A Vlasov-Maxwell Solver to Study Microbunching Instability in the FERMI@ELETTRA First Bunch Compressor System

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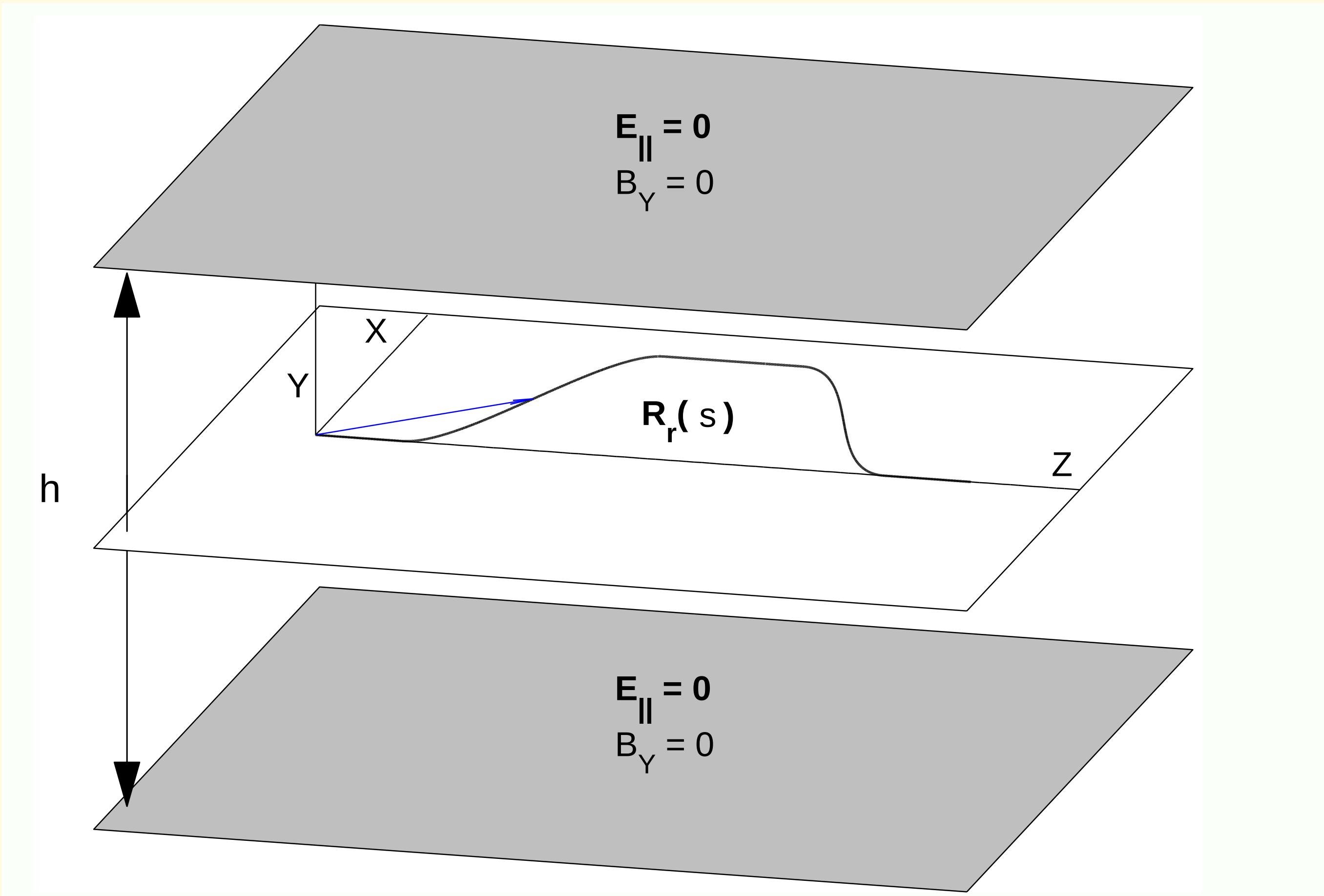
1. Self Consistent Vlasov-Maxwell Treatment
2. Field Calculation
3. Self Consistent Monte Carlo Method
4. Microbunching Instability Studies

* Thanks to Office of Naval Research Global for support

Self Consistent Vlasov-Maxwell Treatment I



Self Consistent Vlasov-Maxwell Treatment II

Wave equation in lab frame with "2D" planar source:

$$(\partial_Z^2+\partial_X^2+\partial_Y^2-\partial_u^2)\mathcal{E} = H(Y)\mathcal{S}(\mathbf{R},u), \quad \mathcal{E}(\mathbf{R},Y=\pm g,u)=0.$$

where $u=ct$, $\mathcal{E}(\mathbf{R},Y,u)=(E_Z,E_X,B)$, $\mathbf{R}=(Z,X)$.

Vlasov equation in beam frame:

$$f_s-\kappa(s)xf_z+F_zf_{p_z}+p_xf_x+[\kappa(s)p_z+F_x]f_{p_x}=0$$

where

$$\begin{aligned} F_z &= \frac{e}{\bar{v}\bar{E}}\mathbf{V}\cdot\mathbf{E}, \\ F_x &= \frac{e}{\bar{E}\bar{\beta}^2}[-\bar{X}'(s)E_Z+\bar{Z}'(s)E_X+\bar{v}B)], \end{aligned}$$

and $\mathbf{V}=\bar{v}(\mathbf{t}(s)+p_x\mathbf{n}(s))$, $\mathbf{E}=(E_Z,E_X)$ and B are evaluated at $\mathbf{R}=\bar{\mathbf{R}}(s)+x\mathbf{n}(s)$ and $u=(s-z)/\bar{\beta}$.

Field Calculation (Lab Frame)

$$\mathcal{E}(\mathbf{R},u) := \langle \mathcal{E}(\mathbf{R},\cdot,u)\rangle = \int_{-g}^{g} H(Y)\mathcal{E}(\mathbf{R},Y,u)dY.$$

averaged field computed much more quickly

$$\mathcal{E}(\mathbf{R},u) = -\frac{1}{2\pi}\sum_{k=0}^{\infty}(-1)^k(1-\frac{\delta_{k0}}{2})\int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta\, \mathcal{S}(\hat{\mathbf{R}},v,k)$$

where $\hat{\mathbf{R}} = \mathbf{R} + \sqrt{(u-v)^2 - (kh)^2}(\cos\theta, \sin\theta)$.

Issues

- localization in θ (angular size of the beam) for $v \ll u - kh$ and in v
- delicate calculation (must be done cum grano salis)

θ integration: superconvergent trapezoidal rule

v integration: adaptive Gauss-Kronrod rule

Beam to Lab Charge/Current Density Transformation

- To solve Maxwell equations in lab frame must express lab frame charge/current density in terms of beam frame phase space density
- To a good approximation lab frame charge/current densities are

$$\rho_L(\mathbf{R}, Y, u) = H(Y)\rho(\mathbf{r}, \beta u) \ ,$$
$$\mathbf{J}_L(\mathbf{R}, Y, u) = \beta c H(Y)[\rho(\mathbf{r}, \beta u)\mathbf{t}(\beta u + z) + \tau(\mathbf{r}, \beta u)\mathbf{n}(\beta u + z)],$$

$$\rho(\mathbf{r}, s) = Q\int dp_z dp_x f(\zeta, s), \quad \tau(\mathbf{r}, s) = Q\int dp_z dp_x p_x f(\zeta, s),$$

where $\zeta = (z, p_z, x, p_x)$

Remark: subtlety in the change of independent variable u=ct $\longrightarrow$ s
Derivation to be published in a forthcoming paper

Self Consistent Monte Carlo Method

Outline and comparison with PIC for Vlasov-Poisson (VP) system fron s to $s+\Delta s$

- From scattered beam frame points at s $\longrightarrow$ smooth/global Lab frame charge/current density via a 2D Fourier method (Charge deposition (+ filtering) in VP PIC).
  1D Example:

  1D orthogonal series estimator of $f(x)$, $x \in [0,1]$

  $$f_J(x) := \sum_{j=0}^{J} \theta_j \phi_j(x), \quad \theta_j = \int_0^1 \phi_j(x) f(x) dx, \quad \phi_0(x) = 1, \phi_j(x) = \sqrt{2}\cos(\pi j x), j = 1, 2, ...$$

  According to the fact that $f(x)$ is a probability density

  $$\theta_j = E\{I_{\{X\in[0,1]\}}\phi_j(X)\}, \quad \text{therefore a natural estimate is} \quad \hat{\theta}_j := \frac{1}{N}\sum_{n=1}^{N} I_{\{X_n\in[0,1]\}}\phi_j(X_n)$$

- Calculate fields at s from history of Lab Frame charge/current density using our field formula (Solve Poisson Equation in VP PIC)

- Use fields at s to move the phase space points to $s+\Delta s$ (Same in VP PIC)

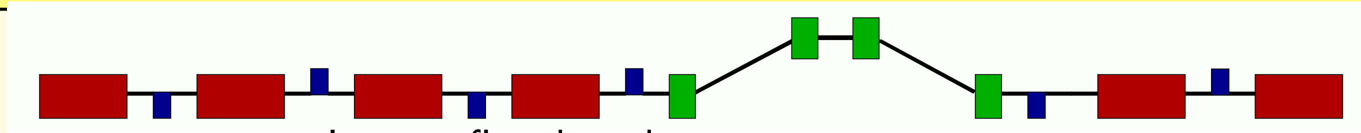
Microbuching in FERMI@ELETTRA First Bunch Compressor

Microbunching can cause an instability which degrades beam quality

This is a major concern for free electron lasers where very bright electron beams are required

FERMI@ELETTRA first bunch compressor system proposed as a benchmark for testing codes at the Workshop on the Microbunching Instability I in Trieste.

FERMI@ELETTRA First Bunch Compressor Parameters

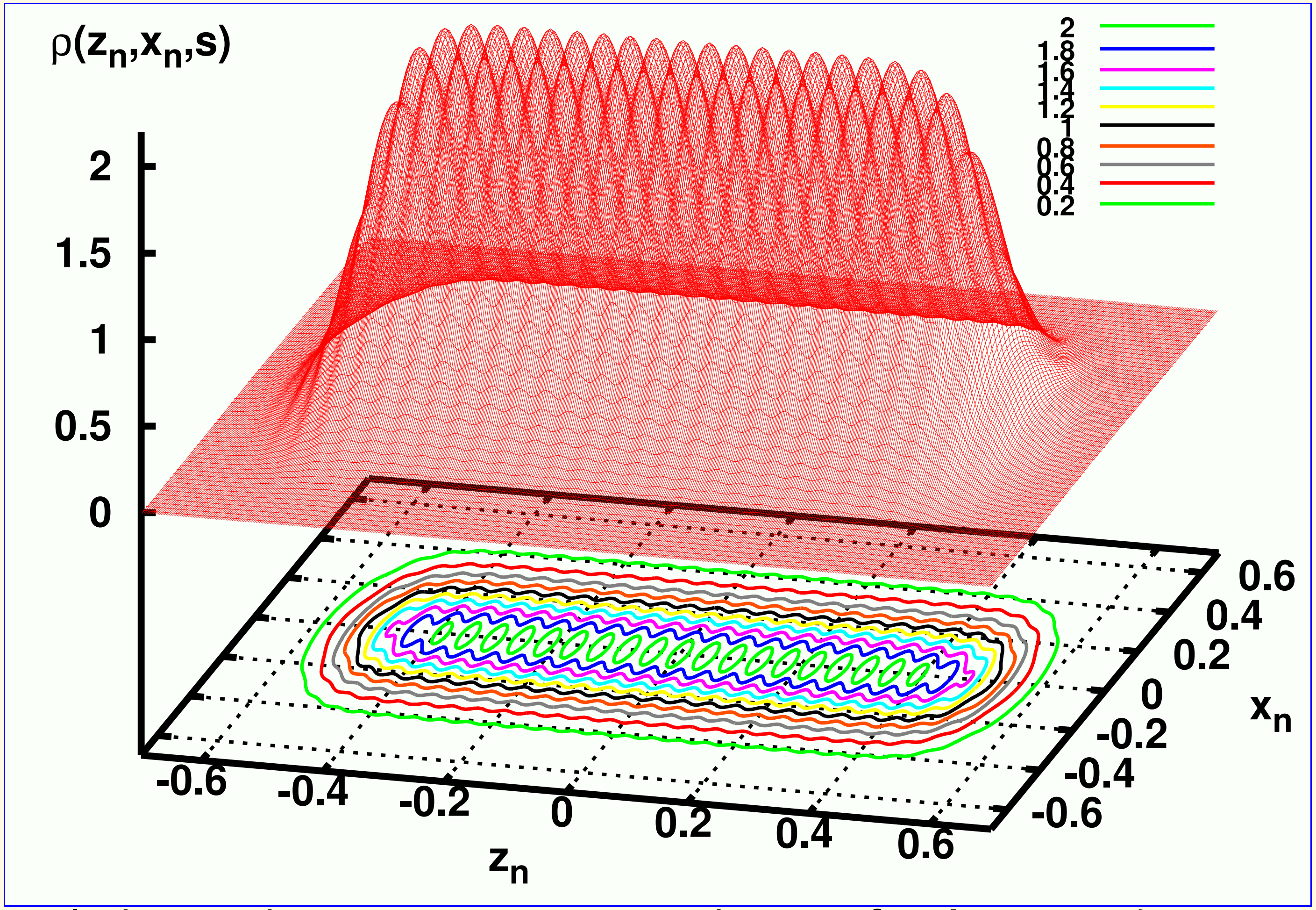


Layout first bunch compressor system

Table 1: Chicane parameters and beam parameters at first dipole

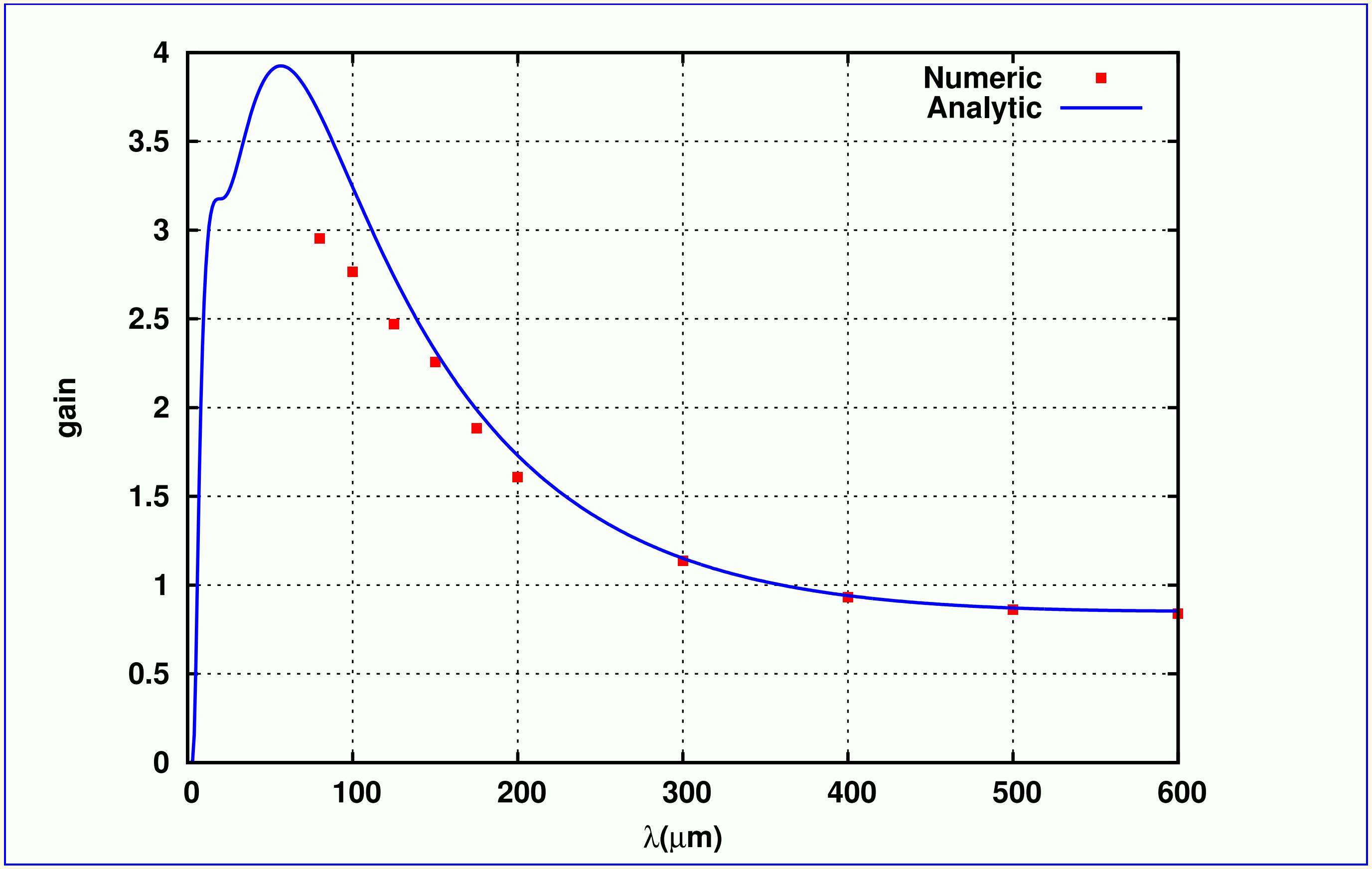
Parameter	Symbol	Value	Unit
Energy reference particle	E_r	233	MeV
Peak current	I	120	A
Bunch charge	Q	1	nC
Norm. transverse emittance	$\gamma\epsilon_0$	1	μm
Alpha function	α_0	0	
Beta function	β_0	10	m
Linear energy chirp	u	-27.5	1/m
Uncorrelated energy spread	σ_E	2	KeV
Momentum compaction	R_{56}	0.0025	m
Radius of curvature	ρ_0	5	m
Magnetic length	L_b	0.5	m
Distance 1st-2nd, 3rd-4th bend	L_1	2.5	m
Distance 2rd-3nd bend	L_2	1	m

FERMI@ELETTRA First Bunch Compressor I



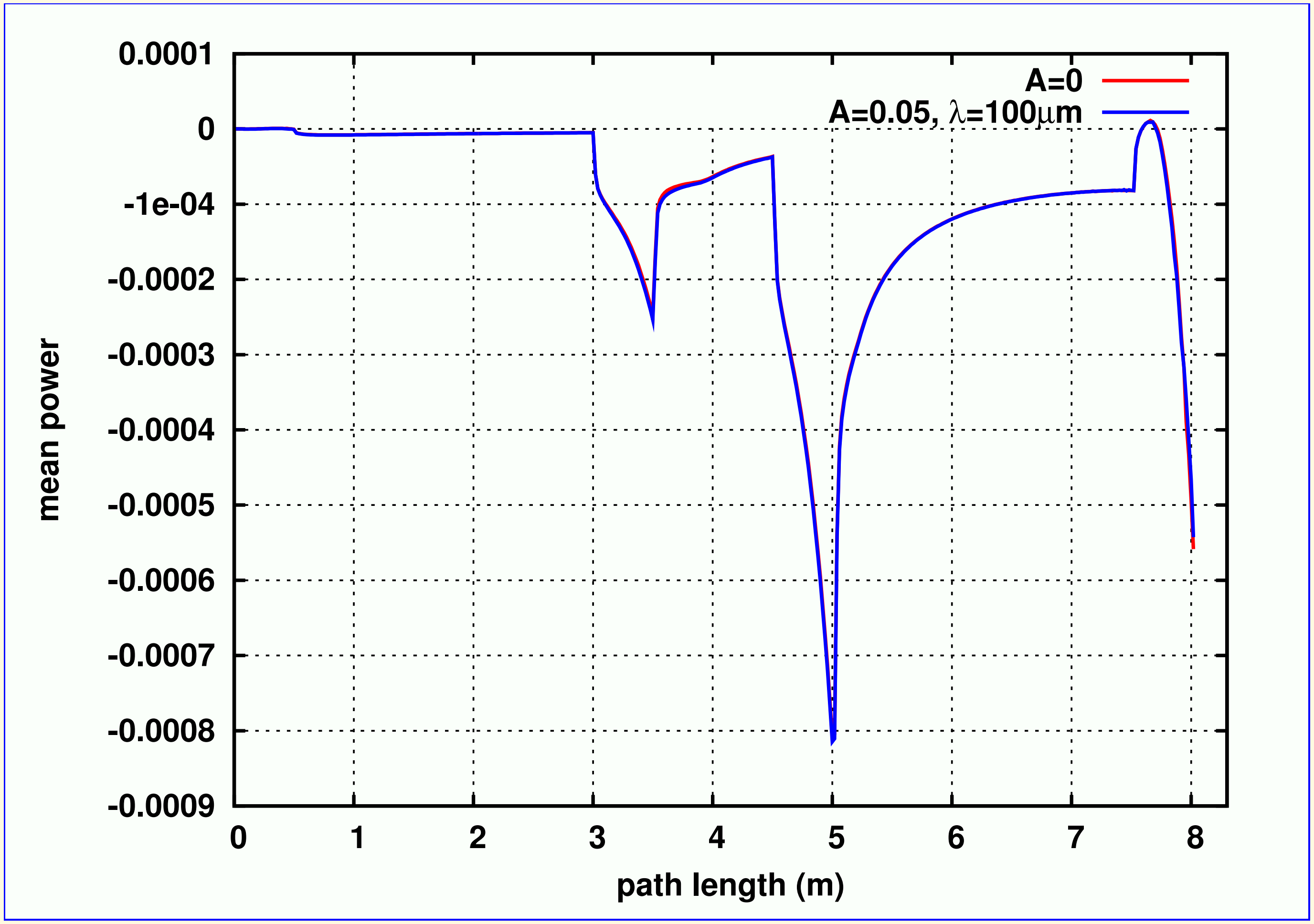
Initial charge density in norm. coordinates for A=0.05, $\lambda = 100\mu$m.
Init. phase space density $= (1 + A\cos(2\pi z/\lambda))\mu(z)\rho_c(z,p_z)g(x,p_x)$.

Gain factor



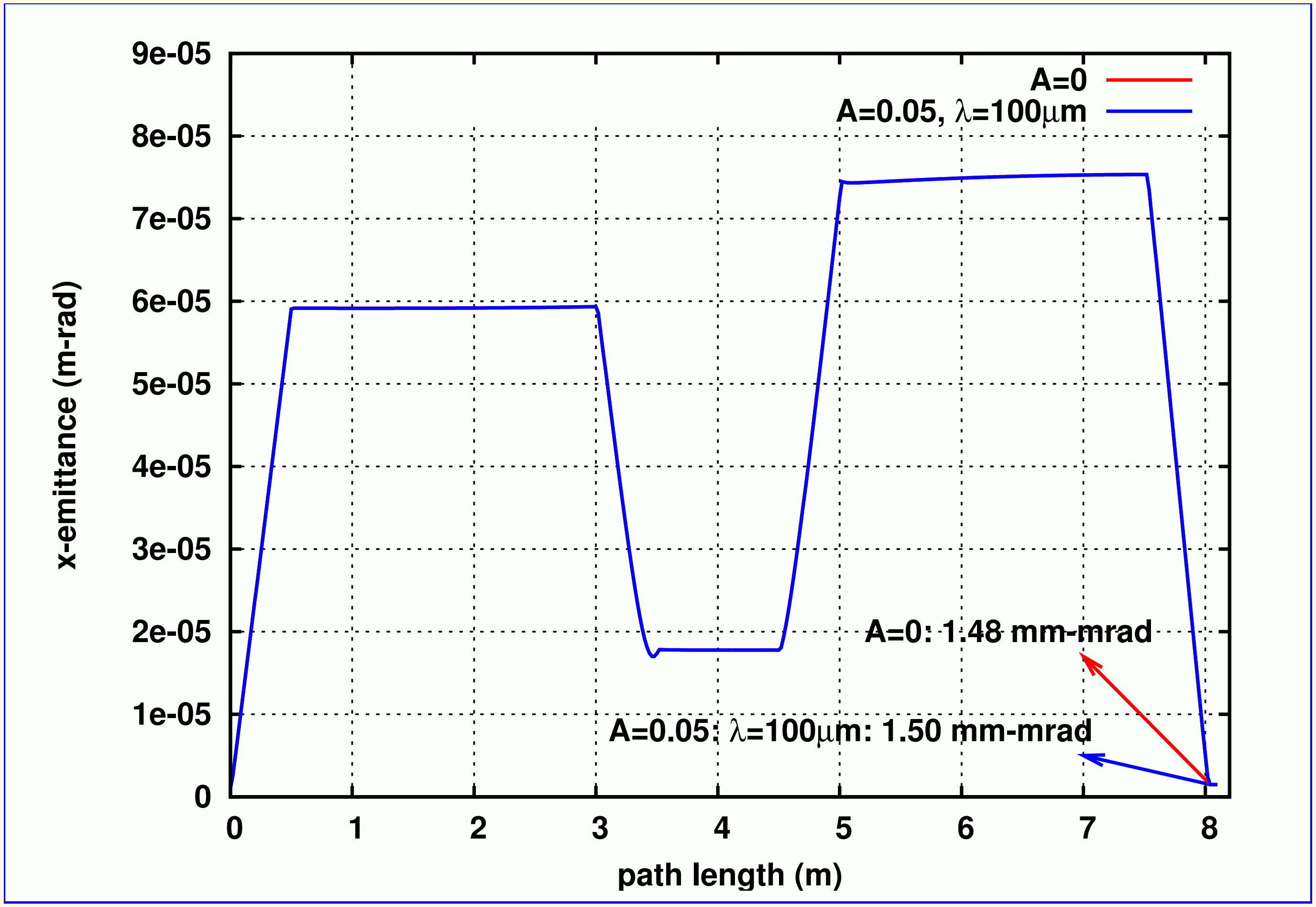
Gain factor $:= \mid b(k_f, s_f)/b(k_0, 0) \mid$, where $b(k,s) = \int dz \exp(-ikz)F(z,s)$ and $k_f = k_0/(1 + uR_{56}(s_f))$ for a given initial wavelength $\lambda = 2\pi/k_0$. Here the compressor factor $C = 1/(1 + uR_{56}(s_f)) = 3.54$, $s_f = 8.029$m.

FERMI@ELETTRA First Bunch Compressor II



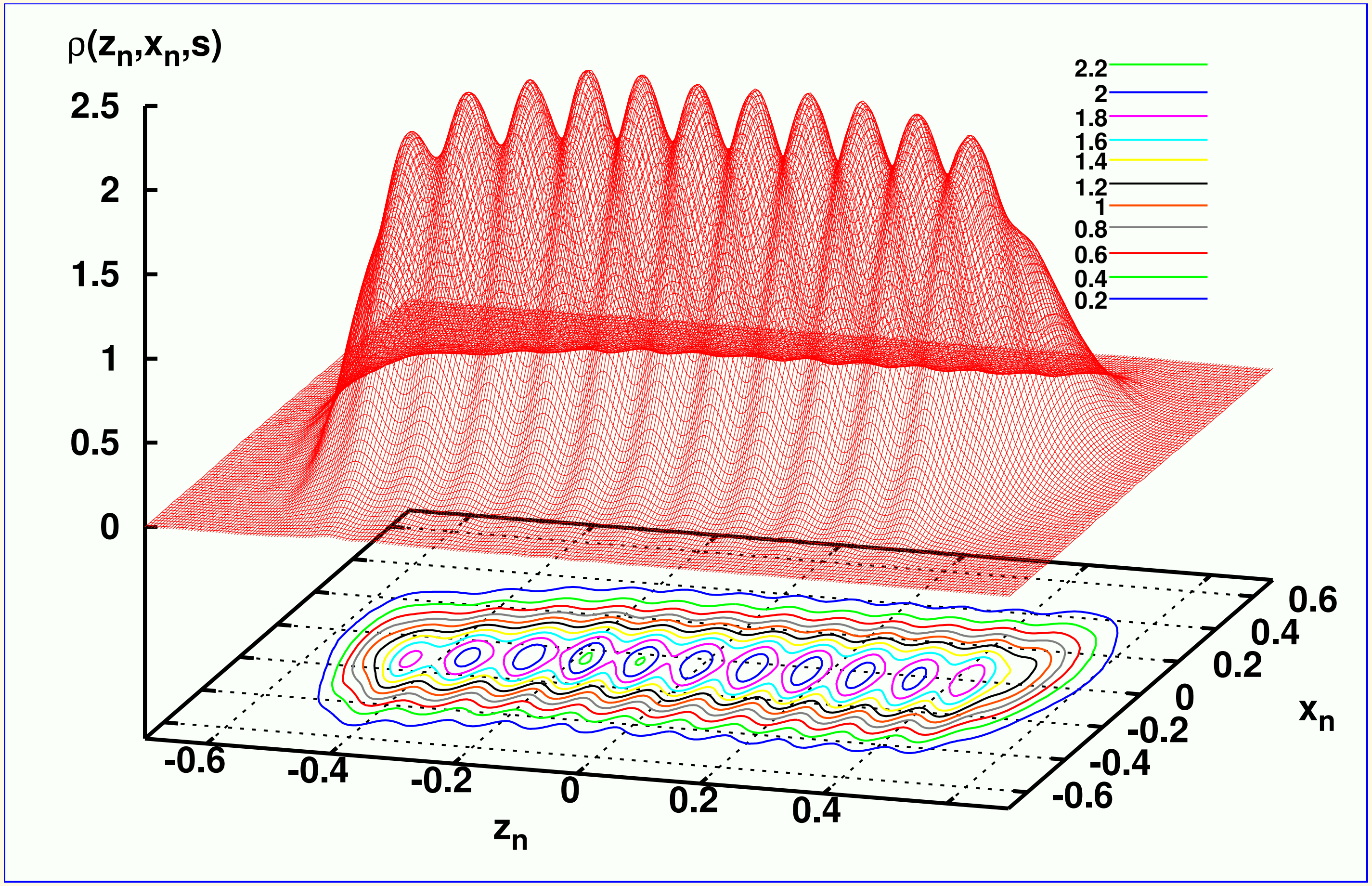
Mean power

FERMI@ELETTRA First Bunch Compressor III



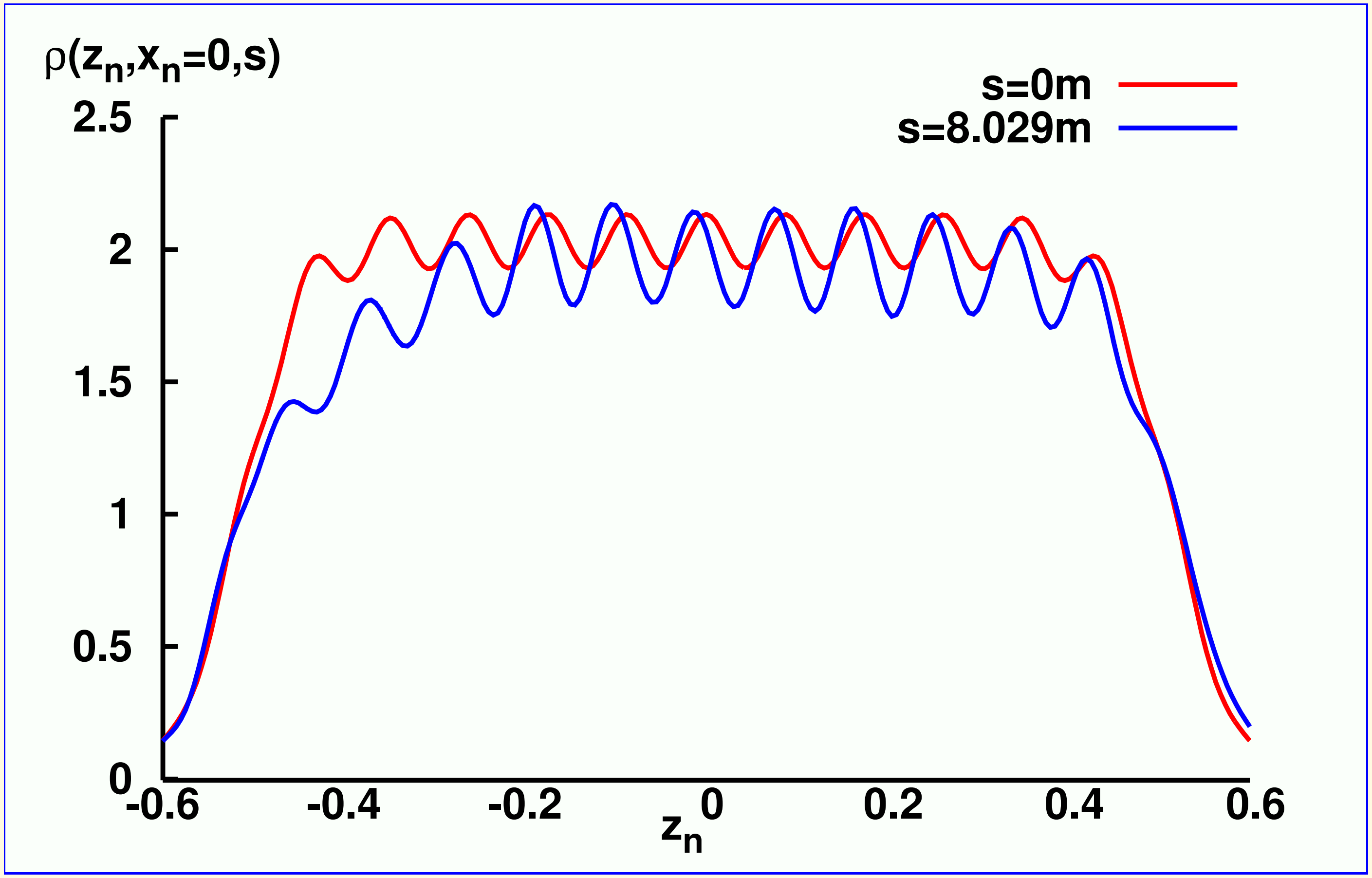
x-emittance

FERMI@ELETTRA First Bunch Compressor IV



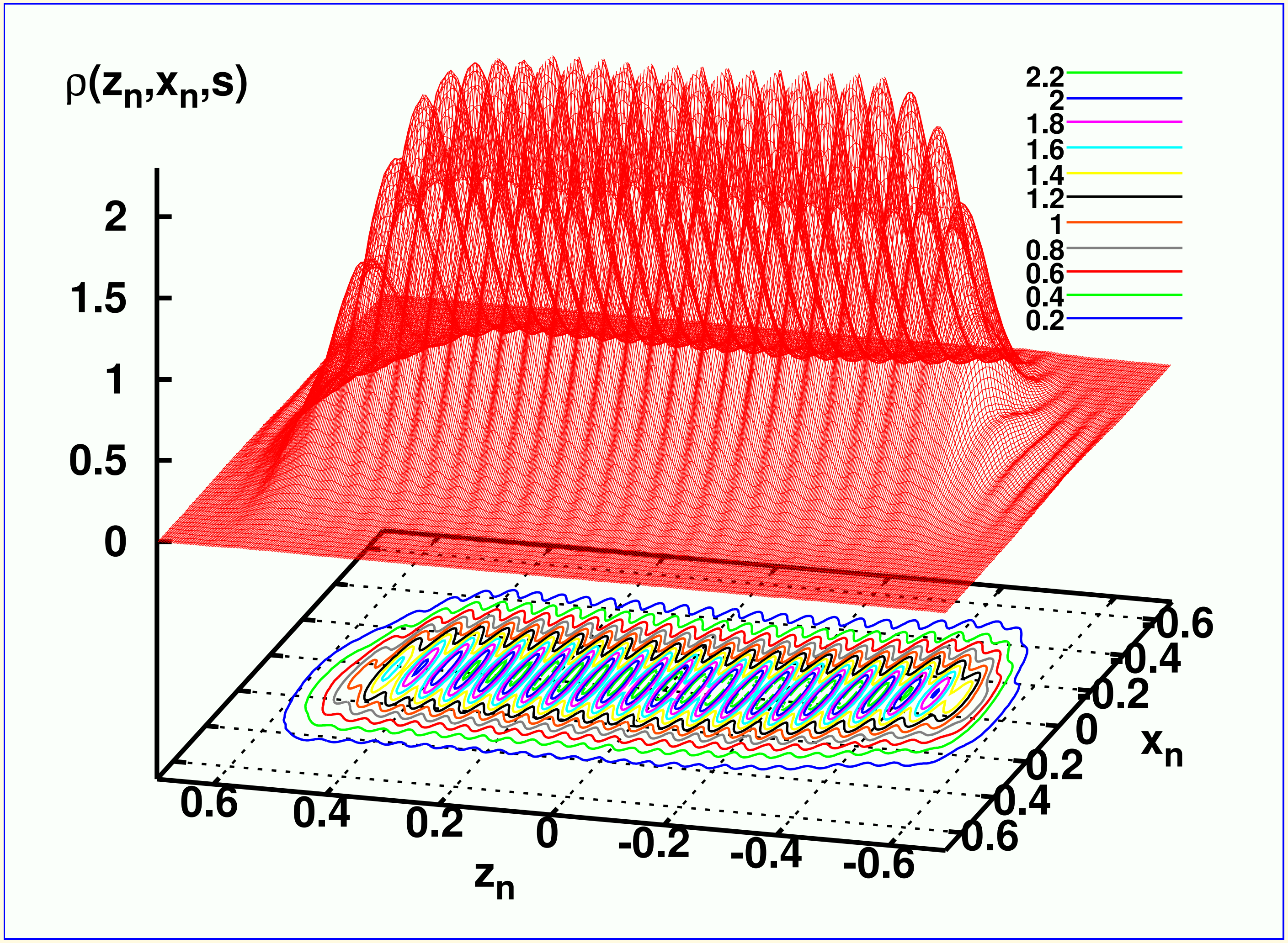
Charge density in normalized coordinates at $s = 8.029$m for $\lambda = 200\mu$m.

FERMI@ELETTRA First Bunch Compressor V



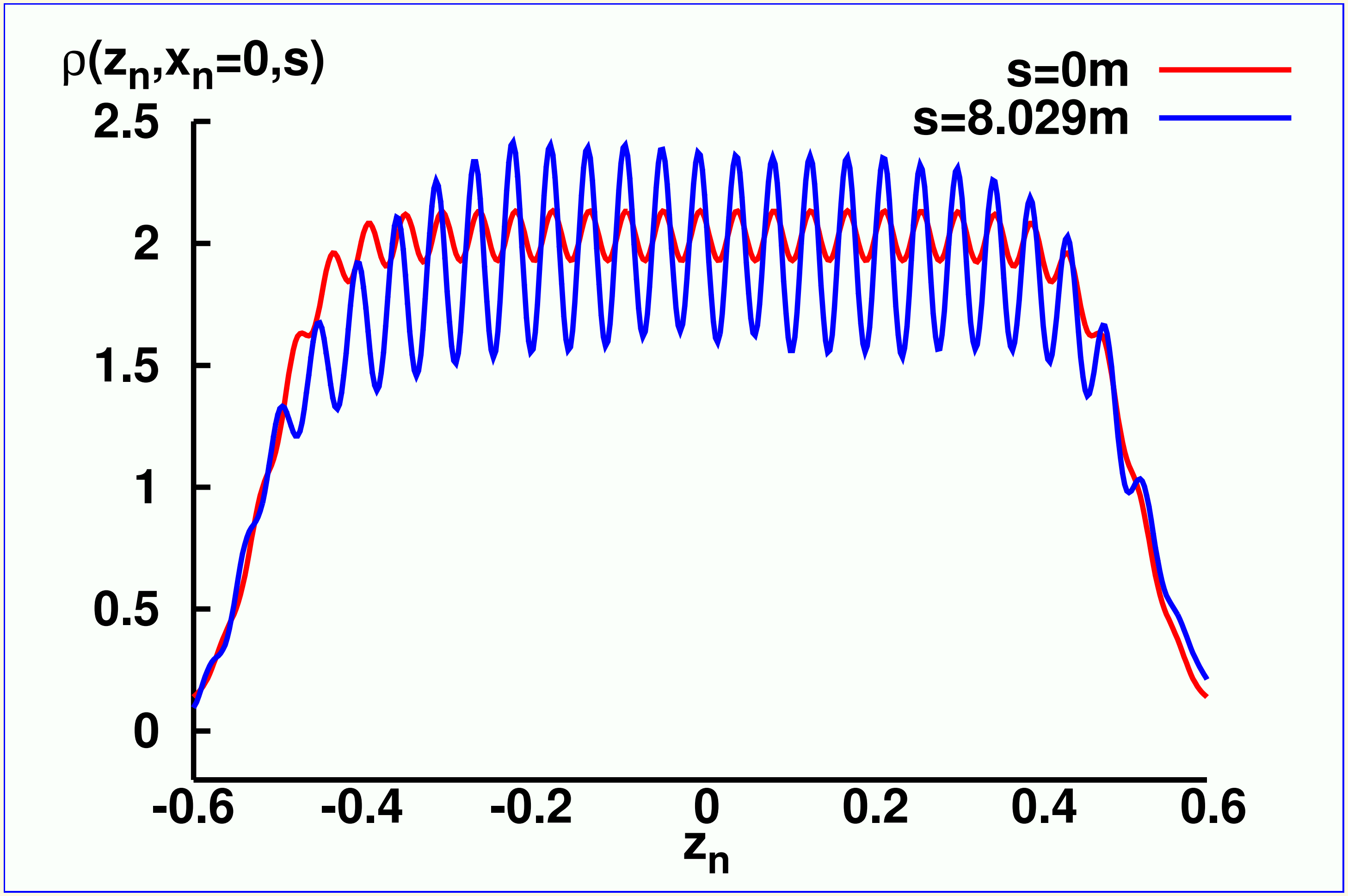
Section of charge density in norm. coord. at $s = 8.029$m for $\lambda = 200\mu$m

FERMI@ELETTRA First Bunch Compressor VI



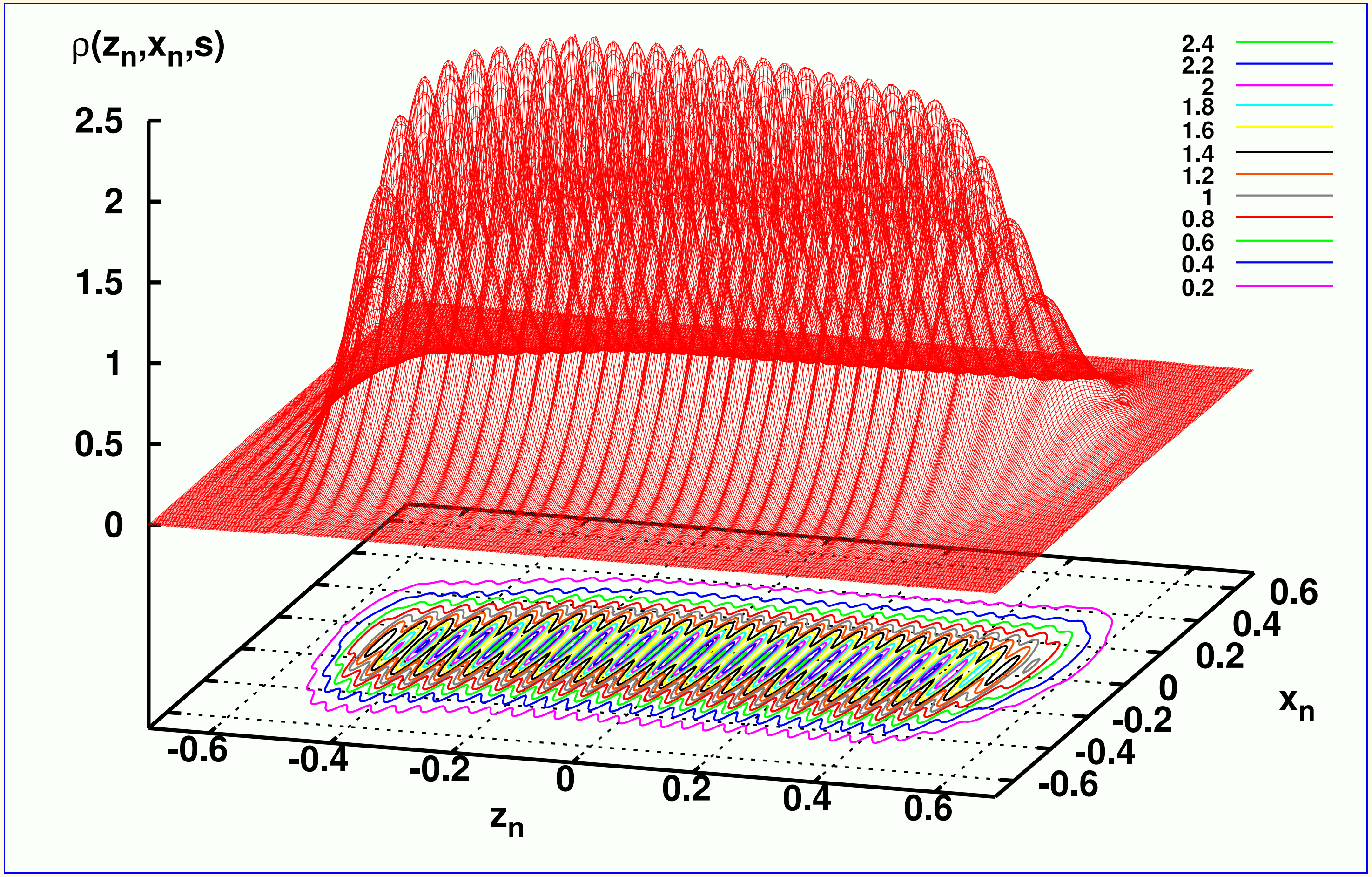
Charge density in norm. coordinates at $s = 8.029\text{m}$ for $\lambda = 100\mu\text{m}$

FERMI@ELETTRA First Bunch Compressor VII



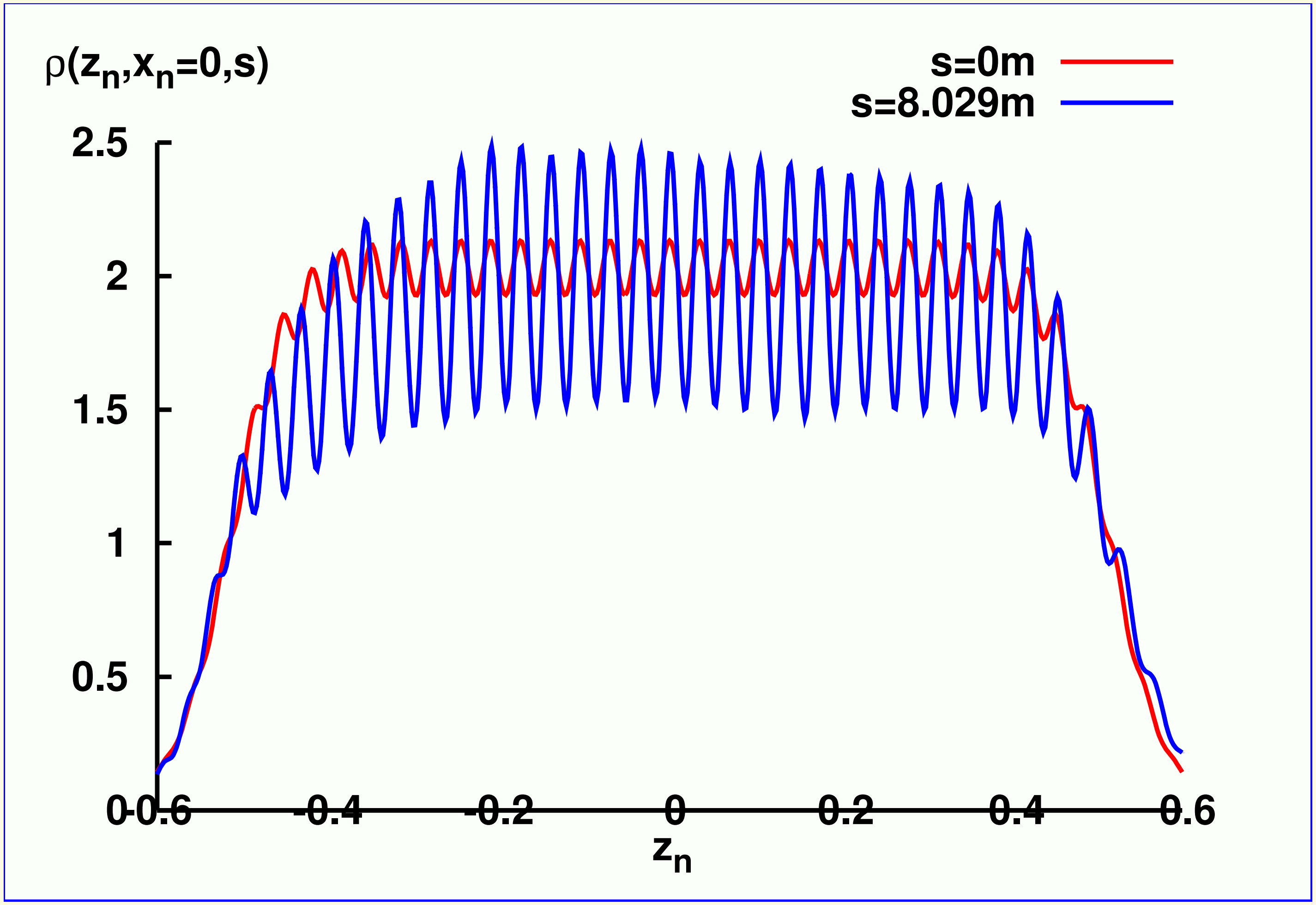
Comparison of $\rho(z_n, 0, s)$ at $s = 8.029$m and $s = 0$m for $\lambda = 100\mu$m

FERMI@ELETTRA First Bunch Compressor VIII



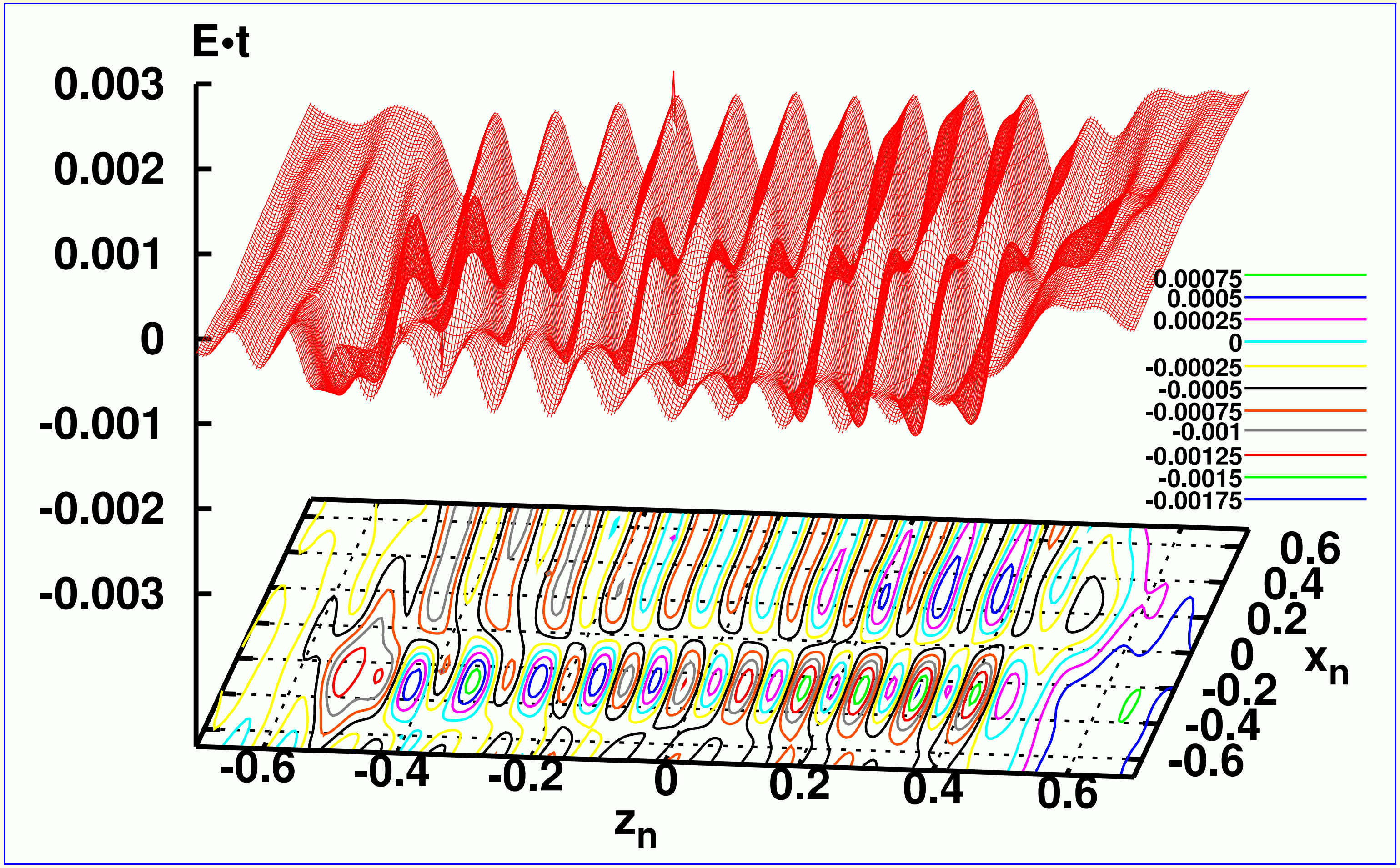
Charge density in norm. coordinates at $s = 8.029$m for $\lambda = 80\mu$m

FERMI@ELETTRA First Bunch Compressor IX



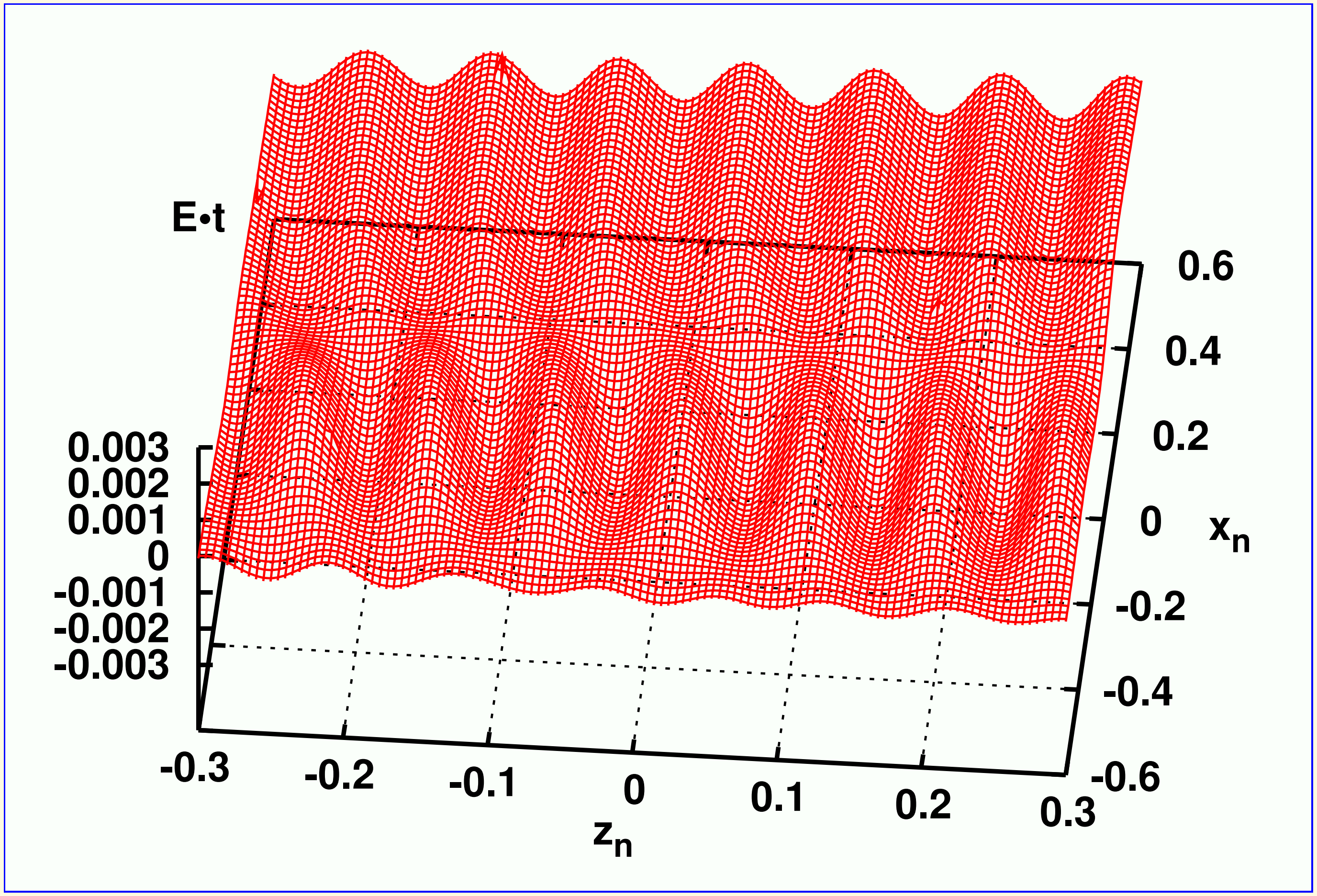
Comparison of $\rho(z_n, 0, s)$ at $s = 8.029$m and $s = 0$m for $\lambda = 80\mu$m

FERMI@ELETTRA First Bunch Compressor X



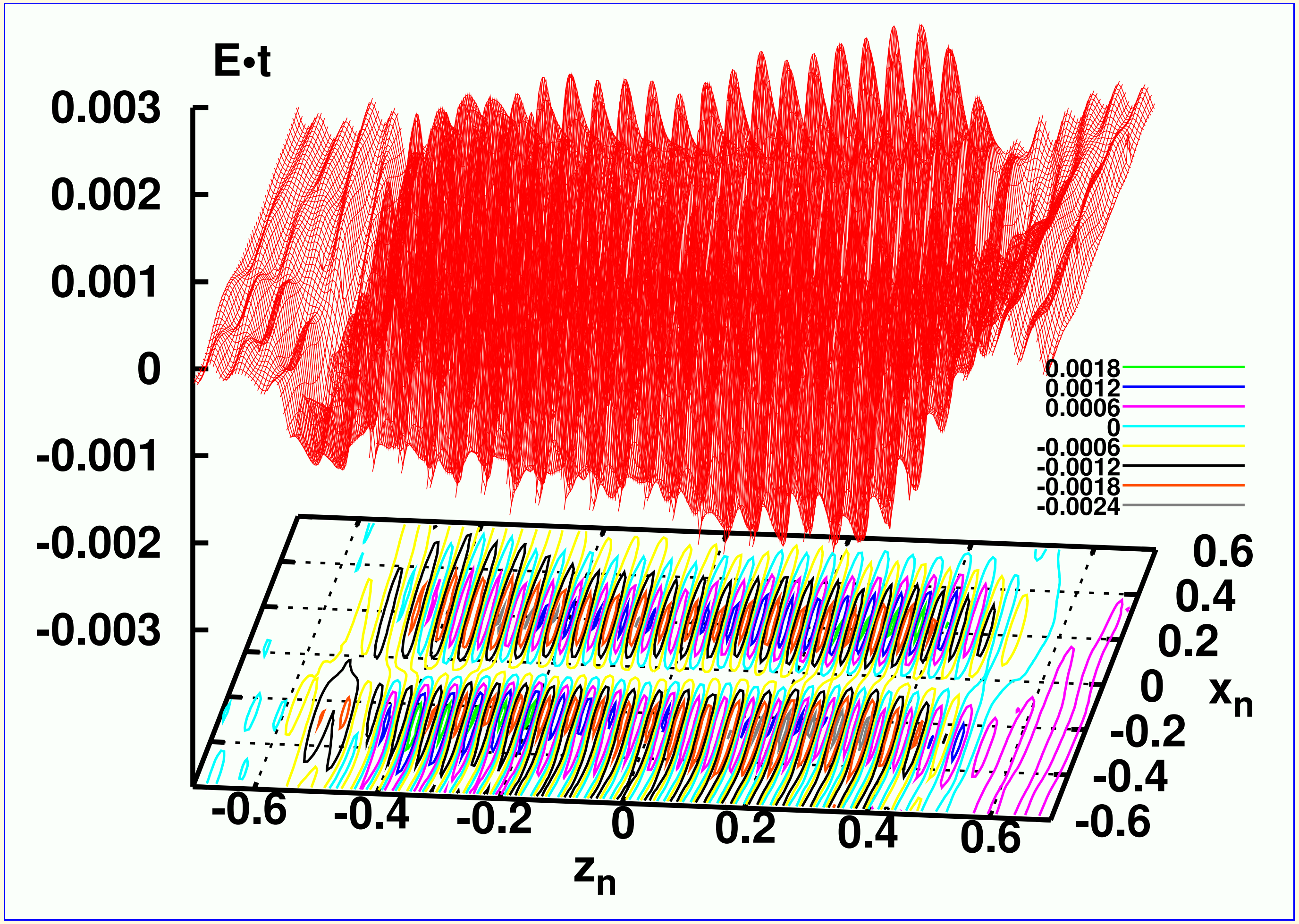
$\mathbf{E}\cdot\mathbf{t}$ in normalized coordinates at s=8.029m for $\lambda = 200\mu$m.

FERMI@ELETTRA First Bunch Compressor XI



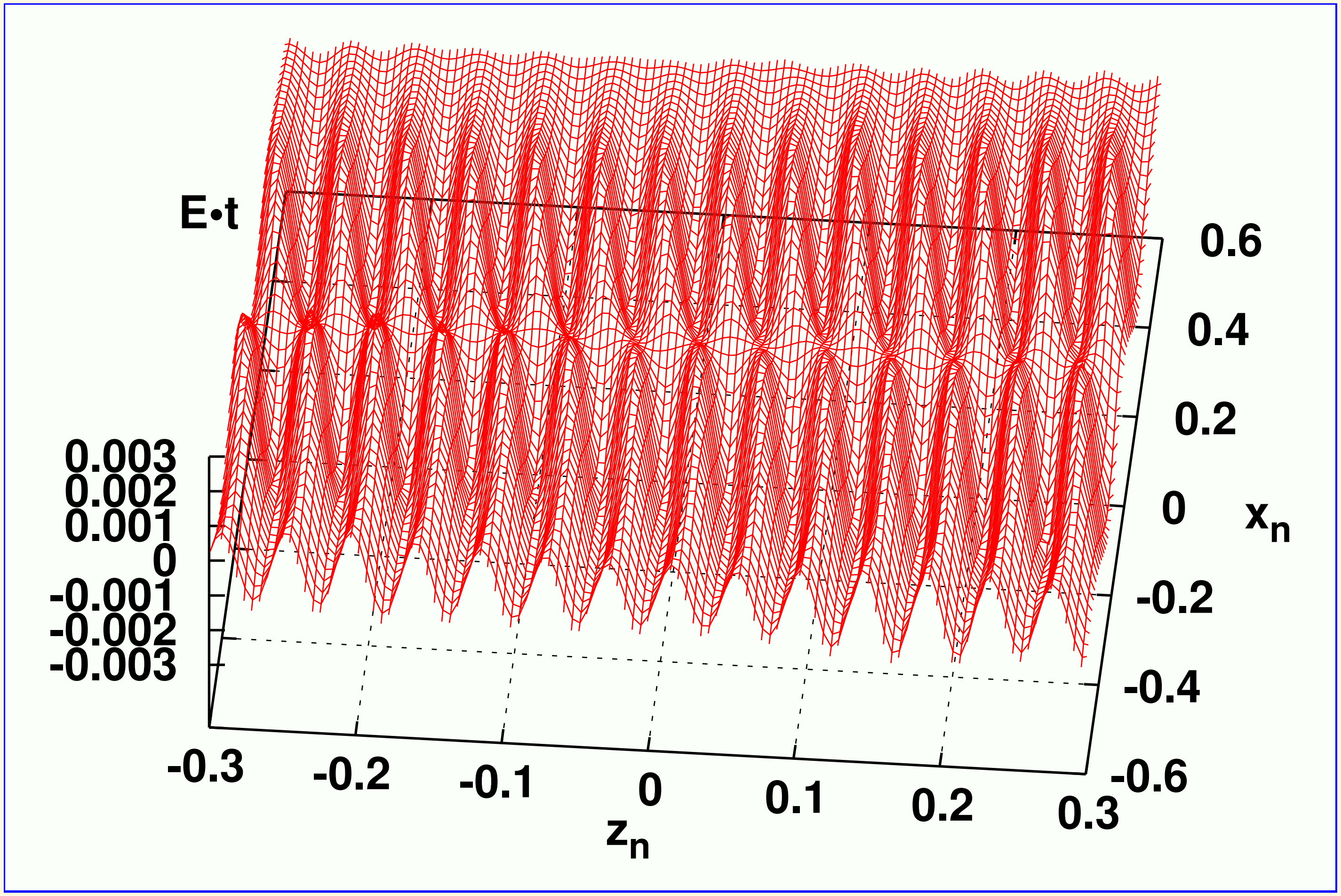
Enlargement of $\mathbf{E} \cdot \mathbf{t}$ in norm. coord. at s=8.029m for $\lambda = 200\mu$m.

FERMI@ELETTRA First Bunch Compressor XII



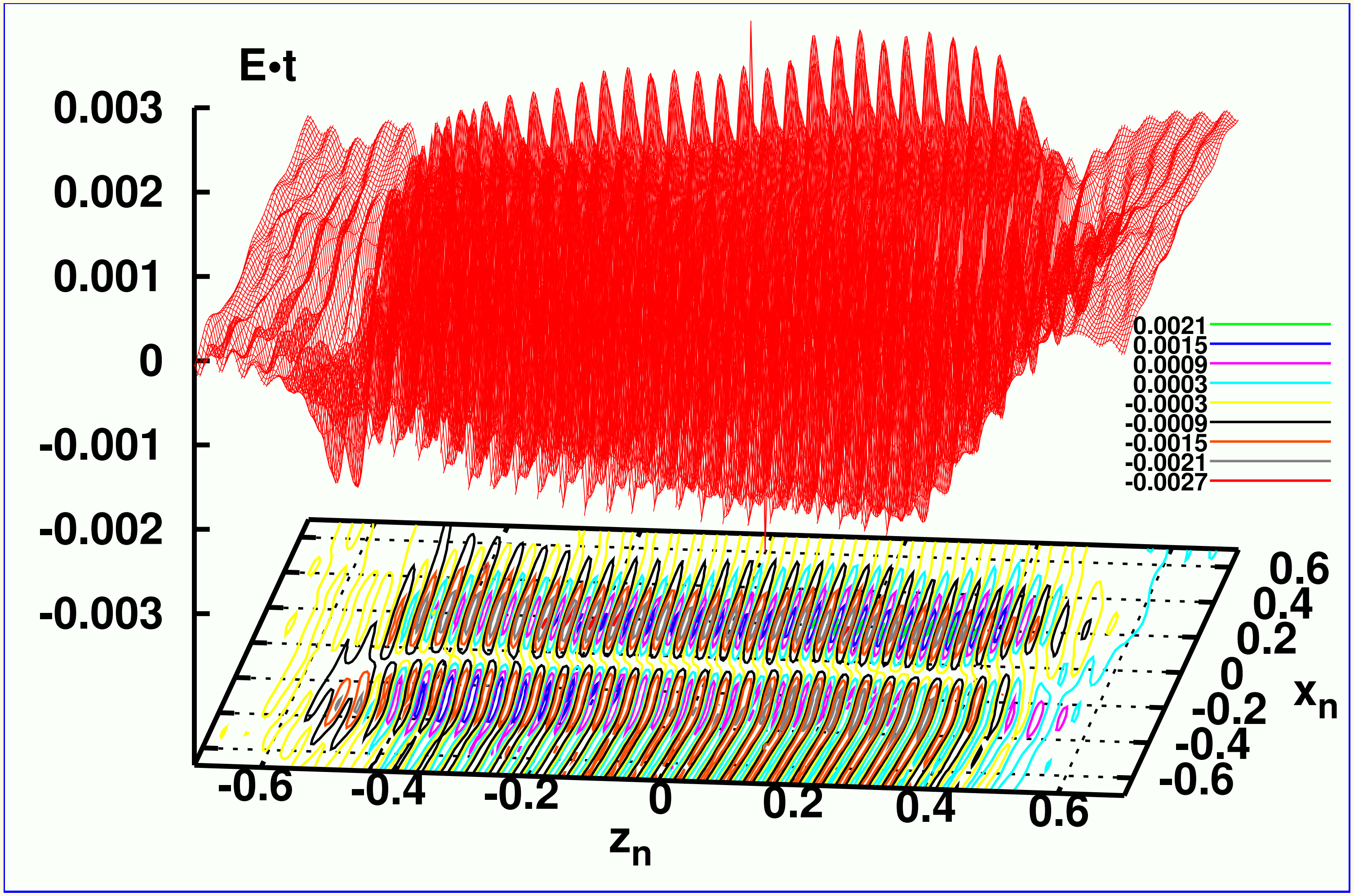
$\mathbf{E} \cdot \mathbf{t}$ in normalized coordinates at s=8.029m for $\lambda = 100\mu$m.

FERMI@ELETTRA First Bunch Compressor XIII



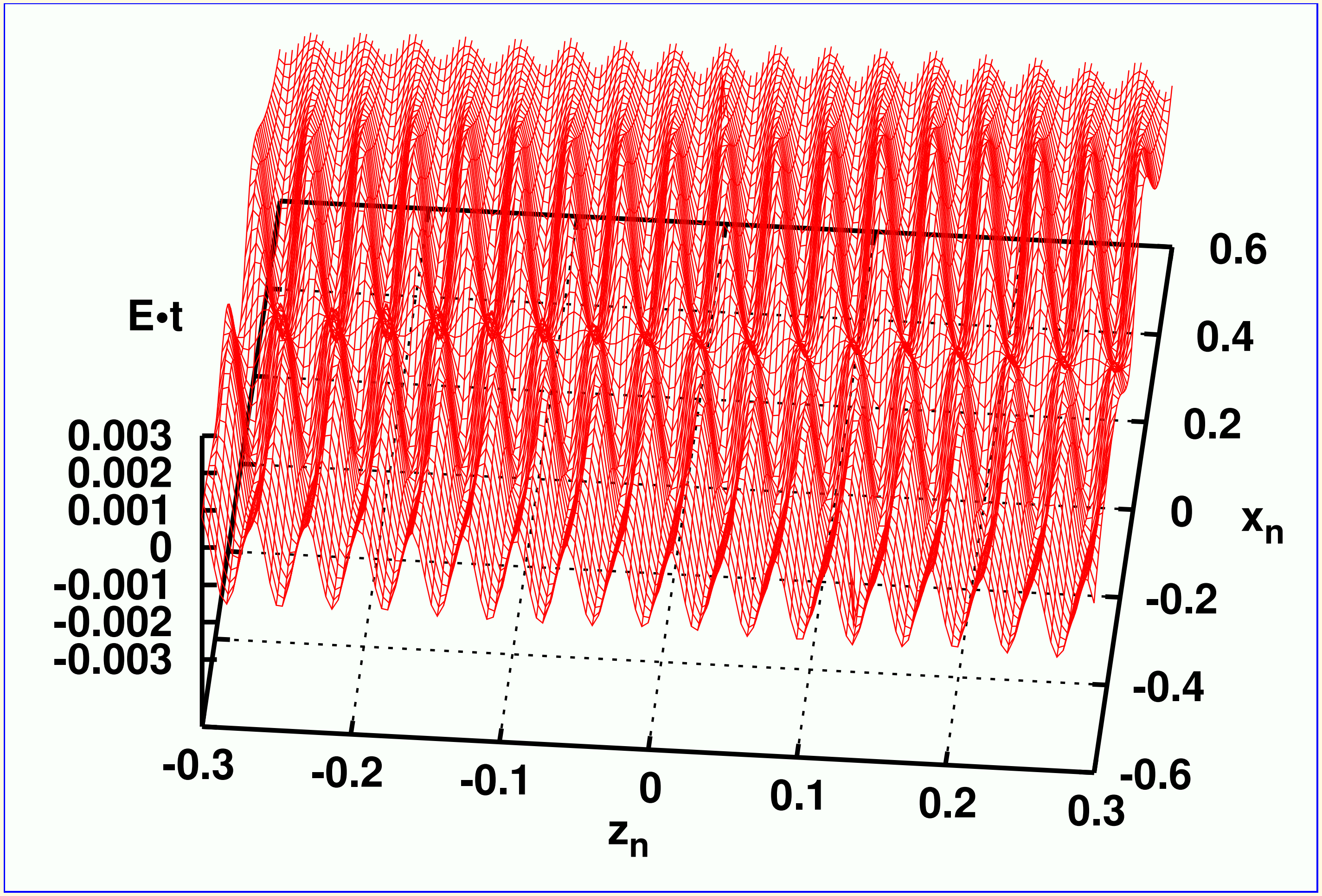
Enlargement of $\mathbf{E}\cdot\mathbf{t}$ in norm. coord. at s=8.029m for $\lambda = 100\mu$m.

FERMI@ELETTRA First Bunch Compressor XIV



$\mathbf{E} \cdot \mathbf{t}$ in norm. coord. at s=8.029m for $\lambda = 80\mu$m.

FERMI@ELETTRA First Bunch Compressor XV



Enlargement of $\mathbf{E}\cdot\mathbf{t}$ in norm. coord. at s=8.029m for $\lambda = 80\mu$m.

Main Issues and Accomplishments

- FERMI@ELETTRA microbunching studies at $\lambda \geq 80\mu$m:
  - Very small effect of μBI on mean power and transverse emittance
  - Gain factor at short wavelengths indicates weaker μBI than predicted by analytical formula
  - Simulations done at the HPC at UNM and on NERSC at LBNL, typical runs on NERSC: N procs = 200-700, N particles = 2×10^7-2×10^8, 10-20 hours of CPU time
- Storage/computational cost very important
  - Analytical work + state of the art numerical techniques: integration, interpolation, density estimation, quasirandom generator
  - Parallel computing
- Delicacy of field calculation, support of charge/phase space density

Future Work

- Study wavelengths shorter than $\lambda = 80\mu$m and different amplitudes of the initial modulation
- Complete studies for benchmark microbunching instability including RF cavities
- A paper will be submitted shortly to PRSTAB EPAC08 Special Issue