Study on Electron Spin Dynamics and Its Application

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Outline

- Electron spin dynamics in the storage ring
- Polarization measurement at Duke storage ring
- Application: Using spin depolarization to measure electron beam energy at Hefei storage ring
Beam polarization

\[ P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = -\frac{8}{5\sqrt{3}} \]
Electron orbital motion and spin motion

- Coordinate system
  Orbital \((\hat{x}, \hat{y}, \hat{z})\)
  Spin \((\hat{m}, \hat{n}, \hat{l})\)

- Closed orbit
  Orbital closed orbit \(X_0\)
  Spin closed orbit \(\hat{n}\)

In an Ideal ring,
spin closed orbit \(\hat{n}\) is anti-parallel to \(\hat{y}\), and
orbital revolution frequency is \(\omega_0\),
spin precession frequency \(\alpha \gamma \omega_0\).
\[
\alpha = 0.001159
\]
• Orbital dispersion, spin chromaticity
  orbital, \( \eta_x, \eta_y \quad x = \eta_x \delta \quad y = \eta_y \delta \)

Spin
  \[ \vec{D}_s = \frac{\partial \hat{n}}{\partial \gamma} \quad \tilde{\alpha}\hat{m} + \tilde{\beta}\hat{l} = \vec{D}_s \delta \]

• Effects of synchrotron radiation
  - Balance between radiation damping and quantum excitation
    \[ \text{emittance} \]
  - Balance between radiation spin flip and spin diffusion
    \[ \text{equilibrium polarization} \]

• Time
  - Damping time: \( \text{order of ms} \)
  - Polarization build up time: \( \text{minutes to hours} \)

• Equation of motion (Classical)
  - Lorentz equation
  - Thomas-BMT equation
Equilibrium polarization and polarization time are

\[ P(t) = P_{dk} \left(1 - e^{-t/\tau_{dk}}\right) - P_0 e^{-t/\tau_{dk}} \]

D-K formula (*)

\[ P_{dk} = -\frac{8}{5\sqrt{3}} \frac{\alpha_-}{\alpha_+} \]

\[ \tau_{dk} = \left(\frac{5\sqrt{3}}{8} \frac{e^2 \gamma^5 \hbar}{m^2 c^2 \alpha_+}\right) \]

\[ \alpha_+ = \frac{1}{2\pi R} \int \frac{ds}{|\rho(s)|^3} \left[ 1 - \frac{2}{9} (\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \gamma \frac{\partial \hat{n}}{\partial \gamma} \right|^2 \right]_s \]

\[ \alpha_- = \frac{1}{2\pi R} \int \frac{ds}{|\rho(s)|^3} \left[ \frac{\hat{v} \times \hat{v}}{|\hat{v}|} \cdot \left( \hat{n} - \gamma \frac{\partial \hat{n}}{\partial \gamma} \right) \right] \]

Thomas-BMT equation (*)

\[ \frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S} \]

\[ \vec{\Omega} = -\frac{e}{m\gamma} \left[ (1 + G\gamma)\vec{B}_\perp + (1 + G)\vec{B}_\parallel + \left( G\gamma + \frac{\gamma}{\gamma + 1} \right) \frac{\vec{E} \times \vec{v}}{c^2} \right] \]

Comments

1) Spin \( \vec{S} \): rest frame; magnetic and electric field: Lab frame

2) Spin precession frequency \( \vec{\Omega} \) is determined by the electromagnetic field seen by the electrons.

3) Direction of \( \vec{\Omega} \) is the direction of spin closed orbit

4) Amplitude of \( \vec{\Omega} \) and spin tune (in the ideal ring)

\[ \nu = \frac{\Omega}{\omega_0} = \alpha \gamma \]

Numerical Algorithm: SLIM

• Use a 8-dimensional vector to designate the state of an electron, additional to 6-D traditional orbital components, two spin components are added to denote the spin motion.

\[
\chi = \begin{bmatrix}
  x \\
  x' \\
  y \\
  y' \\
  z \\
  \delta \\
  \alpha \\
  \beta
\end{bmatrix}
\]

(*) A.W.Chao, NIM 180 (1981) 29

(#) A.W.Chao, AIP Proc. 87 (1981) 395
Characteristics of SLIM

Using eigenvectors and eigenvalues of a matrix, to

- study a general, linear coupled accelerator lattice.
- calculate coupled orbital motions in the 6-D phase space.
- calculate coupled damping, coupled beam size and coupled emittance;
• include coupling of orbital motion on the spin motion, calculate spin closed orbit and spin chromaticity.

• Calculate polarization and polarization time according to D-K formula.

• Seven sets of resonance in SLIM.

\[
\nu = n \\
\nu = n \pm \nu_x \\
\nu = n \pm \nu_y \\
\nu = n \pm \nu_z
\]
Two example applications of SLIM

- Hefei storage ring
- Duke storage ring
4 periods; 3 dipoles, 8 quadupoles, 4 sextupoles, in each period; C=66.1308 m
Optics functions of Hefei storage ring.  
(Top) Thick lens model;  (Bottom) thin lens model (used in SLIM)
Layout of Duke accelerator; there are 2 symmetric arcs, two straight sections, south straight section is for wigglers on the storage ring.
Calculation results of SLIM, in the energy 1.15 GeV, beam polarization is safe

Beam polarization

energy [GeV]

spin tune

$\gamma_{\text{RMS}} = 2.38 \text{ mm}$

$\nu_x = 0.3740$

$\nu_y = 0.1332$

$\nu_z = 0.0093$
Experiments to measure electron beam polarization
Polarimeters

- Two types of polarimeters (*)
  - Moller polarimeters, $e \leftrightarrow e$ scattering, Jlab;
  - Compton polarimeters, $e \leftrightarrow \gamma$ scattering, SLAC.
- Both polarimeters are of high accuracy, but the set up are complicated and the devices are expensive.
- Simple and inexpensive method ??? Even if the accuracy is not high?

(*) A.Chao, M.Tigner, Handbook of accelerator physics and engineering
Touschek beam loss

- Moller cross section $\leftrightarrow$ polarization;
- Touschek beam loss $\leftrightarrow$ polarization.
- For a flat, polarized electron beam,
  \[
  \frac{1}{\tau_{\text{touP}}} = \frac{1}{\tau_{\text{tou0}}} (1 - AP^2)
  \]
- $A$ is a function of momentum acceptance, etc.

$A=0.15$ for Duke storage ring.
If we could produce two beam with same status except that one beam is polarized, and another one is unpolarized,

- then the relative total beam loss is equal to relative Touschek beam loss:

\[
\frac{1/\tau_0 - 1/\tau_P}{1/\tau_0} = \frac{1/\tau_{tou0} - 1/\tau_{tauP}}{1/\tau_{tau0}} = AP^2
\]

The expected relative total beam loss is 13%, for the 92.38% polarized 1.15 [GeV] electron beam in duke storage ring.
Experiment feasibility study

Check whether we could produce two unpolarized beam with the beam status

- Turn on longitudinal feedback system;
- Produce two unpolarized beam, measure their lifetime with the same current;
- Comparing the orbit, RF voltage during the two experiments;
- comparing beam size, bunch length, during the experiments.
Orbits and beam size and bunch length of two unpolarized beam; beam is repetitive and machine reproducibility is good.
Lifetime comparison in two runs
Procedures to measure polarization

• Measure lifetime of an unpolarized beam.
• Measure lifetime of a polarized beam
  – Use the final 120mA of unpolarized beam, as the start current of the polarized beam, we measure lifetime of polarized beam.
  – So the start beam of the polarized beam carries some initial polarization.
• Beam lifetime of the unpolarized electron beam and polarized electron beam
Equilibrium polarization is 88.81 ± 3.76%
Initial polarization is 10.05 ± 4.01%
Polarization time is 59.62 ± 2.52 [min]
Analysis of the experiment results

- Maximum error of measured polarization is 18%, but we only use the trend of measured polarization to find polarization.

- The contribution of initial polarization to the loss rate is due to,
  - we didn't measure beam lifetime at one specific time, but we measure the average lifetime in 5 minutes, some level of polarization can build up in these 5 minutes.
  - The measurement error from lifetime.

- The fitted polarization is not of high accuracy, but can tell us information of the equilibrium polarization, this information is sufficient for the experiment, i.e., to measure beam energy using resonant depolarization.
Resonant depolarization (RD) to measure beam energy
Experiment principle
Based on spin tune and spin resonant condition

- Spin tune is defined as
  \[ \nu = \alpha \gamma = \alpha \frac{E}{E_0} \]
- Add an horizontal, RF magnetic field on the beam, to drive vertical spin resonance,
  \[ \nu = n \pm \nu_{\text{dep}} \]
- So beam energy is:
$E = \left(1 - \frac{\omega_{\text{dep}}}{\omega_0}\right) \frac{E_0}{\alpha}$

- Since the known spin tune is corresponding to the nominal energy, so we need to sweep RF frequency to get the find the real beam energy.
- RF field frequency is of high accuracy, so measured beam energy is of high accuracy.

$10^{-4}$ to $10^{-5}$
Experiment flowchart

Signal generator → striplines

BLM

→ electron beam
model of the stripline cavity in OPERA
Distribution of on axis depolarization field

**Local X coord**: 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0

**Local Y coord**: 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0

**Local Z coord**: -70.0, -42.0, -14.0, 14.0, 42.0, 70.0

**Component**: BX, Integral = 2.0341281262602
Depolarization time, on axis depolarization field, input power

\[ \tau_{\text{dep}} = \Delta \omega_{\text{dep}} \frac{Z}{2P} \cdot \left( \frac{<B_z> \cdot C \cdot 4\pi r}{\omega_s l_{\text{dep}} |F(s_{\text{dep}})|} \right)^2 \]


BLM in resonant depolarization experiment

- Very sensitive to beam loss

- response time of beam loss monitor should be short.

- Average cost and requirement, plastic scintillation detector is a good choice.
Energy spectrum of the secondary gamma photon outside the vacuum chamber (EGSnrc)
Angular distribution of the secondary gamma photon outside the vacuum chamber (EGSnrc)
Scintillation detector
Control system of the experiment

- Separate control system, from the EPICS
- Using LabView on a computer, to control the RF scan and record the beam loss rate from the beam loss monitor.
Control panel of the RF experiment, RF scan control panel
Record panel of beam loss rate

In order to coincidence with the DAO time of emulation and current and lifetime were taken, the data taken time is also set to 2 second.
Summary

- **Theoretical:**
  Study on spin dynamics

- **Experimental:**
  Using a simple method to measure electron beam polarization.

- **Instruments:**
  Build an experiment set up to measure beam energy, using the resonant depolarization method.
Thank you
Backup slides
Approximation of D-K formula

- If \( \hat{n} = -\hat{y} \) and \( \hat{n} \perp \vec{v} \), then

\[
P = -\frac{8}{5\sqrt{3}} \frac{1/\tau_0}{1/\tau_0 + 1/\tau_{\text{dep}}} \quad \tau = \tau_0 \frac{1/\tau_0}{1/\tau_0 + 1/\tau_{\text{dep}}}
\]

\[
\tau_{\text{dep}} = \left( \frac{5\sqrt{3}}{8} \frac{e^2 \gamma^5 \hbar}{m^2 c^2 2\pi R} \int \frac{ds}{\rho(s)^3} \frac{11}{18} \left| \frac{\gamma}{\partial \gamma} \frac{\partial \hat{n}}{\partial \gamma} \right|^2 \right)^{-1}
\]

- Equilibrium polarization and polarization time are Proportional to the corresponding ideal values, with the same Proportionality constant.
Touschek beam loss

- Moller cross section $\leftrightarrow$ polarization;
- Touschek beam loss $\leftrightarrow$ polarization.
- For a flat, polarized electron beam,

$$\frac{1}{\tau_{\text{touP}}} = \frac{1}{\tau_{\text{tou0}}} (1 - AP^2)$$

- $A$ is a function of momentum acceptance, etc.

$A=0.15$ for Duke storage ring.

Can't measure Touschek loss, only can measure total beam loss.
Electron beam loss

- Electron beam Loss is mainly composed of 3 parts:
  \[
  \frac{1}{\tau} = \frac{1}{\tau_q} + \frac{1}{\tau_{\text{vac}}} + \frac{1}{\tau_{\text{tou}}}
  \]
  \[
  \tau_{\text{vac}} = \tau_{\text{vac}}(I, \text{other parameters})
  \]
  \[
  \tau_{\text{tou}} = \tau_{\text{tou}}(I, P, \text{other parameters})
  \]

- \(\tau_q\) quantum lifetime,
- \(\tau_{\text{vac}}\) vacuum lifetime,
- \(\tau_{\text{tou}}\) Touschek lifetime.

Current dependent is good, this is the key point we use in our experiment design.
Touschek lifetime of a flat, polarized beam

\[
\frac{1}{\tau_p} = \frac{1}{\tau_0} (1 - A P^2)
\]

\[
A = \frac{\langle a F(\epsilon) \rangle}{\langle C(\epsilon) \rangle}
\]

\[
a = \frac{\sqrt{\pi} c e^2}{\gamma^3 V \sigma_{x'} (\Delta p_m/p)^2}
\]

\[
\epsilon = \left( \frac{\delta P_m/P}{\gamma \sigma_{x'}} \right)^2
\]

\[
C(\epsilon) = \epsilon \int_{\epsilon}^{\infty} \frac{1}{u^2} \left\{ \left( \frac{u}{\epsilon} \right) - \frac{1}{2} \ln \left( \frac{u}{\epsilon} \right) - 1 \right\} e^{-u} \, du
\]

\[
F(\epsilon) = \frac{\epsilon}{2} \int_{\epsilon}^{\infty} \frac{1}{u^2} \ln \frac{u}{\epsilon} e^{-u} \, du
\]


SLIM results of HLS

![Graph showing beam polarization and spin tune with specified parameters: \( x_{RMS} = 18.685 \text{ mm} \), \( y_{RMS} = 2.739 \text{ mm} \), \( \nu_x = 0.492 \), \( \nu_y = 0.479 \), \( \nu_z = 0.002 \).]
4 conditions

Produce an unpolarized and a polarized beam, and with
- Instability is weak
- Machine status is repeatable
- Beam is reproducible
- Lifetime measurement error is small

Beam loss difference between these two beam, at the same current, are only depends on polarization....
## Current methods to measure beam energy

<table>
<thead>
<tr>
<th>Method</th>
<th>Characteristics</th>
<th>Device</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hall probe</td>
<td>Low accuracy $10^{-2}$</td>
<td>Hall probe</td>
</tr>
<tr>
<td></td>
<td>Simple devices,</td>
<td></td>
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<tr>
<td></td>
<td>Traditional method</td>
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<tr>
<td>Compton backscattering</td>
<td>High accuracy $10^{-4}$</td>
<td>Laser, high purity Ge detector, optical</td>
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<tr>
<td></td>
<td>Complicated and</td>
<td>system, etc</td>
</tr>
<tr>
<td></td>
<td>expensive devices,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not very popular.</td>
<td></td>
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<tr>
<td>Spin resonant depolarization</td>
<td>High accuracy $10^{-4}$ to $10^{-5}$</td>
<td>Signal generator,</td>
</tr>
<tr>
<td></td>
<td>Simple and inexpensive device;</td>
<td>Power amplifier,</td>
</tr>
<tr>
<td></td>
<td>Popular in recently years</td>
<td>Scintillation detectors.</td>
</tr>
</tbody>
</table>
## Beam loss monitors (BLM)

<table>
<thead>
<tr>
<th>Type of Beam Loss Monitor</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long ionization chamber</td>
<td>Can give position sensitivity</td>
<td>Expensive and complex electronics</td>
</tr>
<tr>
<td>Short ionization chamber</td>
<td>Linear over many decades</td>
<td>Measurement of very low currents is very expensive</td>
</tr>
<tr>
<td>Scintillator + Photomultiplier (PM)</td>
<td>Simple and cheap</td>
<td>Long term degradation of Scintillator and drift of PM</td>
</tr>
<tr>
<td>Pin Photo-diode</td>
<td>Simple and cheap</td>
<td>Limited count rate</td>
</tr>
</tbody>
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