Investigations of Electromagnetic Space-Charge Effects in Beam Physics

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Outline

- **Motivations**
- **Existing Space-Charge Modeling**
- **Space-Charge Effects in RF Photoinjectors**
- **Space-Charge Effects with Transverse Currents**
- **Summary**
- **Future Plans**

Motivations

- **Future accelerator-based experiments demand high-brightness electron beams from photoinjectors**

- **The electron beam has a low energy so space-charge forces can be important relative to external magnetic and rf fields**

- **There are two main challenges for simulations of high-brightness photoinjectors:**

  - Resolution of small length/time scale space-charge fields relative to long length/time scales of injector, e.g. 1-10 ps bunch lengths for 1.3-2.8 GHz

  - Removal of unphysical simulation effects such as numerical grid dispersion and numerical Cherenkov effects in FDTD methods

Existing Space Charge Modeling

Existing Electrostatic Algorithm

- **SCHEFF (Space Charge EFFect)**

  - breaks up the macro-particles into a set of annular rings

  - calculates the electrostatic space-charge forces in the beam rest frame and Lorentz transforms to the lab frame

- **PARMELA (Phase And Radial Motion in Electron Linear Accelerators)**

  - "Workhorse" of photoinjector design codes

  - imposes external rf-fields computed from SUPERFISH and external magnetic fields computed from POISSON

  - cannot calculate wakefields self-consistently

- **ASTRA (A Space charge TRacking Algorithm)**

Existing Electromagnetic Algorithms

- **Yee/PIC algorithm (FDTD method)**

  - solves Maxwell's equations on the two interleaved E and B grids
- **MAFIA**
- **Numerical Dispersion**
- **Numerical Cherenkov Radiation**

- **TREDI (Three Dimensional Injectors)**

  - Lienard-Wiechert Potentials

  - no conducting boundaries

Other EM SC Method

- **Mode analysis and Series Expansions (Salah *et al*)**

  - solves wave equations using series expansions and Fourier transformation in normal modes.

  - calculates the space-charge fields to arbitrary accuracy for given beam charge and current densities

  - needs a sufficient amount of eigenmodes and Fourier modes

Space Charge Effects in RF Photoinjector

Electromagnetic Space-Charge Potentials and Fields

- **The relations of EM fields and potentials**

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

- **Lorenz Gauge**

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$

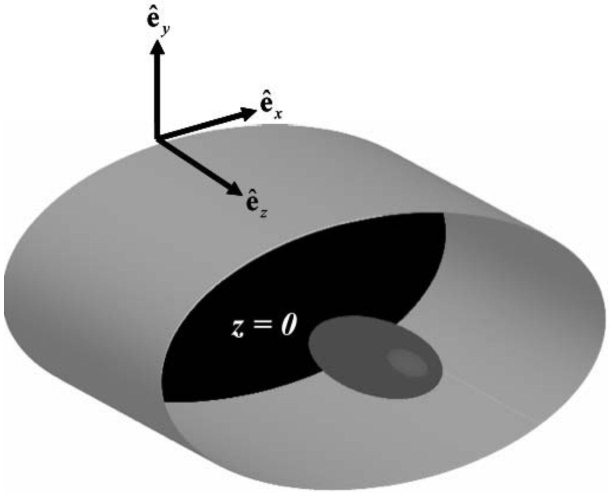
- **Wave Equations**

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\begin{Bmatrix}\phi \\ \mathbf{A}\end{Bmatrix} = -\begin{Bmatrix}\rho/\varepsilon_0 \\ \mu_0 \mathbf{J}\end{Bmatrix}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$



*PRST-AB, Mark Hess *et al*

Time-Dependent Green's Functions

Boundary Conditions: $\phi\big|_{boundary} = 0, \quad A_z\big|_{side\ wall} = 0, \quad \left.\dfrac{\partial A_z}{\partial z}\right|_{cathode} = 0$

$$G_\phi\big|_{boundary} = 0 \quad , \quad G_A\big|_{side\ wall} = 0 \quad , \quad \left.\frac{\partial G_A}{\partial z}\right|_{cathode} = 0$$

- **For the special case of currents in the axial direction in an pipe with a cathode, the potentials are given by**

$$\phi(\mathbf{r},t) = \frac{1}{\varepsilon_0}\int_{-\infty}^{t} dt' \int d^3\mathbf{r}'\, G_\phi(\mathbf{r},t;\mathbf{r}',t')\,\rho(\mathbf{r}',t') \qquad A_z(\mathbf{r},t) = \mu_0 \int_{-\infty}^{t} dt' \int d^3\mathbf{r}'\, G_A(\mathbf{r},t;\mathbf{r}',t') J_z(\mathbf{r}',t')$$

- **Time Dependent Green's Function:** $\left[\nabla^2 - \dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}\right]\begin{Bmatrix} G_\phi \\ G_A \end{Bmatrix} = -\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')$

$$\begin{Bmatrix} G_\phi \\ G_A \end{Bmatrix} = \frac{c}{2\pi a^2}\sum_{mn} \frac{J_m\left(\dfrac{j_{mn} r}{a}\right) J_m\left(\dfrac{j_{mn} r'}{a}\right)}{J_{m+1}^2(j_{mn})} e^{im(\theta-\theta')}\left[J_0(k_{\perp n}\lambda_-)\theta(\lambda_-^2) \mp J_0(k_{\perp n}\lambda_+)\theta(\lambda_+^2)\right]$$

Electromagnetic Space-Charge Fields

- **Arbitrary pipe cross-section with longitudinal currents**
- **Rectangular pipe cross-section with arbitrary currents**
- **Circular pipe cross-section with longitudinal currents**

$$\mathbf{E}_\perp(\mathbf{r},t) = -\frac{c}{2\varepsilon_0}\sum_{mn}\nabla_\perp\psi_{mn}(\mathbf{r}_\perp)\int_{-\infty}^{t}\int\psi^*_{mn}(\mathbf{r}'_\perp)[\Gamma_- - \Gamma_+]\rho(\mathbf{r}',t')d^3\mathbf{r}'dt'$$

For circular pipe,

$$\psi_{mn}(\mathbf{r}_\perp) = \frac{1}{a\sqrt{\pi}}\frac{J_m(j_{mn}r/a)e^{im\theta}}{\left|J_{m+1}(j_{mn})\right|}$$

$$\Gamma_\pm = J_0(j_{mn}\lambda_\pm/a)\theta(\lambda_\pm^2) \quad \textbf{where} \quad \lambda_\pm^2 = c^2(t-t')^2 - (z \pm z')^2$$

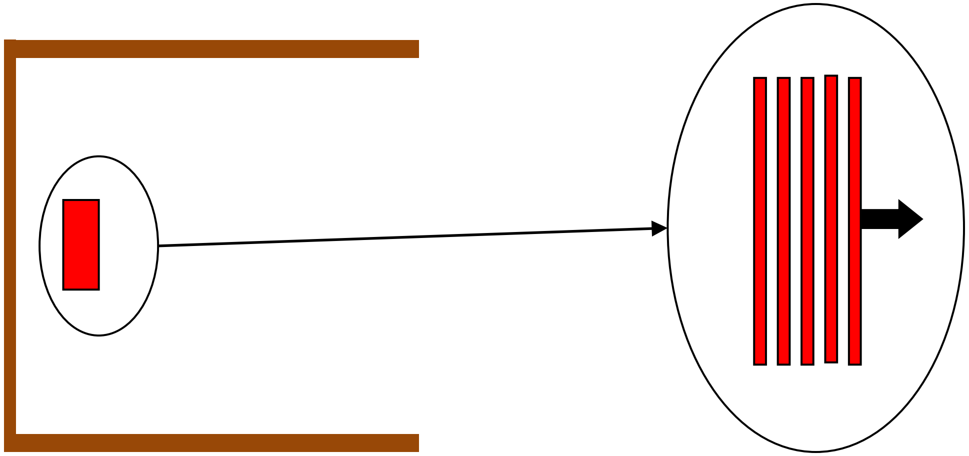
Numerical Implementation

- **IRPSS (Indiana RF Photocathode Source Simulator)**
- **IRPSS is a 3-D electromagnetic particle/slice pushing algorithm**
- **Handles metallic boundary conditions self-consistently (boundary will consist of cathode, circular side walls)**
- **Uses electromagnetic Green's function formalism for solving fields**
- **Time-dependent Green's function formulation has the advantageous property that electromagnetic waves from tight electron bunches**

Specify trajectory and charge density of the beam → **Compute E and B due to space-charge** → **Simulate beam dynamics of test particles**

Bunched Beam Model

- **The multi-sliced bunch model is introduced to generalize the finite size bunch length.**
- **We set up uniformly spaced (in time), equally charged slices, which form one complete bunch.**
- **Needs an enough number of slices**



Equation for charge density (Circled)

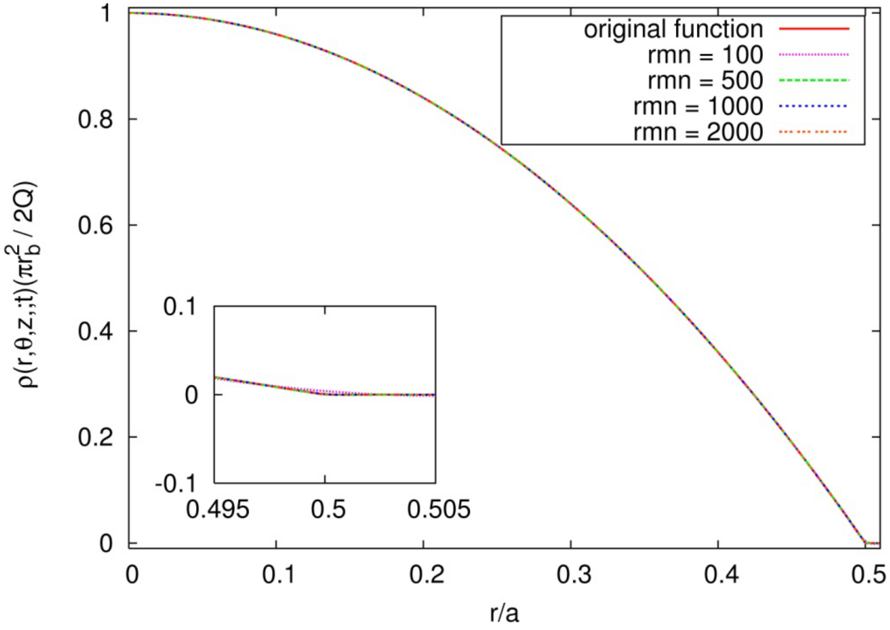
$$\rho(\vec{r},t) = \sum_i \frac{2Q}{\pi r_{b,i}^2}\Theta(r_{b,i}-r)\left(1-\frac{r^2}{r_{b,i}^2}\right)\delta\left[z-z_i''(t)\right]$$

Eigenmode Summations

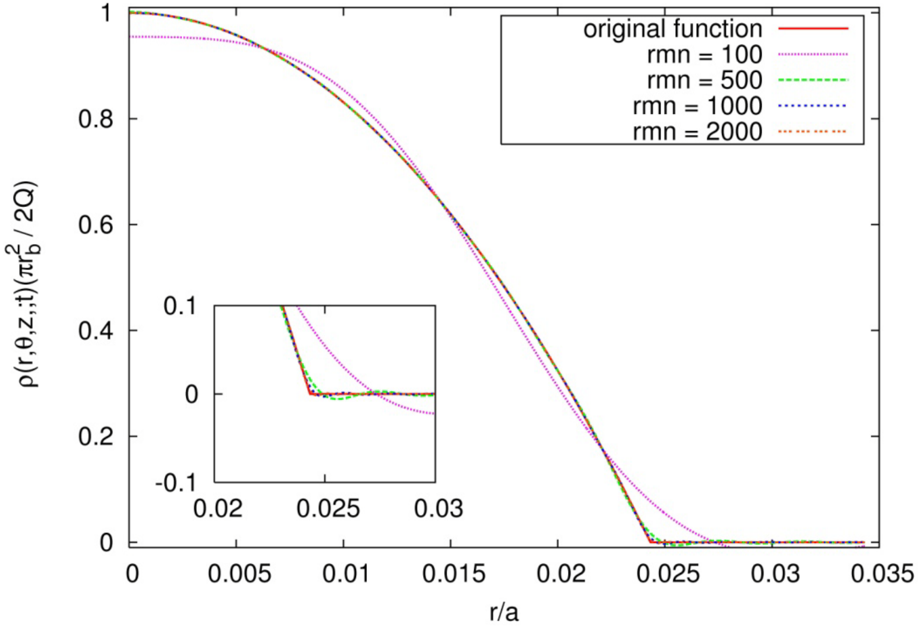
The required eigenmode numbers can be estimated from the expansion of the charge density of the beam. The number of transverse eigenmodes necessary for accurately determining the fields is inversely proportional to the transverse size of the beam.

$$\rho(\vec{r},t) = \sum_i \frac{2Q}{\pi r_{b,i}^2} \Theta(r_{b,i}^2 - r)\left(1 - \frac{r^2}{r_{b,i}^2}\right)\delta\left[z - z_i''(t)\right]$$

$$= \sum_i \sum_{mn} \rho_{mn,i}(z,t)\psi_{mn,i}(\vec{r}_\perp)$$



r_b /a = 0.5



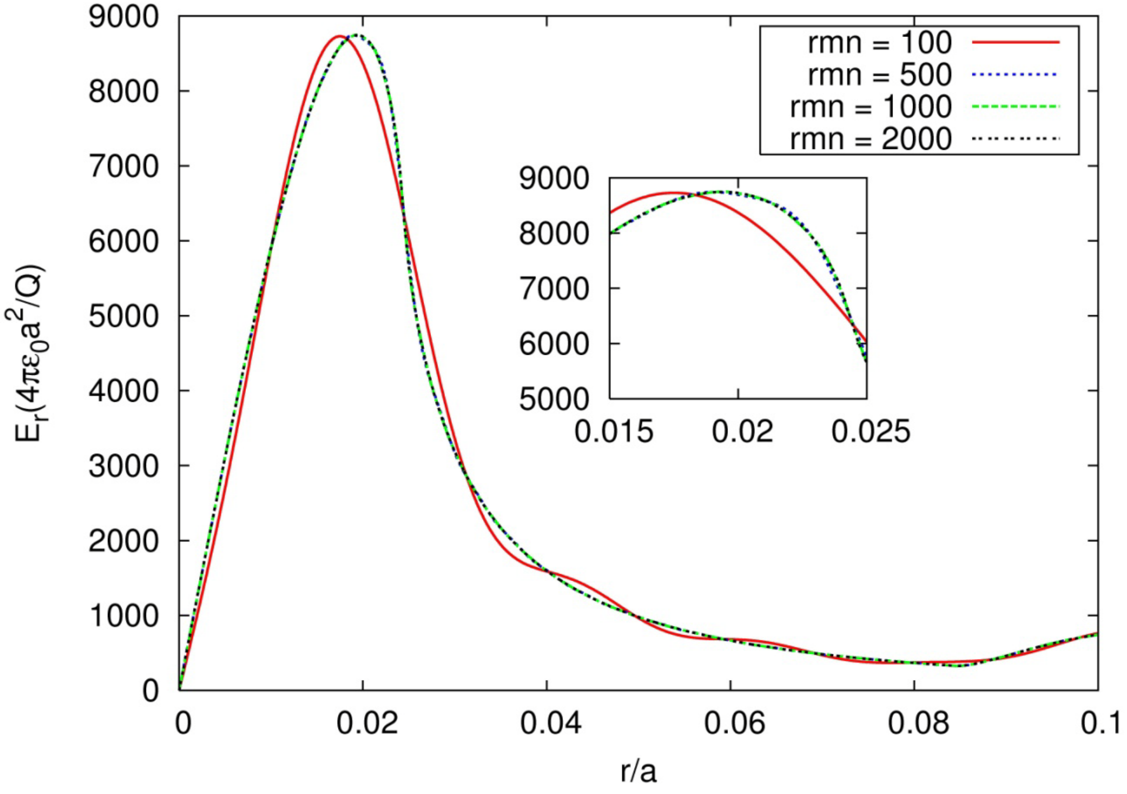
r_b /a = 0.011

Eigenmode Summation (cont'd)

- **From the expansion of the beam density function, we can get the inequality,**

$$j_{0M}\,\frac{r_b}{a} >> 2\pi$$

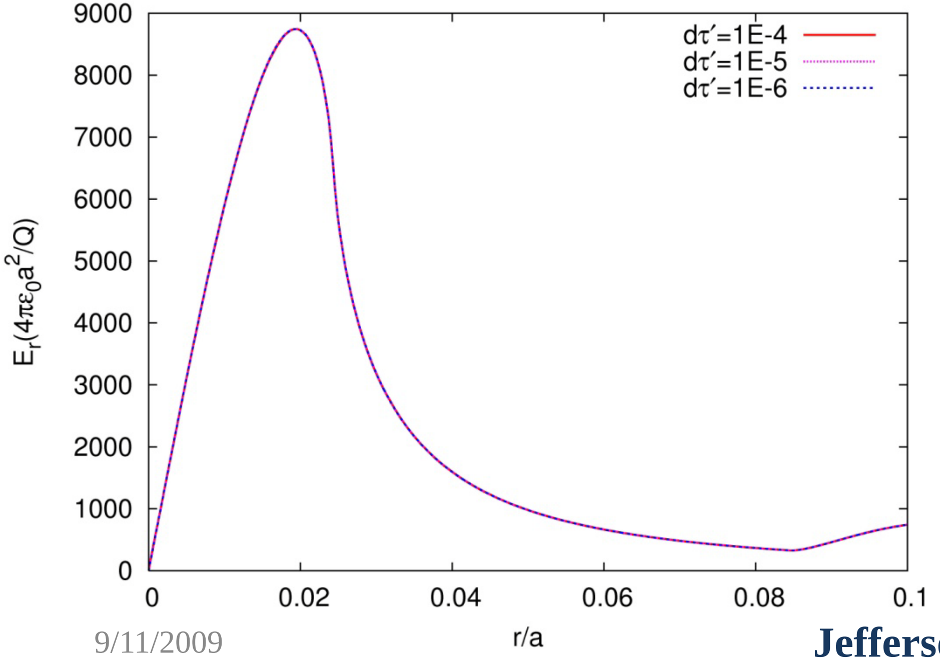
- **In order to model the fields within 1% accuracy, it is necessary to sum over at least 2000 modes corresponding to BNL 2.856 GHz Photocathode gun.**

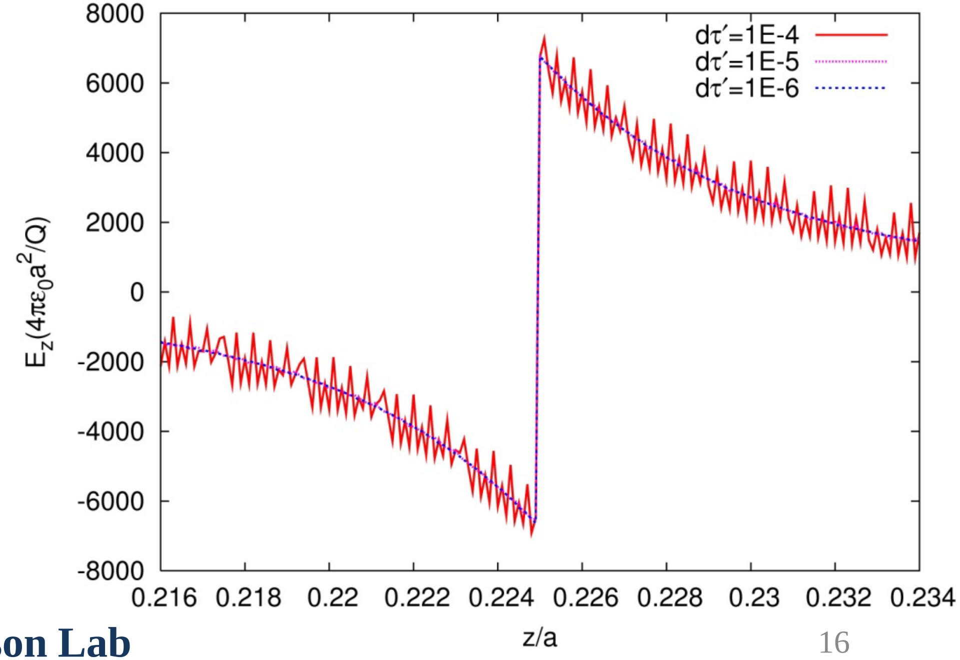


Numerical Time Integration

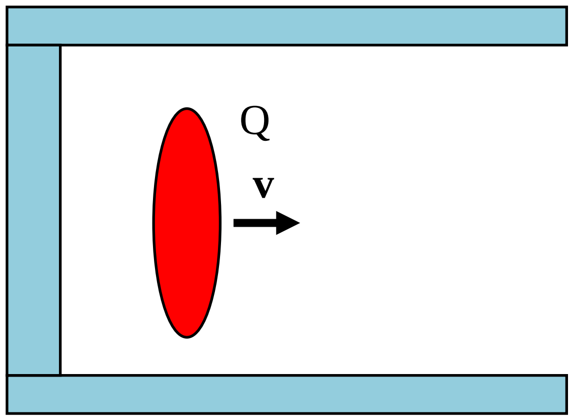
In the Ez calculations, the longitudinal field strongly depends on the beam trajectory. The oscillation periods of the Green's functions are determined by the transverse eigenmode number, M, and the integration step size, Δt'. A smaller Δt' reduces the amplitude of the oscillation.

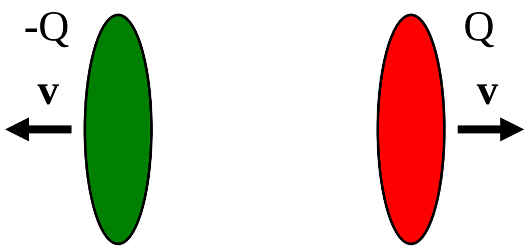
$$\Delta\tau' = \frac{c\Delta t'}{a} << 0.01\frac{r_b}{a}$$





Benchmark Modeling



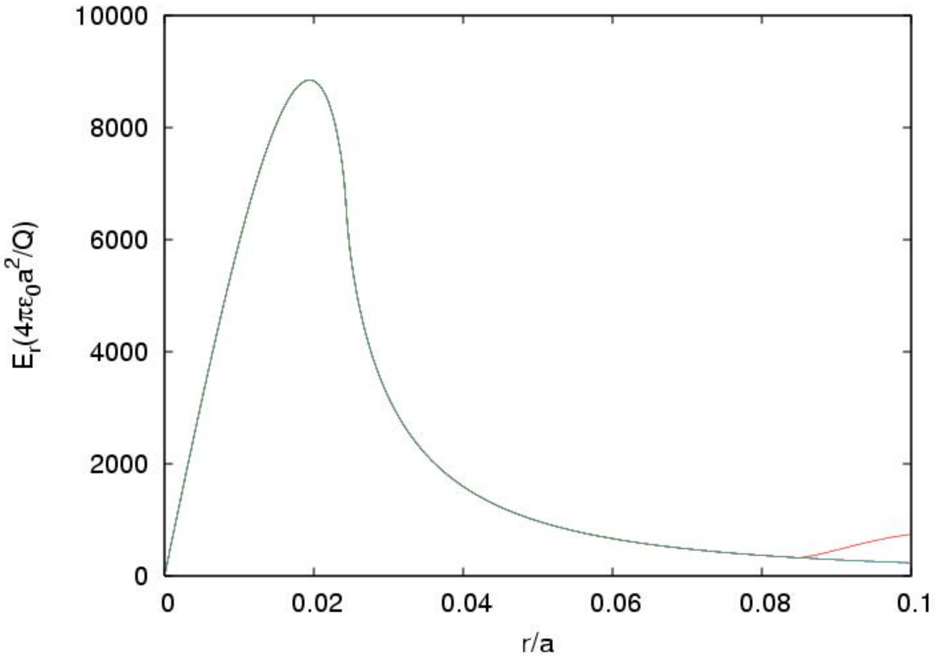


- **IRPSS simulation of a disk bunch of charge emitted at time t = 0 from the cathode surface moving uniformly with speed v**

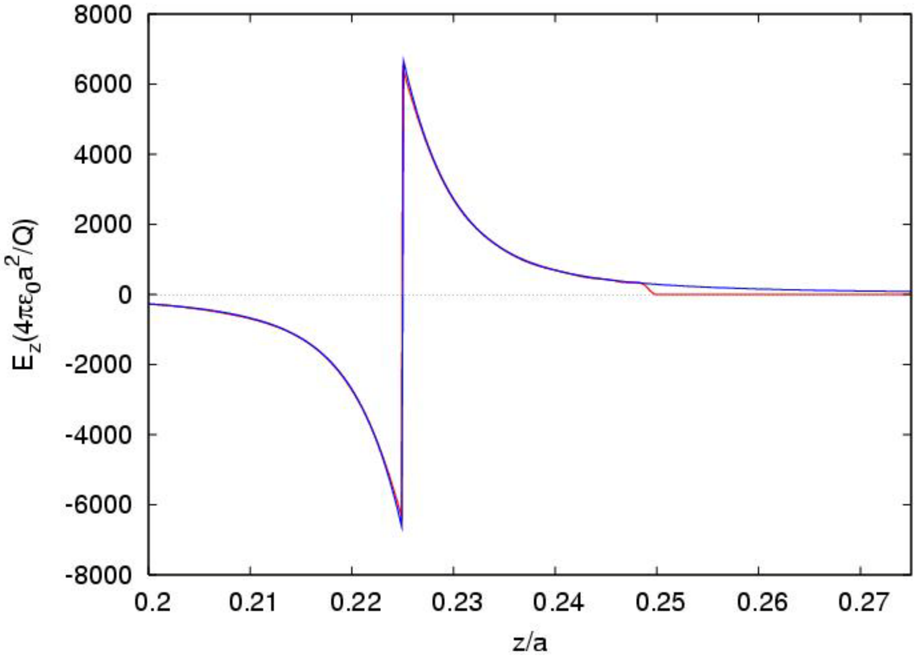
- **Analytical model of two disks of charge moving uniformly in opposite directions for all time and intersecting at t = 0**

Benchmark Comparison

E_r vs. r



E_z vs. z



Blue: Benchmark **Red: IRPSS**

Simulating Beam Dynamics with ANL AWA 1.3 GHz RF Photoinjector

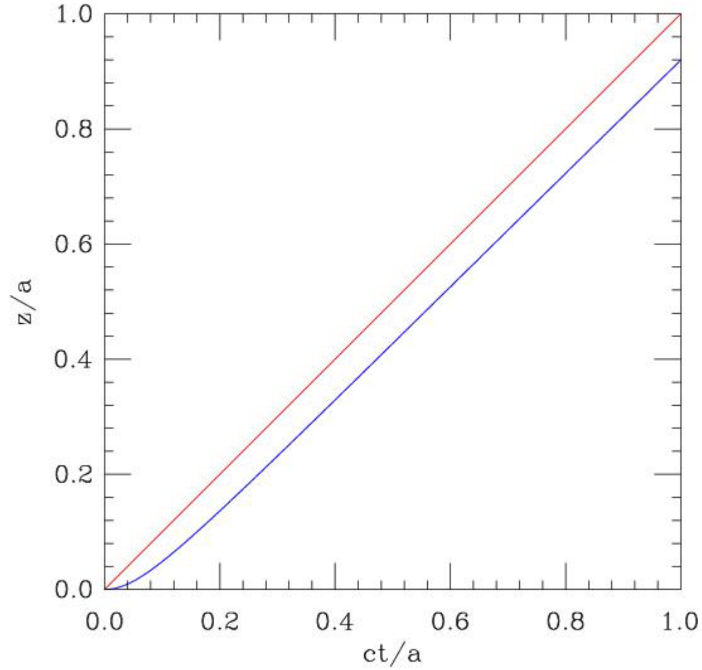
- **The rf fields within AWA gun are approximately**

$$E_z = E_0 \cos(k_z z)\sin(\omega t + \varphi_0)$$

$$E_r = \frac{E_0 r k_z}{2}\sin(k_z z)\sin(\omega t + \varphi_0)$$

$$B_\theta = \frac{E_0 r k_z}{2c}\cos(k_z z)\cos(\omega t + \varphi_0)$$

- **The longitudinal rf fields make beam trajectories**

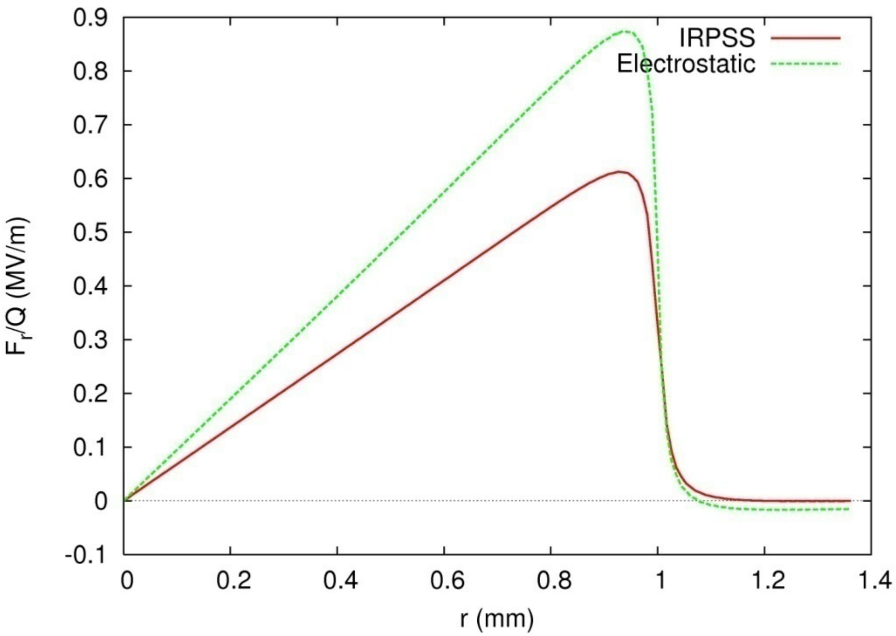


f_{rf}	E_0	φ_0	Q_b	r_b	t_{laser}
1.3 GHz	50 MV/m	65 deg	100 pC – 1 nC	1 mm	1.2 ps

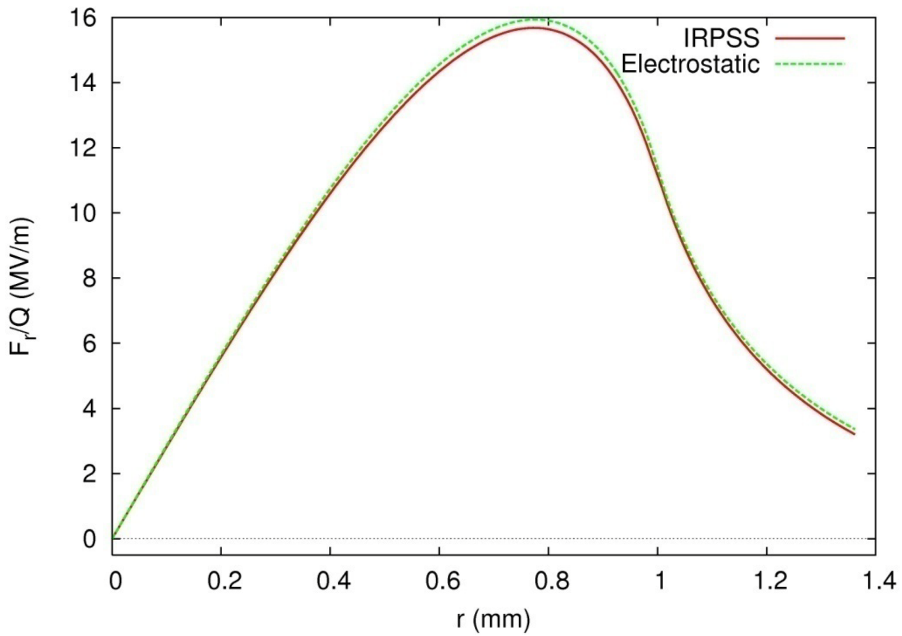
Transverse Space-Charge Forces

- **Large discrepancies for earlier time**
- **Small differences for later time**

$$\mathbf{F}/Q = \mathbf{E} + \boldsymbol{\beta} \times (c\mathbf{B})$$



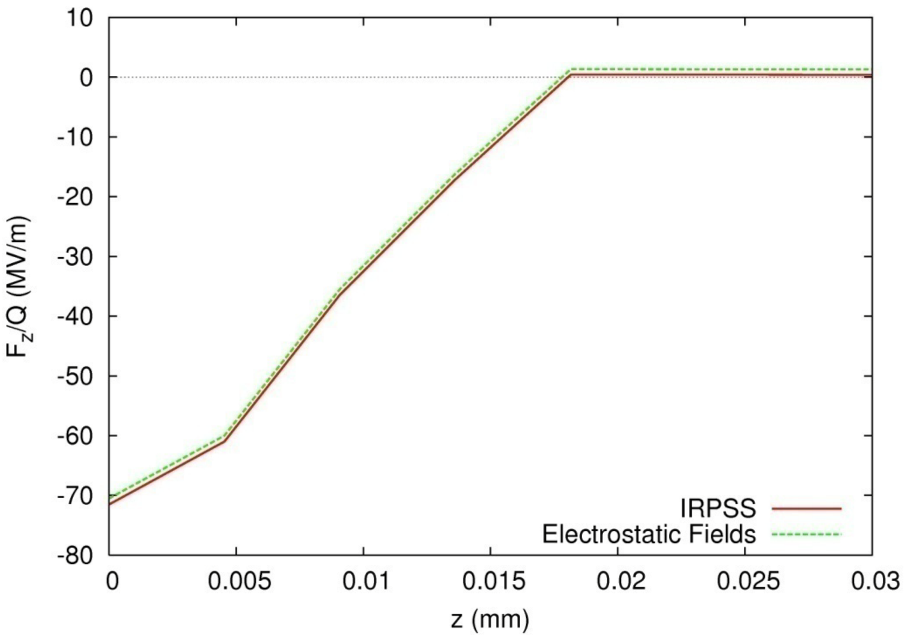
ct/a = 0.005



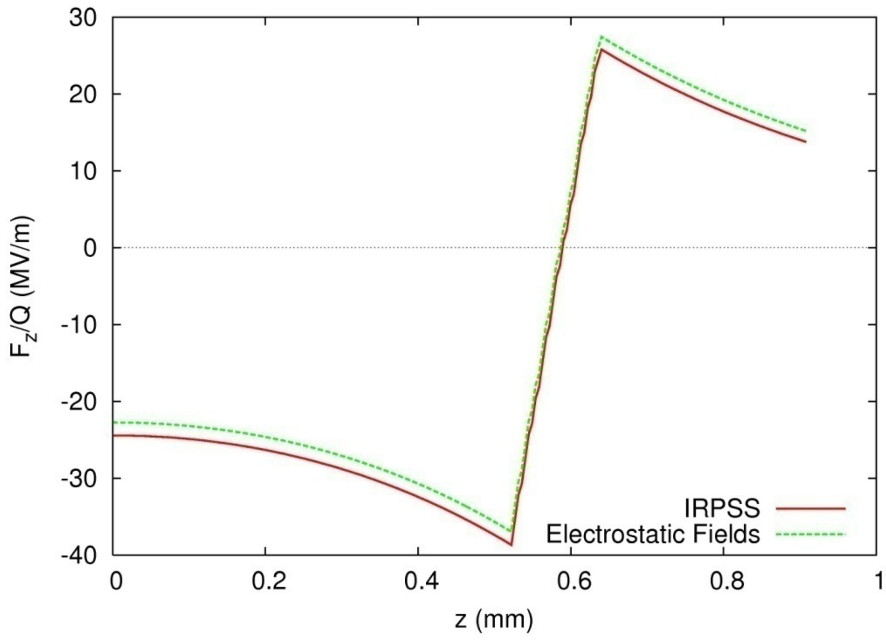
ct/a = 0.040

Longitudinal Space-Charge Forces

- **The difference is smaller than transverse case**
- **For longer bunch length, this will be increased**



ct/a = 0.005

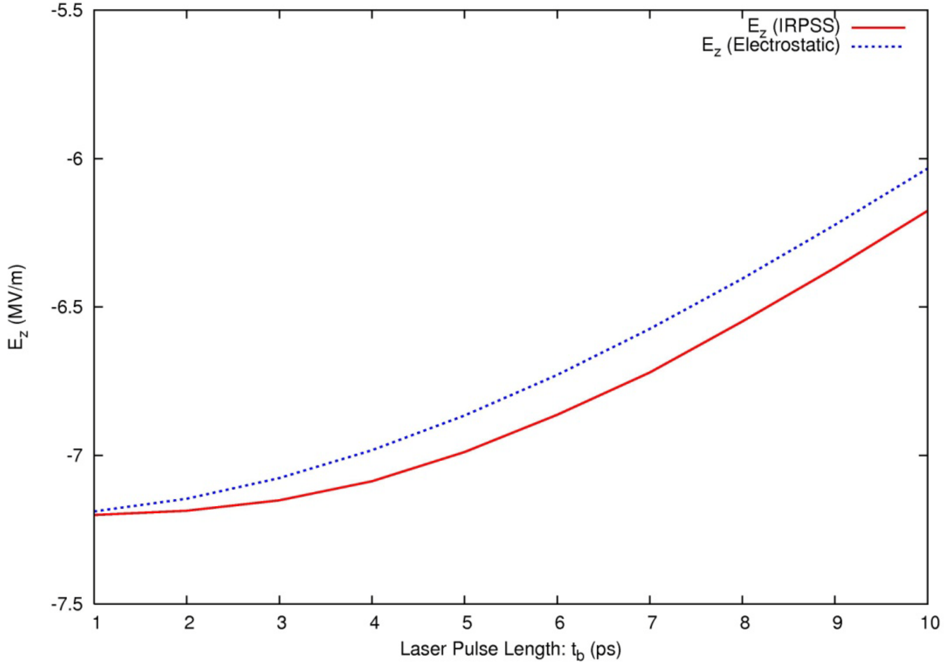


ct/a = 0.040

E_z Space-Charge Fields

- A key result is that as the laser pulse length is increased, the discrepancy between electrostatic and electromagnetic SC fields is increased at the cathode when the back of the bunch is emitted.

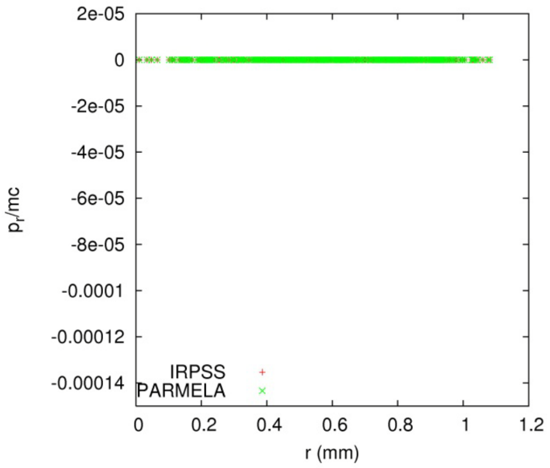
- The size of the discrepancy also depends on the beam radius. Qualitatively, as the beam radius is increased, i.e., the beam becomes more pancake like, and the discrepancy becomes smaller.



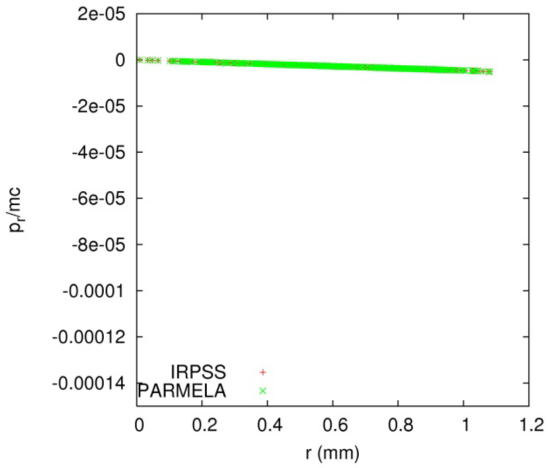
Beam Dynamics w/o Space-Charge Effects

- **With only rf fields, particles are radially focused near the cathode**

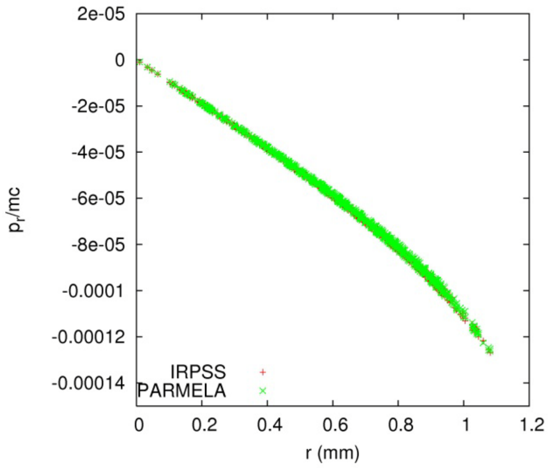
P_r/mc vs. r phasespace



z = 0.0 cm

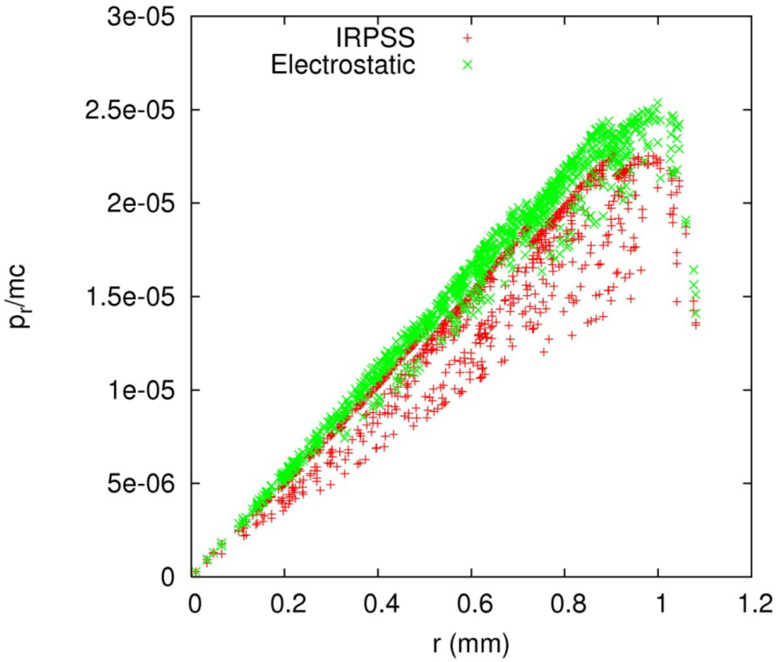


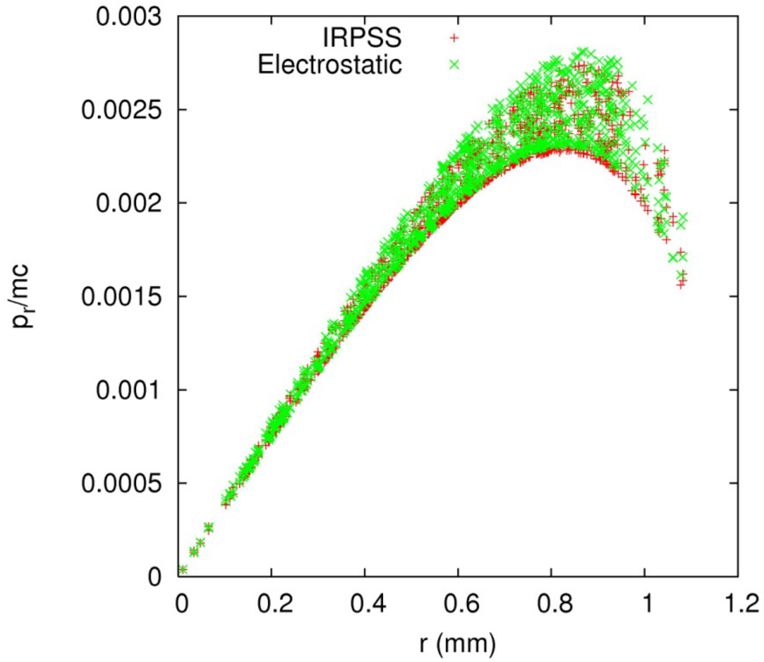
z = 0.003 cm



z = 0.03 cm

Beam Dynamics with Space-Charge Effects



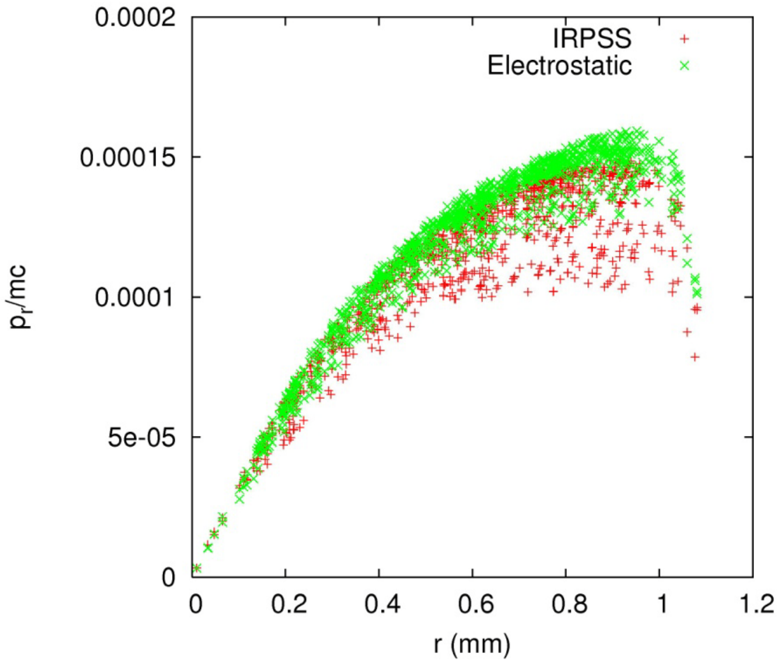


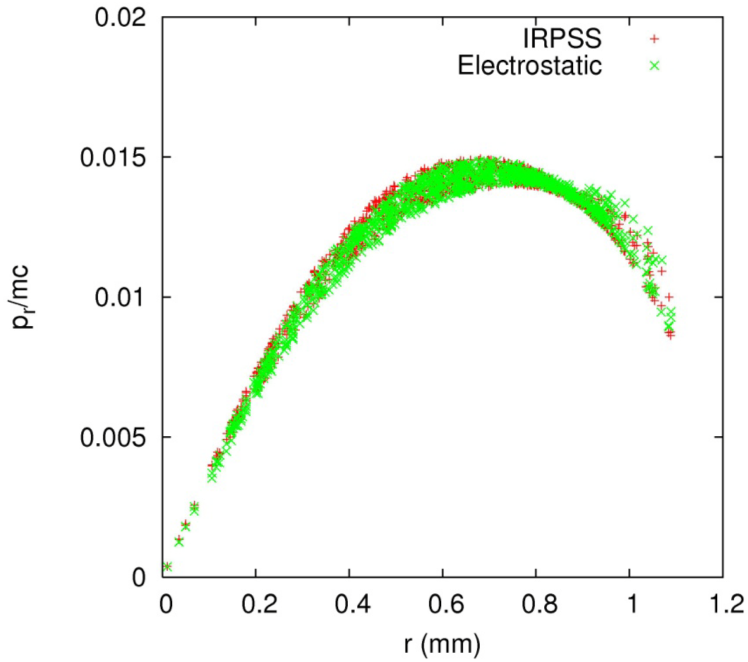
z = 0.003 cm

z = 0.03 cm

Q = 100 pC

Beam Dynamics with Space-Charge Effects





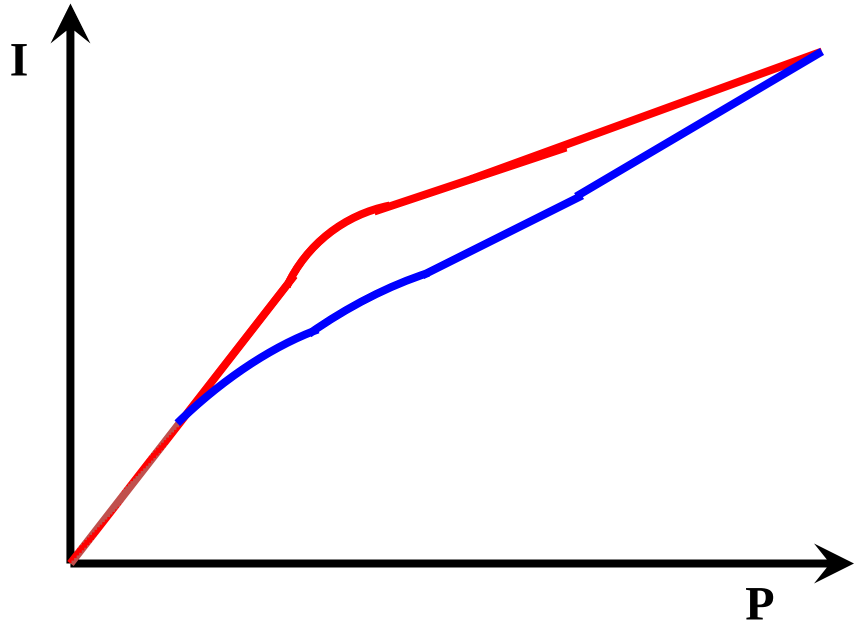
z = 0.003 cm

z = 0.03 cm

Q = 500 pC

Beam Loss Measurements

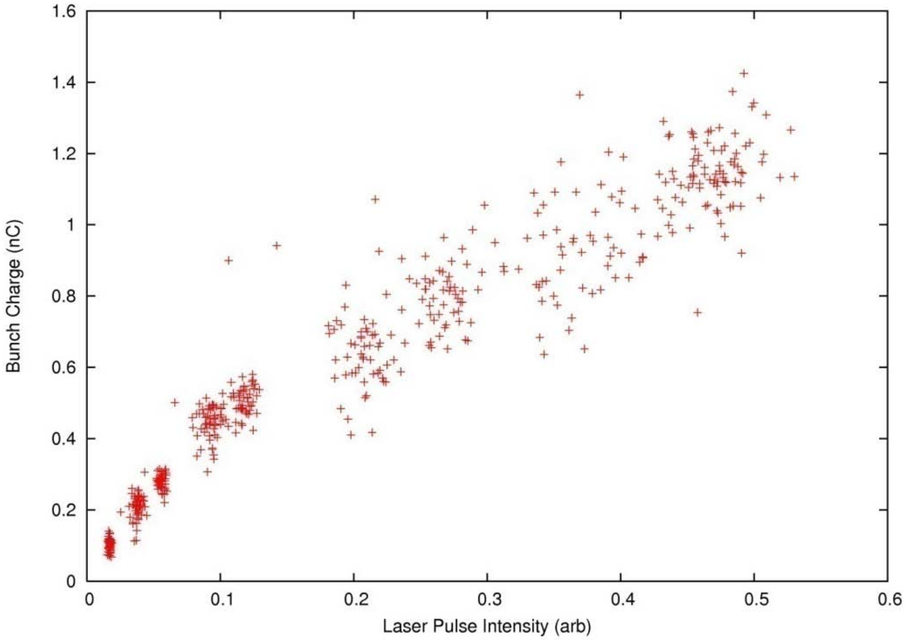
- By varying the amount of laser power, P, (proportional to bunch charge)
- We measure the beam current, I



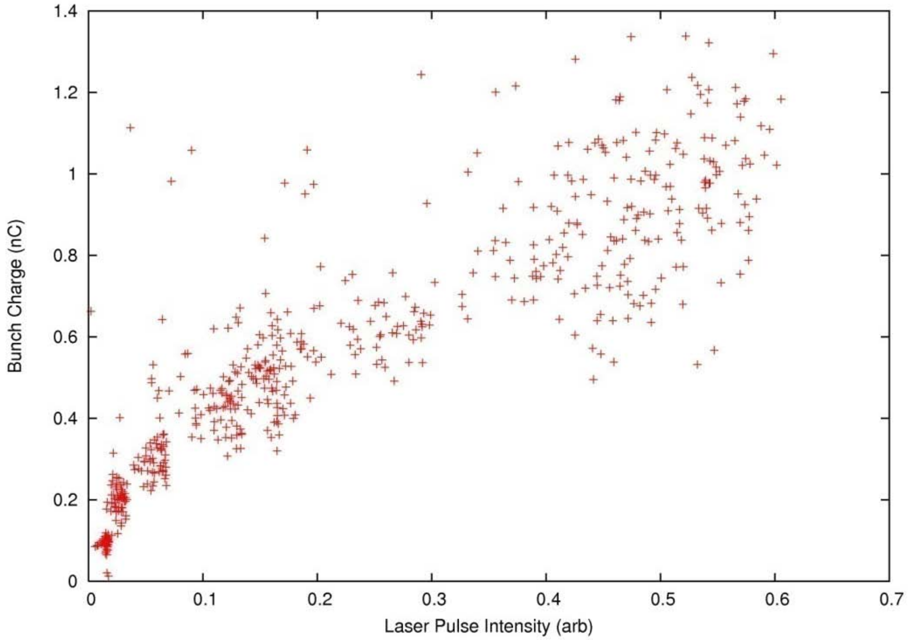
Blue: Electrostatic
Red: Electromagnetic

Experimental Measurements

- **Performed an experimental beam loss measurement on the 1.3 GHz rf gun at the ANL AWA experiment. Below are plots of measured beam charge vs. measured laser pulse intensity. If no beam loss was to occur then the plot should be linear with a uniform slope. However, at a critical bunch charge. i.e., $E_z(rf) = E_z(critical)$, for fixed laser pulse length and radius, one would expect beam loss to occur and a reduction in the slope of the curve.**



t_b = 3.4 psec



t_b = 10.4 psec

Space Charge Effects with Transverse Currents

Electromagnetic Field for Circularly Symmetric Sources

- **Generalize the exact formalism for the SC fields of a cylindrically symmetric beam in a circular conducting pipe**
- **Include the effect of the transverse currents**
- **Construct electromagnetic SC fields using the time-dependent Green's function method in the cylindrical conducting boundary conditions**
- **Can model the high SC dominated systems, such as high-power microwave sources**
- **Compare to Electrostatic (ES) result which is frequently used to model high-power microwave sources, such as klystron**

Expansions of Charge and Current Densities

- **Beam source and the system are cylindrically symmetric***

$$\rho=\sum_{m=1}^{\infty}\rho_m(z,t)J_0\left(\frac{j_{0m}r}{a}\right)$$

$$J_r=\sum_{m=1}^{\infty}J_{rm}(z,t)J_1\left(\frac{j_{0m}r}{a}\right)$$

$$J_\theta=\sum_{m=1}^{\infty}J_{\theta m}(z,t)J_1\left(\frac{j_{1m}r}{a}\right)$$

$$J_z=\sum_{m=1}^{\infty}J_{zm}(z,t)J_0\left(\frac{j_{0m}r}{a}\right)$$

⇒

$$\rho_m(z,t)=\frac{2}{a^2J_1^2(j_{0m})}\int_0^a dr\, r\rho J_0\left(\frac{j_{0m}r}{a}\right)$$

$$J_{rm}(z,t)=\frac{2}{a^2J_1^2(j_{0m})}\int_0^a dr\, rJ_rJ_1\left(\frac{j_{0m}r}{a}\right)$$

$$J_{\theta m}(z,t)=\frac{2}{a^2J_0^2(j_{1m})}\int_0^a dr\, rJ_\theta J_1\left(\frac{j_{1m}r}{a}\right)$$

$$J_{zm}(z,t)=\frac{2}{a^2J_1^2(j_{0m})}\int_0^a dr\, rJ_zJ_0\left(\frac{j_{0m}r}{a}\right)$$

Continuity equation: $\dfrac{\partial\rho_m}{\partial t}+\dfrac{j_{0m}}{a}J_{rm}+\dfrac{\partial J_{zm}}{\partial z}=0$

*Submitted to IEEE Transaction

TM Mode Space-Charge Fields

- **EM fields generated by J_r and J_z**

$$E_r = \sum_{m=1}^{\infty} E_{rm}(z,t) J_1\left(\frac{j_{0m} r}{a}\right) \Leftarrow \qquad E_{rm} = \int_{-\infty}^{t} dt' \int dz' \left(\frac{j_{0m}\rho_m}{\varepsilon_0 a} - \mu_0 \frac{\partial J_{rm}}{\partial t'}\right) G_{0m}(z,t,z',t')$$

$$G_{0m}(z,t,z',t') = \frac{c}{2} J_0\left(\frac{j_{0m}}{a}\lambda\right)\theta(\lambda^2)$$

Maxwell's Equations:

$$B_{rm} = -\frac{a}{j_{1m}}\frac{\partial B_{zm}}{\partial z}, \quad E_{\theta m} = -\frac{a}{j_{1m}}\frac{\partial B_{zm}}{\partial t}, \quad \frac{j_{0m}}{a}E_{rm} + \frac{\partial E_{zm}}{\partial z} = \frac{\rho_m}{\varepsilon_0}$$

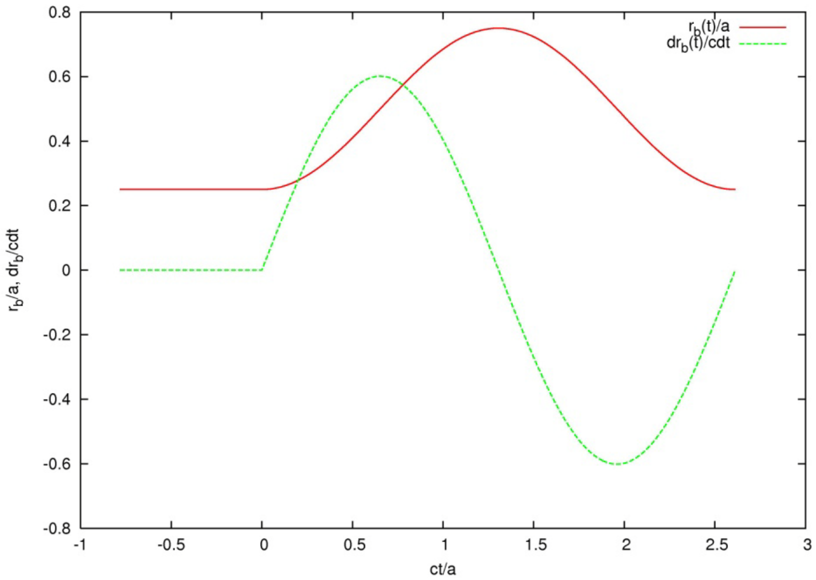
$$\frac{\partial E_{rm}}{\partial z} + \frac{j_{0m}}{a}E_{zm} = -\frac{\partial B_{\theta m}}{\partial t}, \quad -\frac{\partial B_{\theta m}}{\partial z} = \mu_0 J_{rm} + \frac{1}{c^2}\frac{\partial E_{rm}}{\partial t}, \quad \frac{j_{0m}}{a}B_{\theta m} = \mu_0 J_{zm} + \frac{1}{c^2}\frac{\partial E_{zm}}{\partial t}$$

Numerical Example

- Simulate radially "breathing beam"
- Radial beam current, but no longitudinal beam current
- Compare EM SC fields with ES SC fields

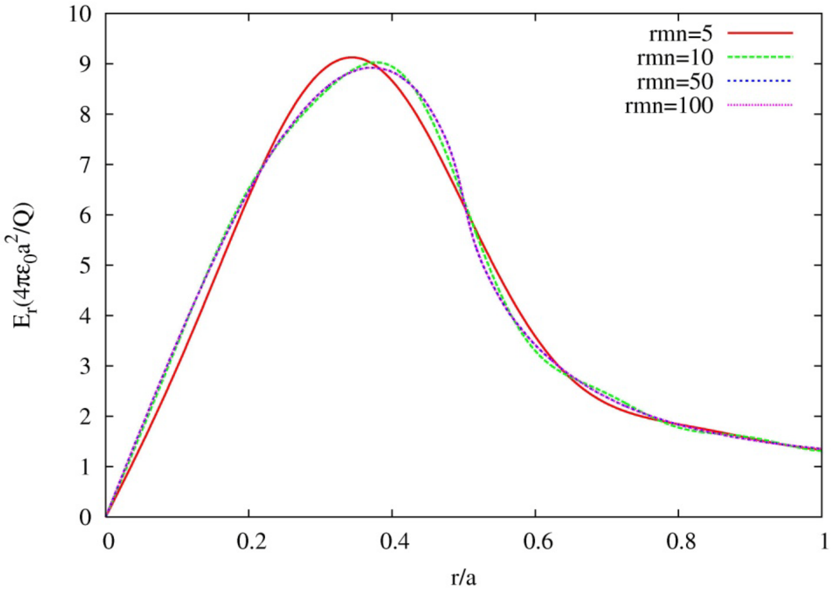
$$\rho(r,z,t) = \frac{2Q}{\pi r_b^2(t)}\left(1 - r^2/r_b^2(t)\right)\theta\left(r_b(t) - r\right) \times \left[\theta(L/2 + z) - \theta(L/2 - z)\right]$$

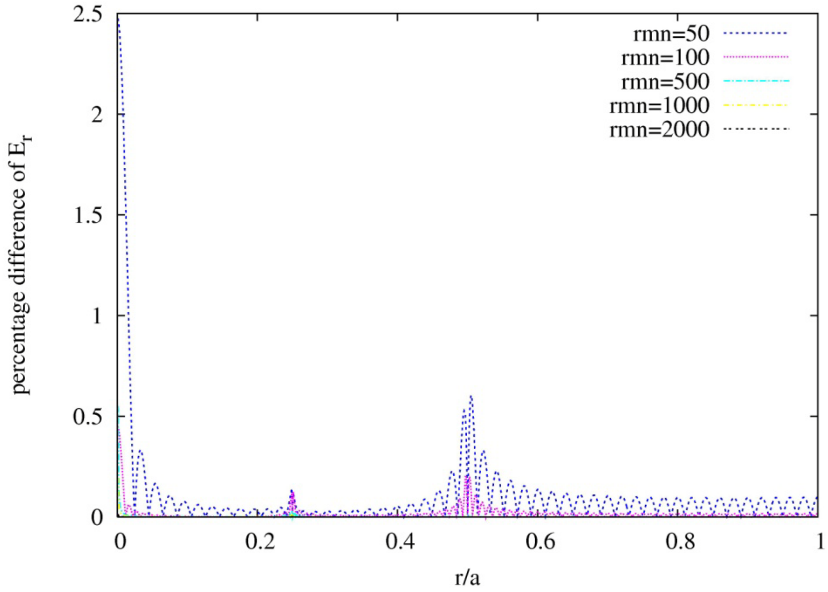
$$r_b(t) = \begin{cases} r_0, & t \le 0 \\ r_0 + \delta r - \delta r \cos(\omega t), & t > 0 \end{cases}$$



Numerical Requirements

- Similar to the photoinjector modeling
- Less number of eigenmodes due to larger beam radius

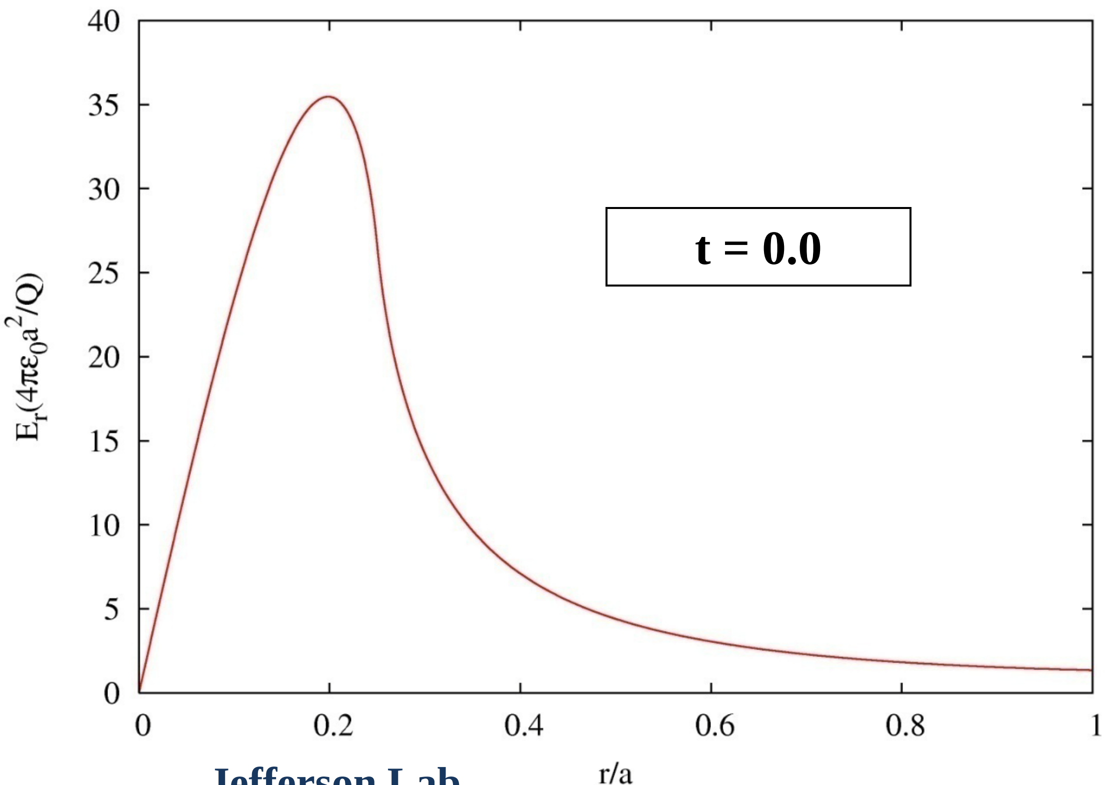




Transverse Space-Charge Fields with Transverse Currents

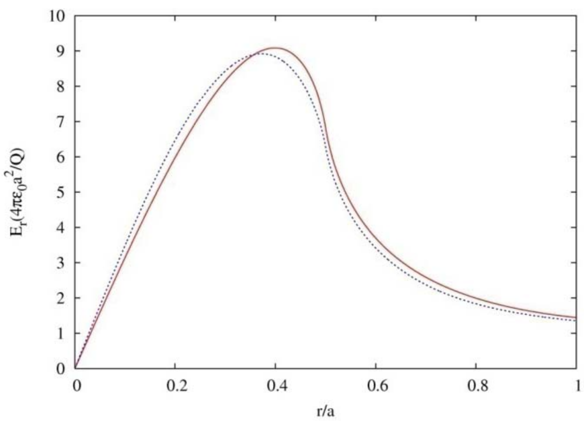
- **ES fields are also found using Green's function method**
- **The beam oscillation starts with the initial beam radius, $r_b/a=0.25$**

- **$a=9.08$ cm**
- **$\omega=j_{01}c/a$**
- **$m=1000$**
- **$\Delta t'=0.0001a/c$**

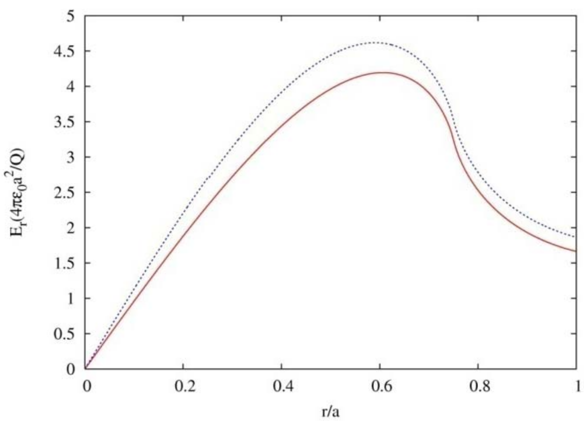


Transverse Space-Charge Fields with Transverse Currents (cont'd)

Blue: Electromagnetic **Red**: Electrostatic

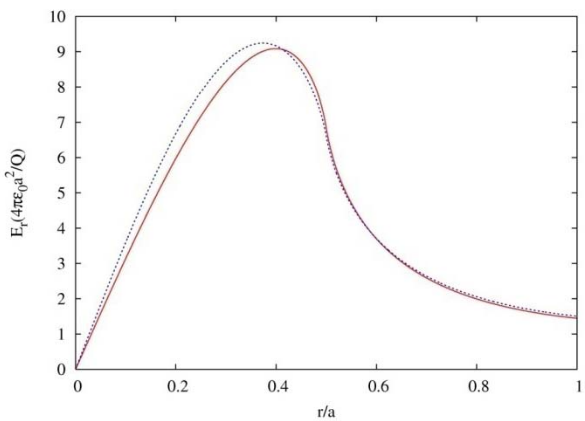


t = 0.25T

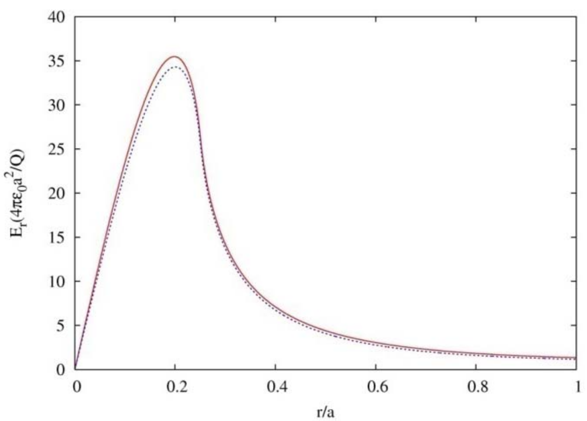


t = 0.50T

t = 0.75T

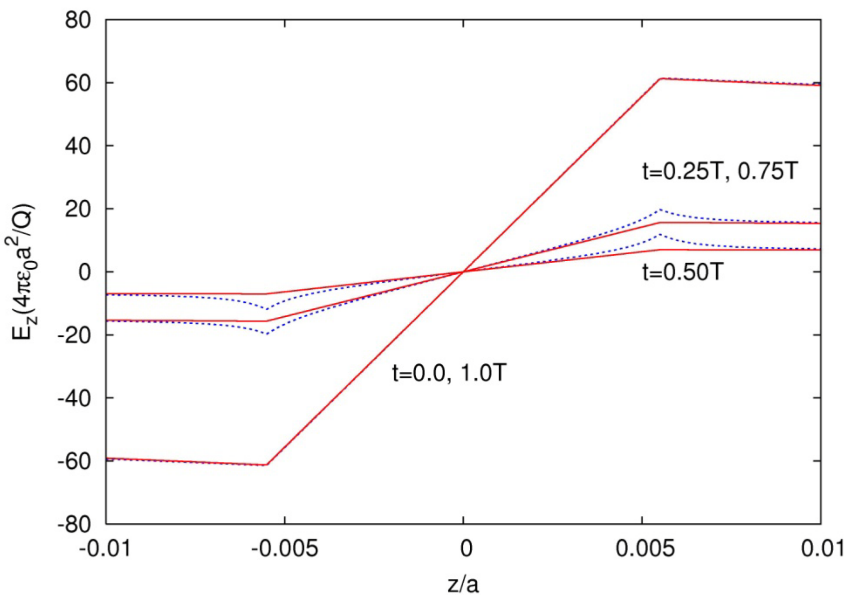


t =T

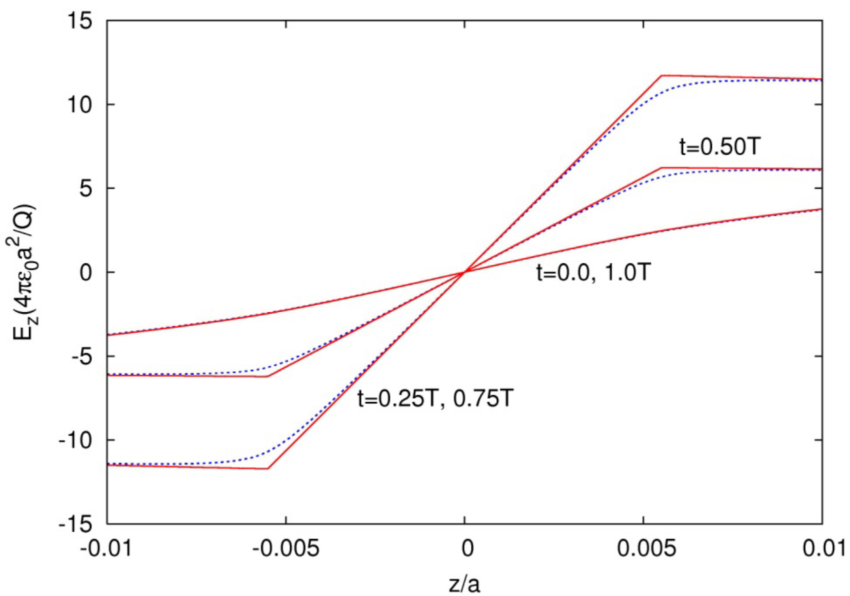


Longitudinal Space-Charge Fields with Transverse Currents

- **The effects of the SC E_z fields are important near the beam edge**



r/a = 0.0



r/a = 0.25

Summary

- **Developed electromagnetic space-charge models of electron beams in the presence of the conducting boundaries**
- **Developed a novel computational code, IRPSS, to compute the SC fields numerically**
- **Is capable of simulating beams with arbitrarily small bunch Lengths, since it uses a Green's function approach**
- **Simulated the beam dynamics of the beam near the cathode in the rf photoinjector**
- **Extended Green's function methods by including the transverse currents**
- **Investigated the electromagnetic SC fields for a radially breathing beam oscillation**

Future Plans

- **Extend IRPSS code by including the effects of iris(es) or discontinuities of the cavity**
- **Improve the code to self-consistently calculate the trajectories due to both the external and SC fields**
- **Study how the beam dynamics are affected due to the SC fields in the designs of magnetic focusing schemes for emittance compensations**
- **Include an arbitrary beam currents, such as azimuthally varying currents**

Thank You!