A New TEM-Type Deflecting/Crabbing Cavity

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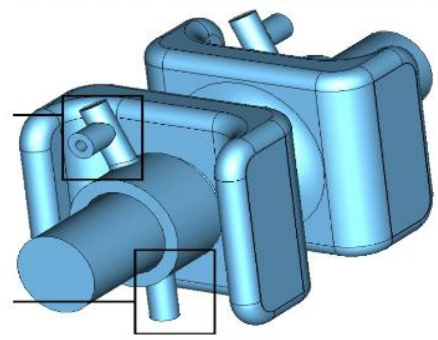
18 February 2009

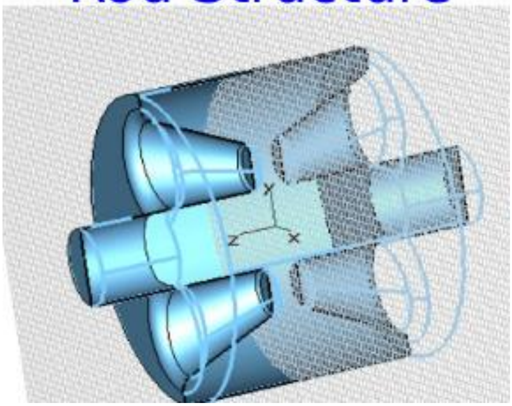
Genesis

- LHC Upgrade would need a small, low frequency (400 MHz) crabbing cavity

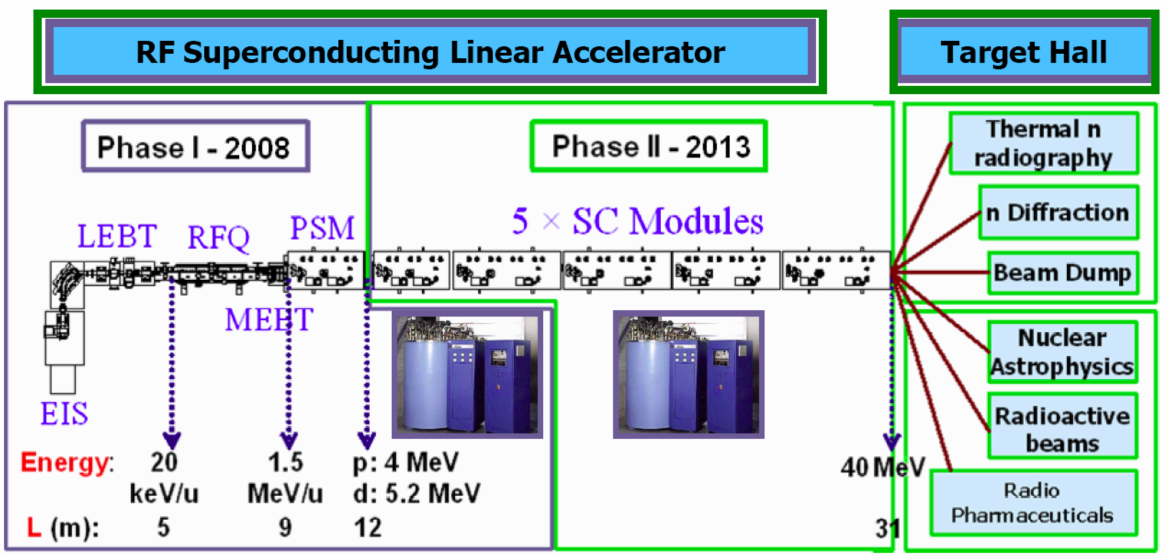
FNAL Mushroom Cavity



EUCard,UK-JLAb Rod Structure



- Low velocity ion accelerators need deflecting cavities at ~150 MHz, β~0.2

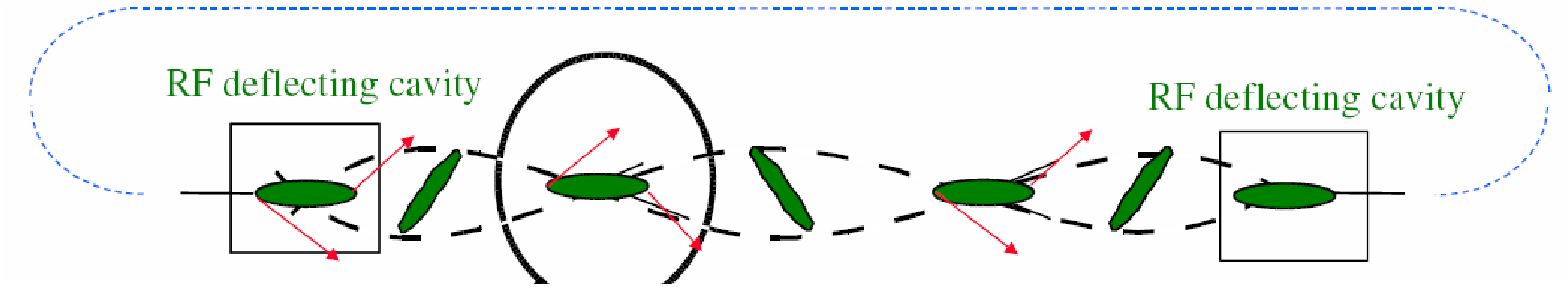


Genesis

- ELIC
  - 24 MV for protons
  - 1.2 MV for electrons (KEK is 1.4 MV)

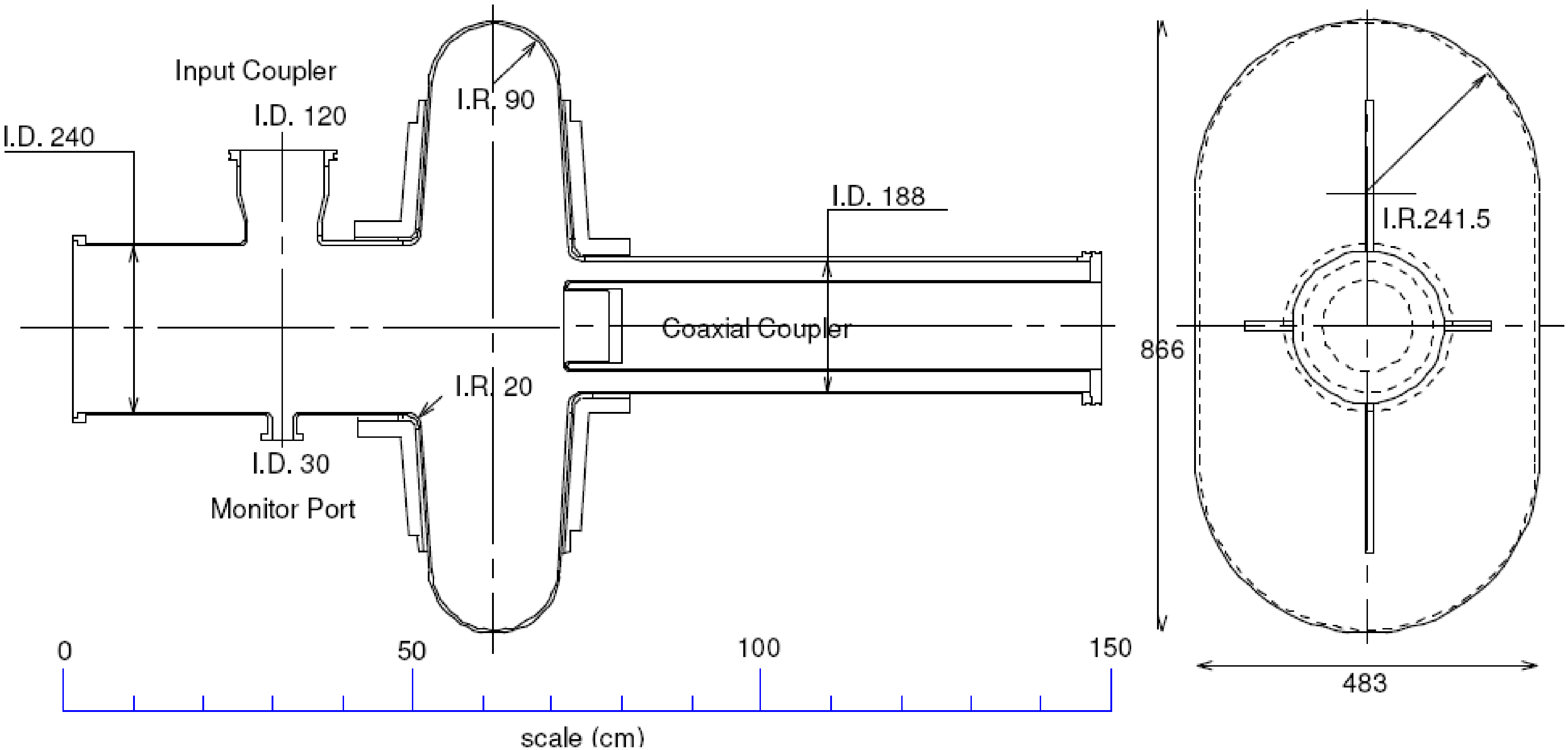
- 11 GeV separator

- Generation of subpicosecond light pulses



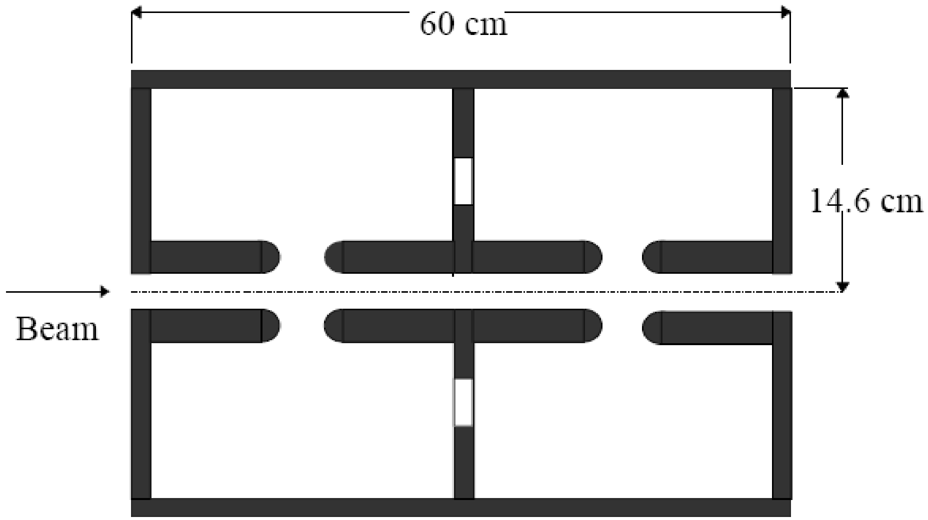
Existing Solutions

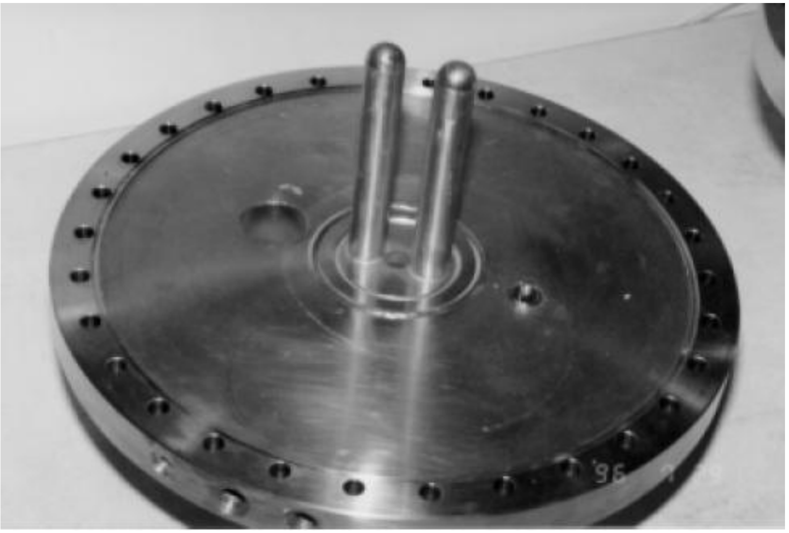
- TM110 cavity (à la KEK) 509 MHz



Existing Solutions

- CEBAF Deflecting cavity, 499 MHz

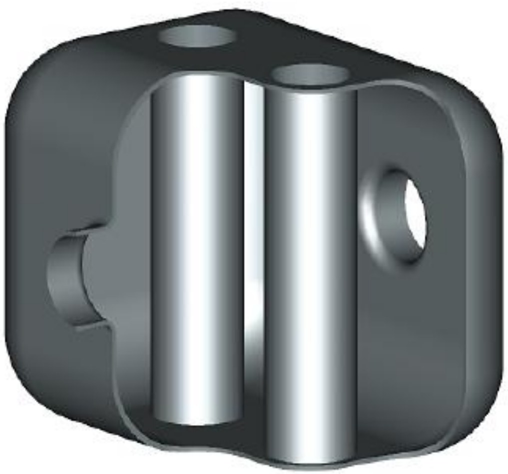




Parallel-Bar Cavity

- Basic cell is made of 2 TEM resonant lines, of length $\lambda/2$, operating in opposite phase

Electromagnetic Fields



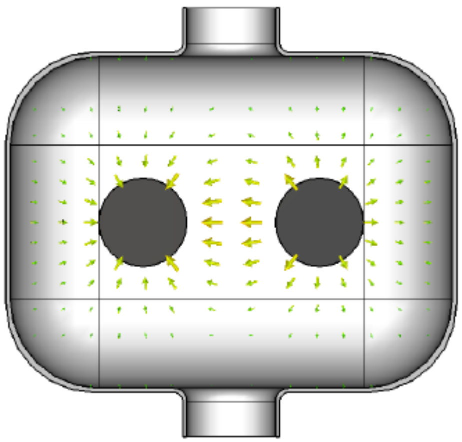


FIG. 2: (Color) Electric field in the mid-plane of the parallel-bar structure operating in the π-mode.

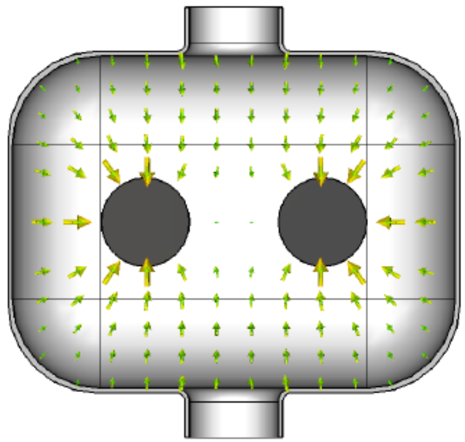


FIG. 4: (Color) Electric field in the mid-plane of the parallel-bar structure operating in the 0-mode.

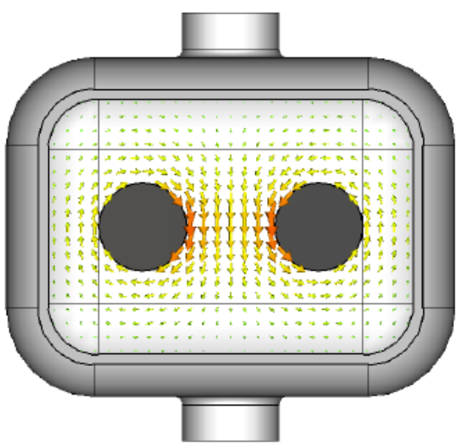


FIG. 3: (Color) Magnetic field in the top plate of the parallel-bar structure operating in the π-mode.

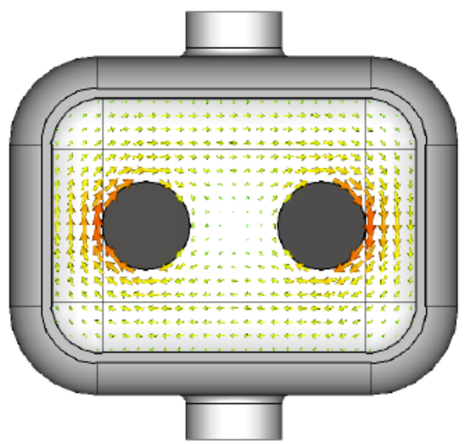
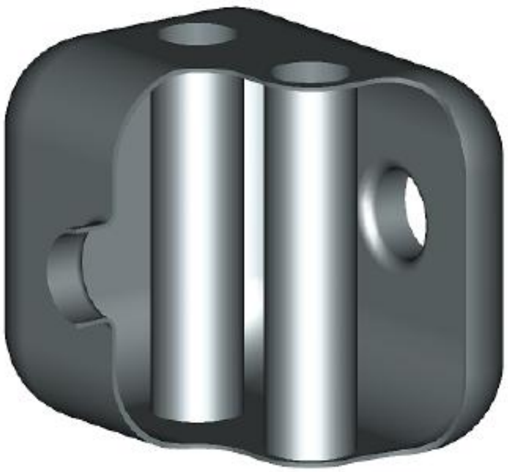


FIG. 5: (Color) Magnetic field in the top plate of the parallel-bar structure operating in the 0-mode.

TEM Modes

- If no beam aperture, no rounded corners

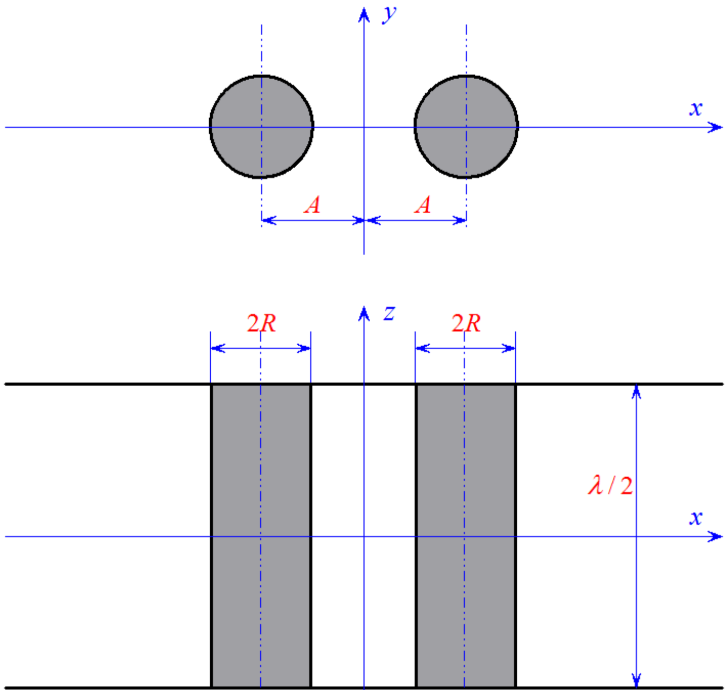


- Translational Invariance
  - No z component
- Multiply connected

$$\vec{E}_t(x,y,z,t) = \vec{E}_t(x,y)\cos\left(\frac{2\pi z}{\lambda}\right)\cos(\omega t)\,, \quad \text{(A.1)}$$

$$\vec{H}_t(x,y,z,t) = \hat{z}\times\frac{\vec{E}_t(x,y)}{Z_0}\sin\left(\frac{2\pi z}{\lambda}\right)\sin(\omega t) \quad \text{(A.2)}$$

Analytical Model



2 circular cylinders between 2 infinite planes

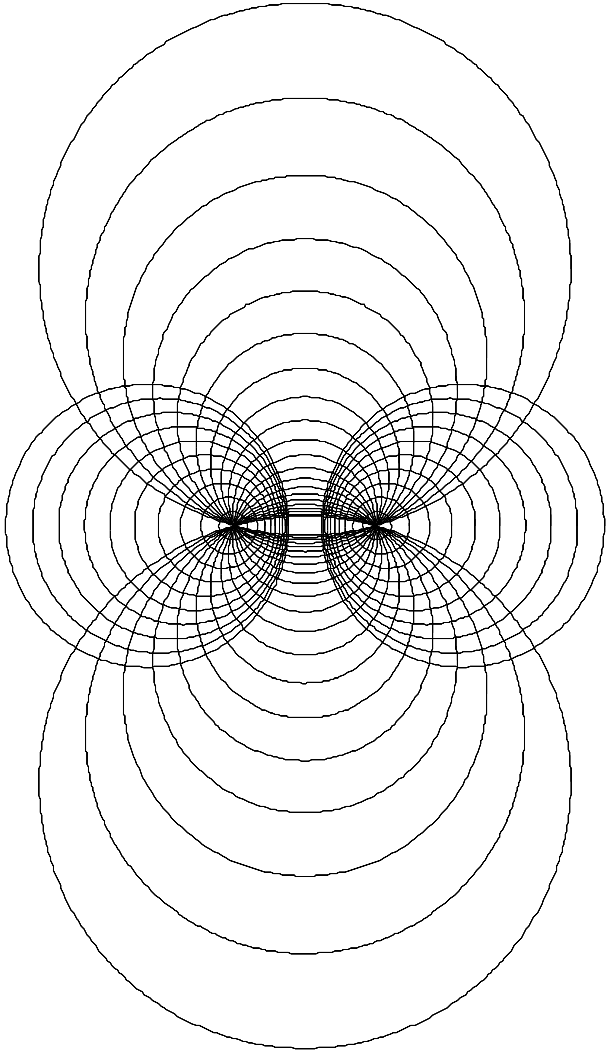
Pure TEM mode
Electrostatic problem of 2 infinitely long, parallel cylinders

$$\vec{E}_t(x,y,z,t) = \vec{E}_t(x,y)\cos\left(\frac{2\pi z}{\lambda}\right)\cos(\omega t)\,, \quad (A.1)$$

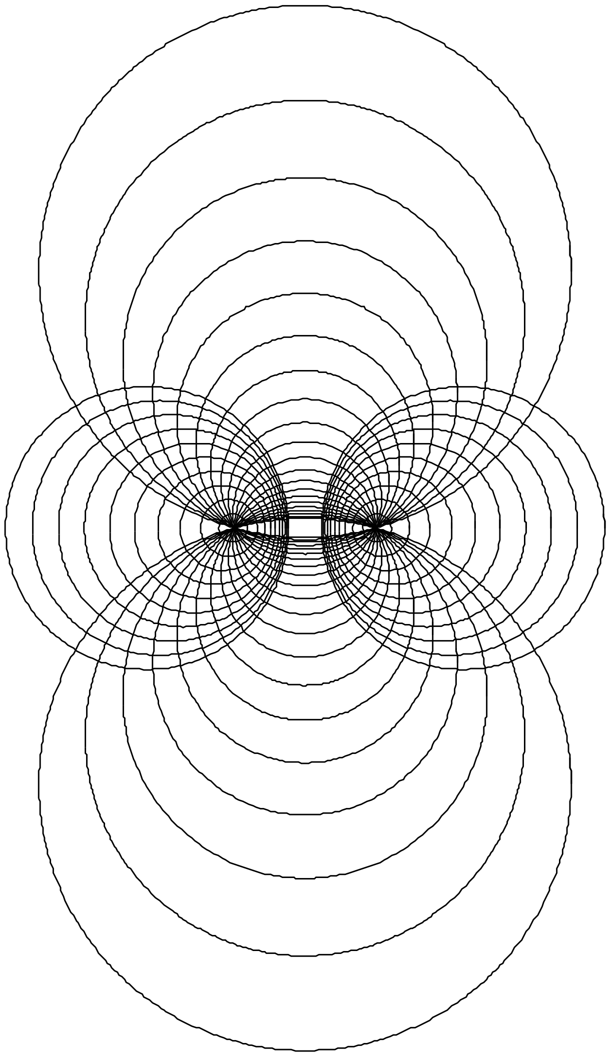
$$\vec{H}_t(x,y,z,t) = \hat{z}\times\frac{\vec{E}_t(x,y)}{Z_0}\sin\left(\frac{2\pi z}{\lambda}\right)\sin(\omega t)\quad (A.2)$$

Analytical Model

- If side wall are sufficiently far from the parallel bars, the electromagnetic properties can be obtained analytically
- Since the electromagnetic mode is TEM, it is an electrostatic problem
- If the cross section of the bars is circular, they can be modeled by 2 parallel, infinitely long, uniformly charged lines



Potential and Fields



Assuming two infinitely long lines, parallel to the z axis, and crossing the (x, y) plane at $x = \pm a, y = 0$, and carrying uniform linear charge per unit length $\pm q$, the potential is given by

$$V(x,y) = \frac{q}{4\pi\epsilon_0} \ln\left(\frac{r_-^2}{r_+^2}\right) , \qquad \text{(A.3)}$$

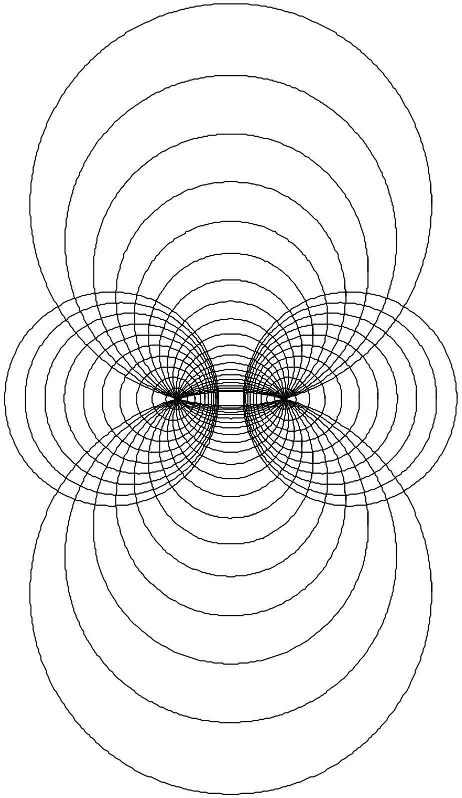
with

$$r_-^2 = (x-a)^2 + y^2 , \qquad \text{(A.4)}$$
$$r_+^2 = (x+a)^2 + y^2 . \qquad \text{(A.5)}$$

and the electric field is

$$E_x(x,y) = -\frac{\partial V}{\partial x} = -\frac{aq}{\pi\epsilon_0}\left[\frac{x^2 - a^2 - y^2}{r_-^2 r_+^2}\right] , \qquad \text{(A.6)}$$
$$E_y(x,y) = -\frac{\partial V}{\partial y} = -\frac{2aq}{\pi\epsilon_0}\left[\frac{xy}{r_-^2 r_+^2}\right] . \qquad \text{(A.7)}$$

Equipotentials



The equipotential surfaces are given by

$$\frac{r_-^2}{r_+^2} = \frac{(x-a)^2 + y^2}{(x+a)^2 + y^2} = e^{2\mu} ,$$

where

$$\mu = \frac{2\pi\epsilon_0 V_0}{q} .$$

This implies

$$(x + a\coth\mu)^2 + y^2 = \left(\frac{a}{\sinh\mu}\right)^2 .$$

Thus two infinite parallel cylinders, at potentials $\pm V_0$, of radius R and axis separation $2A$, can be modelled by two infinite parallel lines, separated by $2a = 2\sqrt{A^2 - R^2}$ and of uniform linear charge $\pm 2\pi\epsilon_0 V_0 / \cosh^{-1}(A/R)$.

Capacitance and Inductance

- Capacitance per unit length

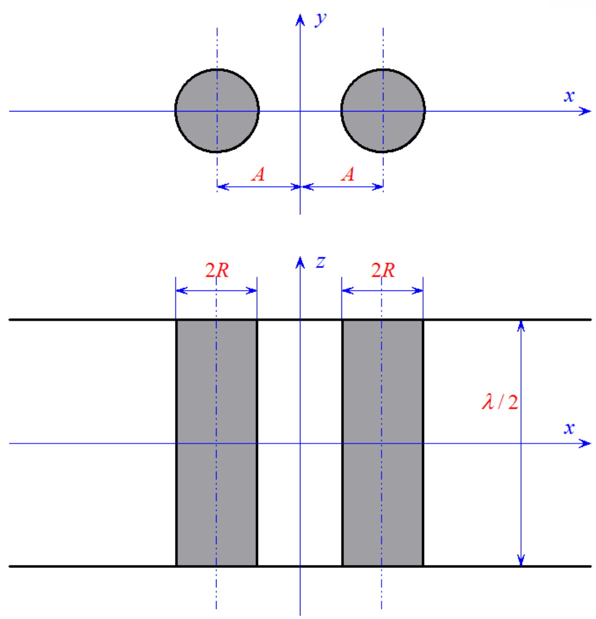
$$C = \frac{\pi \epsilon_0}{\cosh^{-1}\left(\frac{A}{R}\right)}$$

- Inductance per unit length

$$L = \frac{\mu_0}{\pi} \cosh^{-1}\left(\frac{A}{R}\right)$$

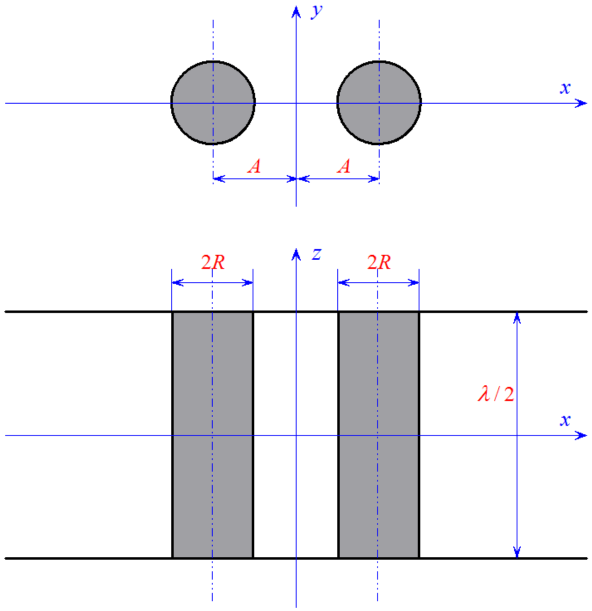
- Transmission line impedance

$$Z = \frac{Z_0}{\pi} \cosh^{-1}\left(\frac{A}{R}\right)$$



Energy Content

- Energy content

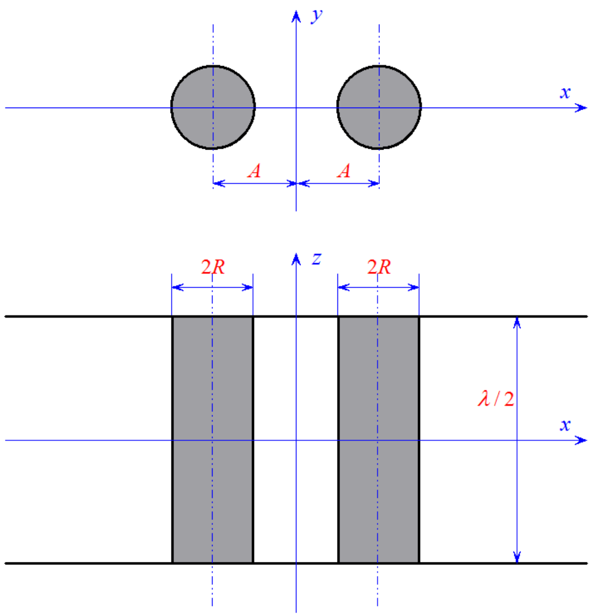


$$
\begin{aligned}
U &= \int_{-\lambda/4}^{\lambda/4} \frac{1}{2} C V^2 \, dz \\
&= \frac{1}{2} C (2V_0)^2 \int_{-\lambda/4}^{\lambda/4} \cos^2\left(\frac{2\pi z}{\lambda}\right) dz \\
&= \frac{\lambda}{2} C V_0^2 = \frac{\lambda}{2} \frac{\pi \epsilon_0}{\cosh^{-1}(\alpha)} V_0^2 \, .
\end{aligned}
$$

Losses in Bars

- Power dissipation in the bars

$$P_b = 2 \times \frac{R_s}{2} \int_{\mathrm{bar}} H^2 \, dS = R_s \int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \cos^2\left(\frac{2\pi z}{\lambda}\right) dz \int_C H^2 \, dl = R_s \frac{\lambda}{4} \int_C H^2 \, dl \, ,$$



$$\int_C H^2 \, dl = \frac{1}{Z_0^2} \int_{C'} E^2 \, dl \, ,$$

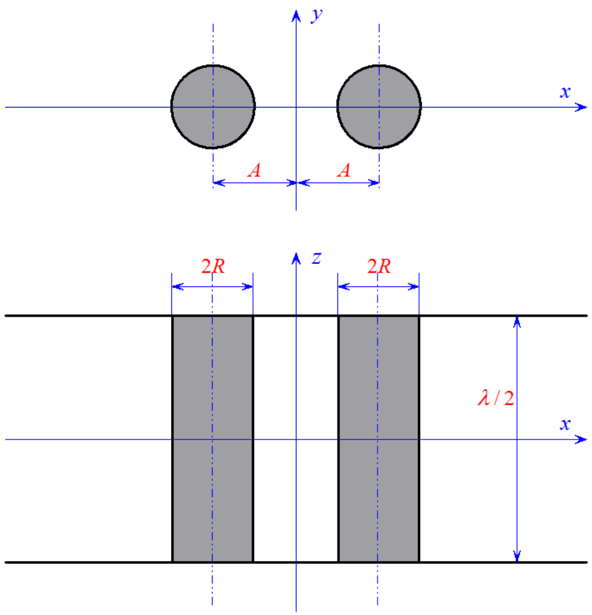
$$E^2(C') = \frac{V_0^2}{R^2} \frac{\alpha^2 - 1}{\left[\cosh^{-1}(\alpha)\right]^2} \frac{1}{(\alpha + \cos\phi)^2} \, ,$$

$$P_b = \frac{\pi}{2} \frac{\lambda}{R} \frac{R_s V_0^2}{Z_0^2} \frac{\alpha}{\left[\cosh^{-1}(\alpha)\right]^2} \frac{\alpha}{\sqrt{\alpha^2 - 1}}$$

Losses in Planes

- Power dissipation in the planes

$$P_p = 2 \times \frac{R_s}{2} \int_{\{P\}-2\{D\}} H^2\, dS$$



$$U = \frac{\mu_0}{2} \int_V H^2\, dV = \frac{\mu_0}{2} \frac{\lambda}{4} \int_{\{P\}-2\{D\}} H^2\, dS$$

$$\int_{\{P\}-2\{D\}} H^2\, dS = \frac{8U}{\lambda \mu_0} = \frac{4\pi}{Z_0^2} \frac{V_0^2}{\cosh^{-1}(\alpha)}$$

$$P_p = 4\pi \frac{R_s V_0^2}{Z_0^2} \frac{1}{\cosh^{-1}(\alpha)}$$

- Total power dissipation

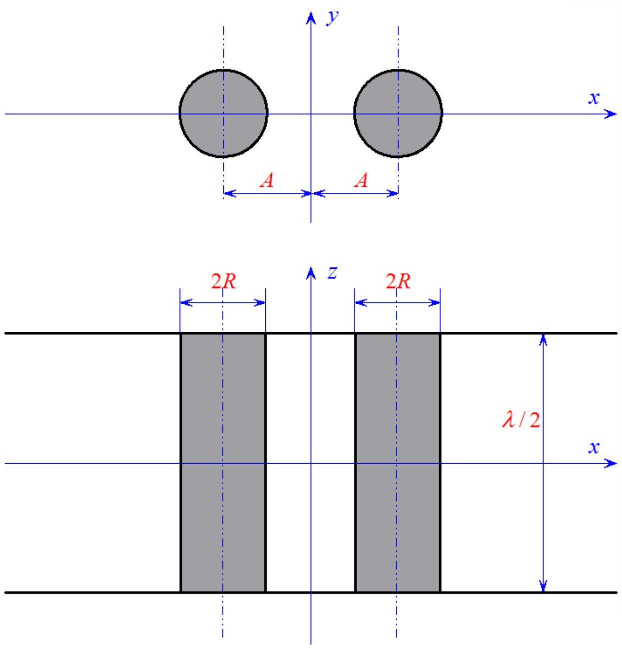
$$P = \frac{R_s V_0^2}{Z_0^2} \frac{1}{\cosh^{-1}(\alpha)} \left[\frac{\lambda}{2R} \frac{1}{\cosh^{-1}(\alpha)} \frac{\alpha}{\sqrt{\alpha^2 - 1}} + 4 \right]$$

Transverse Voltage and Gradient

- Transverse voltage and gradient

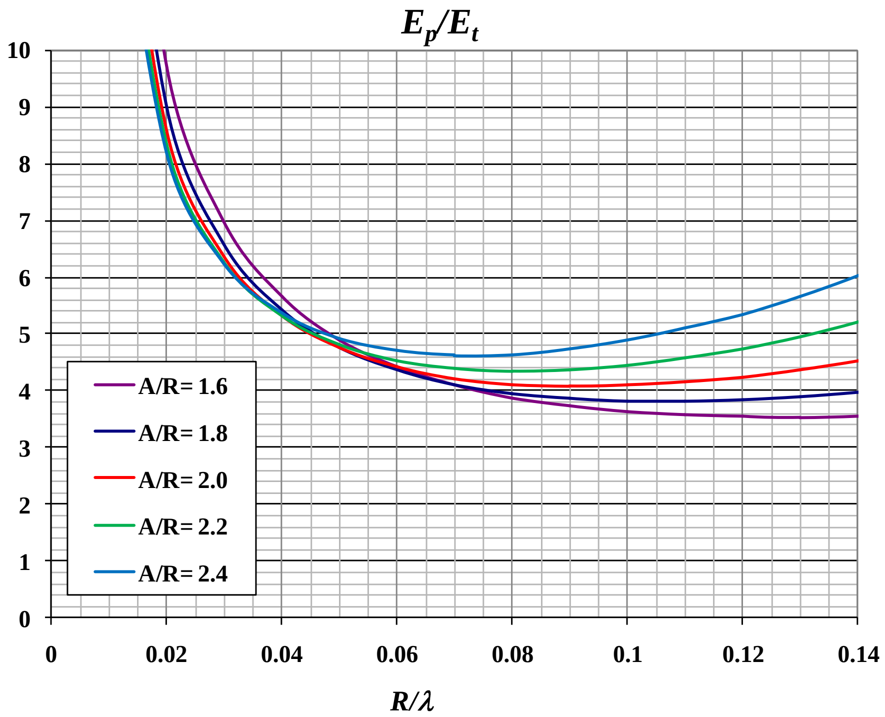
$$\overline{E}_x(y) = \sum_{k=-\infty}^{k=\infty} (-1)^k E_x(0, y + k\lambda/2)$$

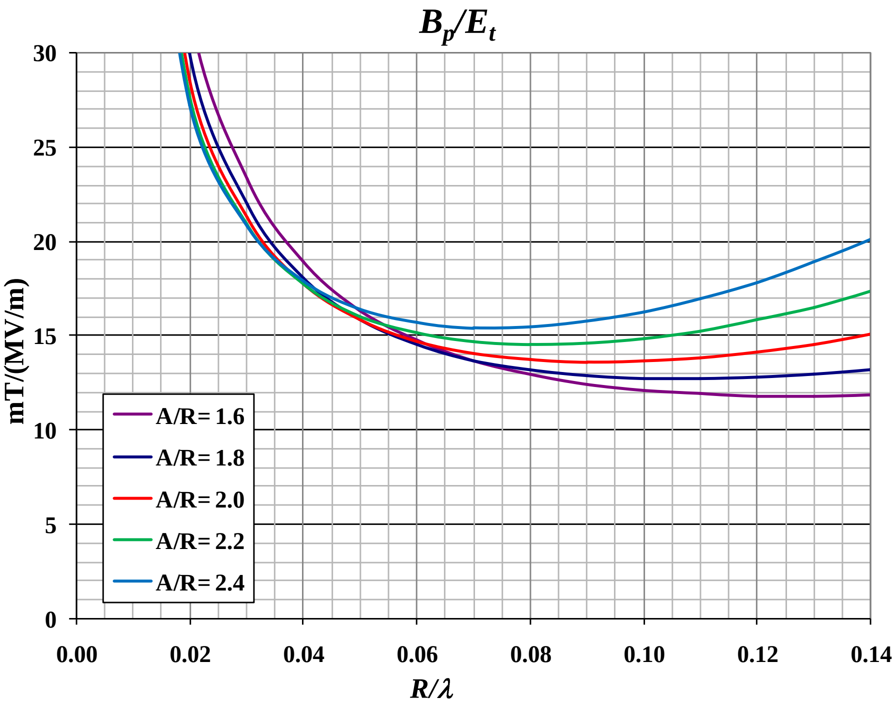
$$E_x(0,y) = \frac{aq}{\pi\epsilon_0}\frac{1}{a^2+y^2}$$



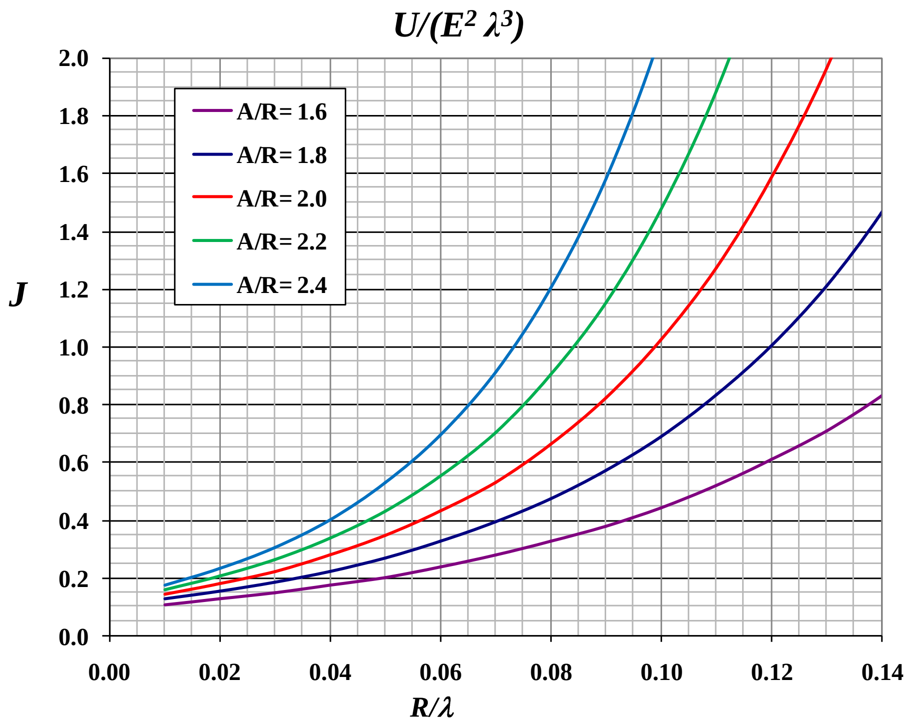
$$\begin{aligned} V_t &= \int_{-\lambda/4}^{\lambda/4} dy\, \overline{E}_x(y)\cos\omega t \\ &= \sum_{-\infty}^{\infty}\int_{-\lambda/4}^{\lambda/4} dy\,(-1)^k E_x\left(0, y+\frac{k\lambda}{2}\right)\cos\left(\frac{2\pi y}{\lambda}\right) \\ &= \int_{-\infty}^{\infty} du\, E_x(0,u)\cos\left(\frac{2\pi u}{\lambda}\right) \\ &= \frac{aq}{\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{du}{a^2+u^2}\cos\left(\frac{2\pi u}{\lambda}\right) = \frac{q}{\epsilon_0}\exp\left[-\frac{2\pi a}{\lambda}\right] \\ &= V_0\frac{2\pi}{\cosh^{-1}(\alpha)}\exp\left[-2\pi\frac{R}{\lambda}\sqrt{\alpha^2-1}\right]. \qquad (A.29) \end{aligned}$$

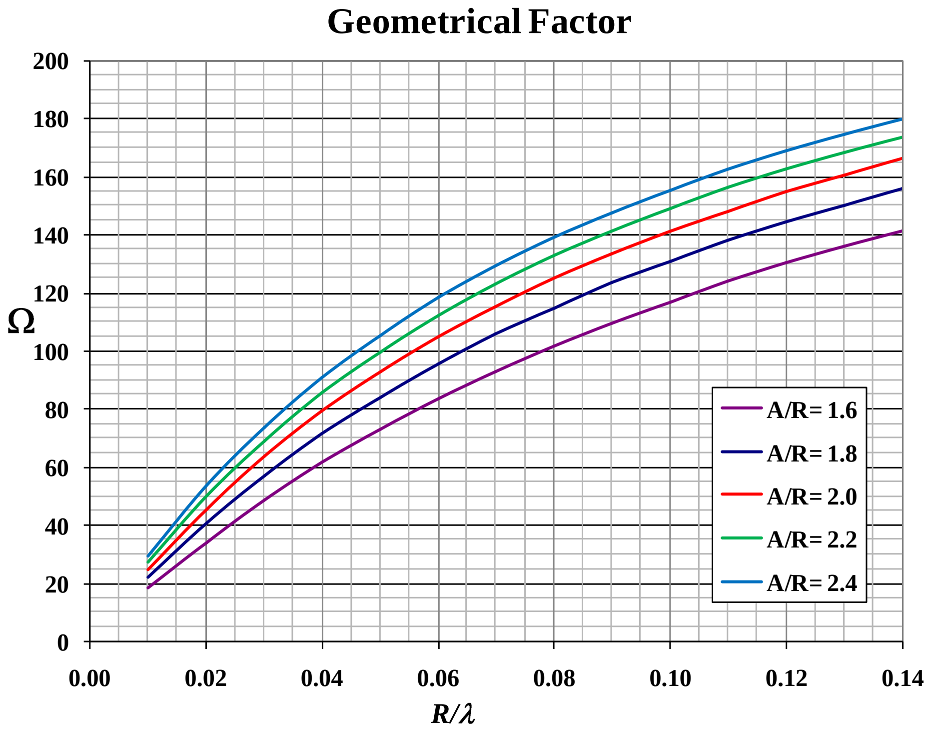
Surface Fields



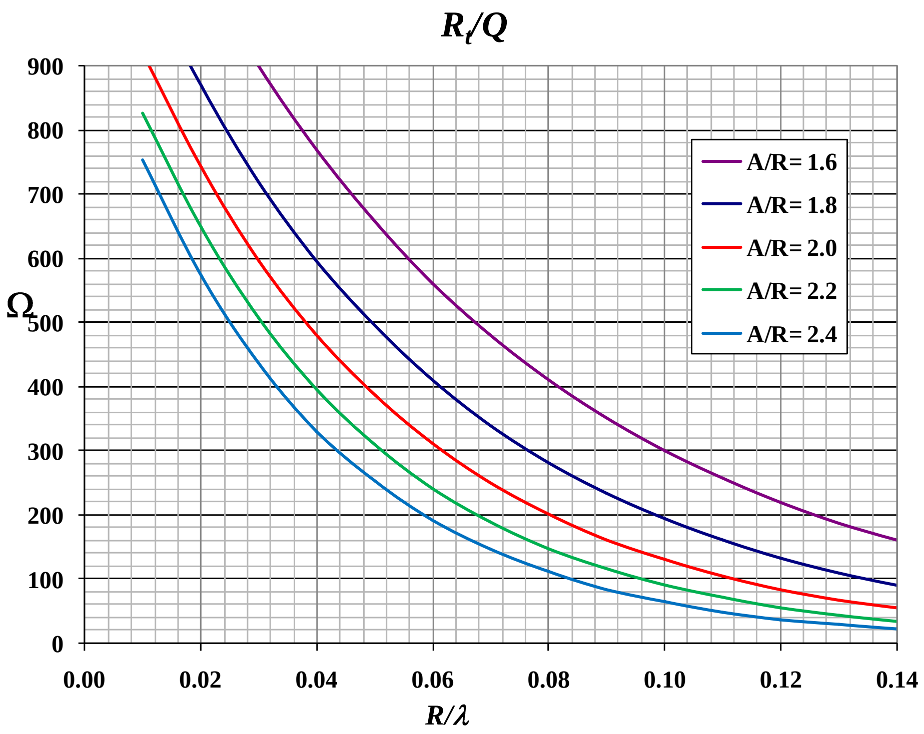


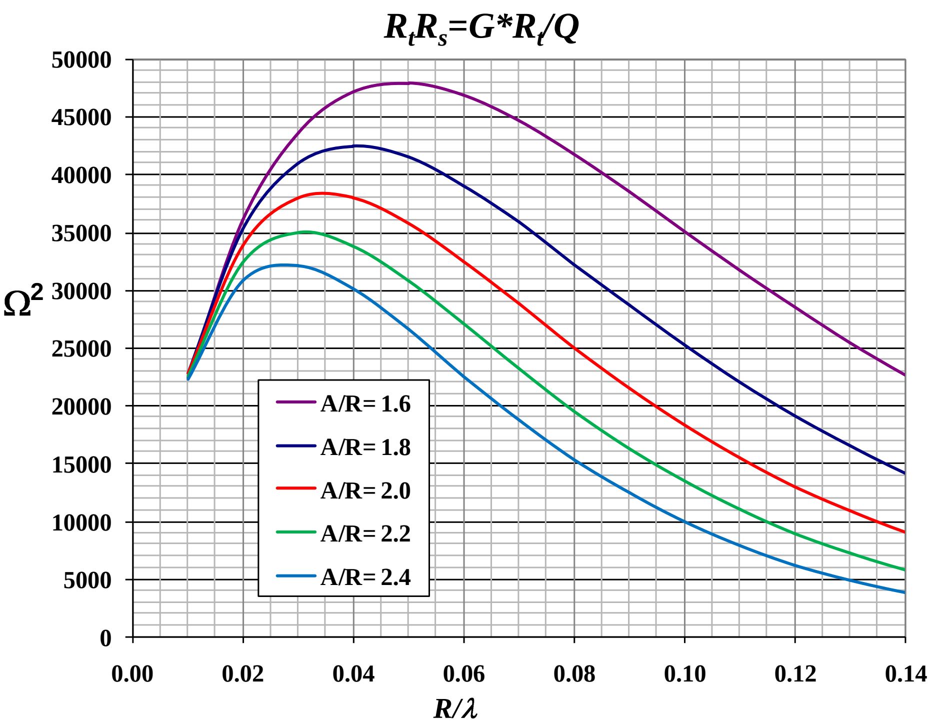
Energy Content and Geometrical Factor



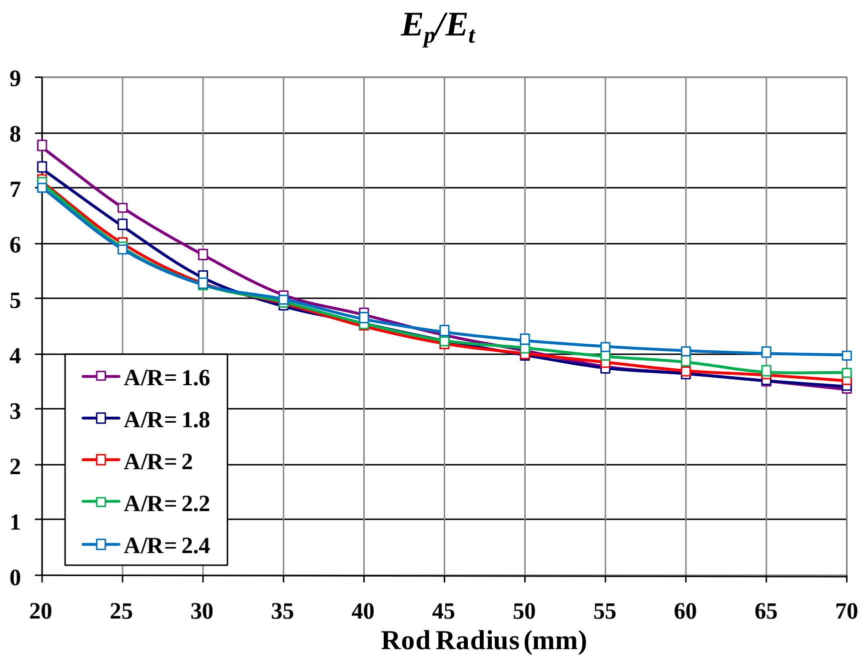


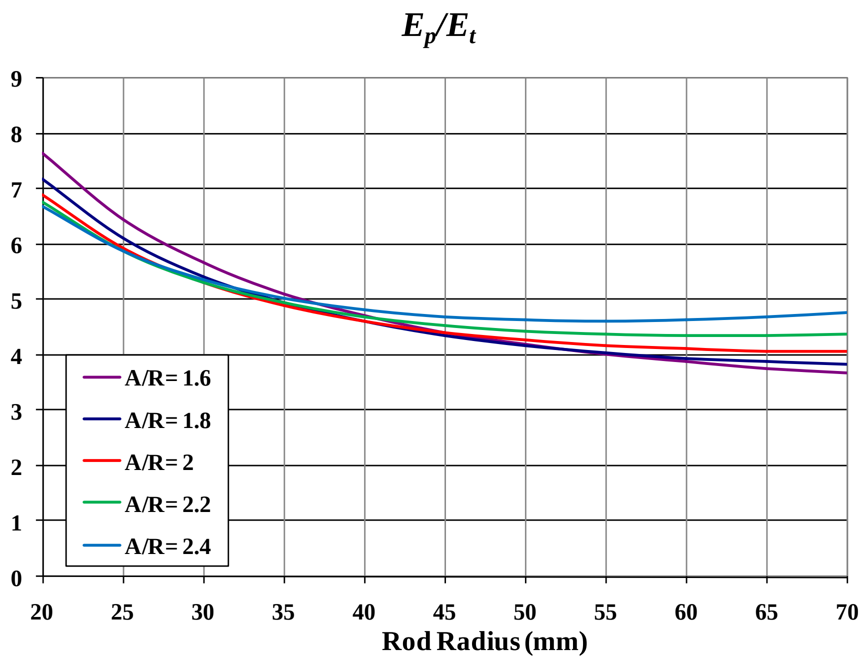
R/Q and Shunt Impedance

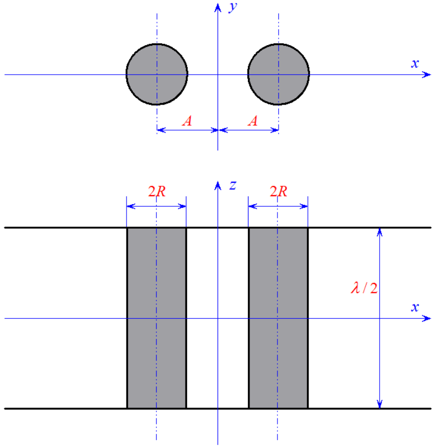




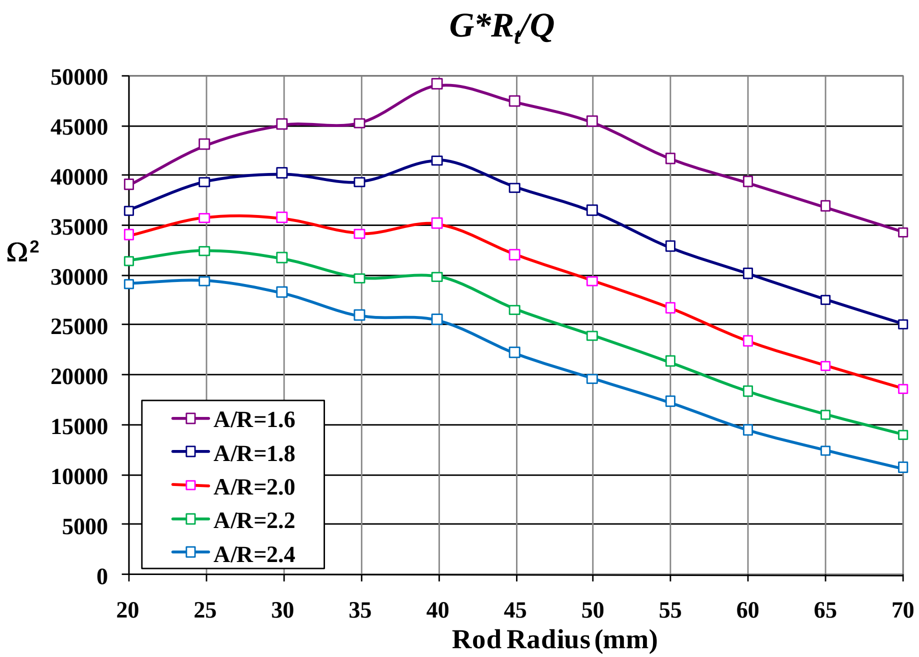
Microwave Studio and Model

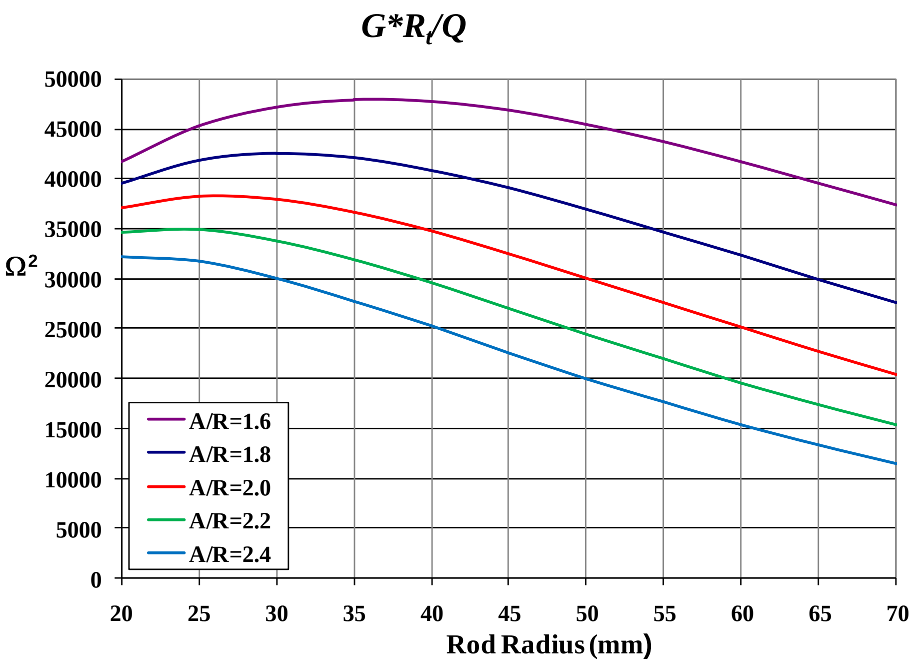


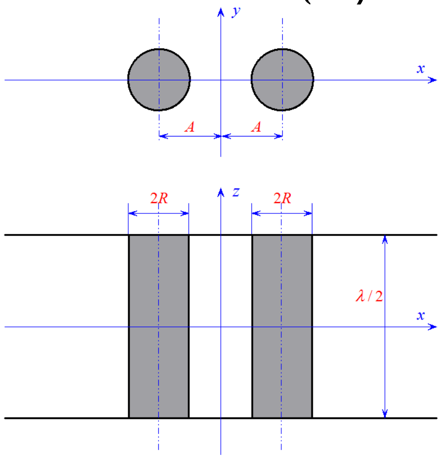




Microwave Studio and Model



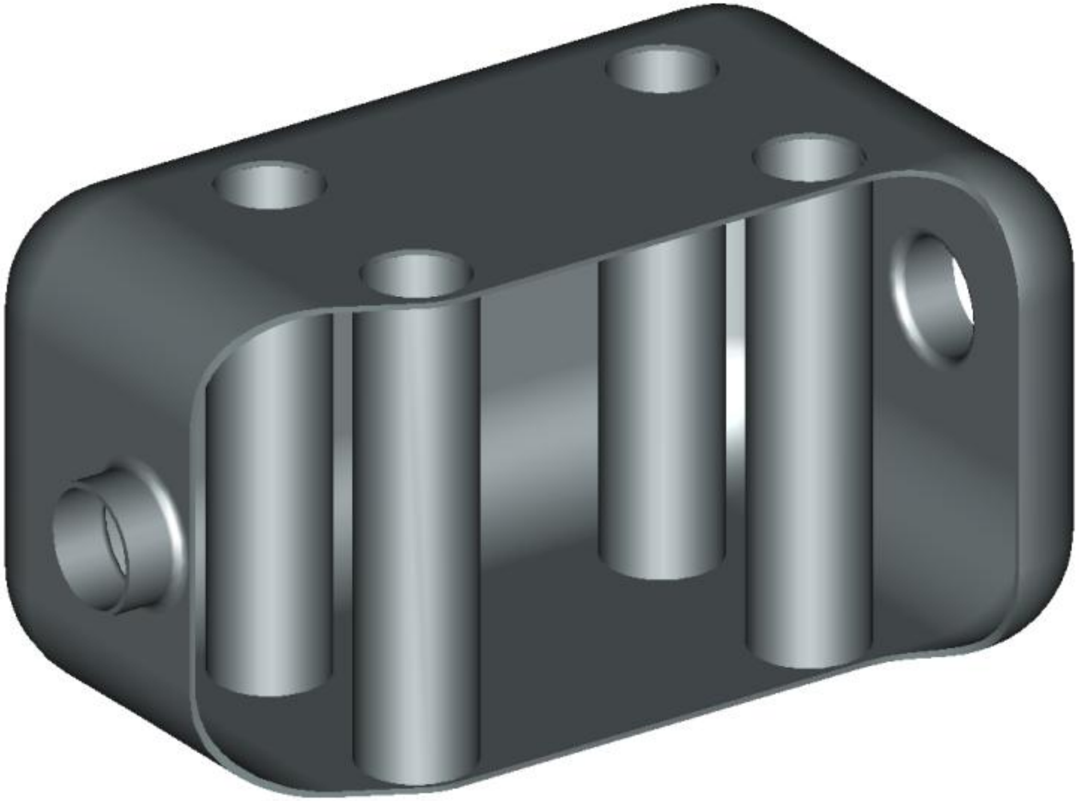




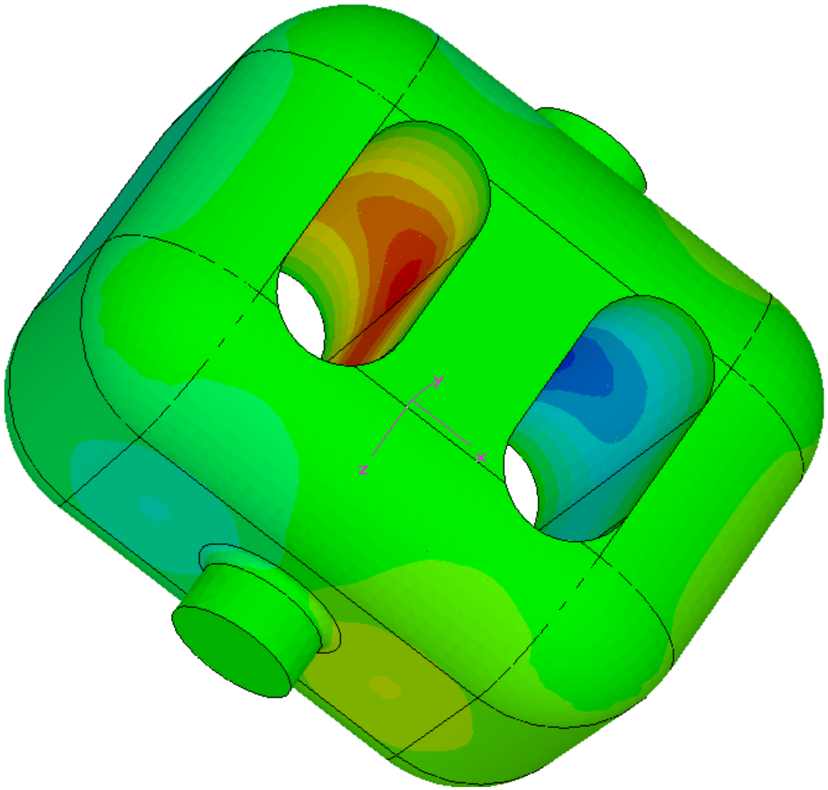
Omega3P and Model

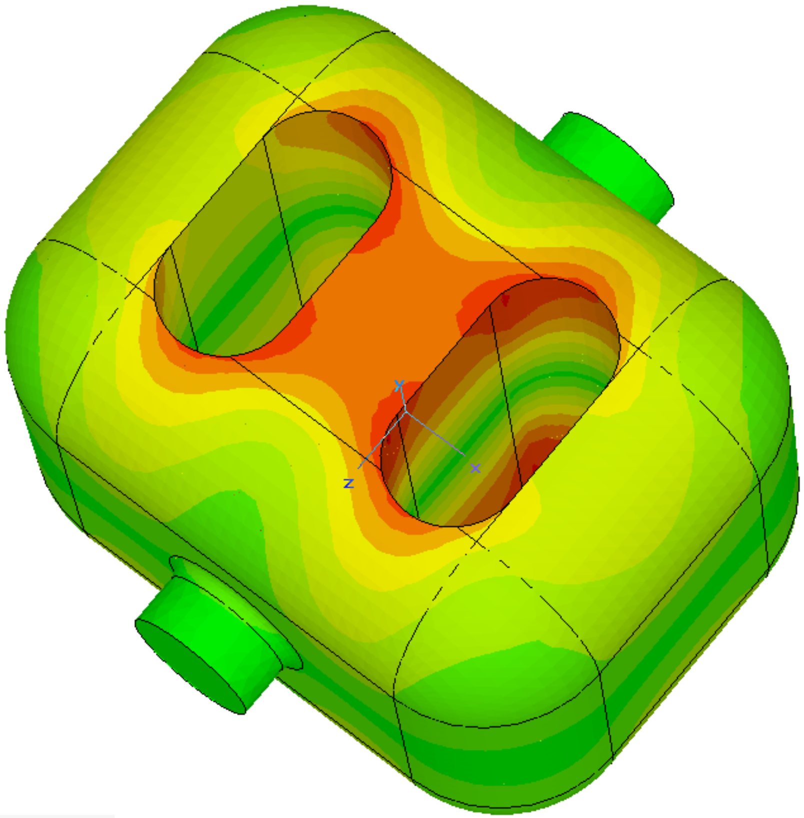
Parameter	Omega3P	Analytical Model	Unit
Frequency of π-mode	400	400	MHz
$\lambda/2$ of π-mode	374.7	374.7	mm
Frequency of 0-mode	414.4	400	MHz
Cavity length	374.7	∞	mm
Cavity width	500	∞	mm
Bars length	381.9	374.7	mm
Bars diameter $(2R)$	100	100	mm
Bars axes separation $(2A)$	200	200	mm
Aperture diameter	100	0	mm
Deflecting voltage $V_t{}^*$	0.375	0.375	MV
Peak electric field $E_p{}^*$	4.09	4.28	MV/m
Peak magnetic field $B_p{}^*$	13.31	14.24	mT
Energy content U^*	0.215	0.209	J
Geometrical factor $G = QR_s$	96.0	112	Ω
R_t/Q	260	268	Ω
R_tR_s	25000	31100	Ω^2
* at E_t=1 MV/m			

Multicell

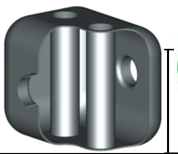
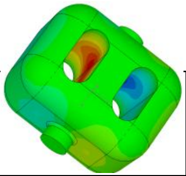
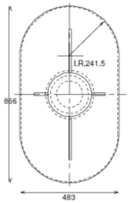


Other Cross-section



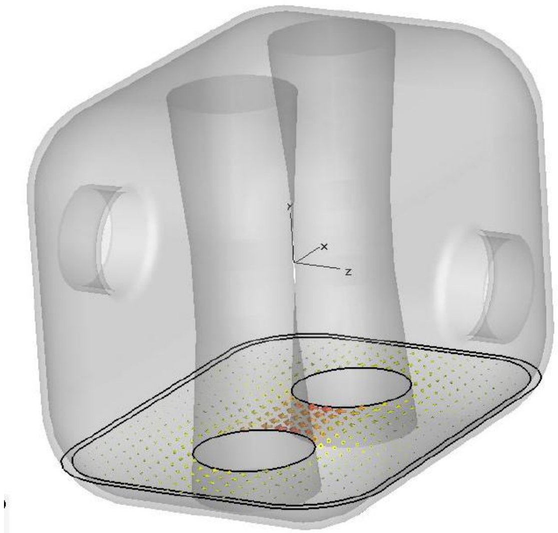


Parallel Bar vs. TM110

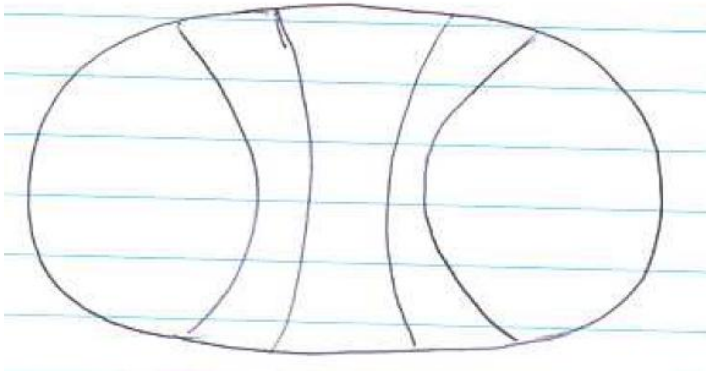
Parameter			Unit	
Frequency of π-mode	400	405.5	MHz	400
$\lambda/2$ of π-mode	374.7	367.8	mm	
Frequency of 0-mode	414.4	427.9	MHz	
Cavity length	374.7	374.7	mm	
Cavity width	500	500	mm	615
Bars length	381.9	382.2	mm	1101
Bars thickness $(2R)$	100	100	mm	
Bars width	100	200	mm	
Bars axes separation $(2A)$	200	200	mm	
Aperture diameter	100	100	mm	305
Deflecting voltage $V_t{}^*$	0.375	0.375	MV	0.375
Peak electric field $E_p{}^*$	4.09	2.90	MV/m	4.25
Peak magnetic field $B_p{}^*$	13.31	8.84	mT	12.24
Energy content U^*	0.215	0.163	J	
Geometrical factor $G = QR_s$	96.0	92.6	Ω	220
R_t/Q	260	336	Ω	46.7
R_tR_s	25000	31100	Ω^2	10274
* at E_t=1 MV/m				

Other Possible Improvements

- Variable cross-section



- Curved resonant lines



Some Numbers (Superconducting)

$B_p / E_t \simeq 8.8$ mT/(MV/m)

Should be able to achieve 80 mT $\Rightarrow E_t = 9.1$ MV/m

$E_p = (9.1)(2.9) = 26.4$MV/m

At 500 MV/m, $\lambda / 2 = 30$ cm $\Rightarrow$ 2.73 MV

At 11 GeV, $\delta\alpha = \dfrac{2.73}{11000} \simeq 248\ \mu$rad

$$P_c = \frac{V^2}{G\ \ R/Q} R_s = \frac{(2.73)^2 10^{12}}{(92.6)(336)} 15 \cdot 10^{-9} \simeq 3.6 \text{ W}$$

$$P_{rf} = VI = (2.73 \cdot 10^6)(400 \cdot 10^{-6}) \simeq 1.1 \text{ kW}$$

CEBAF deflecting cavity produces 42 μrad at 1 kW

Next

- Optimize bar geometry
- Multicell
- Higher order modes
- Coupling
  - Fundamental
  - HOM