BETACOOL Program for Simulation of Beam Dynamics in Storage Rings

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2. Physical motivation
3. BETACOOL algorithms
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6. Software structure, code benchmarking
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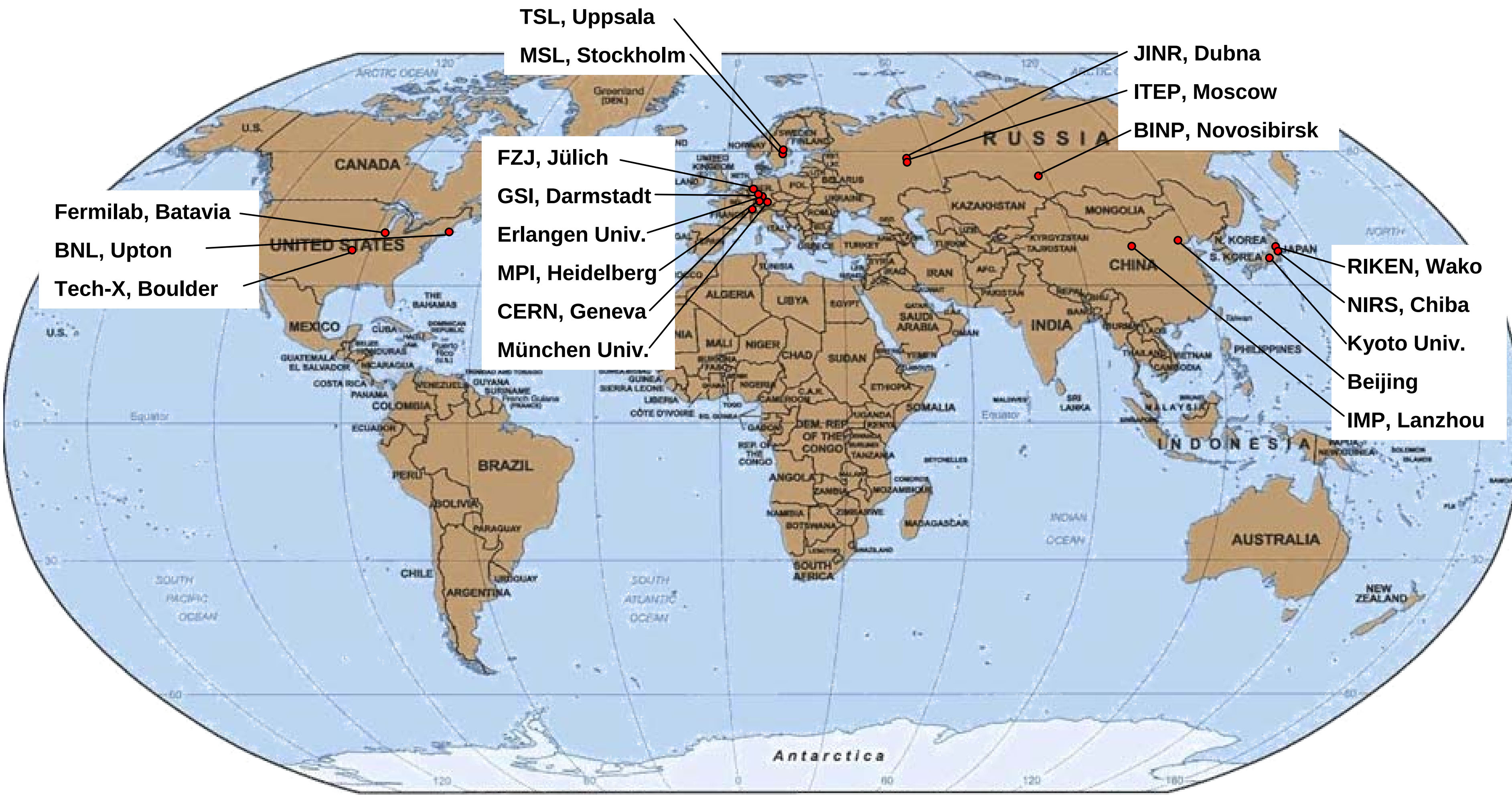
Collaboration with Scientific Centers

- BNL (USA)
- Fermilab (USA)
- RIKEN (Japan)
- NIRS (Japan)
- Kyoto Univ. (Japan)
- CERN (Switzerland)
- ITEP (Russia)
- BINP (Russia)
- Juelich (Germany)
- GSI (Germany)
- Erlangen Univ. (Germany)
- Uppsala Univ. (Sweden)

BETACOOL application over the world

(since 1995)



http://lepta.jinr.ru/betacool/betacool.htm

Physical motivation

Accelerator design, beam stability investigation can be provided using:

MAD, CERN
UAL (Unified Accelerator Library), BNL …..

General goal of the BETACOOL program is
to simulate long term processes
(in comparison with the ion revolution period)
leading to variation of the ion distribution function
in 6 dimensional phase space.

Competitive programs:
MOCAC (Monte-Carlo Code)
ITEP, Moscow, P. Zenkevich, A. Bolshakov

SIMCOOL (Simulation of Cooling), **TRUBS** –
BINP, Novosibirsk, V. Parkhomchuk, V. Reva

BETACOOL assumptions

- The ion beam motion inside a storage ring is supposed to be stable and it is treated in linear approximation.
- Ion beam is presented by rms parameters of the distribution function or by array of model particles
- Each effect calculates characteristic times of emittance variation and kick of the ion momentum components and changes the particle number

Basic models

Kit of algorithms:

-Evolution of rms parameters
-Evolution of distribution function
- Tracking

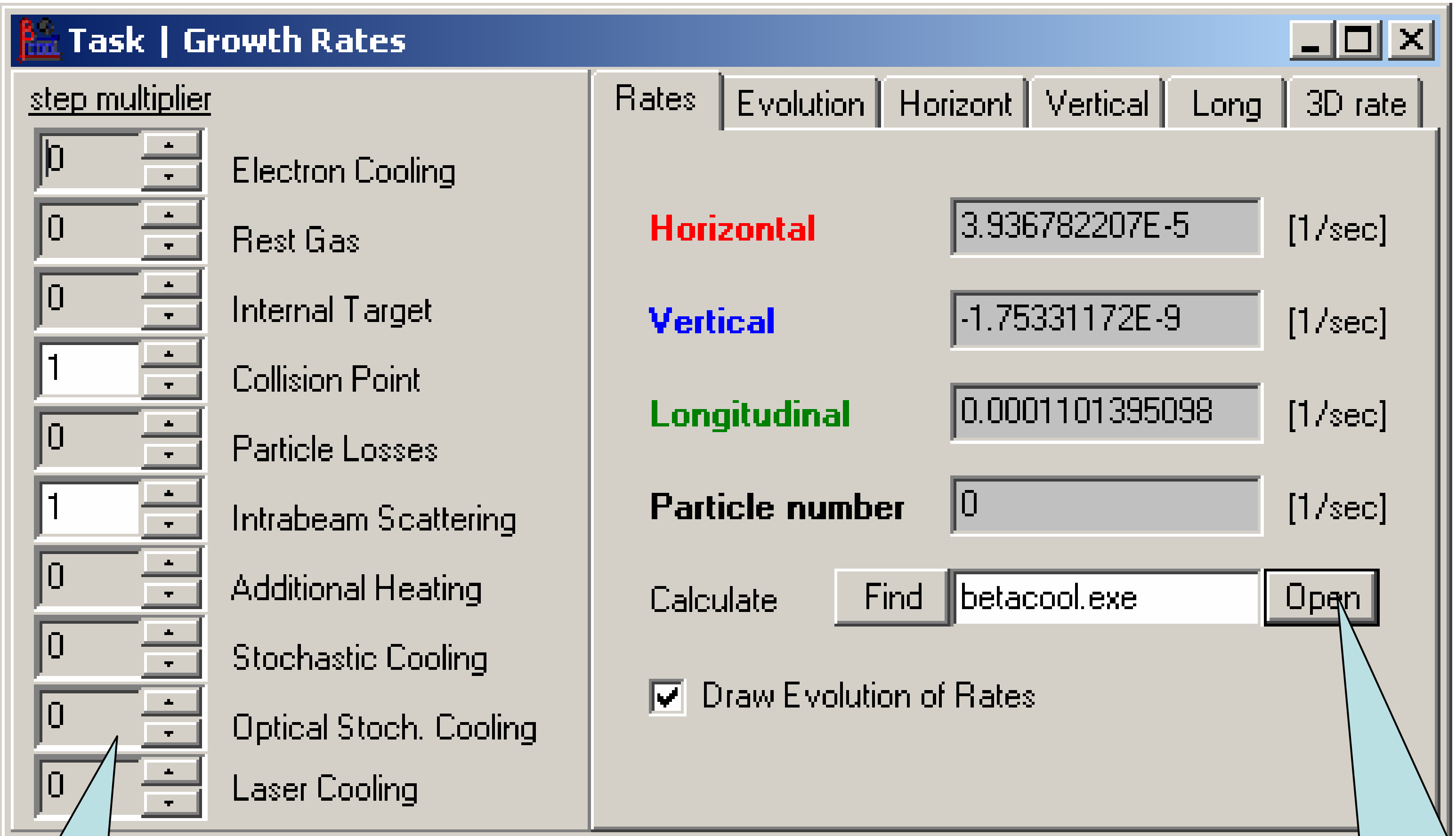
Library of effects:

- IntraBeam Scattering,
- Interaction with internal target and rest gas,
- Beam-beam effect,
- Electron cooling,
- Stochastic cooling,
- Laser cooling,
- External heating

…

Models of storage ring and ion beam

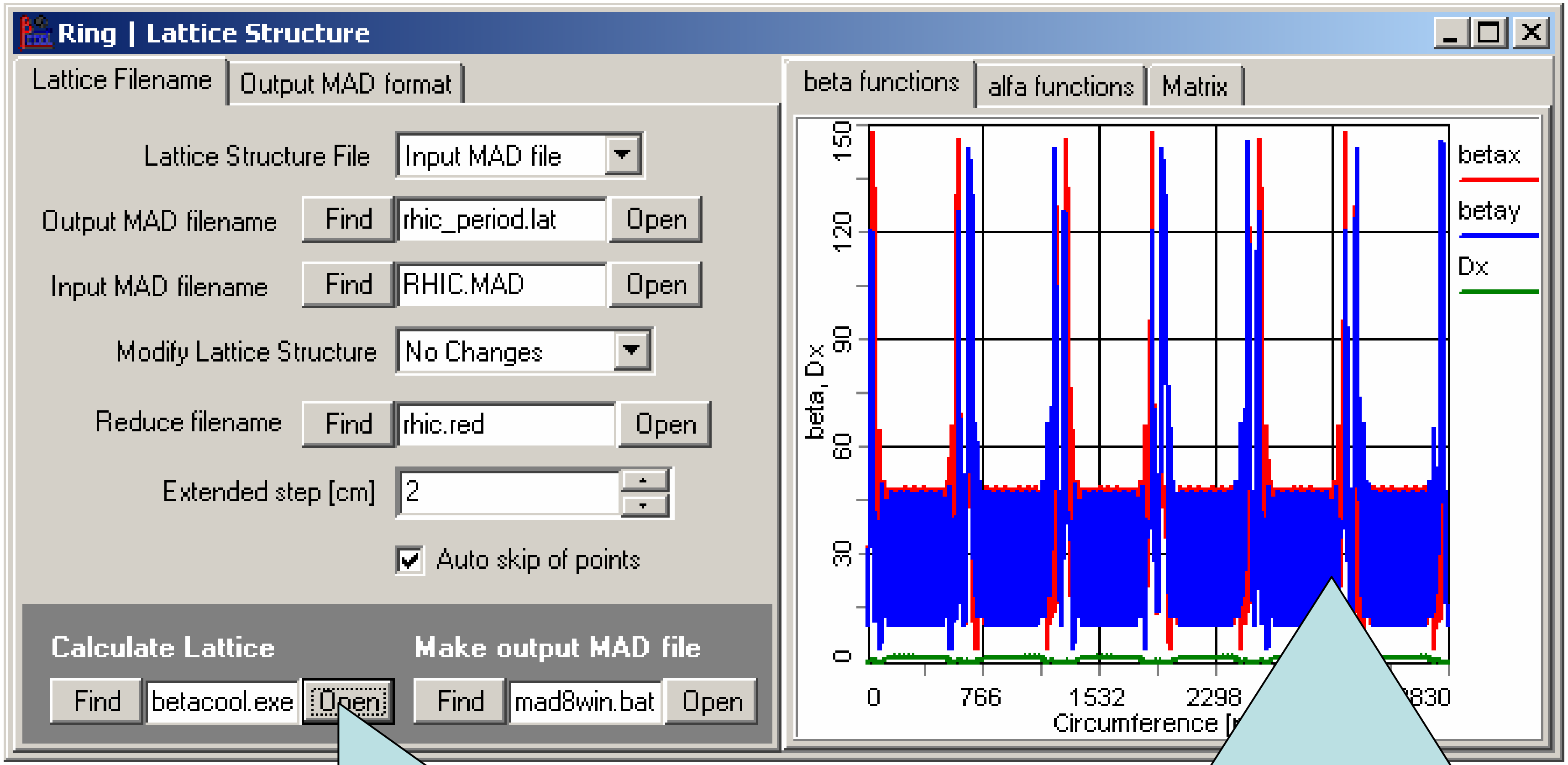
Physical Effects involved in BETACOOL program



Active effects

Calculate of growth rates

Lattice Structure using MAD files



Calculate of lattice functions

Horizontal and Vertical beta-functions, Horizontal dispersion for RHIC

BETACOOL Algorithms

- **RMS Dynamics** – evolution of RMS parameters of ion beam (Gaussian distribution)
- **Model Beam** – Monte-Carlo method with modeling particles
- **Tracking** – particles dynamics over the real lattice with using Molecular Dynamics technique

RMS Dynamics

$$\begin{cases} \dfrac{dN}{dt} = N\sum_j \dfrac{1}{\tau_{life,j}}, \\ \dfrac{d\varepsilon_x}{dt} = \varepsilon_x \sum_j \dfrac{1}{\tau_{x,j}}, \\ \dfrac{d\varepsilon_y}{dt} = \varepsilon_y \sum_j \dfrac{1}{\tau_{y,j}}, \\ \dfrac{d\varepsilon_s}{dt} = \varepsilon_s \sum_j \dfrac{1}{\tau_{s,j}} \end{cases}$$

- **Ion beam has Gaussian distribution during the evolution**
- **Algorithm is considered as a solution of the equations for R.M.S. parameters**
- **Maxima of all the distribution functions coincide with equilibrium orbit**
- **Real lattice structure is used for IBS calculation only**

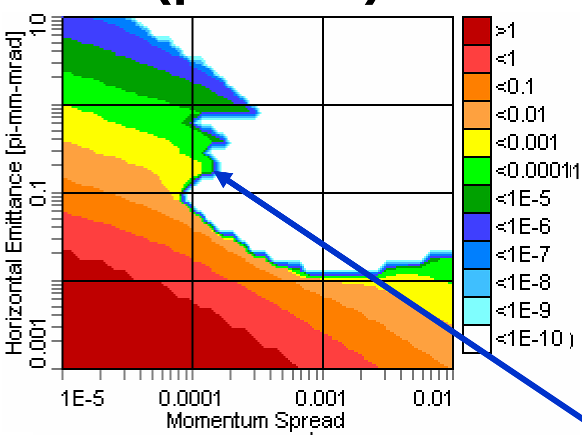
3D Diagrams for HESR

heating and cooling rates

IBS
(positive)

ECOOL
(negative)

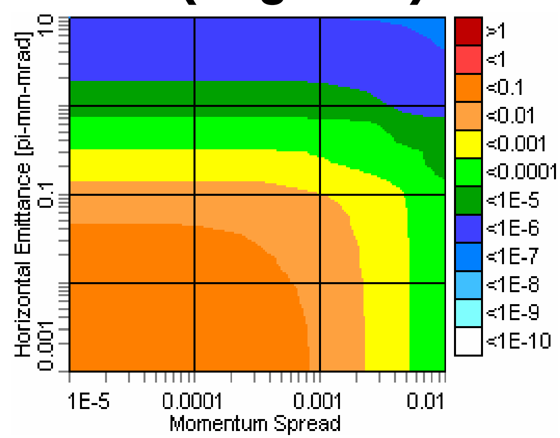
emittances



momentum spread

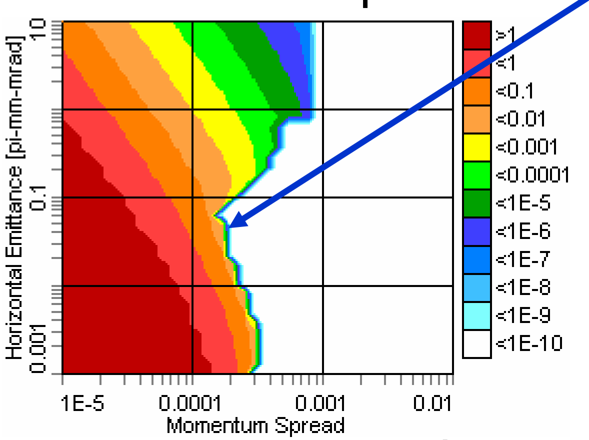
$$\tau^{-1}_{hor}$$

transverse component



Equilibrium between IBS and ECOOL

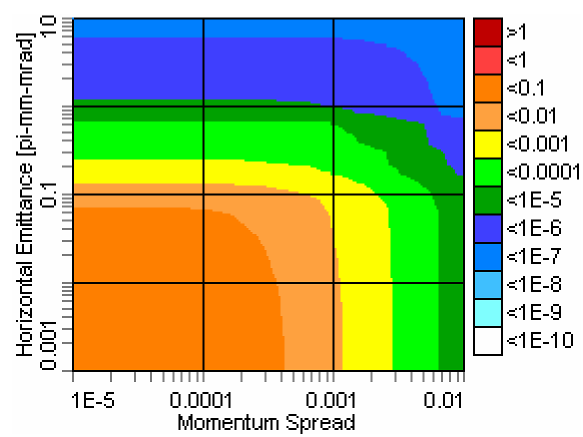
emittances



momentum spread

$$\tau^{-1}_{lon}$$

longitudinal component

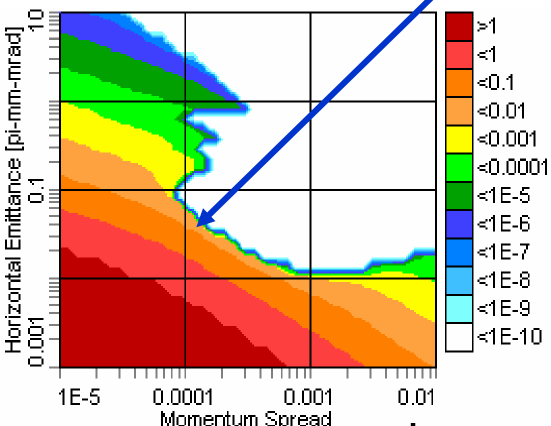


RMS Dynamics for HESR
(ECOOL+IBS)

3D Diagrams

Equilibrium point

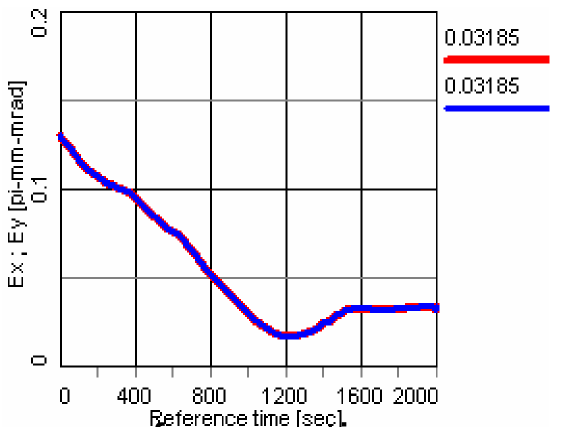
emittances



momentum spread

Beam evolution

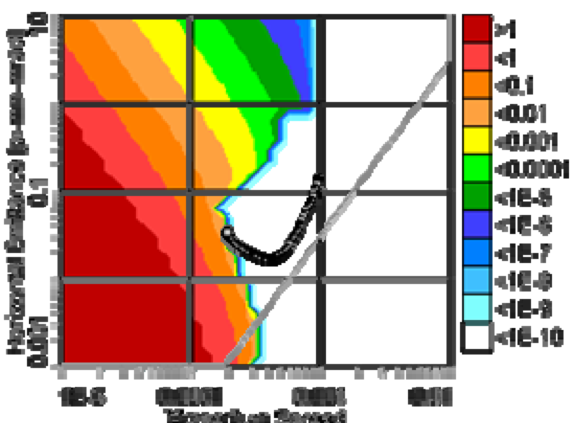
emittances



reference time

transverse

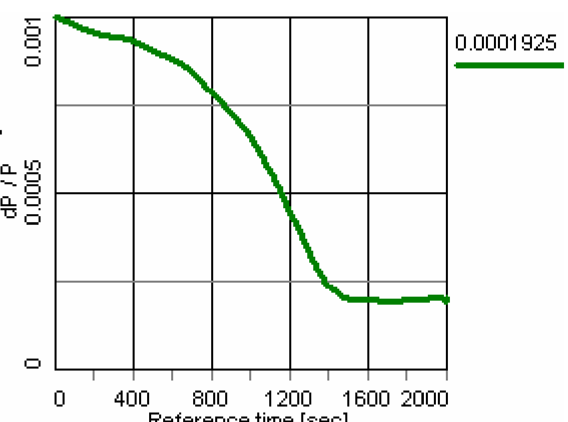
emittances



momentum spread

longitudinal

momentum spread



reference time

Model Beam algorithm

Ion beam is presented by array of model particles.

For each model particle the program solves Langevin equation:

$$P_i(t+\Delta t) = P_i(t) - F_i \Delta t + \sqrt{\Delta t} \sum_{j=1}^{3} C_{i,j} \xi_j$$

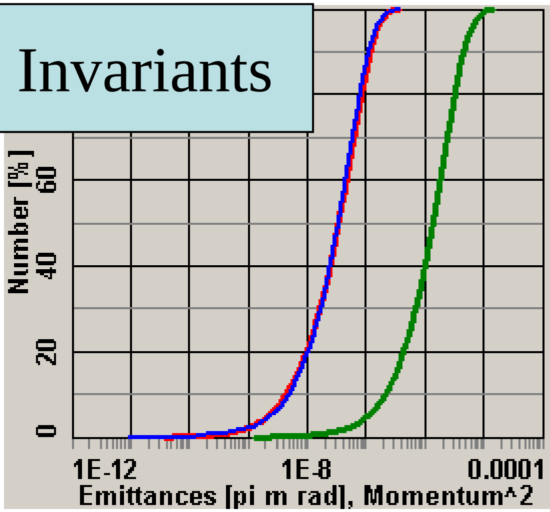
ξ_j are independent Gaussian random numbers.

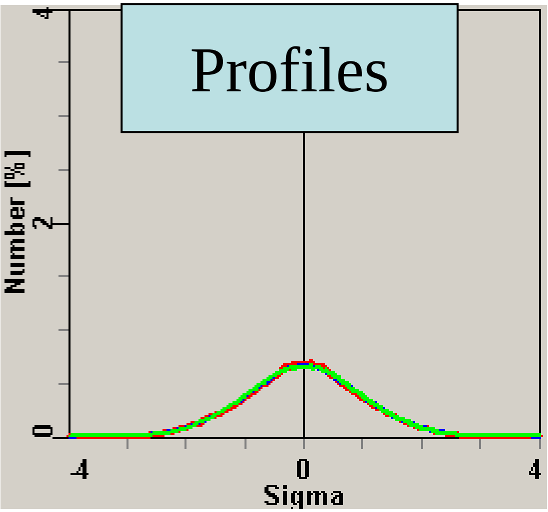
The algorithm is equivalent to solution of Fokker-Plank equation, if

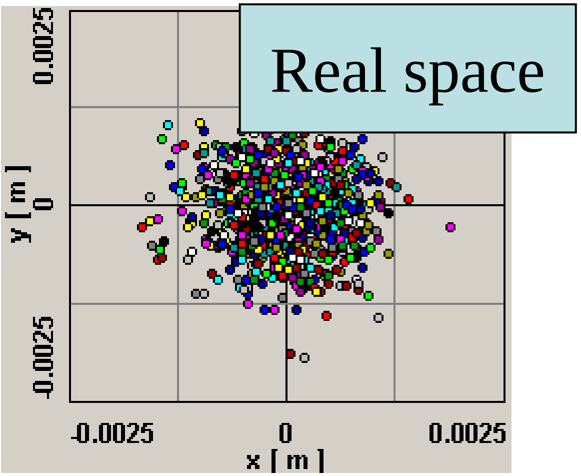
$$\sum_{k=1}^{3} C_{i,k} C_{j,k} = D_{i,j}$$

Each effect calculates a kick of the ion momentum components and changes the particle number

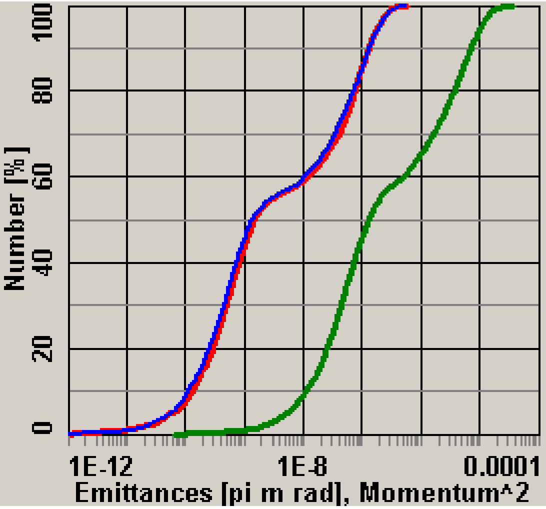
Initial distribution for RHIC

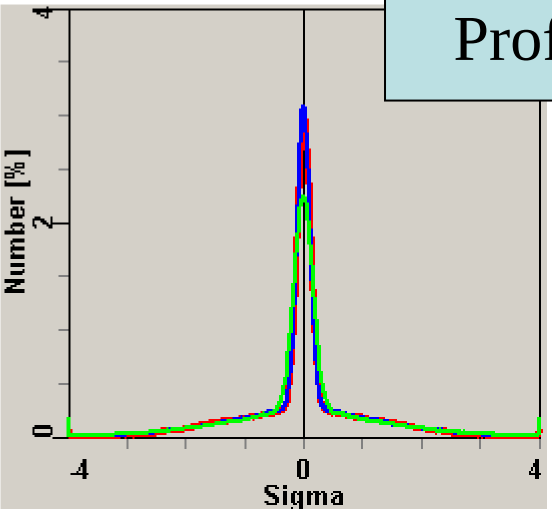


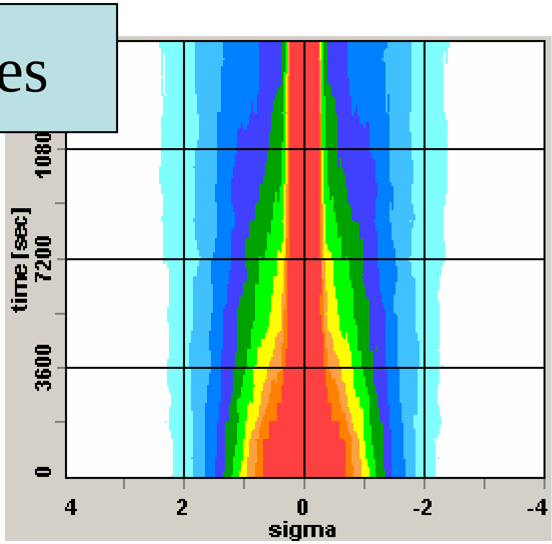




Distribution after 4 hours of cooling







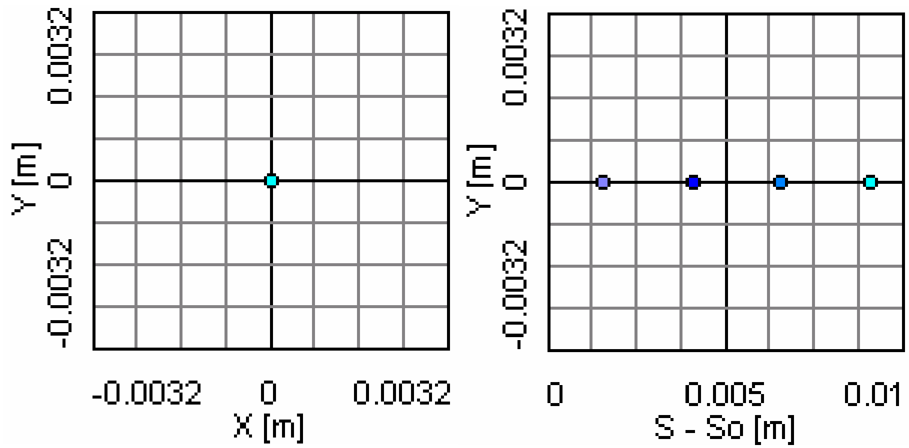
Tracking procedure

Ion beam is presented by array of real or macro particles

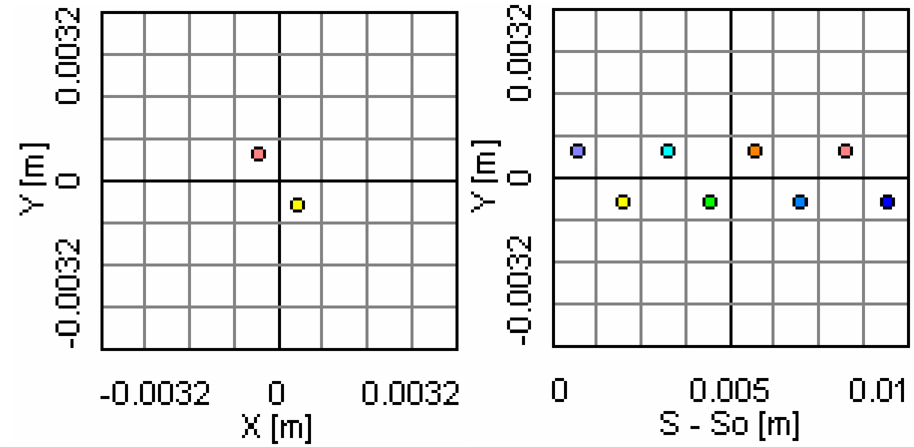
- **Each effect is related with some optic element**
- **The effect works as a transformation map**
- **IBS is calculated as a Coulomb scattering using Molecular Dynamics technique**
- **The ring structure is imported from modified input MAD file**

MD simulation of crystalline beams

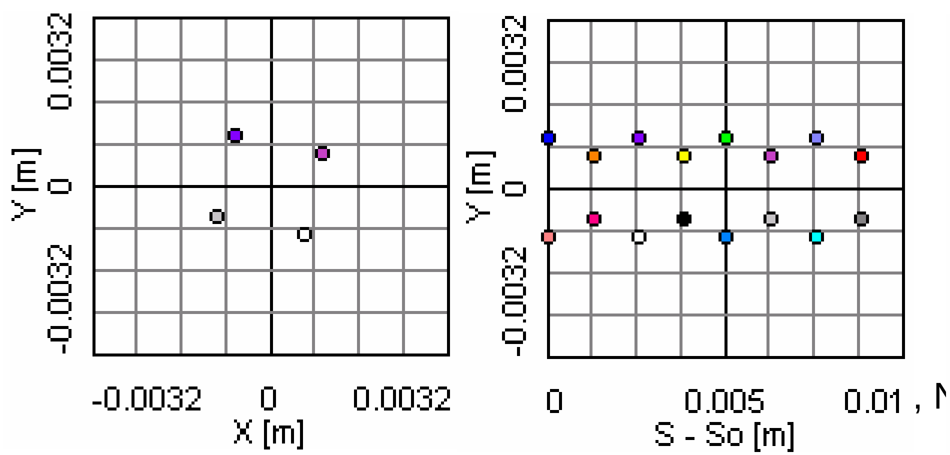
String (λion < 0.709)



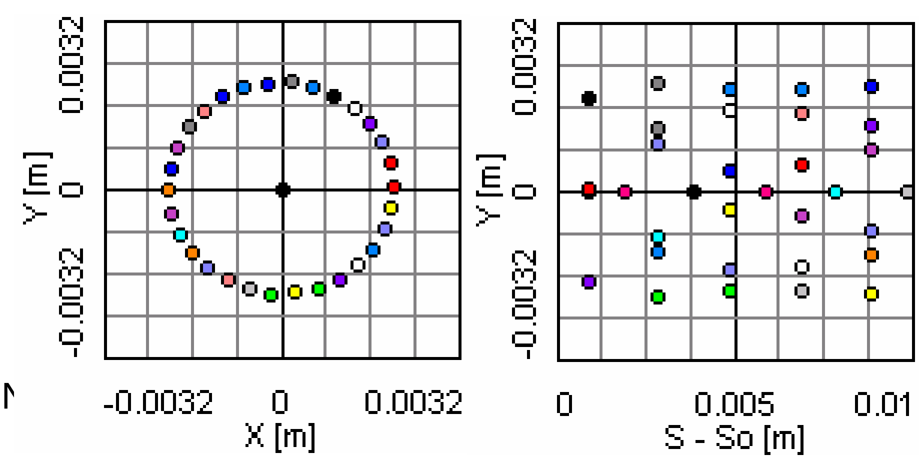
Zigzag (0.709 < λion < 0.964)



Helix or Tetrahedron (0.964 < λion < 3.10)



Shell + String (3.10 < λion < 5.7)



Intrabeam scattering simulation

RMS dynamics

For uncoupled transverse motion at zero vertical dispersion
the heating rates are calculated in accordance with:

M. Martini "Intrabeam scattering in the ACOOL-AA machines", CERN PS/84-9 AA, Geneva, May 1984.

For uncoupled motion at non-zero vertical dispersion:

M.Venturini, "Study of intrabeam scattering in low-energy electron rings", Proceedings of the 2001 PAC, Chicago (J.D. Bjorken, S.K. Mtingwa, "Intrabeam scattering", Particle Accelerators, Vol. 13, p.115, 1983.)

The models require lattice functions of the ring

+ a few simplified models to speed up the calculations

Intrabeam scattering simulation

Model Beam

-**Simplified kinetic model:**
Constant diffusion and friction linearly depending on the ion velocity.
The friction coefficient and the diffusion tensor are calculated
in accordance with Venturini model.

-**Local model**

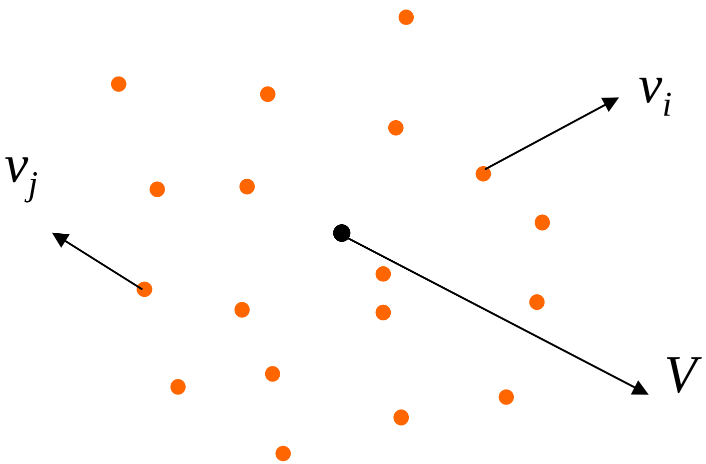
-**“Core-Tail” model (**Bi-Gaussian distribution**)**

Tracking

IBS is calculated as a Coulomb scattering using
Molecular Dynamics technique

The models require optic structure of the ring

Local model for IBS



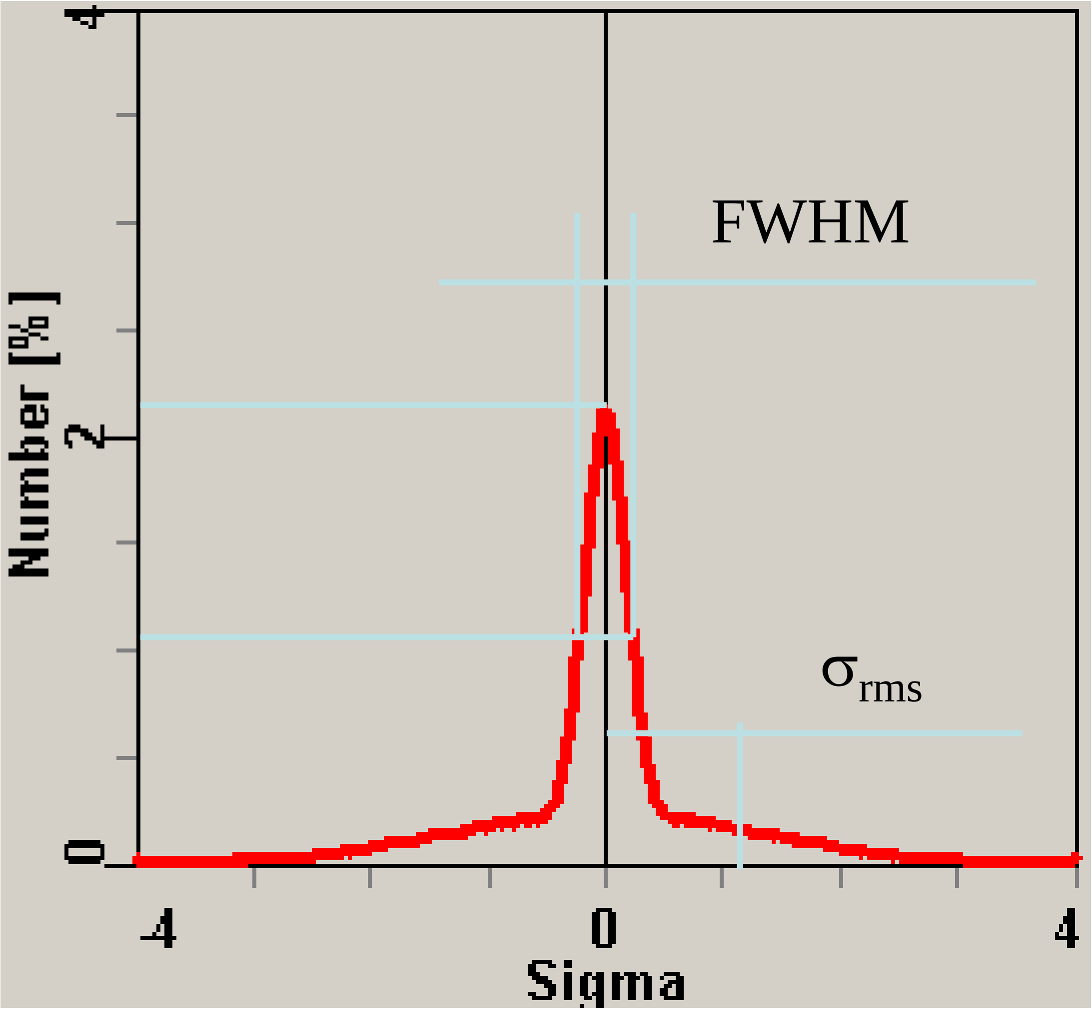
"Test" particle moves inside a cloud of "field" particles

$$\vec{U} = \vec{V} - \vec{v}$$

$$\vec{F} = \frac{\langle \Delta \vec{p} \rangle}{\Delta t} = -\frac{4\pi n e^4 Z_t^2 Z_f^2}{\left(\dfrac{m_f m_t}{m_f + m_t} \right)} \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}} \right) \frac{\vec{U}}{U^3} f(v) d^3 v$$

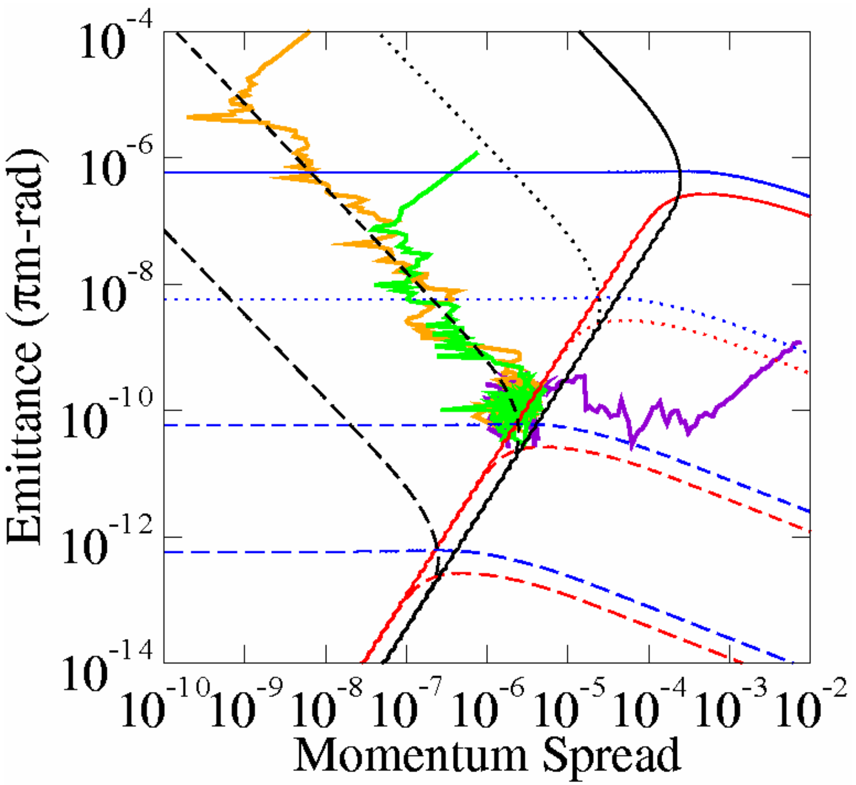
$$D_{\alpha,\beta} = \frac{\langle \Delta p_\alpha \Delta p_\beta \rangle}{\Delta t} = 4\pi n e^4 Z_t^2 Z_f^2 \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}} \right) \frac{U^2 \delta_{\alpha,\beta} - U_\alpha U_\beta}{U^3} f(v) dv$$

Core-tail model

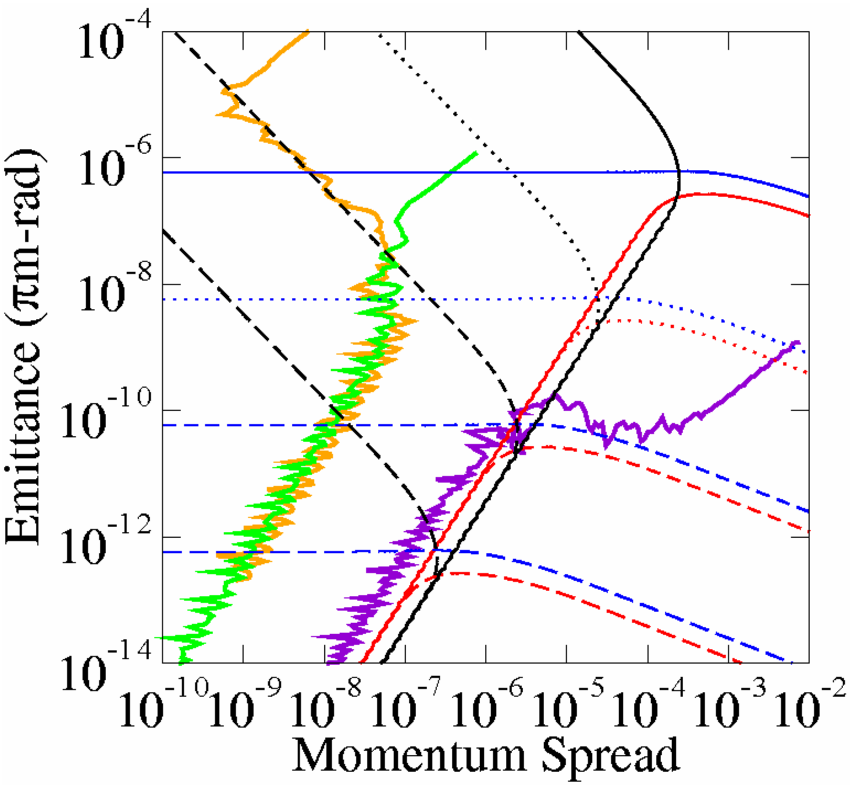


Theoretical and MD simulation for ESR

Equilibrium between ECOOL and IBS



Ordered state of ion beam



Map of Electron Cooling system

Ion co-ordinates at the entrance

Model of cooler

Solution of the ion motion equations

Transformation of the ion co-ordinates

to the frame referenced to the electron beam orbit

Magnetic field errors

Electron beam space-charge

Thin lens

Cooler at non zero length

Electron beam model

Transformation of the ion velocity to PRF

friction force components to LRF

Calculation of local electron density and temperature

Uniform cylinder

Gaussian cylinder

Gaussian bunch

Hollow beam

Array of electrons

Friction force library:

Calculation of

force components in PRF,

dP_{loss}/ds

Non-magnetized,

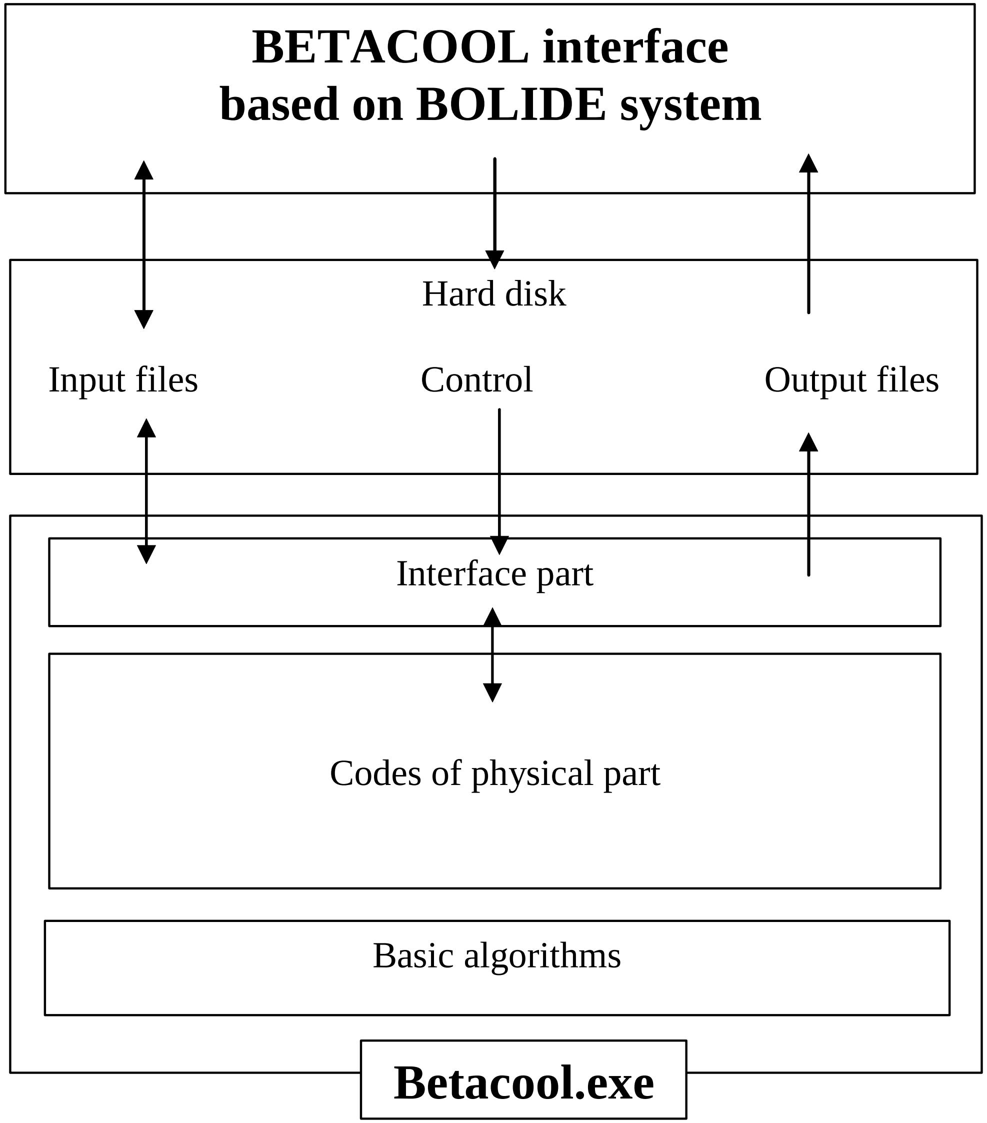
by Parkhomchuk,

Derbenev-Skrinsky,

Erlangen University

Ion coordinates at the exit, loss probability

Software structure



Platforms of C++ Compilers

- ✓Borland C++Builder (Windows)
- ✓Borland C++BuilderX (Windows / LINUX)
- ✓Microsoft Visual Studio (Windows)
- ✓GNU (LINUX)

Physics guide of BETACOOL code,
http://www.agsrhichome.bnl.gov/AP/ap_notes/ap_note_262.pdf

User guide is in preparation now – will be ready this year

The code benchmarking

1. Comparison with competitive codes:
   MADX – Intrabeam scattering simulations
   MOCAC
   SimCool, TRUBS

2. Comparison with experimental data:
   - Equilibrium beam parameters at ESR, TSR, COSY, CELSIUS, RECYCLER, LEIR…
   - Interaction with internal target in experiments at ESR, COSY, CELSIUS
   - Stochastic cooling + Internal target at COSY

3. Dedicated experiments:
   - Electron cooling – COSY, CELSIUS, RECYCLER
   - Intrabeam scattering – RHIC
   - Ion beam ordering – COSY, S-LSR

Possible applications for Electron-Ion Collider design

- Ion beam life-time due to interaction with residual gas, requirements to vacuum conditions
- IBS rates estimation, luminosity life-time without cooling
- Electron cooling system design. Requirements to the electron beam intensity and quality, accuracy of magnetic field in the cooling section, beam alignment etc.
- Luminosity evolution in time. An opposite (electron) bunch can be imported from external program in the forms: RMS parameters, parameters of bi-Gaussian distribution or as array of particles.