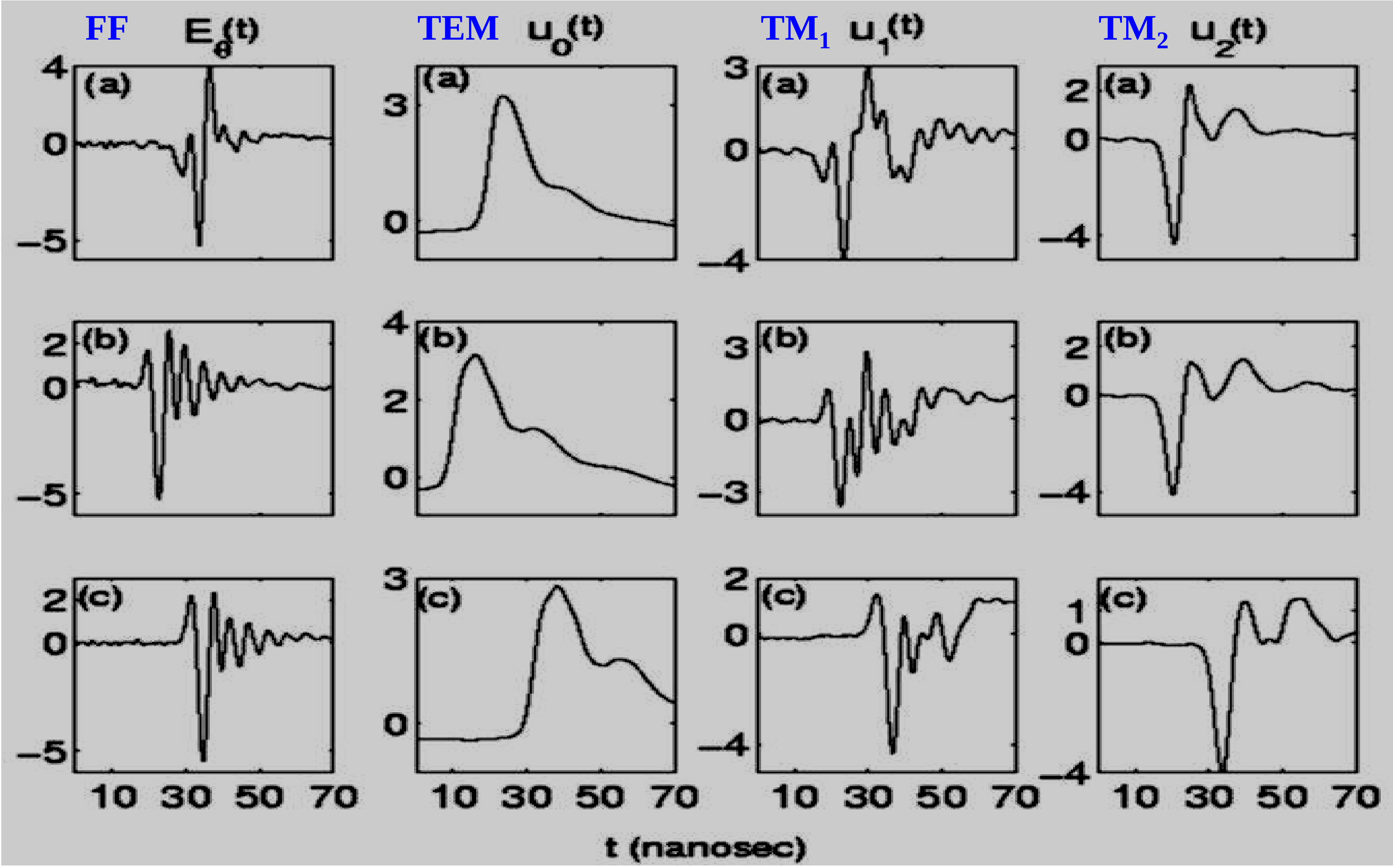
$E_{int} \sim 40$ J **$E_{loss} \sim 9$, 12 , 15 J for $\beta = 10$, 15 , 20**

Visual Inspection

(a). β = 10

(b). β = 15

(c). β = 20



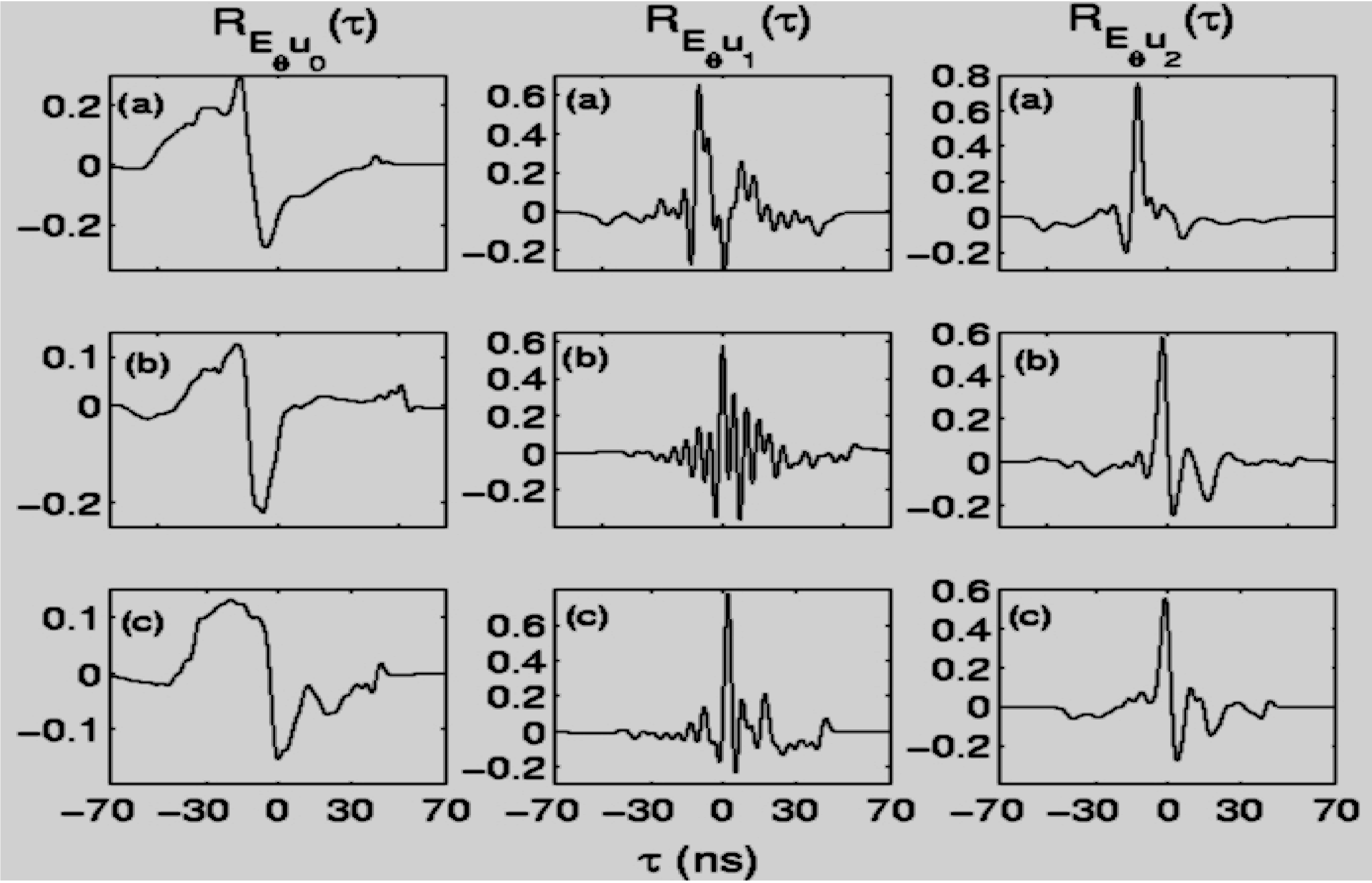
Far-field (FF): Strong correlation -- TM_1 & TM_2 ; Weak correlation -- TEM

Cross-Correlation

(a). β = 10

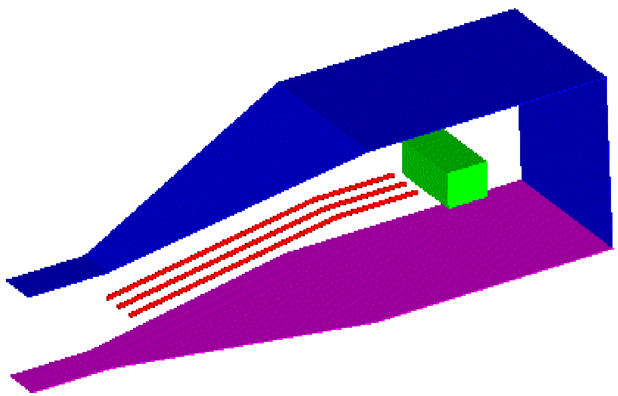
(b). β = 15

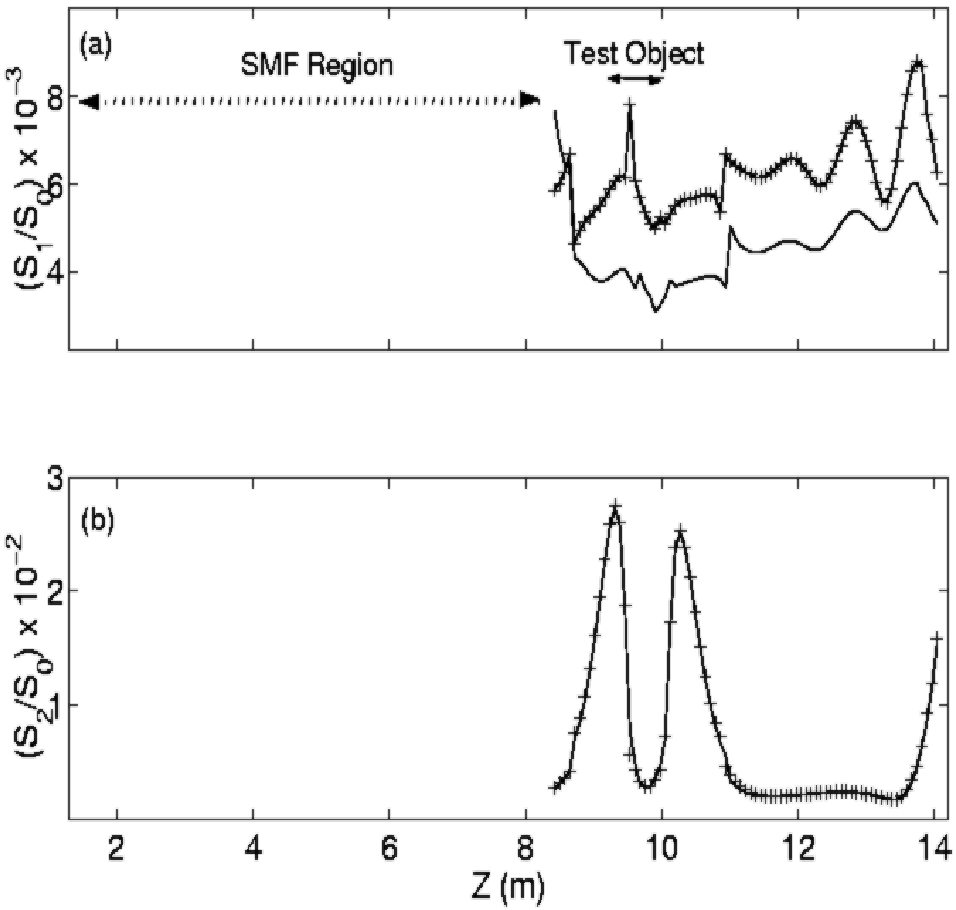
(c). β = 20



FF : Short time weak correlation -- TEM ; Long time strong correlation -- TM_1 , TM_2

HOM Suppression





Conclusions

- **Extract modes inside complex structure using FDTD and SVD methods.**

- **Quantitative measurement of modal magnitudes.**

- **Prediction of radiation leakage using modal time waveforms.**

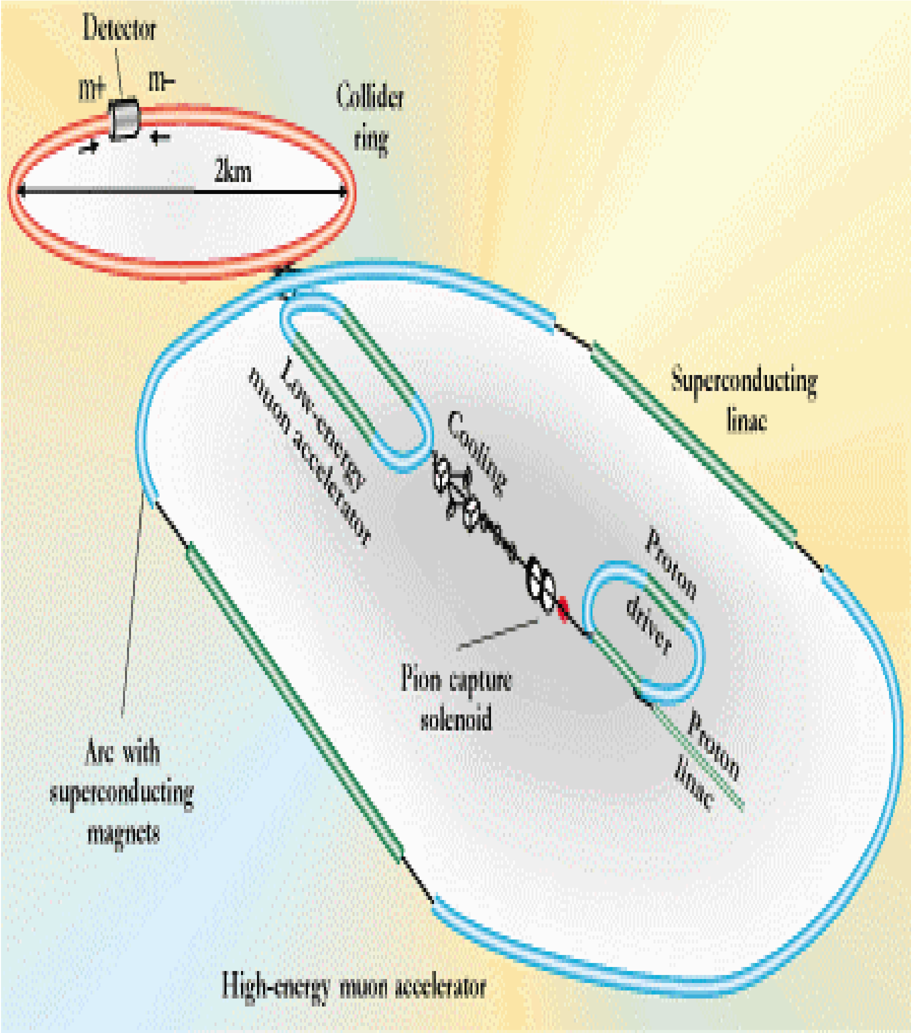
- **Useful study for designing HOM suppressor.**

Beam Matter Interaction

μ^+–μ^- Collider

Important parameter:

luminosity $$L \propto \frac{P_b}{E_{c.m.}} \times \frac{N_\mu}{\sigma_x \sigma_y}$$

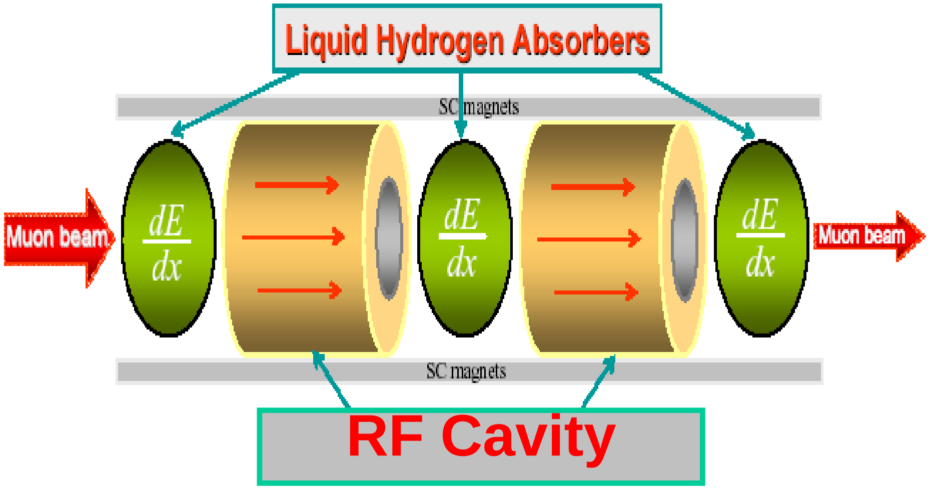


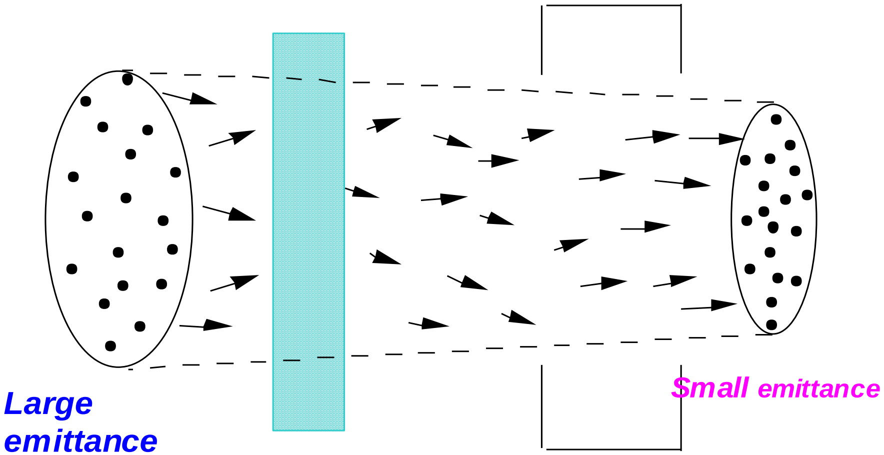
Cooling for small cross-section.

Fast cooling scheme required (Small τ_μ = 2.2 μs).

Ionization Cooling

Ionization Cooling Principle





Absorber **RF**

Loss ($P_{||}$, $P_\perp$) **regain $P_{||}$**

$(P_\perp / P_{||})_{final}$ < $(P_\perp / P_{||})_{initial}$

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2}\left|\frac{dE_\mu}{ds}\right|\frac{\varepsilon_N}{E_\mu} + \frac{\beta_\perp (0.014\,\mathrm{GeV})^2}{2\beta^3 E_\mu m_\mu X_0}$$

Cooling Heating

$$\varepsilon_{x,N,equil.} = \frac{\beta_\perp (0.014\,\mathrm{GeV})^2}{2\beta m_\mu X_0 \left|\frac{dE_\mu}{ds}\right|}$$

To optimize cooling:

- Low $\beta_\perp$ (strong focusing)
- Large X_0 (low Z)
- Low E_μ (typ. 150 < p_μ < 400 MeV/c)

Motivation

Absorber ionized (plasma -- ions, e^-) by several bunches in cooling.

Long recombination rate (~ μs or longer).

How does plasma interact with an incoming beam (**collective effect**)?

Important effects:

- Excitation of plasma wave and wakefield for μ^- and μ^+ beam propagating through plasma.
- Effects of various densities of plasma on incoming beam.
- Effects of external magnetic field.

XOOPIC

Electromagnetic code based on particle-in-cell method, developed by Pla

- **2-dimensional spatial grid**
- **Plasma and beam emission / interaction**
- **Space charge physics**
- **Full EM field solver**
- **Supports distributed computing**
- **Application:** Microwave devices, Plasma sources, Beam optics, Laser/beam plasma interaction, Wakefield Accelerators.

Parameters

Final end of cooling channel

Beam:

Gaussian muons (μ^-, μ^+)

N_b = number of beam particles = 1 x 10^{12} per bunch

σ_r = bunch radius = 3 mm

σ_z = bunch Length = 40 mm

P = reference momentum = 200 MeV/c

m_μ = rest mass of muon = 105.7 MeV/c^2

β = 0.88

Υ = 2.1

τ_p = pulse length = 151 ps

E_{tot} = 226 MeV

Q_b = total charge = 160 nC

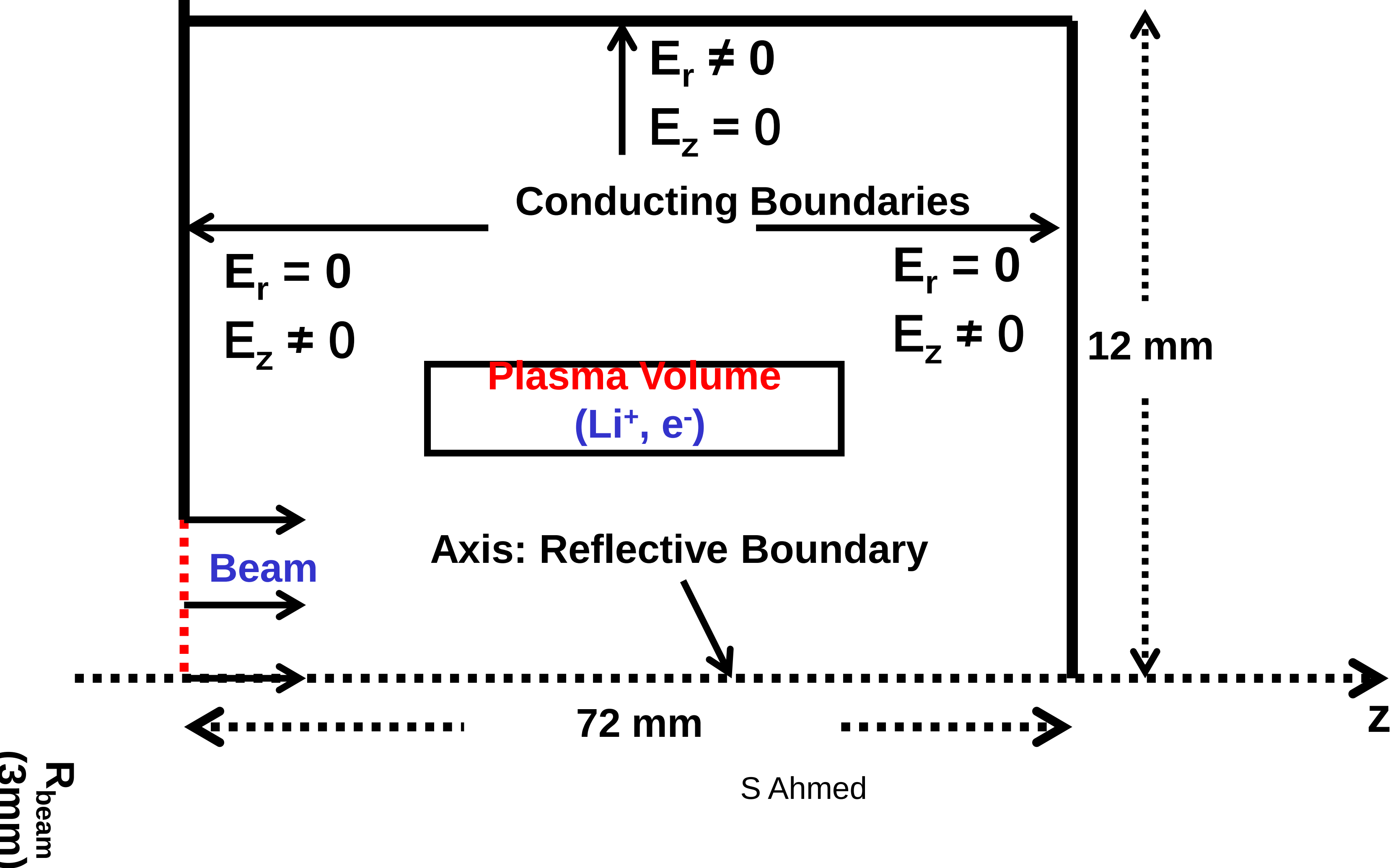
n_b = peak beam density ~ 10^{18} m^{-3}

Simulation Setup

2D Cylindrical Symmetry

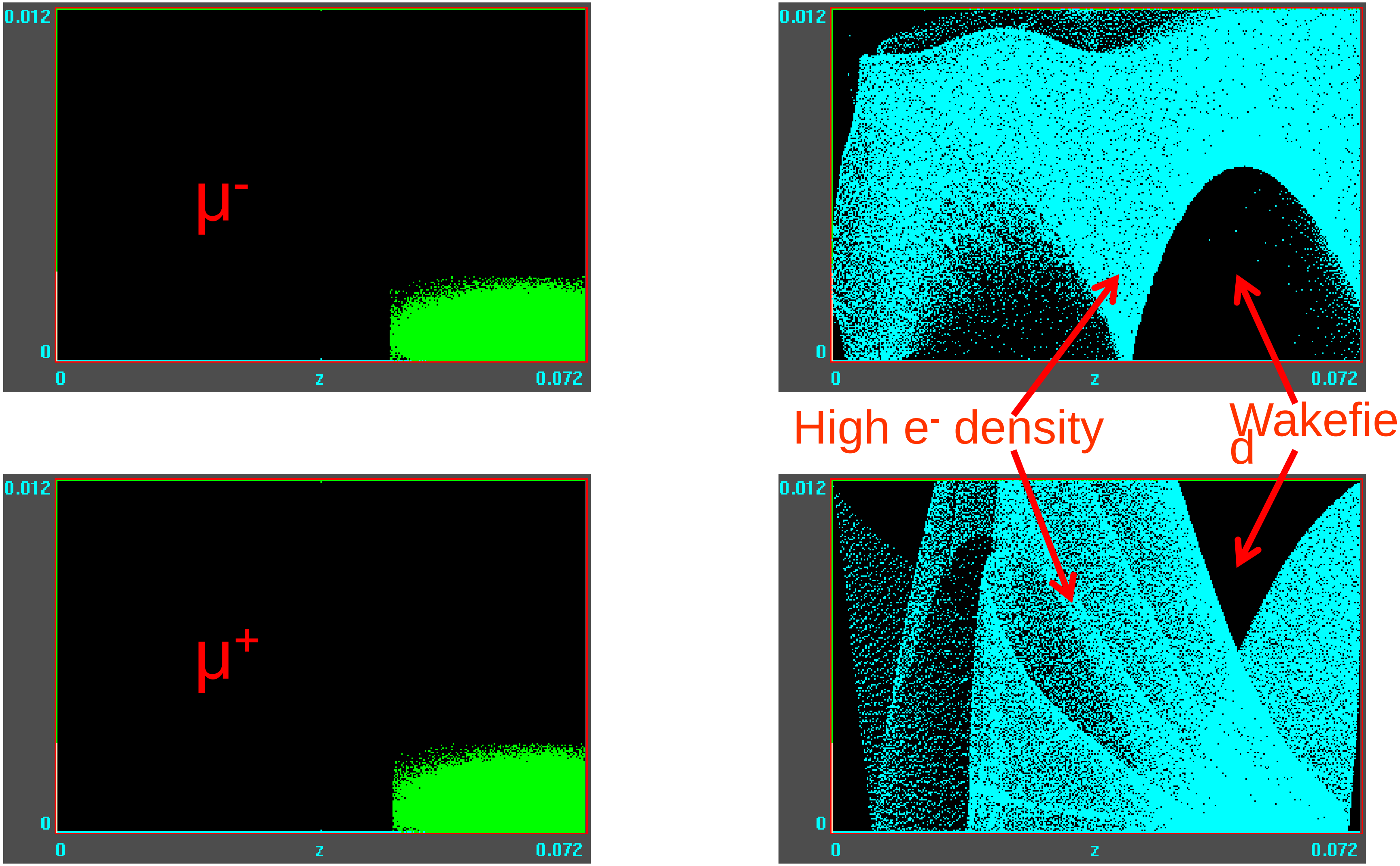
Grid Space (N_r x N_z) = 80 x 480

Time Step-size (Δt) = 0.2 ps (stability condition)



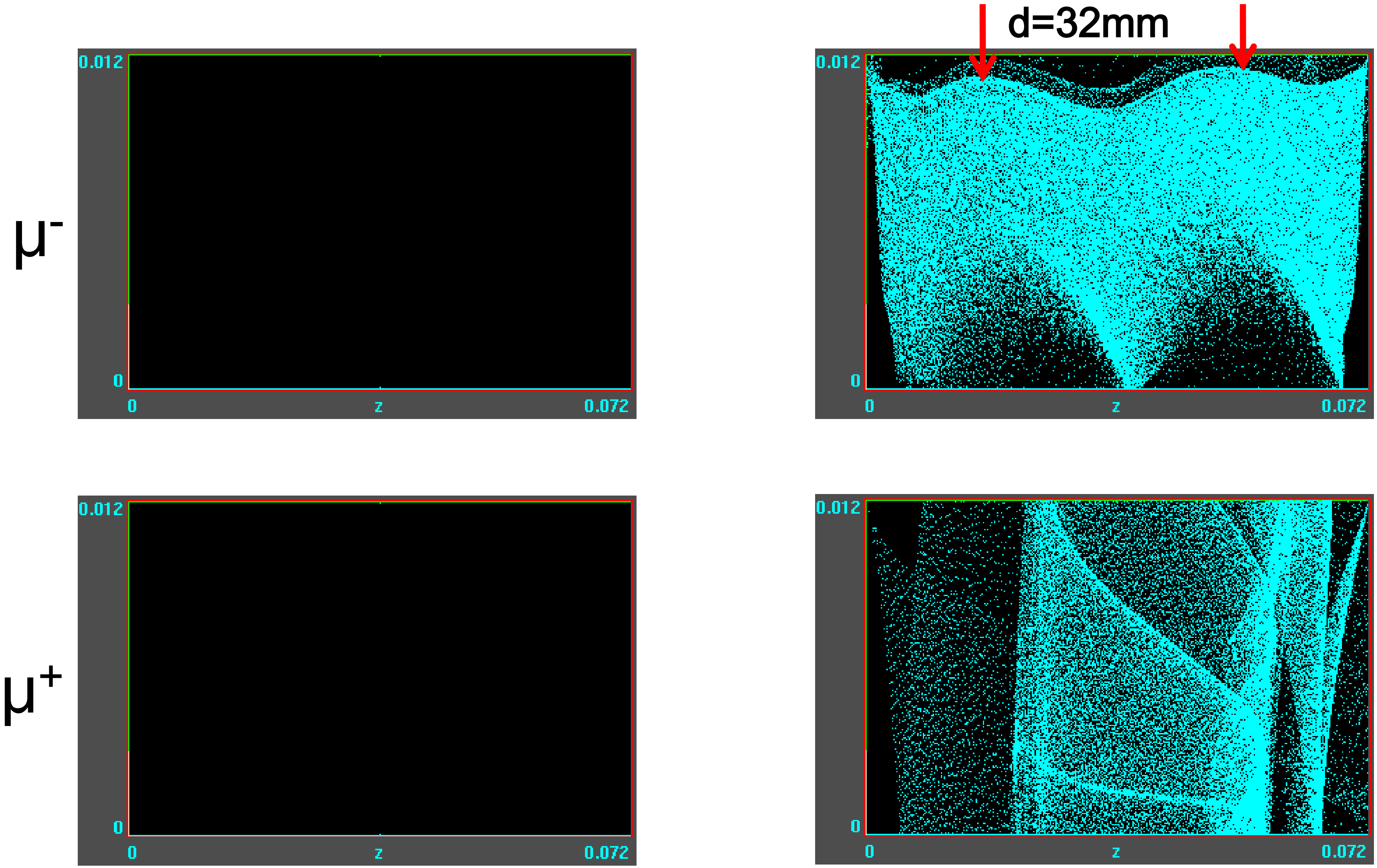
Plasma Wakefield Excitation by μ⁻ & μ⁺ Beam

Snapshot @ t = 300 ps, pulse head @ 80 mm



Muon Beam **Electron Plasma**

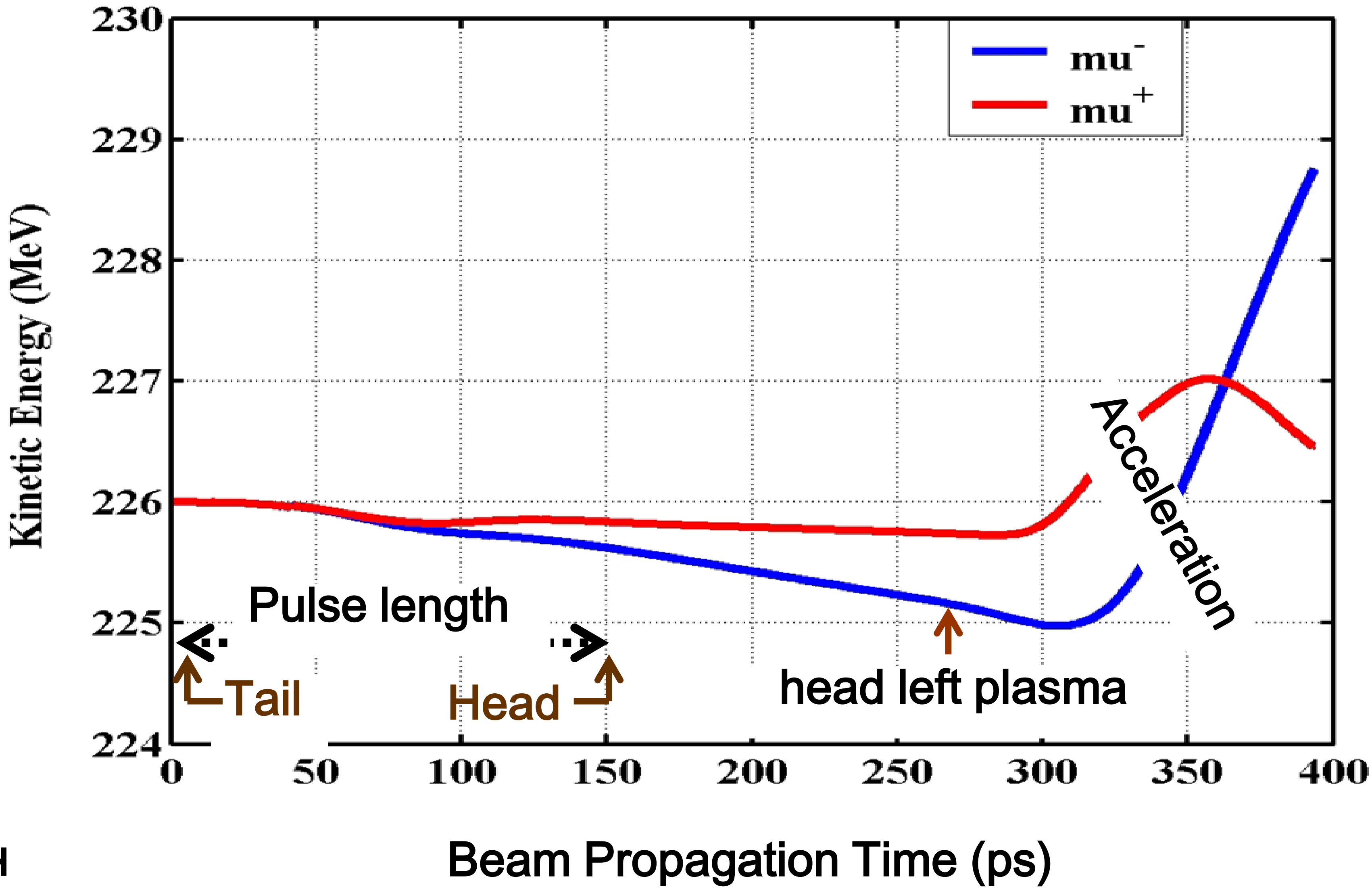
Evolution of μ^- - μ^+ beam in e^- Plasma



wavelength of plasma wave = 32 mm for $n_e = 10^{18}\ m^{-3}$

Plasma wave is excited, no need to worry

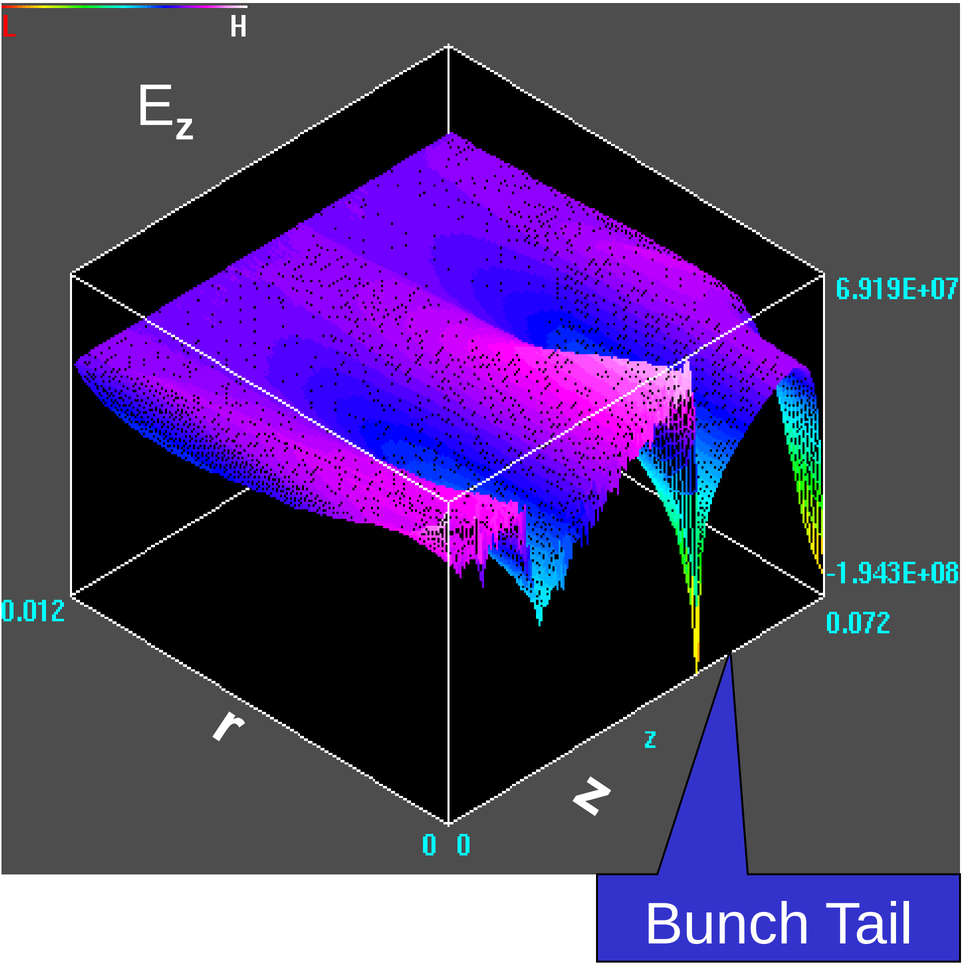
Total Energy of μ⁻ and μ⁺ Beam

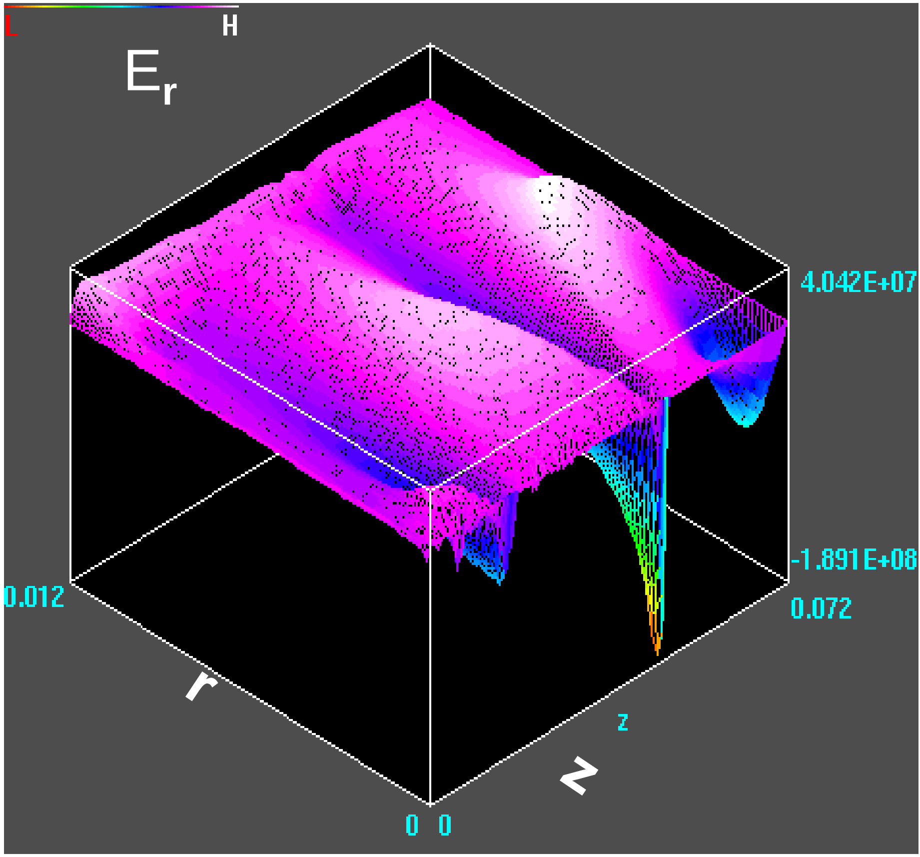


Total

3-D Wake E-field Structures for μ⁻

Snapshot @ t = 325 ps (~ 85 mm) Beam-length = 40 mm

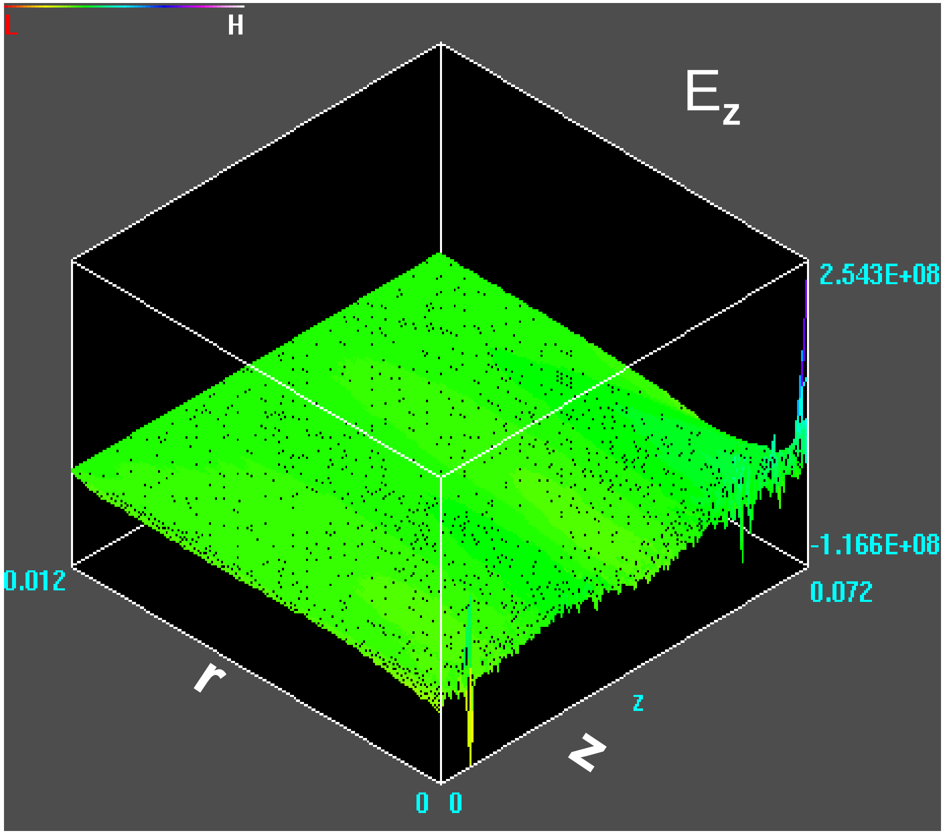


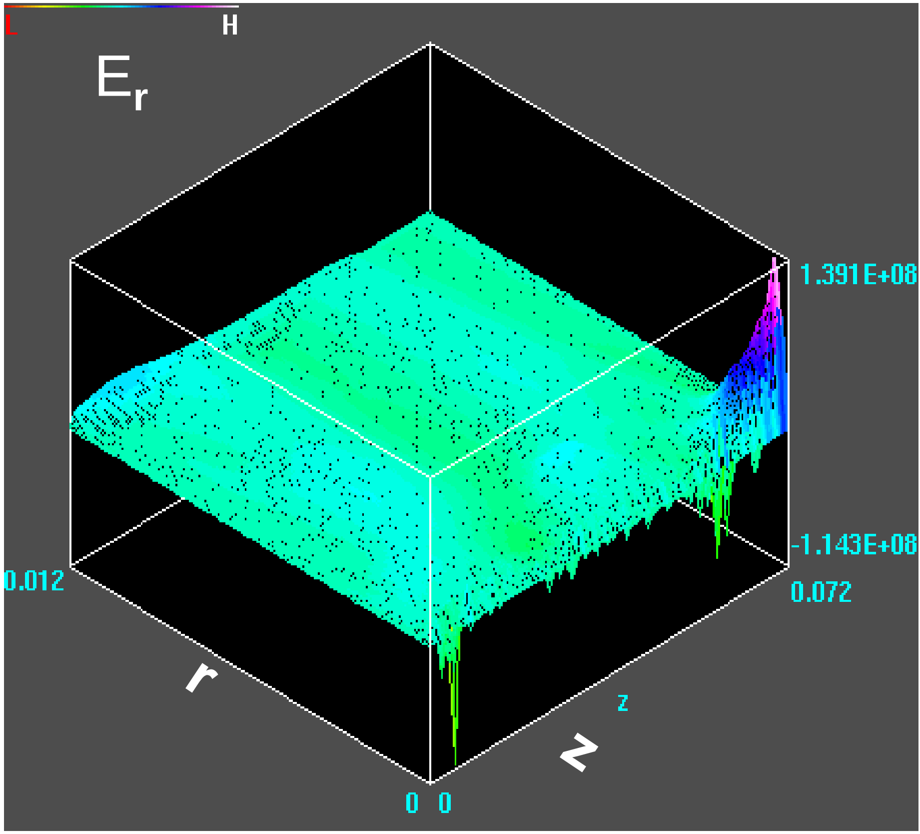


Bunch head generates strong negative wakefield (E_z) -- tail accelerated.

3-D Wake E-field Structures for μ^+

Snapshot @ t = 325 ps (~ 85 mm)   Beam-length = 40 mm

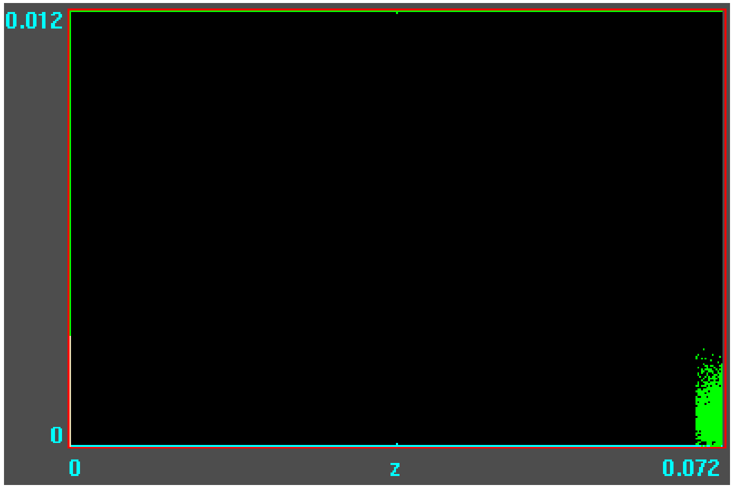




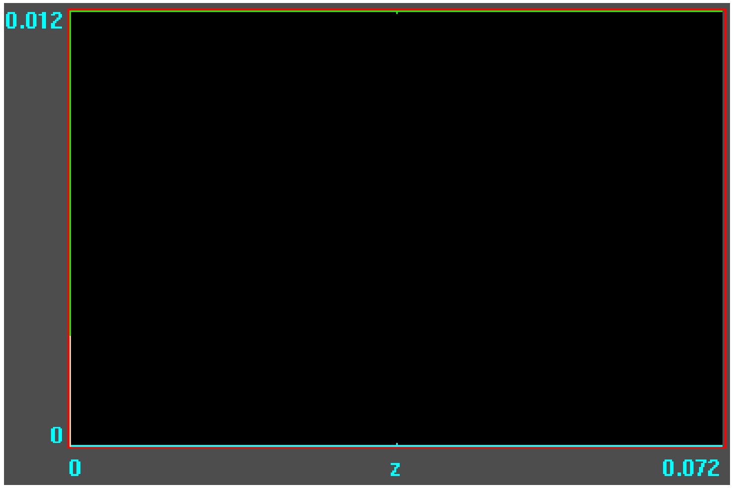
Weak wakefield

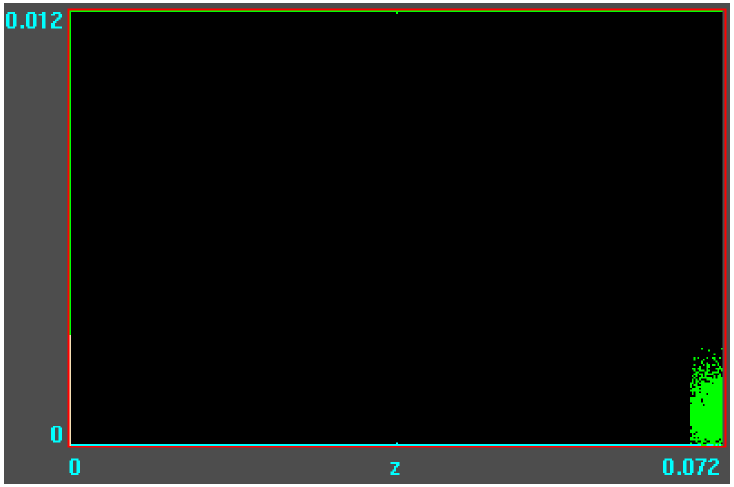
Evolution of μ⁻ beam in Under and Over Dense Plasma

Beam Window — $n_b \sim 10^{18}$ — **Plasma Window**

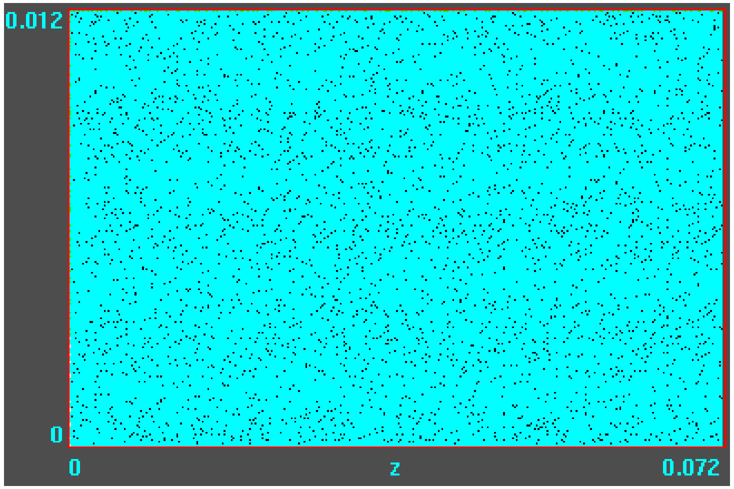


$n_{pe} \sim 10^{16}$





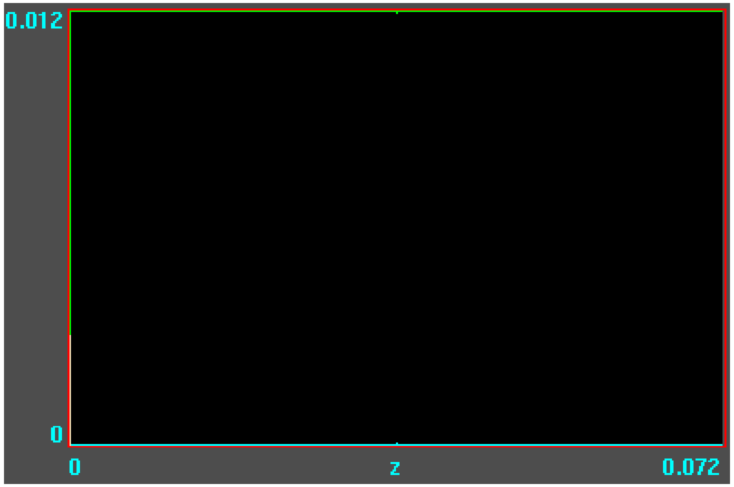
$n_{pe} \sim 10^{22}$



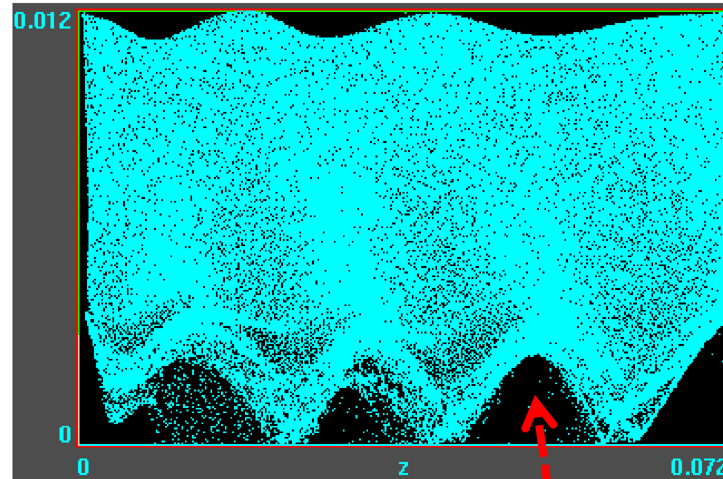
Under dense plasma expelled, over dense plasma unaffected

μ^- Beam and External Magnetic field (B_z)

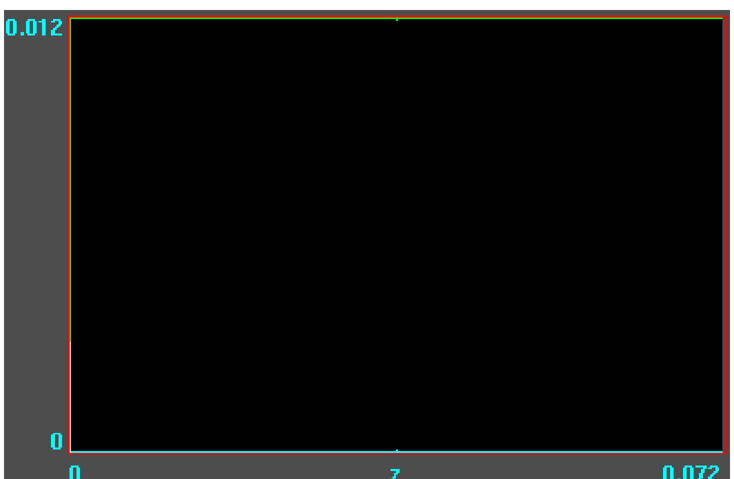
For μ collider, cooling channel is placed in a strong B-field



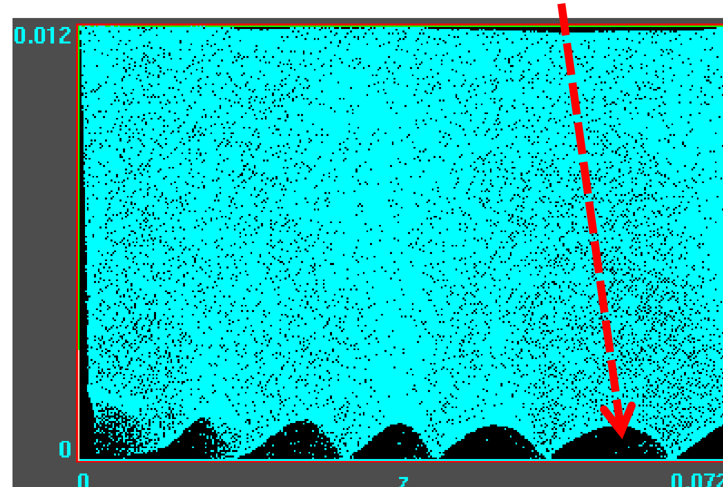
B_z=.5 T



$n_b \sim 10^{18}$, $n_{pe} \sim 10^{18}$



B_z=1 T



S Ahmed

Magnetic field suppresses wakefield

Space Charge Effect

Test space charge implementation in G4beamline:

- G4beamline (GEANT4 based) user friendly accelerator design software developed by Muons, Inc.
- New feature collective computation (stepping in time) allows space charge computation.
- No experimental results available for test.
- Using text book example and particle-in-cell code.

Ref: Theory and Design of Charged Particle Beams – Martin Reiser

Benchmark

Cylindrical beam with parallel beam particles propagating in a drift tube (i.e. no external field).

Parameters of study:

Particles = 2.0 x 10^{10} electrons/bunch

Bunch length = 200.0 mm

Initial radius = 2.0 mm

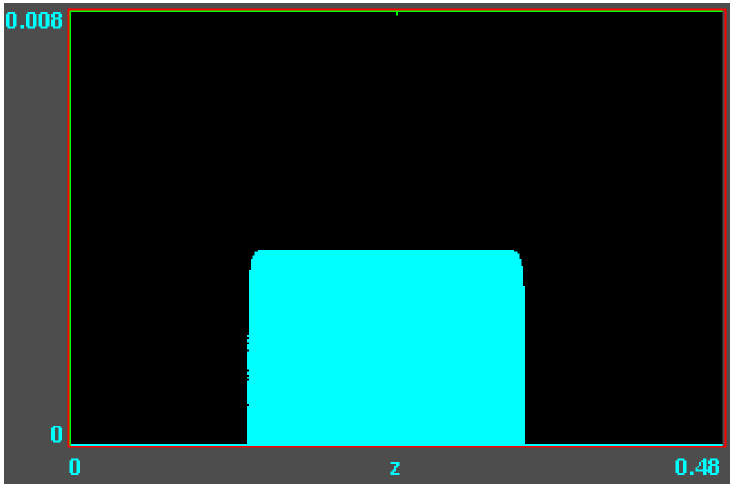
P_z = 0.4 MeV/c

β = 0.615

γ = 1.27

Time Evolution of Beam Envelope

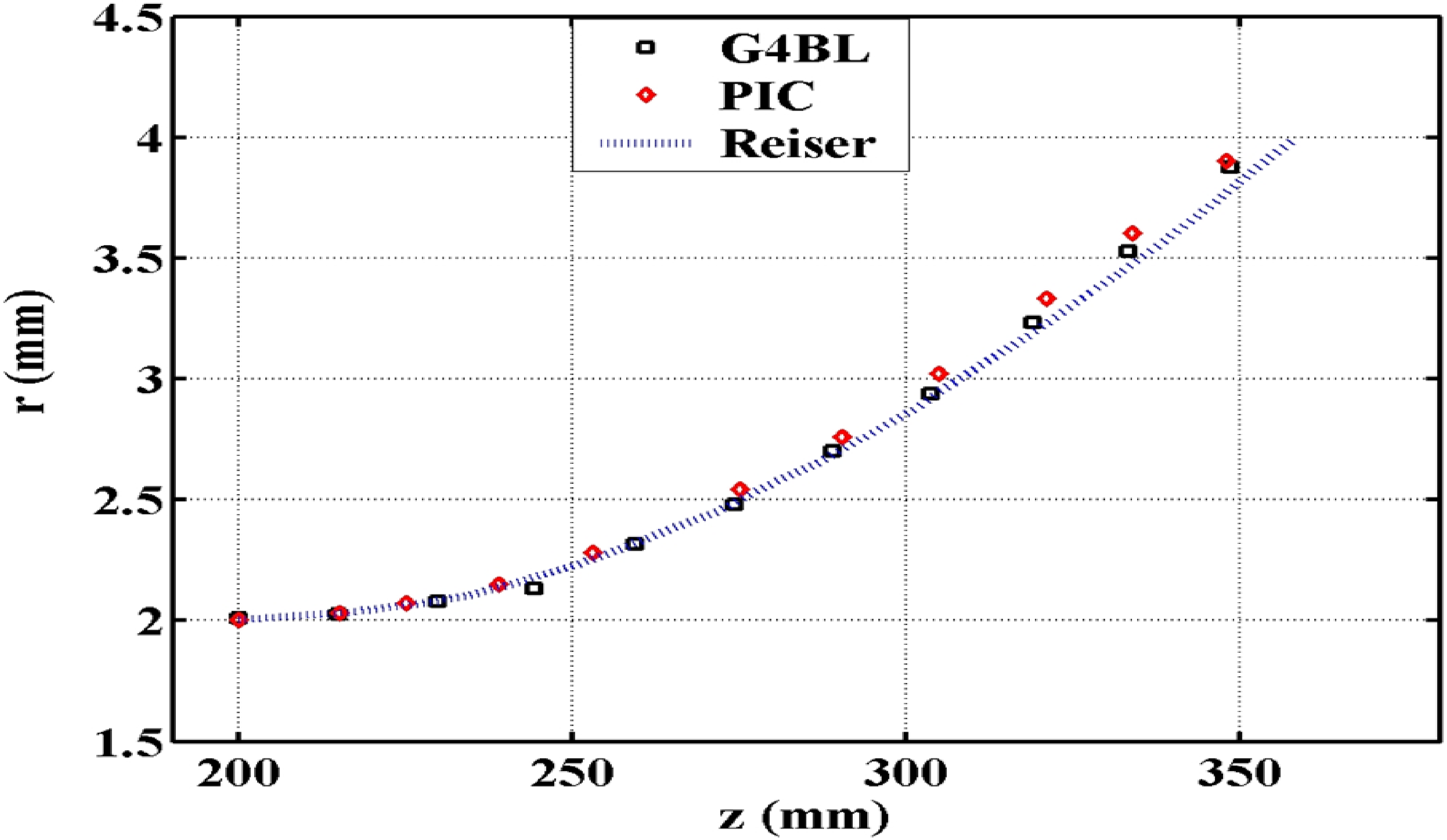
PIC Simulation



For beam head at z = 348 mm:

r = 4 mm by Martin Reiser
r = 3.9 mm by PIC simulation.

Comparison of Results



PIC and G4BL Simulations agree well with analytical result.

Conclusions

- Particle-in-cell simulations of beam interaction with plasma reveal detailed wakefield structures which depend on beam and plasma densities, applied field strength, polarity of beam particle, etc.
- Plasma wakefield excitation is important when peak density of beam is comparable with plasma density, consistent with the other plasma wakefield accelerator simulations --- **polarization of medium and wakefield does not degrade beam.**
- Negatively charged beam experiences net acceleration. However, acceleration is weak for positively charged beam. These results are consistent with SLAC and Max Planck Institute wakefield accelerator simulation results. **Wakefield due to μ^+ is weaker than μ^-.**
- External magnetic field can suppress wakefield. Final cooling channel has 50 T magnetic field -- **should not worry about wakefield**.
- Present simulations reveal that **collective effects are not important** for present design parameters.
- Comparison of space charge physics model used in G4beamline matches well with analytical result and PIC simulation – **a good benchmark.**