Study on Electron Spin Dynamics and Its Application

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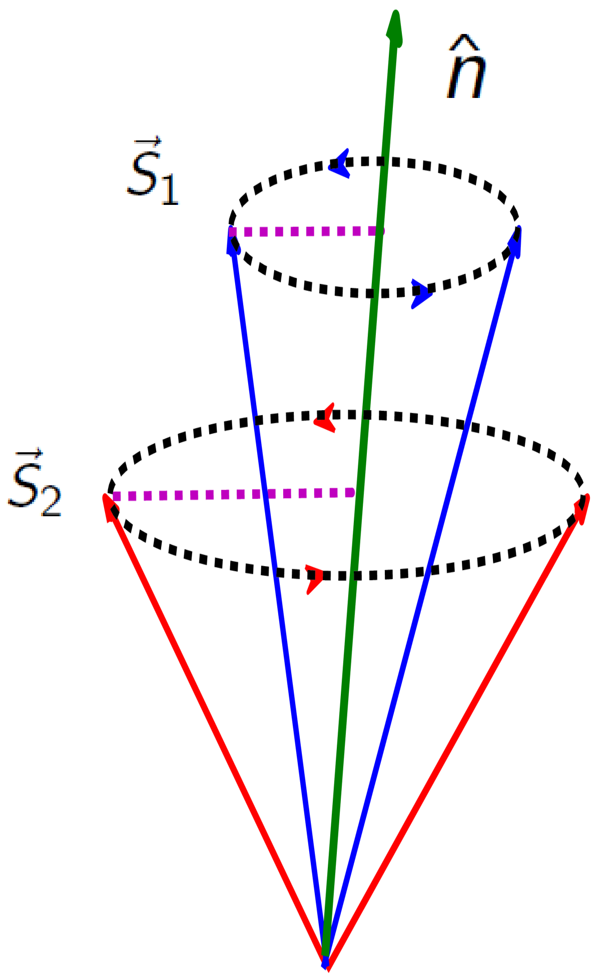
Outline

- Electron spin dynamics in the storage ring
- Polarization measurement at Duke storage ring
- Application:

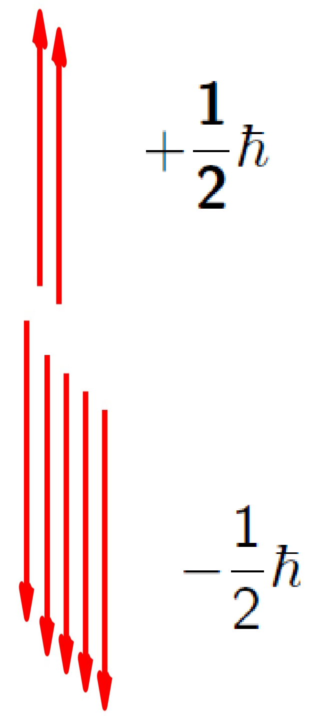
  Using spin depolarization to measure electron beam energy at Hefei storage ring

Beam polarization

classical



quantum mechanical



$$P = \frac{(N_{\uparrow} - N_{\downarrow})}{(N_{\uparrow} + N_{\downarrow})} = -\frac{8}{5\sqrt{3}}$$

Electron orbital motion and spin motion

- Coordinate system

  Orbital $(\hat{x}, \hat{y}, \hat{z})$

  Spin $(\hat{m}, \hat{n}, \hat{l})$

- Closed orbit

  orbital closed orbit X_0

  spin closed orbit $\hat{n}$

  In an Ideal ring,

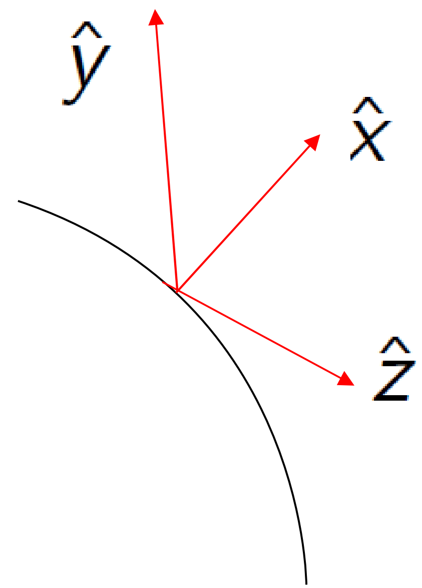
  spin closed orbit $\hat{n}$ is anti-parallel

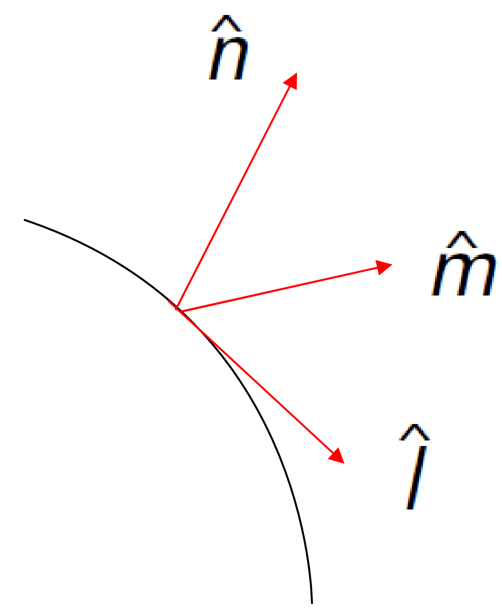
  to $\hat{y}$, and

  orbital revolution frequency is ω_0 ,

  spin precession frequency $\alpha\gamma\omega_0$.

  $\alpha = 0.001159$



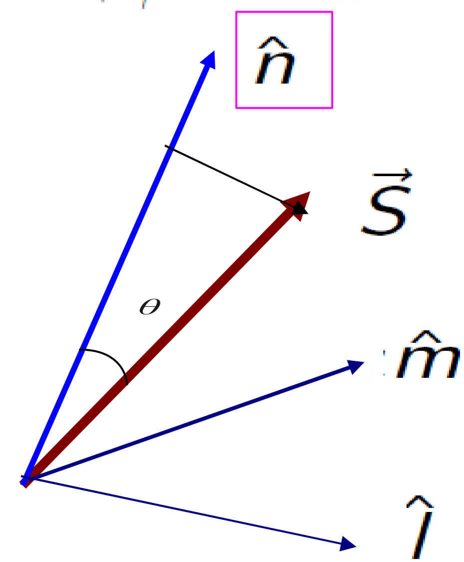


$$\vec{S} = \hat{n} + \bar{\alpha}\hat{m} + \bar{\beta}\hat{l}$$

- Oribtial dipsersion, spin chromaticity

orbital $, \eta_x, \eta_y$ $x = \eta_x \delta$ $y = \eta_y \delta$

Spin $\vec{D}_s = \boxed{\gamma \frac{\partial \hat{n}}{\partial \gamma}}$ $\bar{\alpha}\hat{m} + \bar{\beta}\hat{l} = \vec{D}_s \delta$



- Effects of synchrotron radiation
  - Balance between radiation damping and quantum excitation
    emittance
  - Balance between radiation spin flip and spin diffusion
    equilibrium polarization
- Time
  - Damping time: **order of ms**
  - Polarization build up time: **minutes to hours**
- Equation of motion (Classical)
  - Lorentz equation
  - Thomas-BMT equation

D-K formula(*)

$$P(t) = P_{\mathrm{dk}}\left(1 - e^{-t/\tau_{\mathrm{dk}}}\right) - P_0 e^{-t/\tau_{\mathrm{dk}}}$$

Equilibrium polarization and polarization time are

$$P_{\mathrm{dk}} = -\frac{8}{5\sqrt{3}}\frac{\alpha_-}{\alpha_+} \qquad \tau_{\mathrm{dk}} = \left(\frac{5\sqrt{3}}{8}\frac{e^2\gamma^5\hbar}{m^2c^2}\alpha_+\right)$$

$$\alpha_+ = \frac{1}{2\pi R}\oint \frac{\mathrm{d}s}{|\rho(s)|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}\left|\gamma\frac{\partial\hat{n}}{\partial\gamma}\right|^2\right]_s$$

$$\alpha_- = \frac{1}{2\pi R}\oint \frac{\mathrm{d}s}{|\rho(s)|^3}\left[\frac{\dot{\hat{v}}\times\hat{v}}{|\dot{\vec{v}}|}\cdot\left(\hat{n} - \gamma\frac{\partial\hat{n}}{\partial\gamma}\right)\right]$$

(*) Ya.S.Derbenev, A.M.Kondratenko, Sov.Phys. JETP. 37(1973)968

Thomas-BMT equation (*)

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}t} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(1+G\gamma)\vec{B}_{\perp} + (1+G)\vec{B}_{\parallel} + \left(G\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right]$$

Comments

1) Spin $\vec{s}$: rest frame; magnetic and electric field : Lab frame

2) Spin precession frequency $\vec{\Omega}$ is determined by the electromagnetic field seen by the electrons.

3) Direction of $\vec{\Omega}$ is the direction of spin closed orbit

4) Amplitude of $\vec{\Omega}$ and spin tune (in the ideal ring)

$$\nu = \frac{\Omega}{\omega_0} = a\gamma$$

(*) J.D. Jackson, "*Classical Electrodynamics*", Wiley, New York (1975)

Numerical Algorithm: SLIM (*) (#)

- Use a 8-dimensional vector to designate the state of an electron, additional to 6-D traditional orbital components, two spin components are added to denote the spin motion.

$$X = \begin{bmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta \\ \bar{\alpha} \\ \bar{\beta} \end{bmatrix}$$

(*) A.W.Chao, NIM 180 (1981) 29

(#) A.W.Chao, AIP Proc. 87 (1981) 395

Characteristics of SLIM

Using eigenvectors and eigenvalues of a matrix, to

- study a general, linear coupled accelerator lattice.
- calculate coupled orbital motions in the 6-D phase space.
- calculate coupled damping, coupled beam size and coupled emittance;

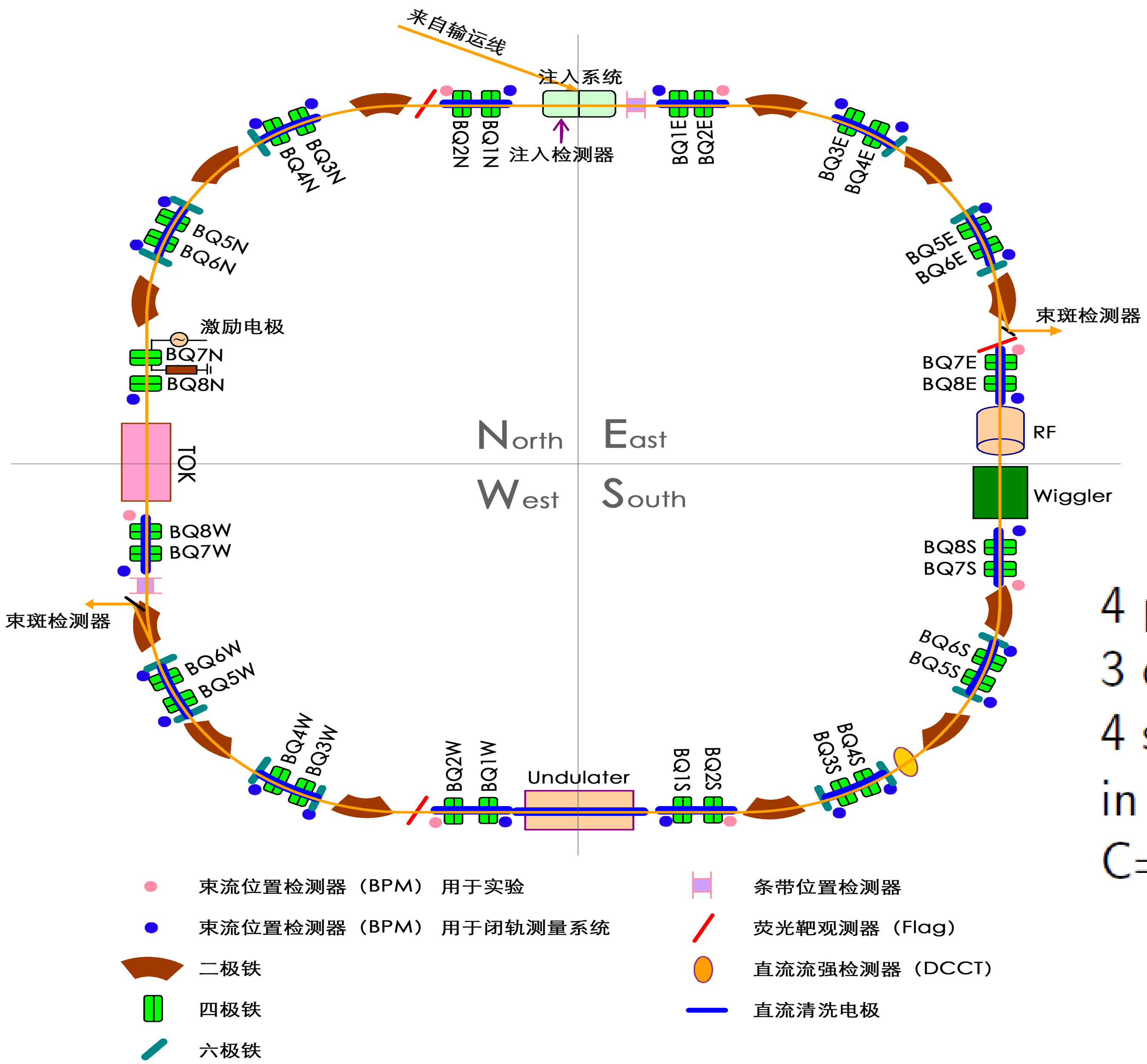
- include coupling of orbital motion on the spin motion, calculate spin closed orbit and spin chromaticity.
- Calculate polarization and polarization time according to D-K formula.
- Seven sets of resonance in SLIM.

$$
\begin{aligned}
\nu &= n \\
\nu &= n \pm \nu_x \\
\nu &= n \pm \nu_y \\
\nu &= n \pm \nu_z
\end{aligned}
$$

Two example applications of SLIM

- Hefei storage ring
- Duke storage ring

Layout of Hefei Storage ring



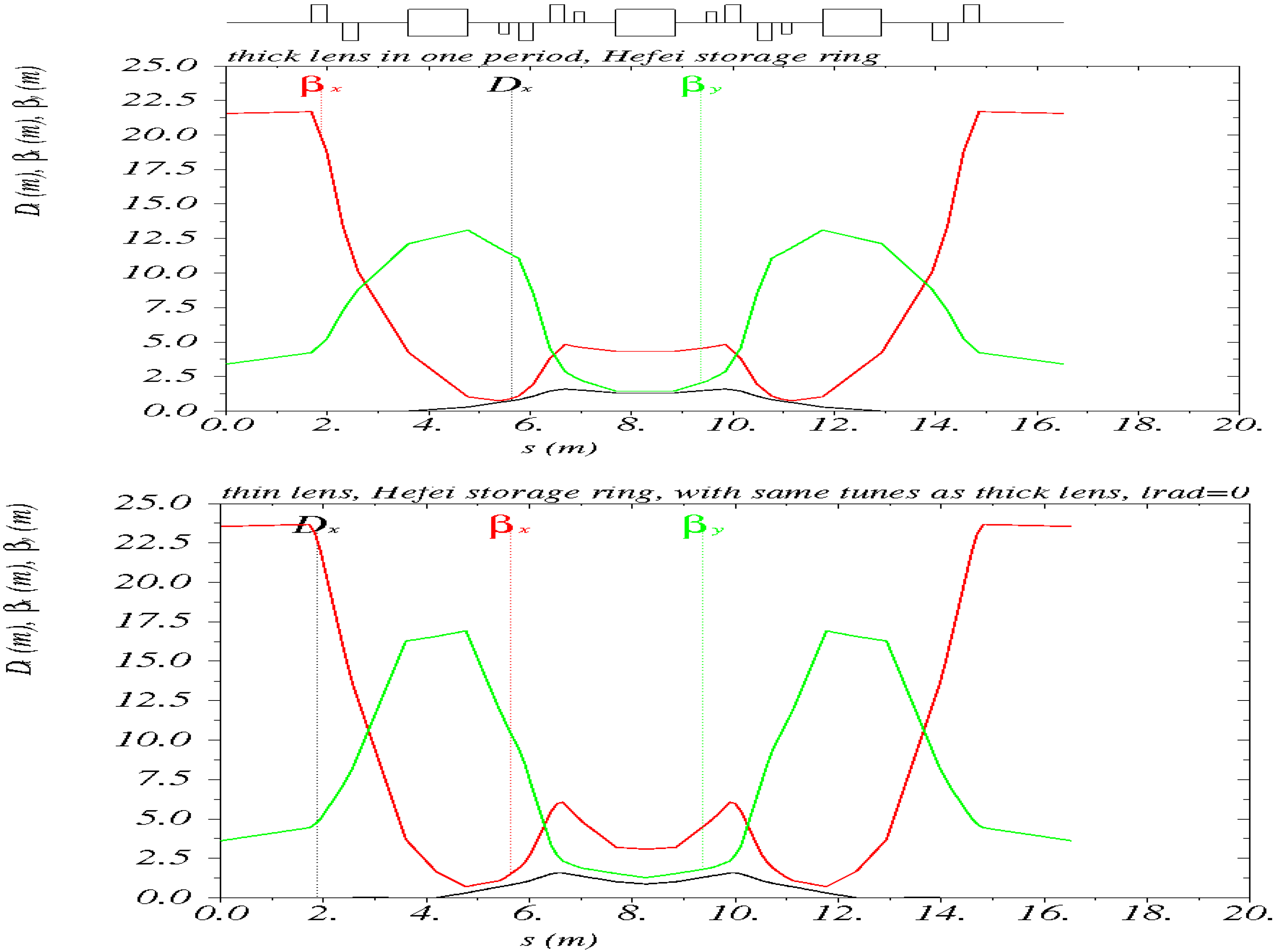
4 periods;

3 dipoles,8 quadupoles,

4 sextupoles,

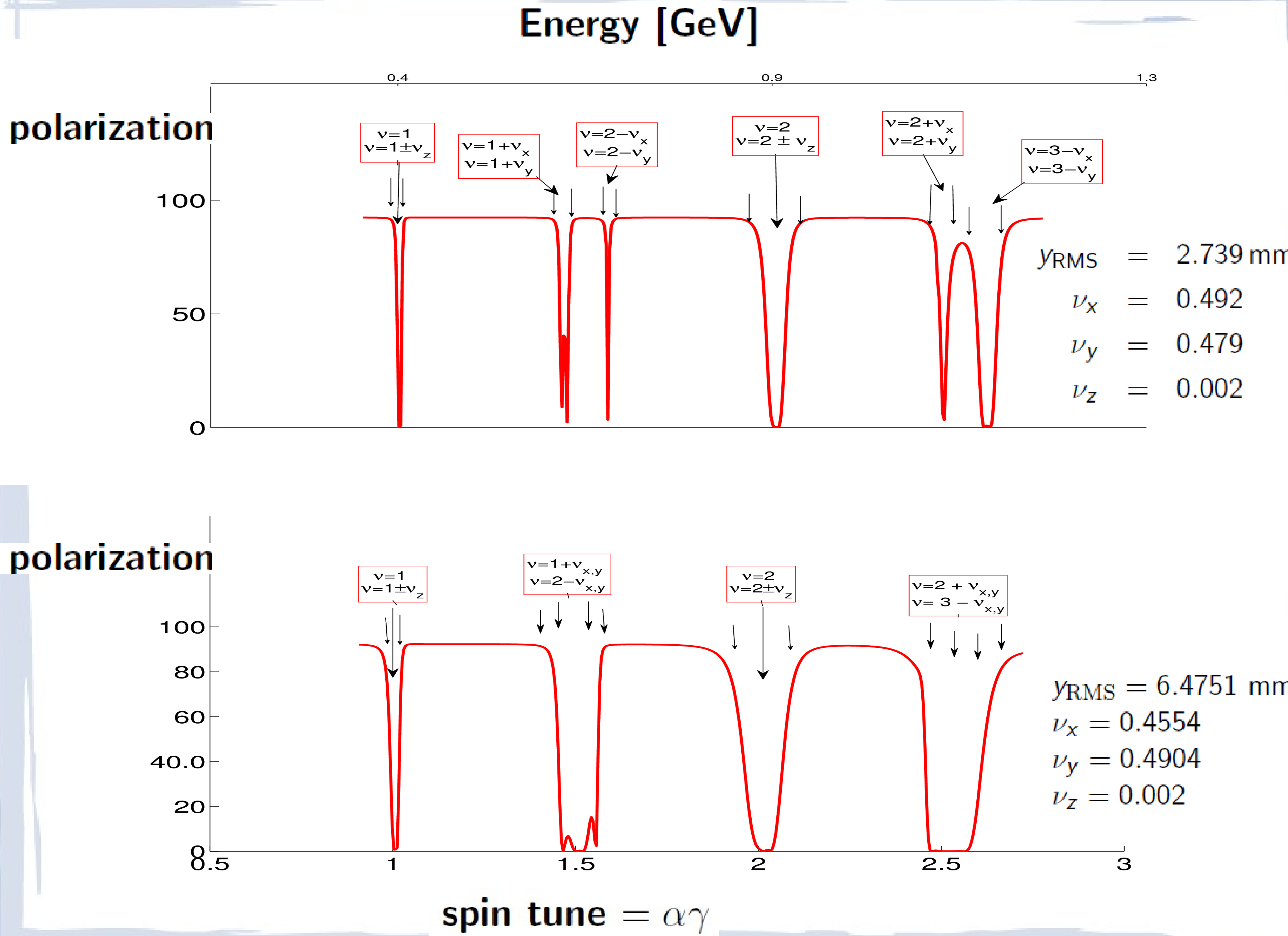
in each period;

C=66.1308 m

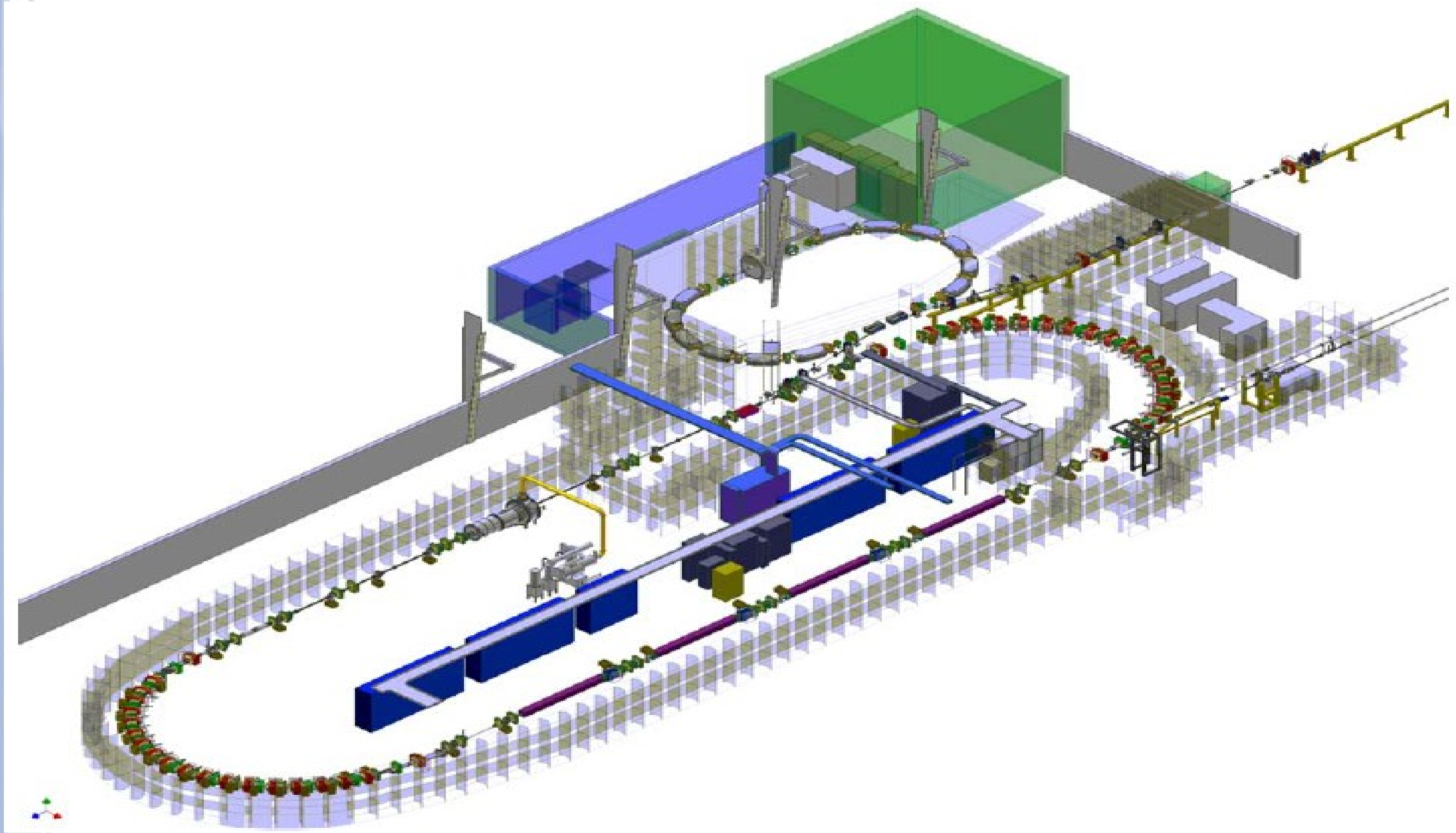


Optics functions of Hefei storage ring.

(Top) Thick lens model ; **(Bottom) thin lens model (used in SLIM)**

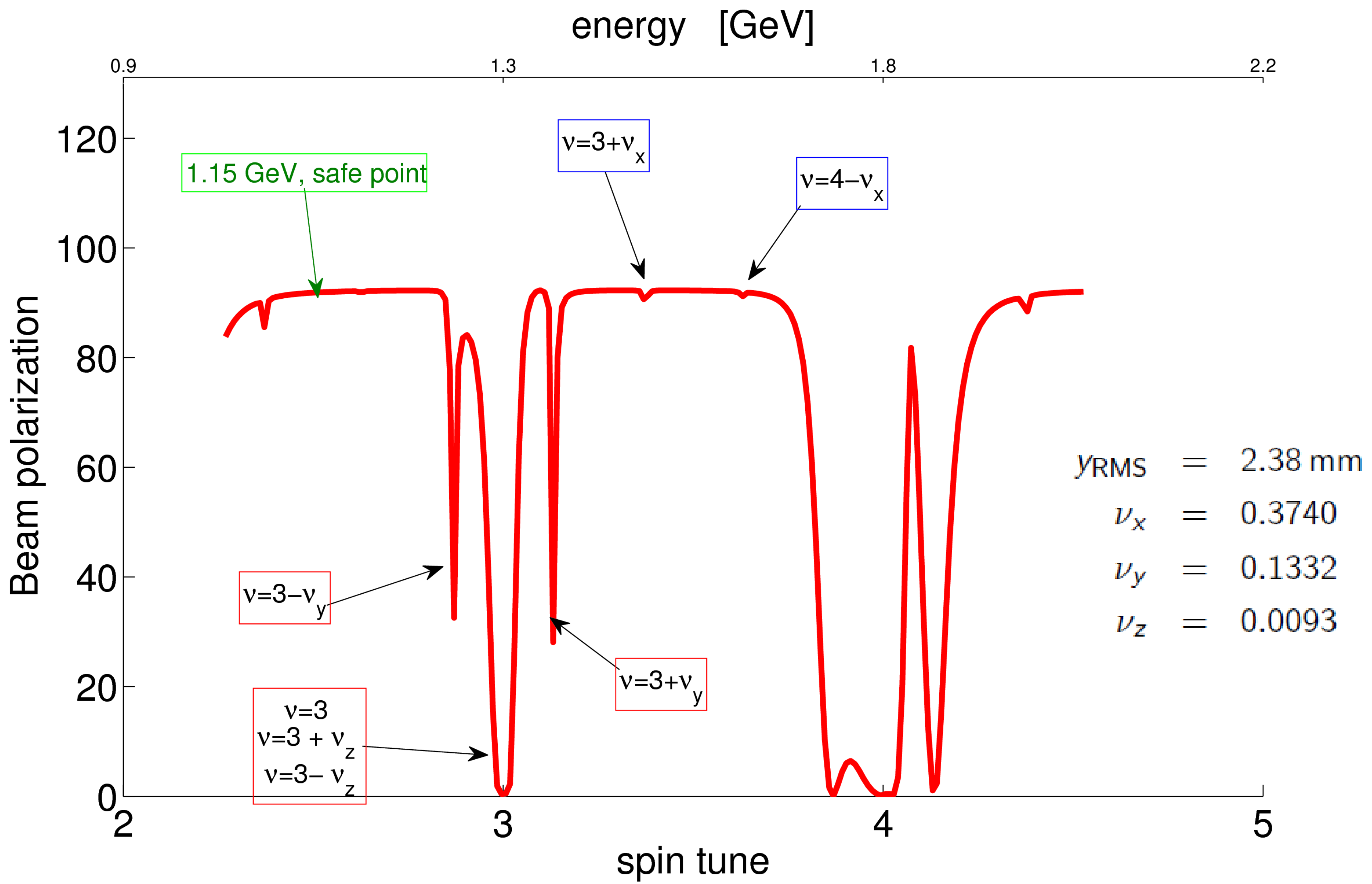


Energy [GeV]
0.4
0.9
1.3
polarization
100
50
0
ν=1
ν=1±ν_z
ν=1+ν_x
ν=1+ν_y
ν=2−ν_x
ν=2−ν_y
ν=2
ν=2 ± ν_z
ν=2+ν_x
ν=2+ν_y
ν=3−ν_x
ν=3−ν_y
y_RMS = 2.739 mm
ν_x = 0.492
ν_y = 0.479
ν_z = 0.002
polarization
100
80
60
40.0
20
0
ν=1
ν=1±ν_z
ν=1+ν_x,y
ν=2−ν_x,y
ν=2
ν=2±ν_z
ν=2 + ν_x,y
ν= 3 − ν_x,y
y_RMS = 6.4751 mm
ν_x = 0.4554
ν_y = 0.4904
ν_z = 0.002
0.5
1
1.5
2
2.5
3
spin tune = αγ



Layout of Duke accelerator;
there are 2 symmetric arcs, two straight sections, south straight section is for wigglers on the storage ring.

Calculation results of SLIM, in the energy 1.15 GeV, beam polarization is safe



Experiments to measure electron beam polarization

Polarimeters

- Two types of polarimeters (*)

  Moller polarimeters, $e \longleftrightarrow e$ scattering, Jlab;

  Compton polarimeters, $e \longleftrightarrow \gamma$ scattering, SLAC.

- Both polarimeters are of high accuracy, but the set up are complicated and the devices are expensive.

- Simple and inexpensive method ???? Even if the accuracy is not high?

(*) A.Chao, M.Tigner, Handbook of accelerator physics and engineering

Touschek beam loss

- Moller cross section ⟸ polarization;
- Touschek beam loss ⟸ polarization.
- For a flat, polarized electron beam,

$$\frac{1}{\tau_{\mathrm{touP}}} = \frac{1}{\tau_{\mathrm{tou0}}}(1 - A\,P^2)$$

- *A* is a function of momentum acceptance, etc.

A=0.15 for Duke storage ring.

If we could produce two beam with same status except that one beam is polarized, and another one is unpolarized,

- then the relative total beam loss is equal to relative Touschek beam loss:

$$\frac{1/\tau_0 - 1/\tau_{\mathrm{P}}}{1/\tau_0} = \frac{1/\tau_{\mathrm{tou0}} - 1/\tau_{\mathrm{tauP}}}{1/\tau_{\mathrm{tau0}}} = A\,P^2$$

The expected relative total beam loss is 13%,

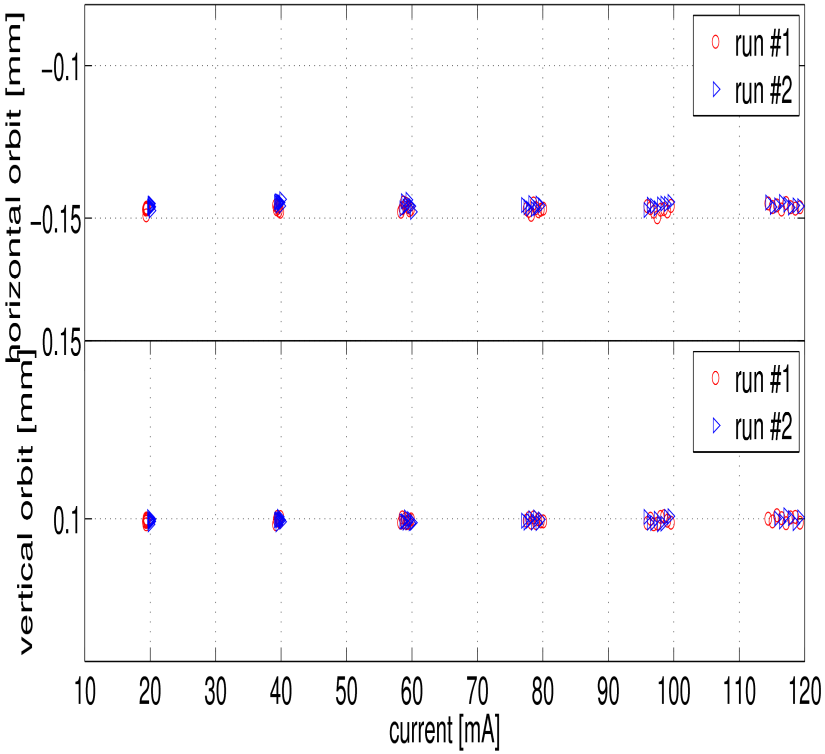
for the 92.38% polarized 1.15 [GeV] electron beam in duke storage ring.

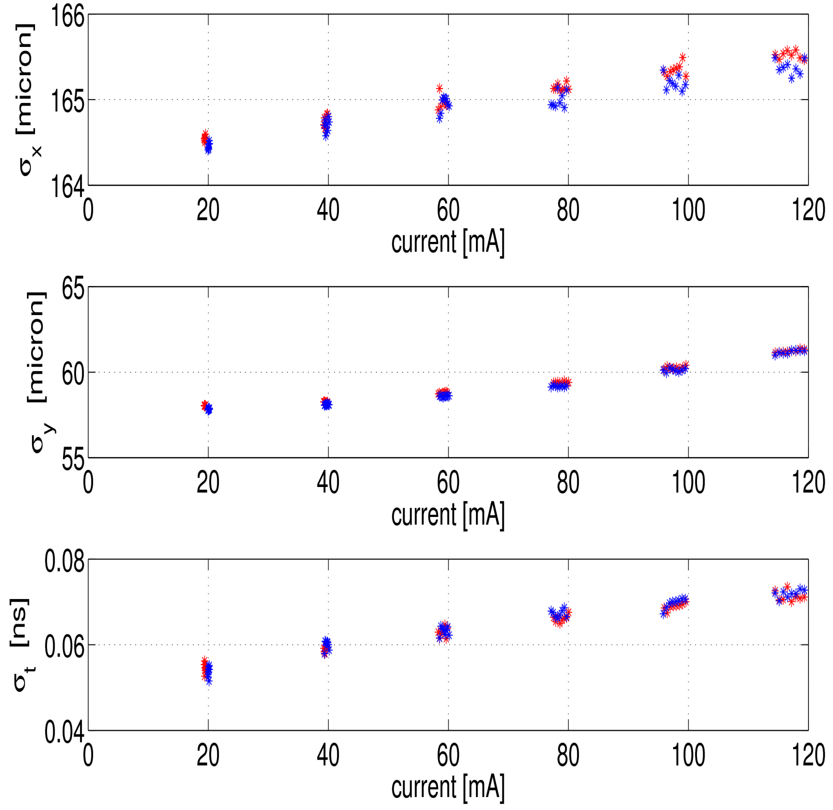
Experiment feasibility study

Check whether we could produce two unpolarized beam with the beam status

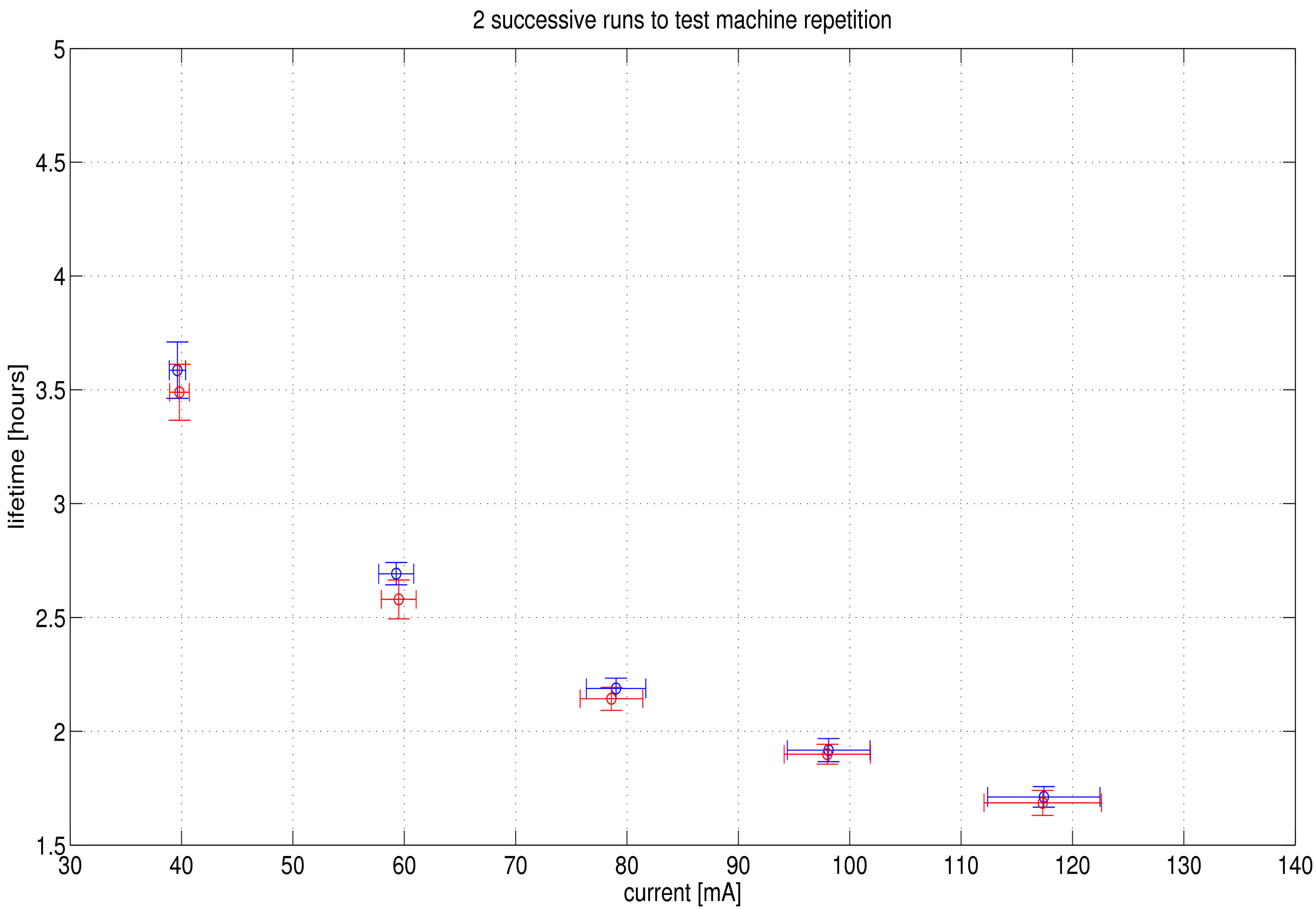
- Turn on longitudinal feedback system;
- Produce two unpolarized beam, measure their lifetime with the same current;
- Comparing the orbit, RF voltage during the two experiments;
- comparing beam size, bunch length, during the experiments.

Orbits and beam size and bunch length of two unpolarized beam; beam is repetitive and machine reproducibility is good





Lifetime comparation in two runs

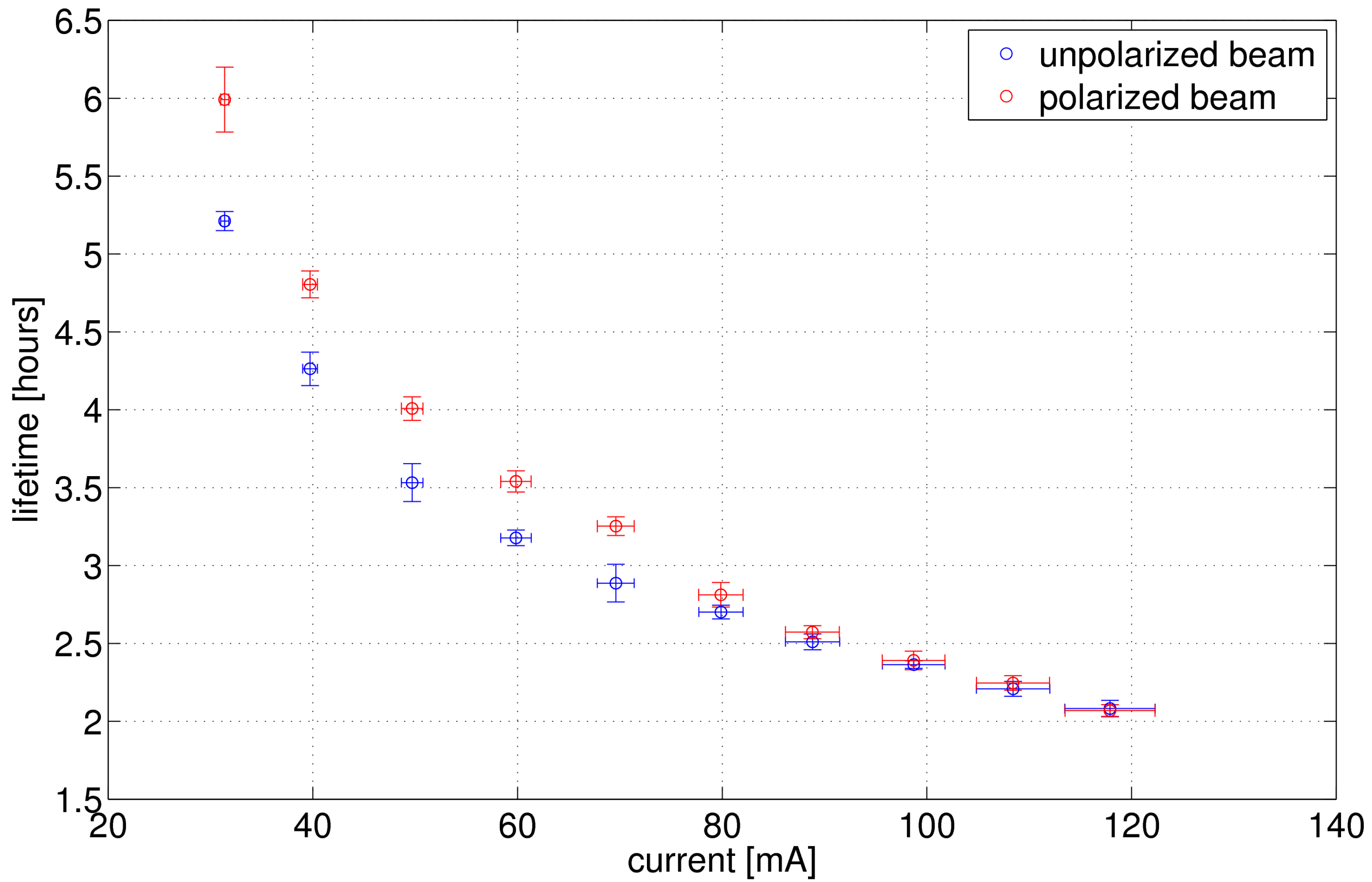


Procedures to measure polarization

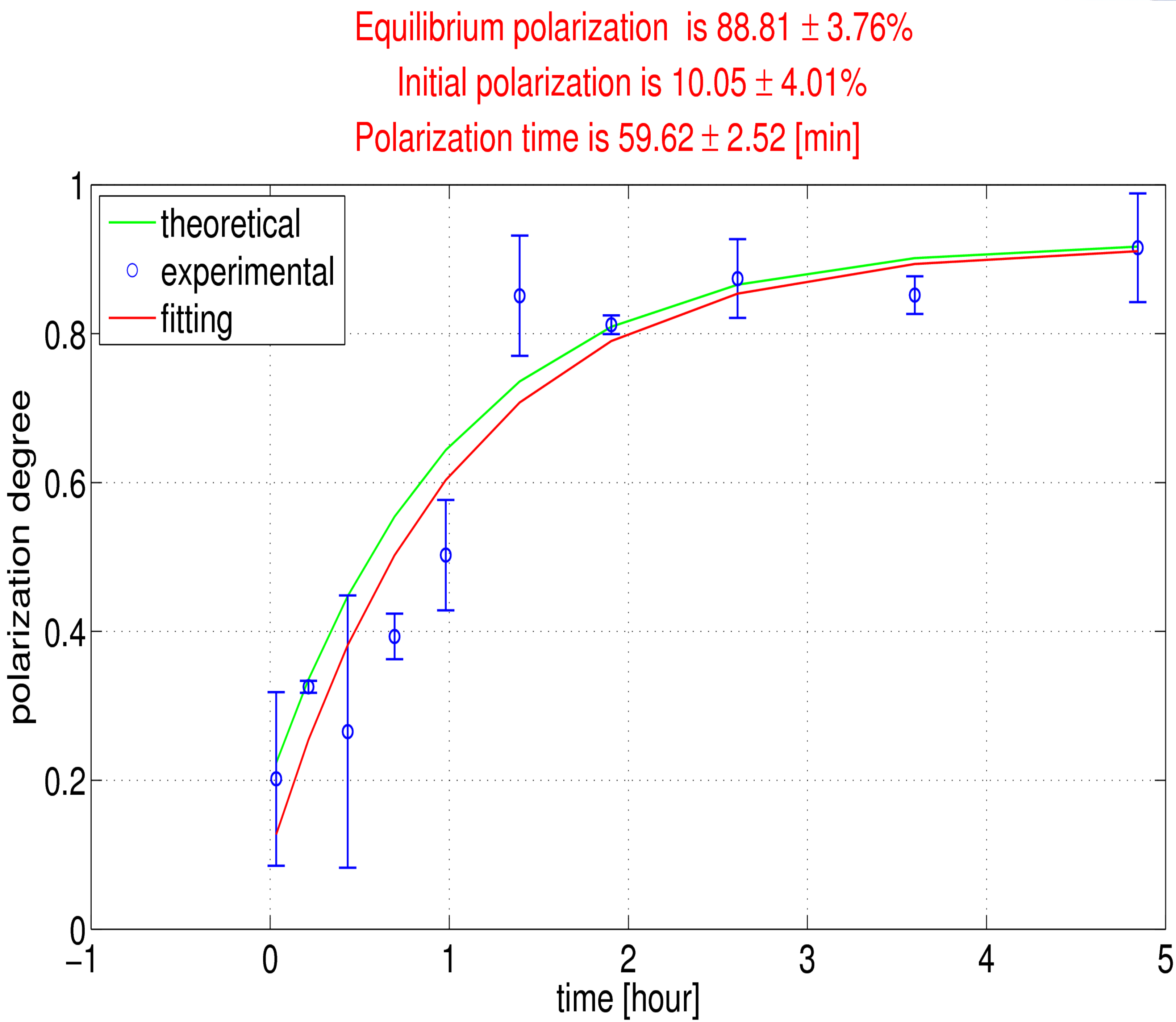
- Measure lifetime of an unpolarized beam.
- Measure lifetime of a polarized beam
  - Use the final 120mA of unpolarized beam,

as the start current of the polarized beam, we measure lifetime of polarized beam.

  - So the start beam of the polarized beam carries some initial polarization.



- Beam lifetime of the unpolarized electron beam and polarized electron beam



Equilibrium polarization is 88.81 ± 3.76%
Initial polarization is 10.05 ± 4.01%
Polarization time is 59.62 ± 2.52 [min]
theoretical
experimental
fitting
polarization degree
time [hour]
0
0.2
0.4
0.6
0.8
1
−1
0
1
2
3
4
5

Analysis of the experiment results

- Maximum error of measured polarization is 18%, but

We only use **the trend** of measured polarization

the find polarization.

- The contribution of initial polarization to the loss rate is due to,
  - we didn't measure beam lifetime at one specific time, but we measure the average lifetime in 5 minutes, some level of polarization can build up in these 5 minutes.
  - The measurement error from lifetime
- The fitted polarization is not of high accuracy, but can tell us information of the equilibrium polarization, this information is sufficient for the experiment, i.e., to measure beam energy using resonant depolarization.

Resonant depolarization (RD) to measure beam energy

Experiment principle

Based on spin tune and spin resonant condition

- Spin tune is defined as $$\nu = \alpha\gamma = \alpha\frac{E}{E_0}$$
- Add an horizontal, RF magnetic field on the beam, to drive vertical spin resonance,

$$\nu = n \pm \nu_{\mathrm{dep}}$$

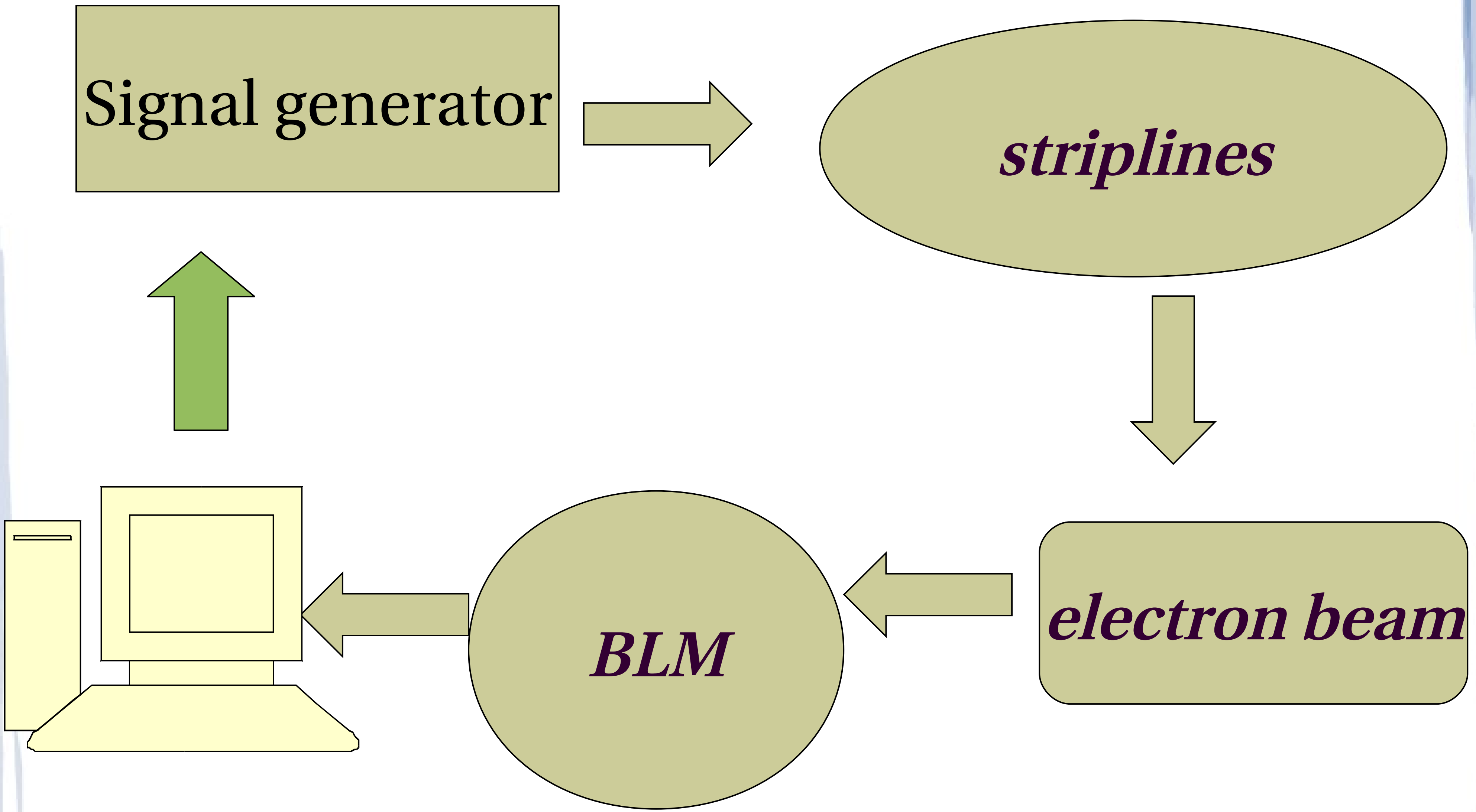
- So beam energy is:

$$E = \left(1 - \frac{\omega_{\mathrm{dep}}}{\omega_0}\right)\frac{E_0}{\alpha}$$

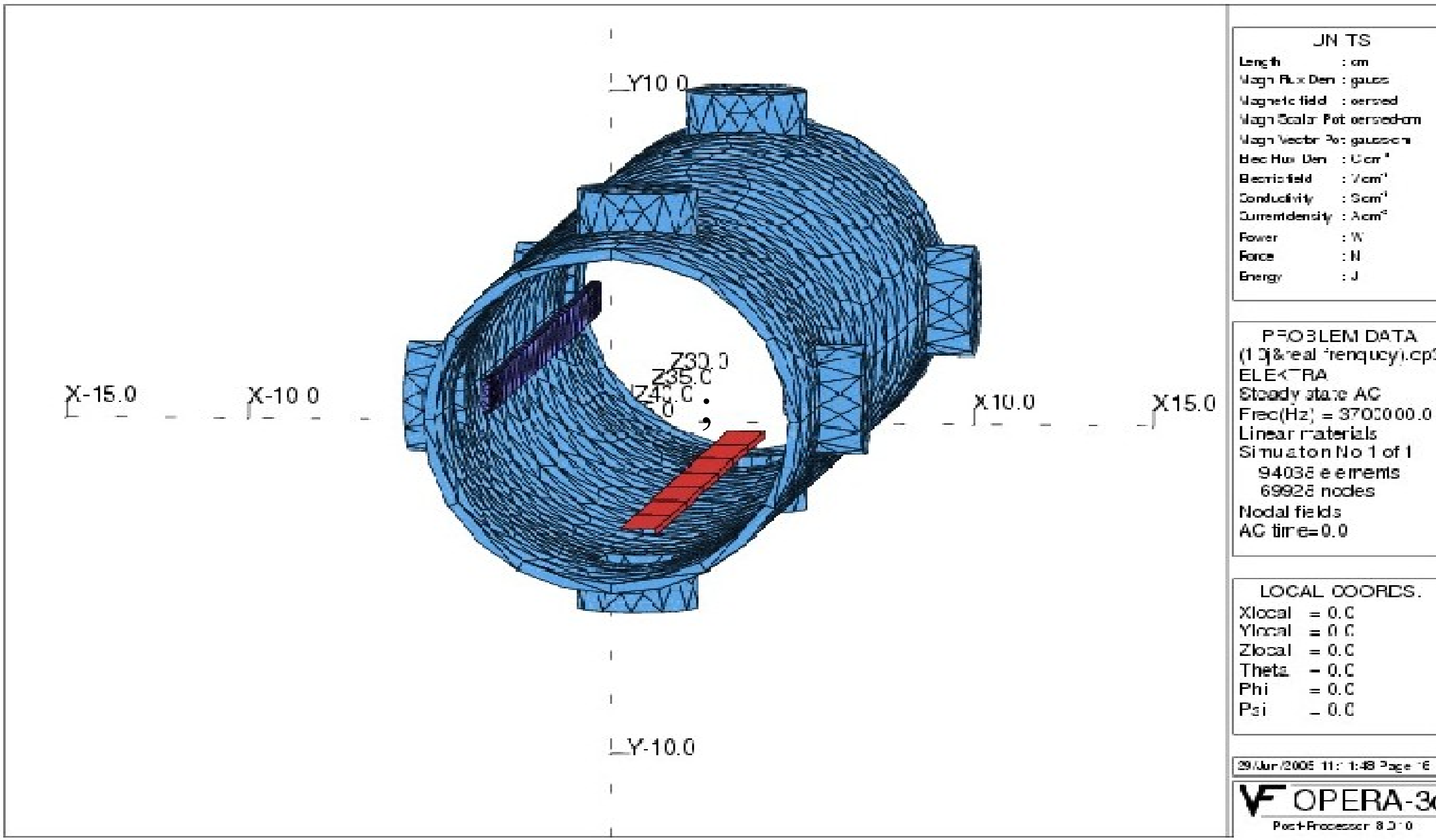
- Since the known spin tune is corresponding to the nominal energy, so we need to sweep RF frequency to get the find the real beam energy.
- RF field frequency is of high accuracy, so measured beam energy is of high accuracy.

10^{-4} to 10^{-5}

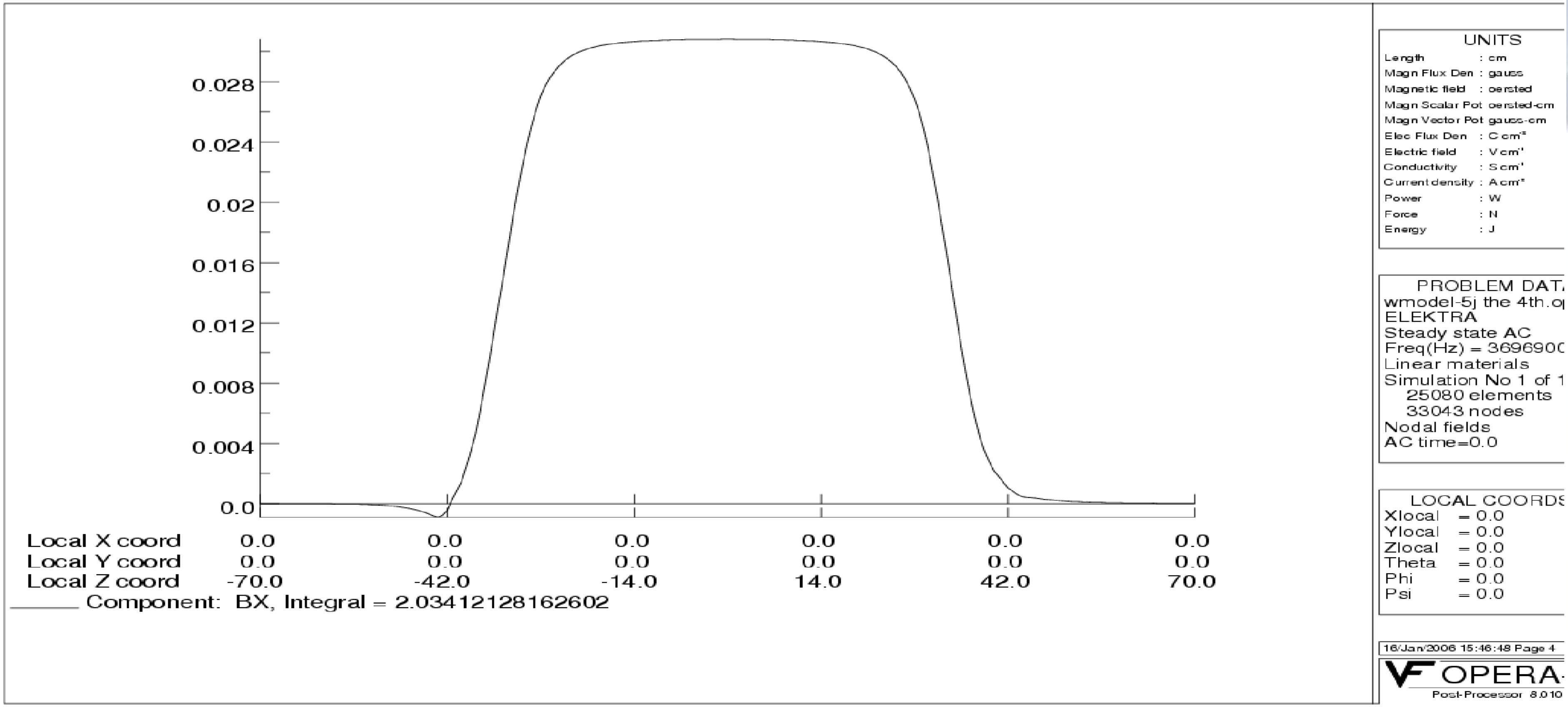
Experiment flowchart



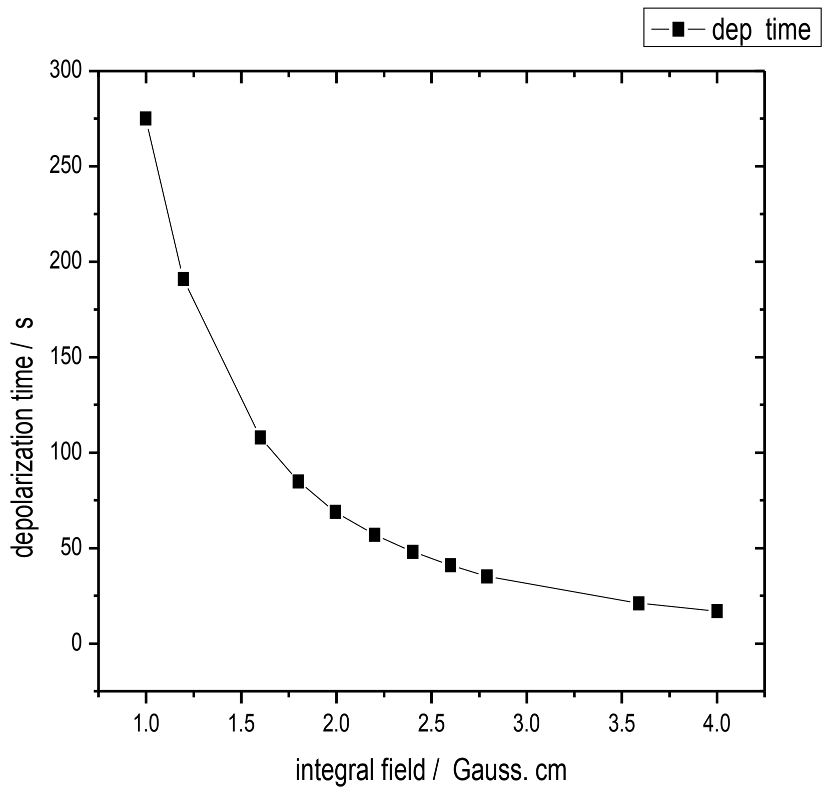
model of the stripline cavity in OPERA

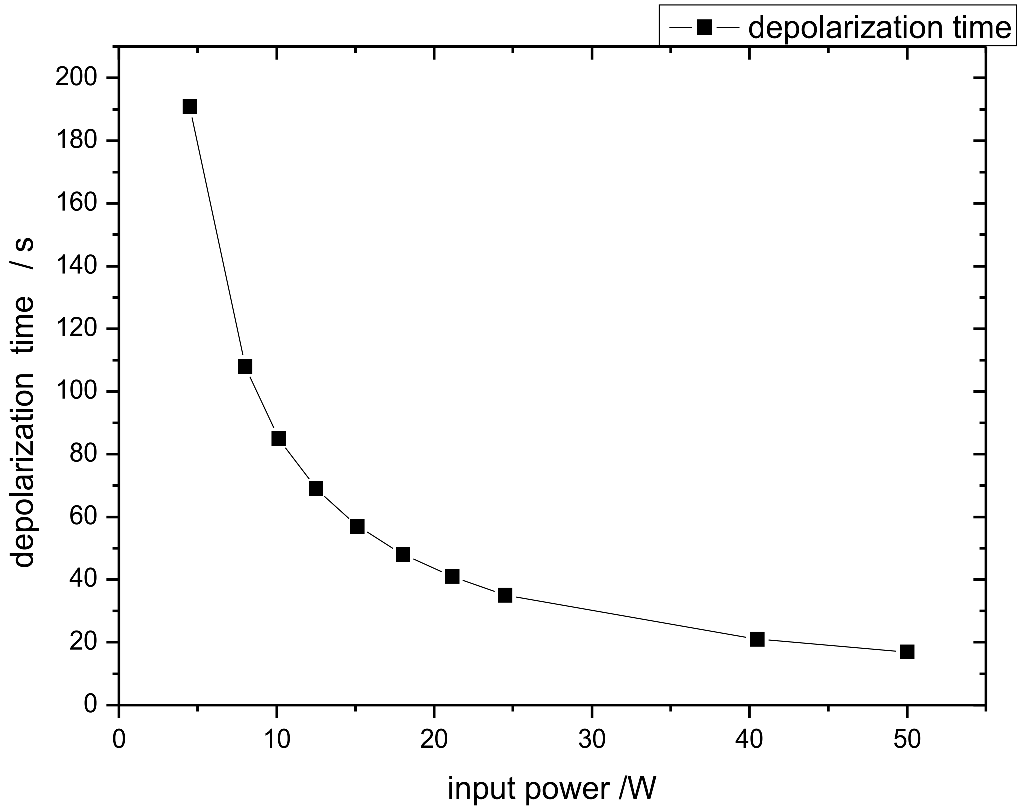


Distribution of on axis depolarization field



Depolarization time, on axis depolarization field, input power





$$\tau_{\mathrm{dep}} = \Delta\omega_{\mathrm{dep}} \frac{Z}{2P} \cdot \left(\frac{< B_z > \ C \, 4\pi r}{\omega_s \ l_{\mathrm{dep}} \ |F(s_{\mathrm{dep}})|} \right)^2$$

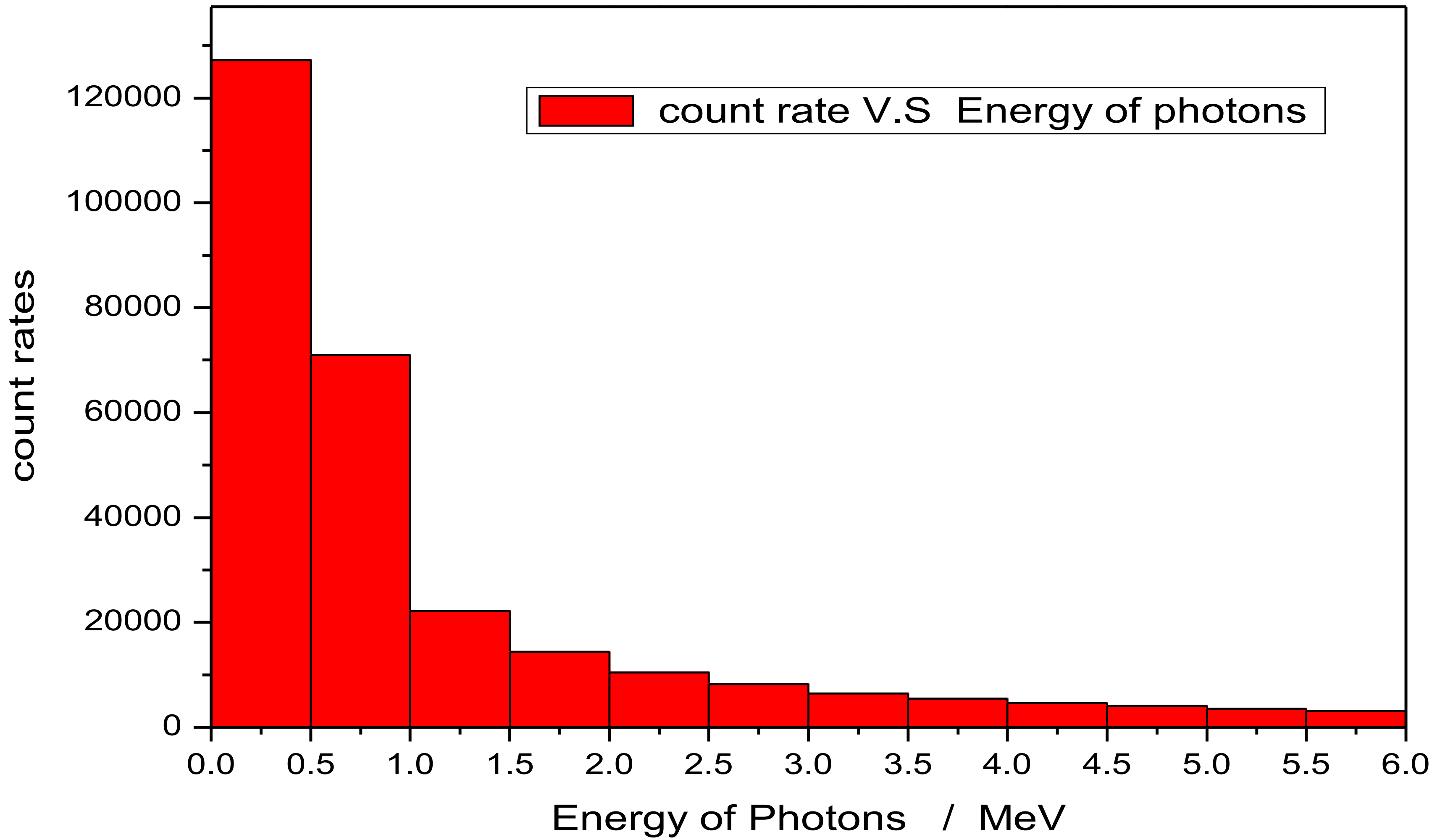
(*) Ya.S.Derbenev, A.M.Kondratenko, A.N.Skrinsky, Particle Accel, 9,247 (1980)

(#) P.Kuske, T.Mayer, Proceedings of EPAC96 (1996)

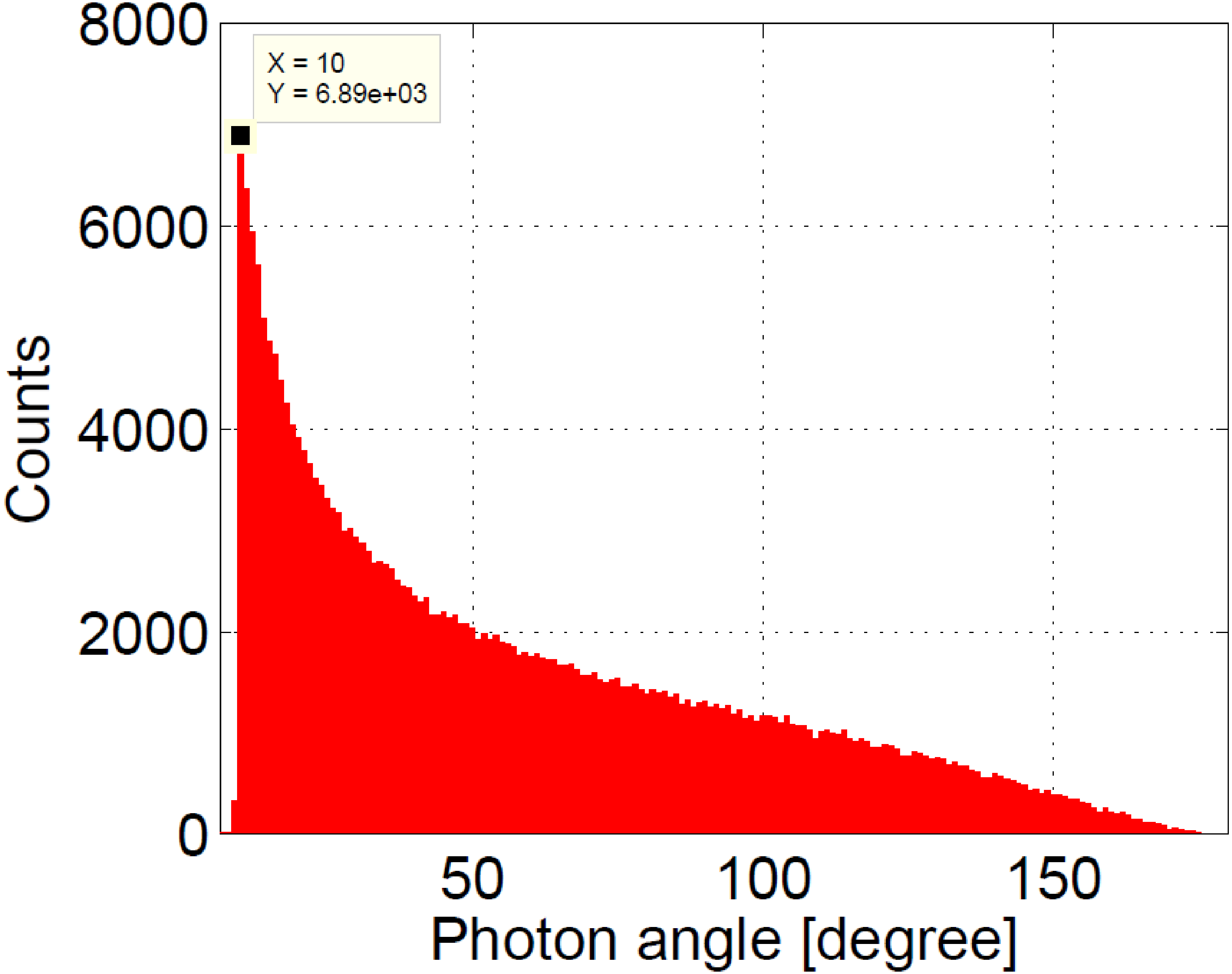
BLM in resonant depolarization experiment

- Very sensitive to beam loss
- response time of beam loss monitor should be short.
- Average cost and requirement, **plastic scintillation detector is a good choice.**

Energy spectrum of the secondary gamma photon outside the vacuum chamber (EGSnrc)



Angular distribution of the secondary gamma photon outside the vacuum chamber (EGSnrc)

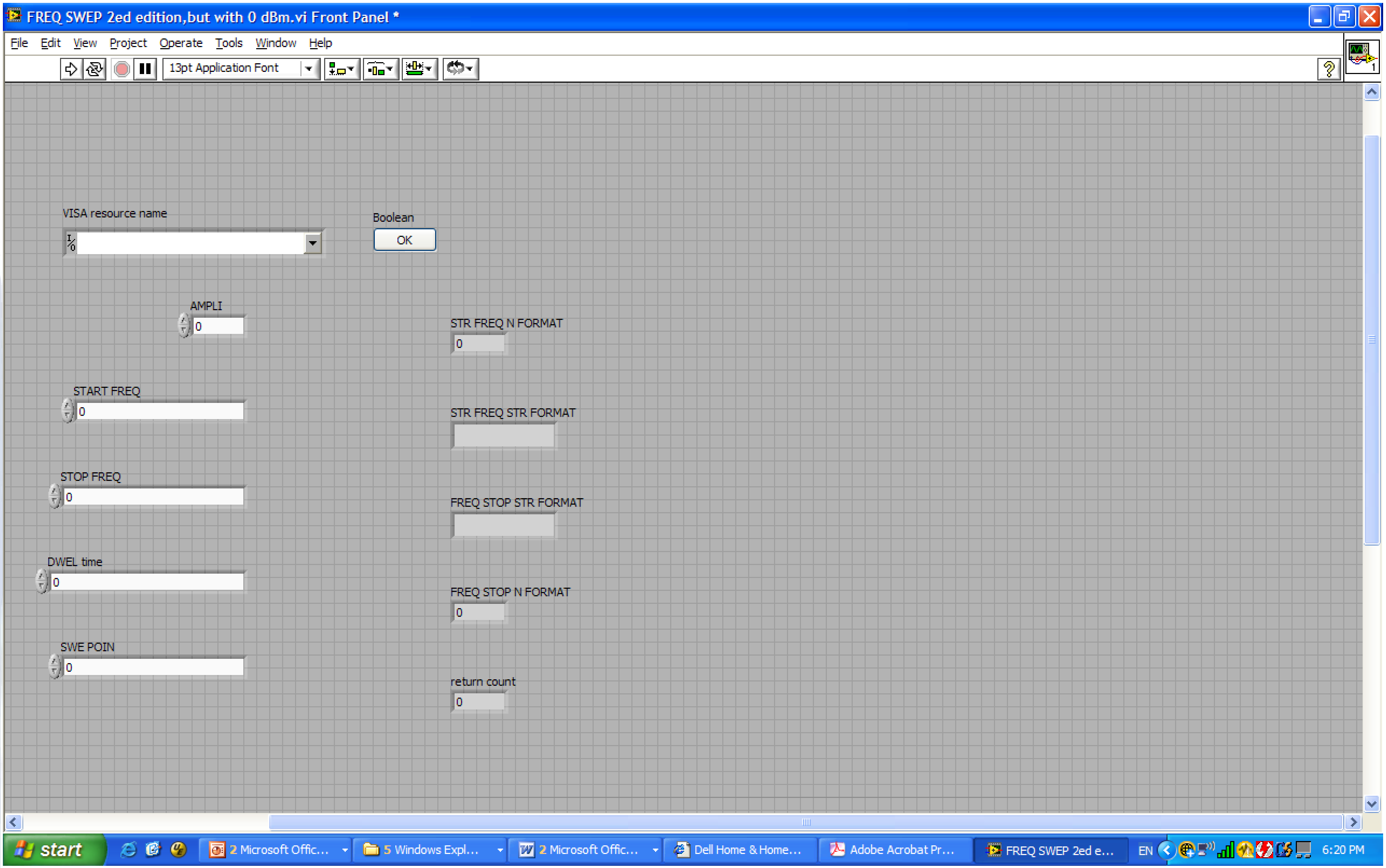


Scintillation detector

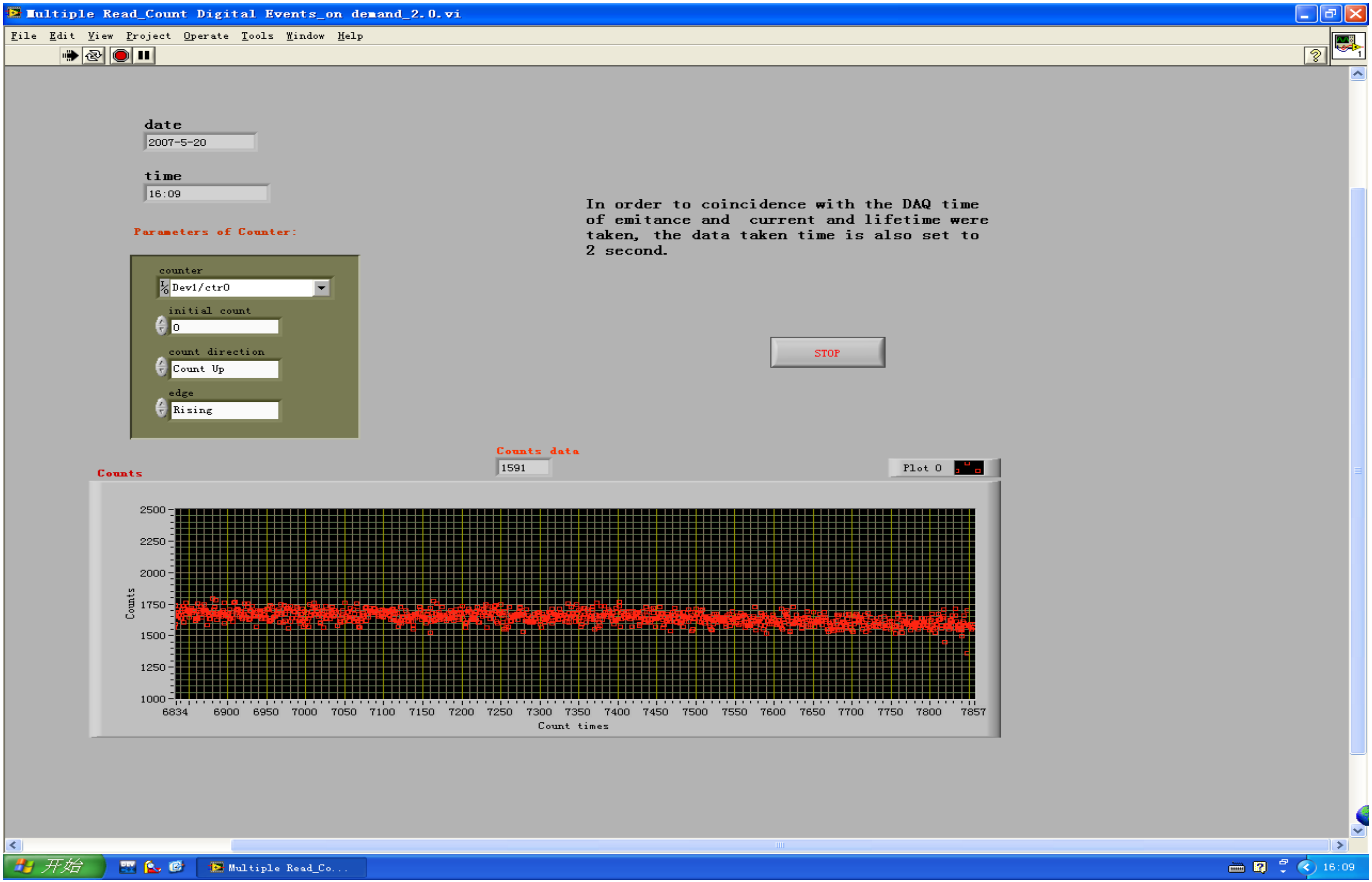
Control system of the experiment

- Separate control system, from the EPICS
- Using LabView on a computer, to control the RF scan and record the beam loss rate from the beam loss monitor.

Control panel of the RF experiment , RF scan control panel



Record panel of beam loss rate



Summary

- **Theoretical:**

  Study on spin dynamics

- **Experimental:**

  Using a simple method to measure electron beam polarization.

- **Instruments:**

  Build an experiment set up to measure beam energy, using the resonant depolarization method.

Thank you

Backup slides

Approximation of D-K formula

- If $\hat{n} = -\hat{y}$ and $\hat{n} \perp \vec{v}$, then

$$P = -\frac{8}{5\sqrt{3}} \frac{1/\tau_0}{1/\tau_0 + 1/\tau_{\rm dep}} \qquad \tau = \tau_0 \frac{1/\tau_0}{1/\tau_0 + 1/\tau_{\rm dep}}$$

$$\tau_{\rm dep} = \left(\frac{5\sqrt{3}}{8} \frac{e^2\gamma^5\hbar}{m^2c^2 2\pi R} \oint \frac{{\rm d}s}{\rho(s)^3} \frac{11}{18} \left| \gamma \frac{\partial \hat{n}}{\partial \gamma} \right|^2 \right)^{-1}$$

- Equilibrium polarization and polarization time are Proportional to the corresponding ideal values, with the same Proportionality constant.

Touschek beam loss

- Moller cross section ⟸ polarization;
- Touschek beam loss ⟸ polarization.
- For a flat, polarized electron beam,

$$\frac{1}{\tau_{\mathrm{touP}}} = \frac{1}{\tau_{\mathrm{tou0}}}(1 - A\,P^2)$$

- *A* is a function of momentum acceptance, etc.

A=0.15 for Duke storage ring.

Can't measure Touschek loss, only can measure total beam loss.

Electron beam loss

- Electron beam Loss is mainly composed of 3 parts:

$$
\begin{aligned}
\frac{1}{\tau} &= \frac{1}{\tau_{\mathrm{q}}} + \frac{1}{\tau_{\mathrm{vac}}} + \frac{1}{\tau_{\mathrm{tou}}} \\
\tau_{\mathrm{vac}} &= \tau_{\mathrm{vac}}(I, \text{other} \quad \text{parameteters}) \\
\tau_{\mathrm{tou}} &= \tau_{\mathrm{tou}}(I,\ P,\ \text{other} \quad \text{parameters})
\end{aligned}
$$

- τ_{q} quantum lifetime,
- τ_{vac} vacuum lifetime,
- τ_{tou} Touschek lifetime.

Current dependent is good, this is the key point we use in our experiment design.

Touschek lifetime of a flat, polarized beam(*) (#)

$$\frac{1}{\tau_{\mathrm{p}}} = \frac{1}{\tau_0}(1 - A P^2)$$

$$A = \frac{\langle a F(\epsilon)\rangle}{\langle C(\epsilon)\rangle}$$

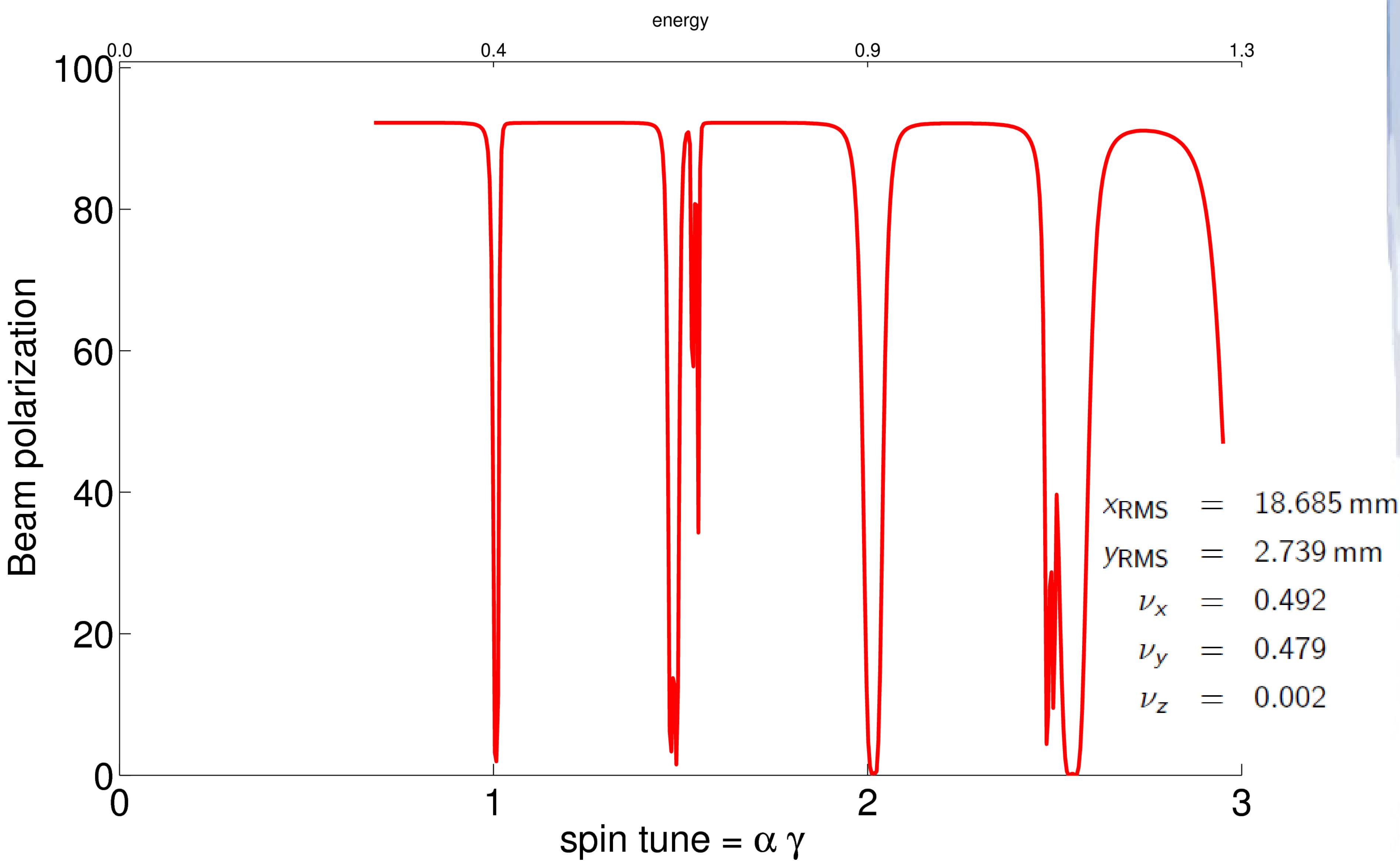
$$a = \frac{\sqrt{\pi} c r_e^2}{\gamma^3 V \sigma_{x'} (\Delta p_m / p)^2} \qquad \epsilon = \left(\frac{\delta P_m / P}{\gamma \sigma_{x'}}\right)^2$$

$$C(\epsilon) = \epsilon \int_\epsilon^\infty \frac{1}{u^2} \left\{\left(\frac{u}{\epsilon}\right) - \frac{1}{2} \ln\left(\frac{u}{\epsilon}\right) - 1\right\} e^{-u} \, \mathrm{d}u$$

$$F(\epsilon) = \frac{\epsilon}{2} \int_\epsilon^\infty \frac{1}{u^2} \ln\frac{u}{\epsilon} e^{-u} \mathrm{d}u$$

(*) A.A.Kresnin and L.N.Rozentsveig, Soviet Physics JETP 5 (1957) 288.
(#) T.-Y. Lee, J.Choi, H.S. Kang,NIMA 554(2005) 85

SLIM results of HLS



4 conditions

Produce an unpolarized and a polarized beam, and with

- Instability is weak
- machine status is repeatable
- beam is reproducible
- lifetime measurement error is small

Beam loss difference between these two beam, at the same current, are only depends on polarization....

Current methods to measure beam energy

method	charateristics	device
Hall probe	Low accuracy 10^{-2} Simple devices, Traditional method	Hall probe
Compton backscattering	High accuracy 10^{-4} Complicated and expensive devices, Not very popular.	Laser, high purity Ge detector, optical system,etc
Spin resonant depolarization	High accuracy 10^{-4} to 10^{-5} Simple and inexpensive device; Popular in recently years	Signal generator, Power amplifier, Scintillation detectors.

Beam loss monitors (BLM)

Type of Beam Loss Monitor	Advantanges	Disadvantages
Long ionization chamber	Can give position sensitivity	Expensive and complex electronics
Short ionization chamber	Linear over many decades	Measurement of very low currents is very expensive
Scintillator +Photomultiplier(PM)	Simple and cheap	Long term degradation of Scintillator and drift of PM
Pin Photo-diode	Simple and cheap	Limited count rate