Strong-Strong Beam-Beam Simulations in Hadron and Lepton Colliders

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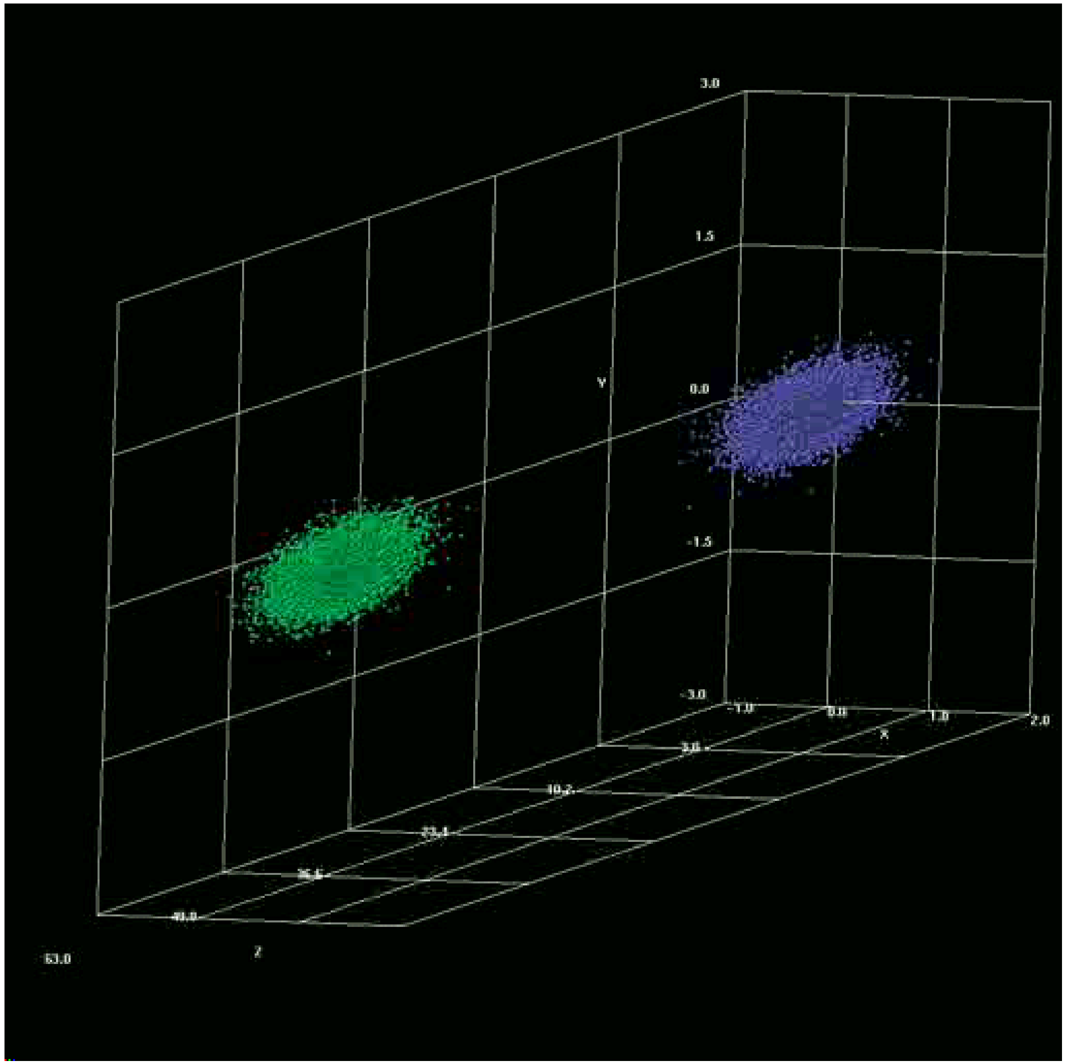
Outline

- Introduction
- Physical model and computational methods
- Parallel implementation
- Applications to studies of emittance growth in hadron machines
- Applications to studies of luminosity evolution in lepton machines

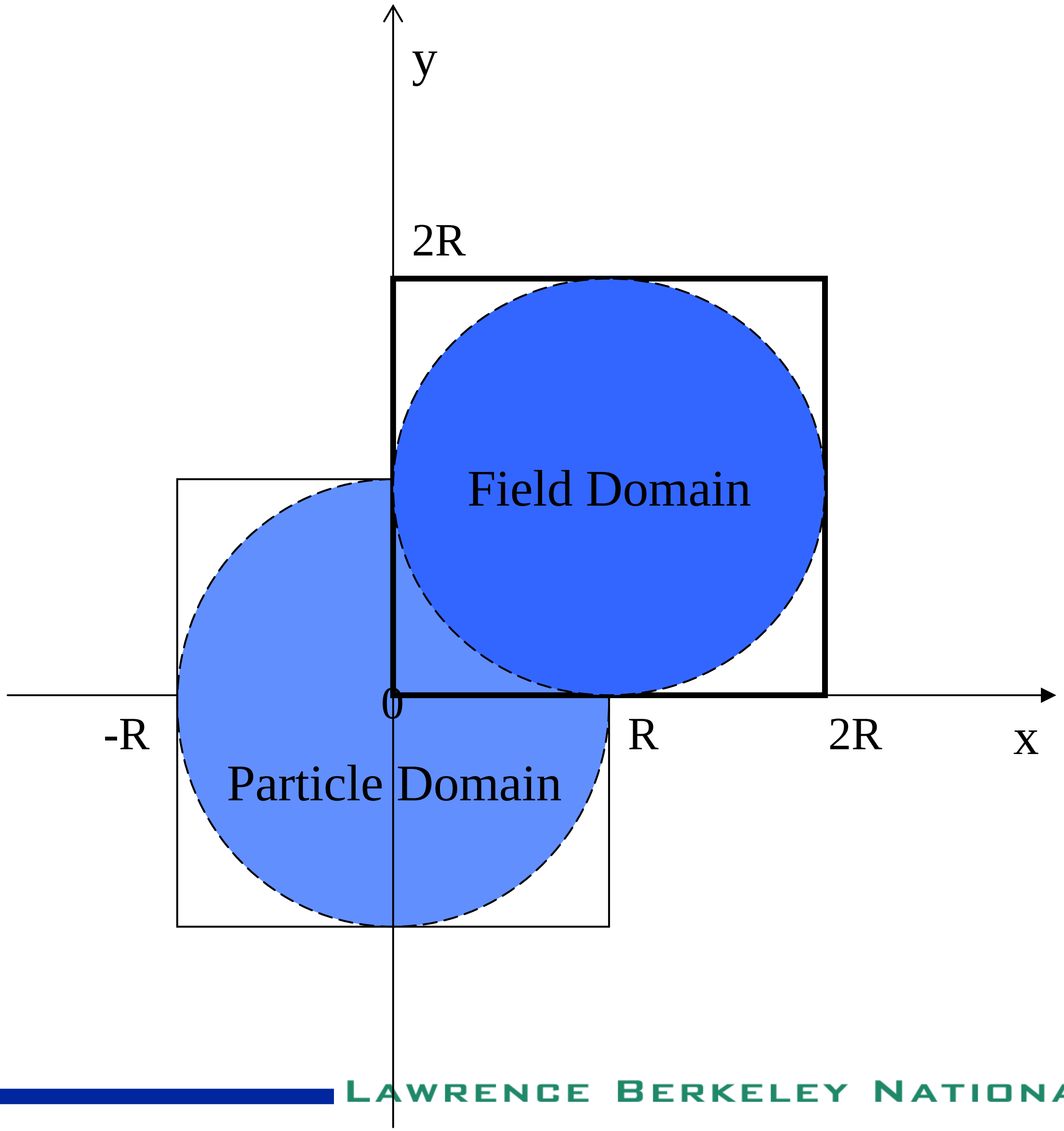
Beam Blow-Up during the Beam-Beam Collision

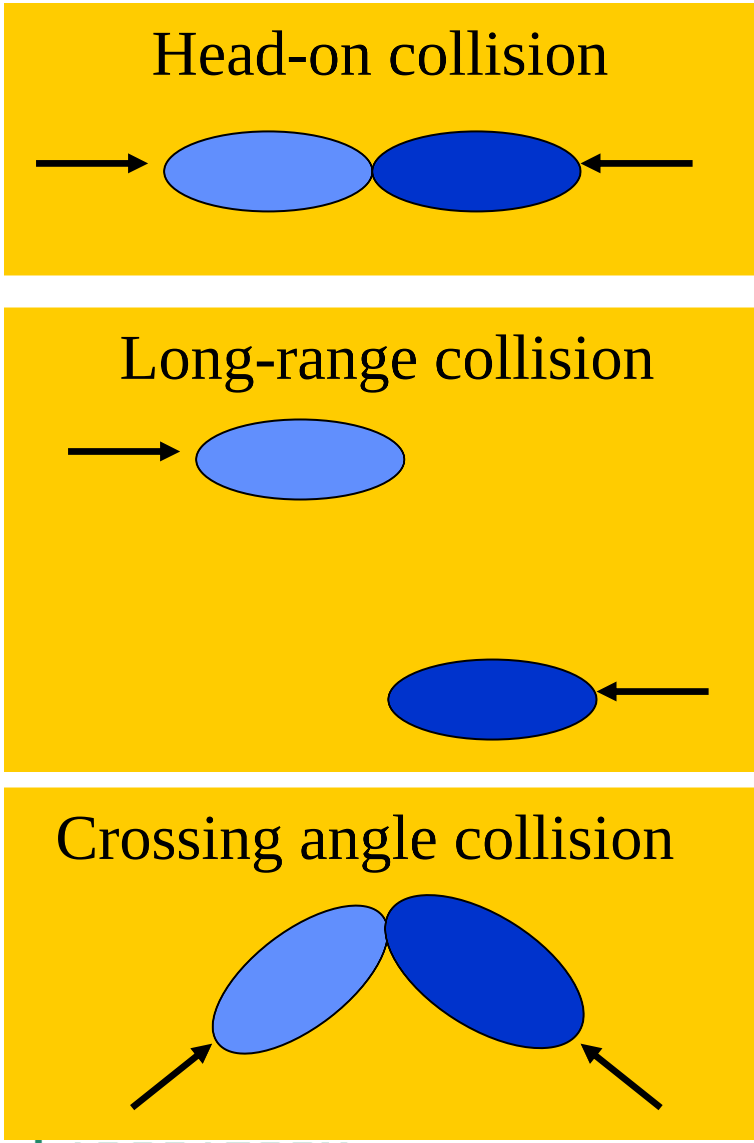


Computational Challenges of Simulation of Colliding Beams

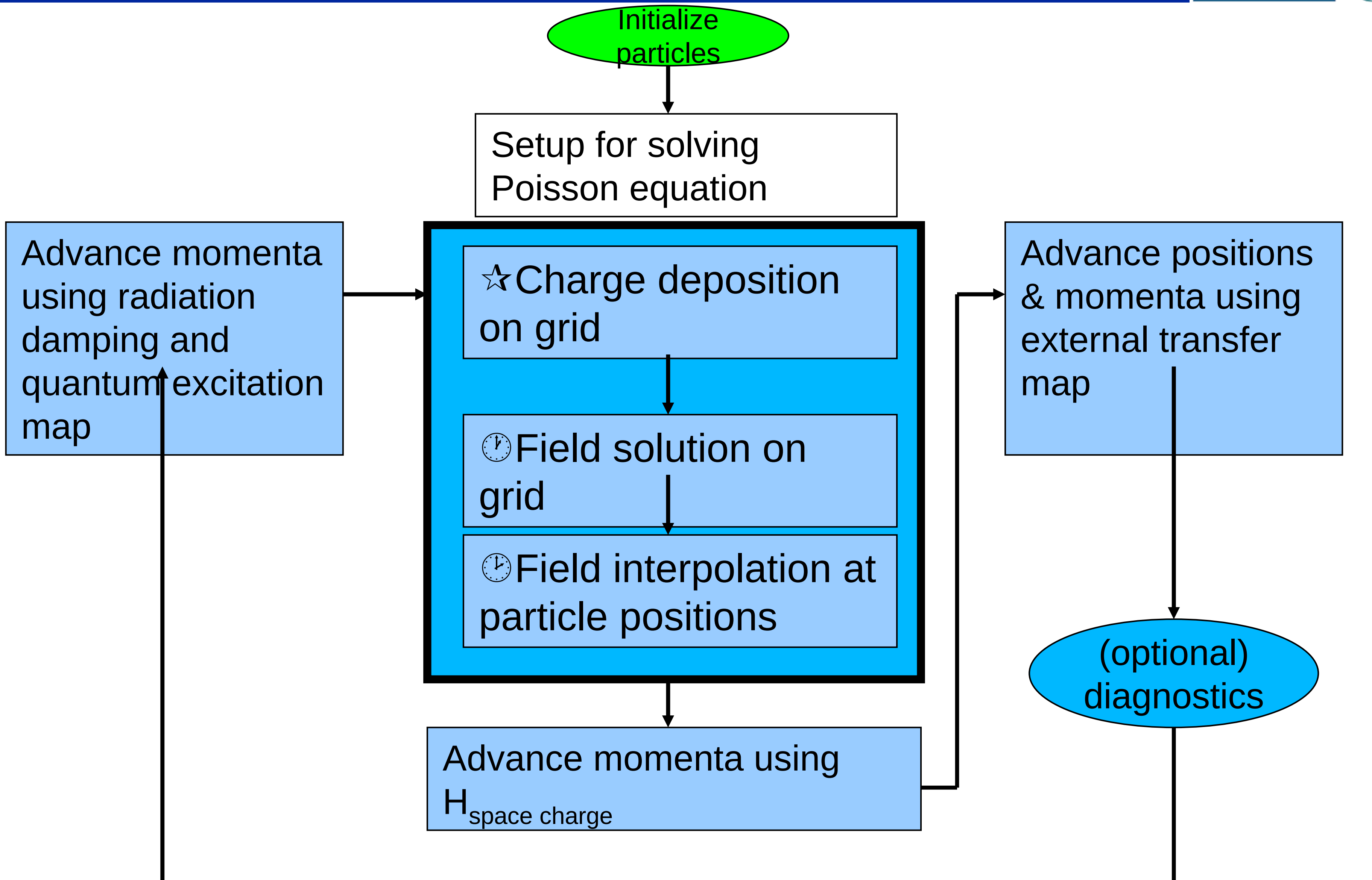
- Multiple physics:
  - Electromagnetic focusing (nonlinear dynamics)
  - Self-consistent beam-beam interaction (Poisson solve in beam frame)
  - Quantum fluctuation and radiation damping
- Long time:
  - Multi-billion revolution turns
- Different geometry:
  - Head-on on-axis collision
  - Crossing angle collision
  - Long range interaction

A Schematic Plot of the Geometry of Two Colliding Beams

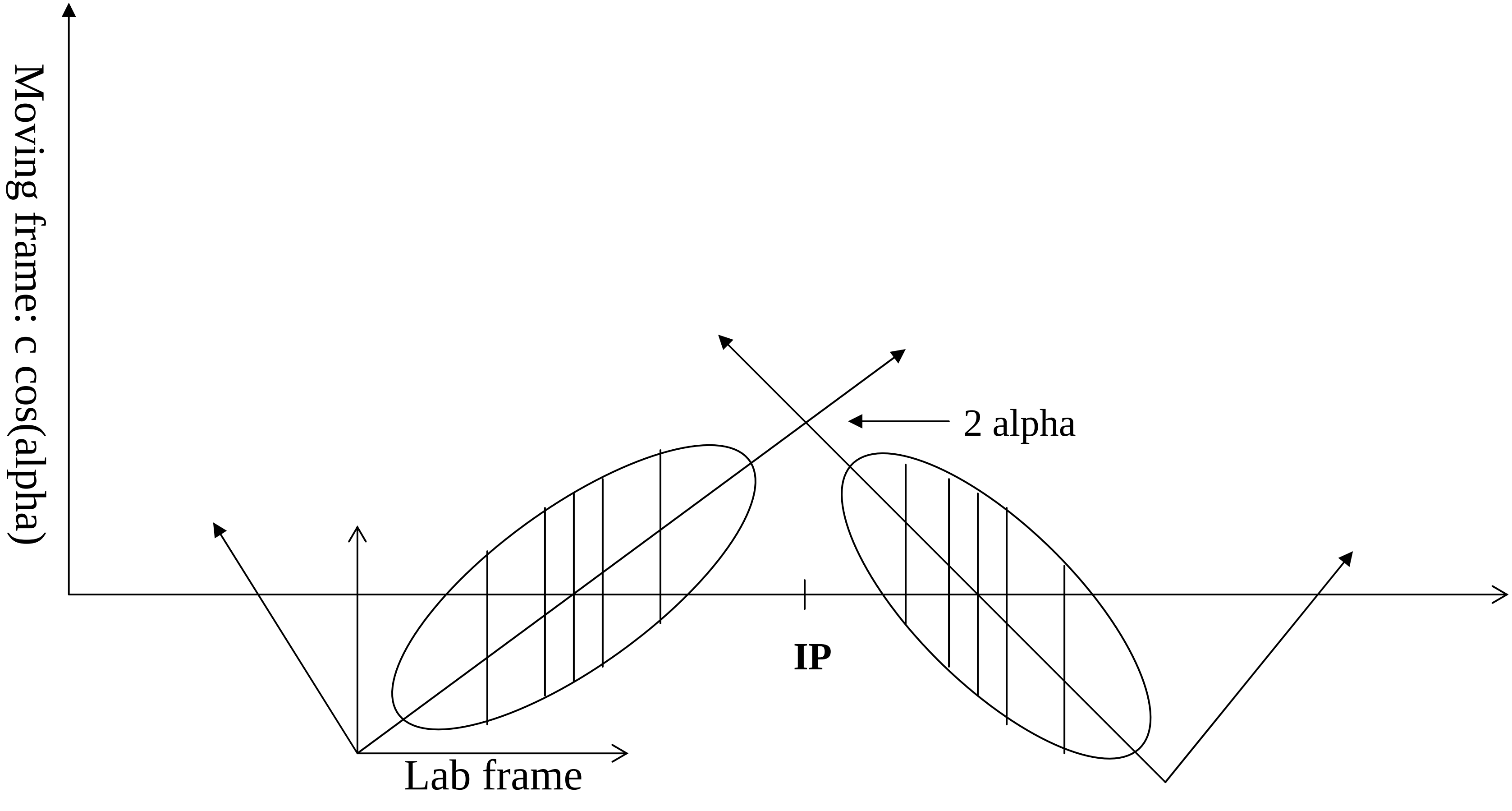




Particle-In-Cell (PIC) Simulation



Two Beam Collision with Crossing Angle Alpha



Computational Issues

- Poisson solver requirements:
  - Able to treat open boundary conditions
  - Able to efficiently treat widely separated beams
  - Able to treat high aspect ratio beams
- Parallelization issue:
  - Significant particle movement between steps
  - Standard domain decomposition not the best choice
- Compared different strategies, utilized hybrid particle/field decomposition for best performance

Green Function Solution of Poisson's Equation

$$\phi(r) = \int G(r, r')\rho(r')dr' \; ; \; r = (x, y)$$

$$\phi(r_i) = h \sum_{i'=1}^{N} G(r_i - r_{i'})\rho(r_{i'})$$

$$G(x, y) = -\frac{1}{2}\log(x^2 + y^2)$$

Direct summation of the convolution scales as N^4 !!!!
N – grid number in each dimension

Green Function Solution of Poisson's Equation (cont'd)

Hockney's Algorithm:- scales as $(2N)^2\log(2N)$

- Ref: Hockney and Easwood, ***Computer Simulation using Particles***, McGraw-Hill Book Company, New York, 1985.

$$\phi_c(r_i) = h\sum_{i'=1}^{2N} G_c(r_i - r_{i'})\rho_c(r_{i'})$$

$$\phi(r_i) = \phi_c(r_i) \quad \text{for i = 1, N}$$

Shifted Green function Algorithm:

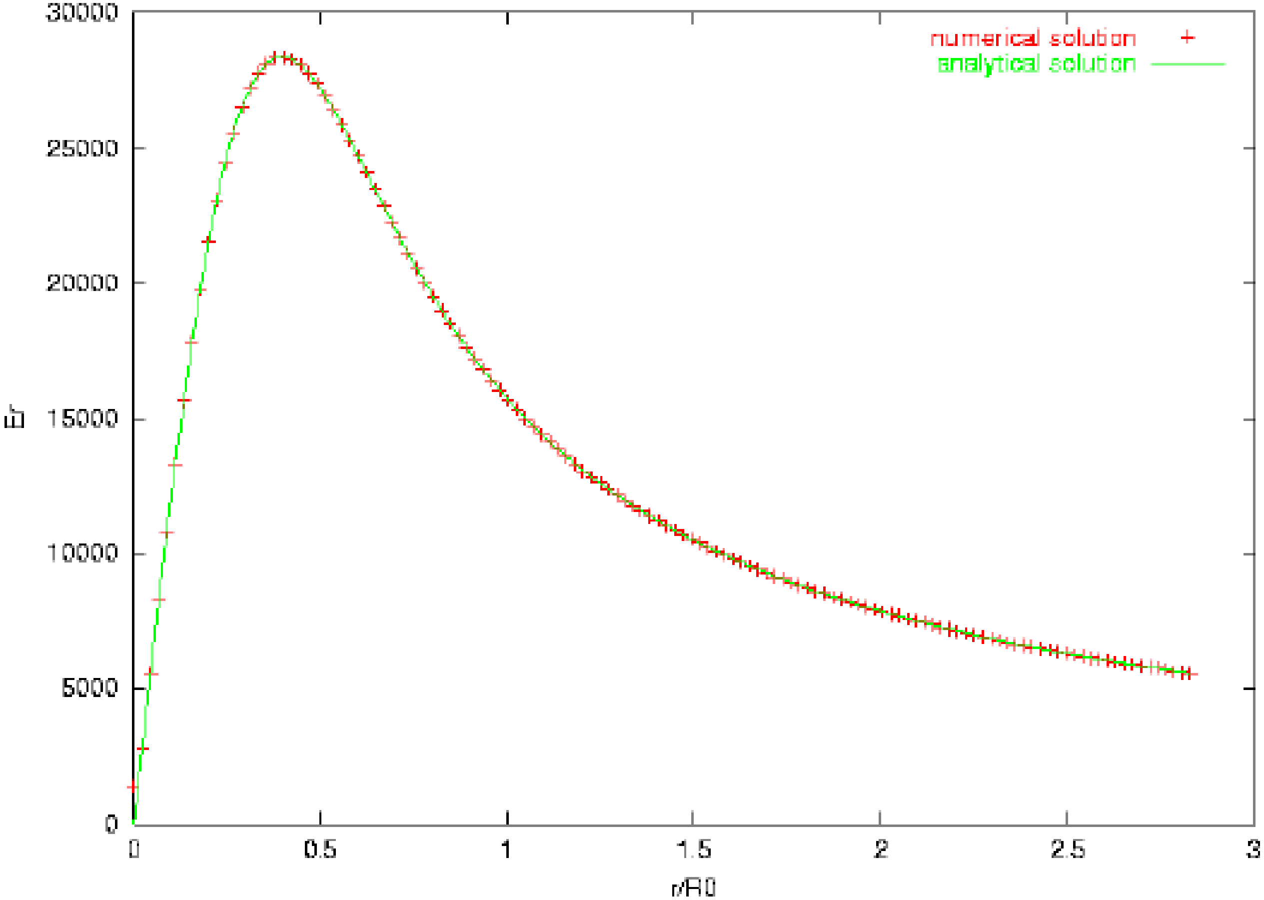
$$\phi_F(r) = \int G_s(r,r')\rho(r')dr'$$

$$G_s(r,r') = G(r + r_s, r')$$

Comparison between Numerical Solution and Analytical Solution

Electric Field vs. Distance inside the Field Domain with Gaussian Density Distribution

E_x



radius

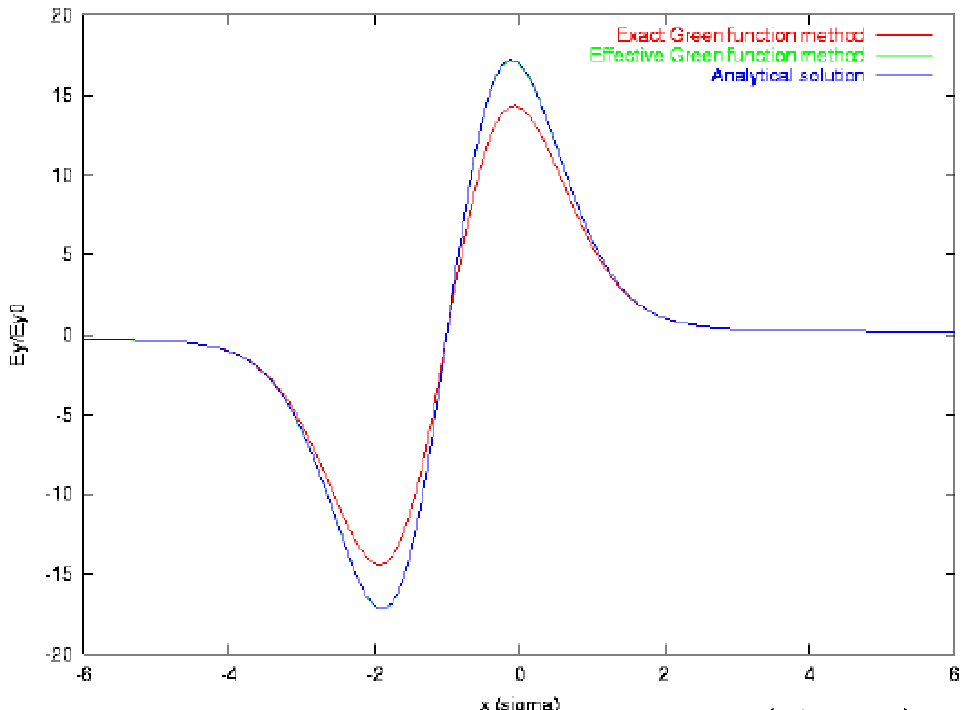
Green Function Solution of Poisson's Equation

<u>*Integrated Green function Algorithm for large aspect ratio:*</u>

$$\phi_c(r_i) = \sum_{i'=1}^{2N} G_i(r_i - r_{i'})\rho_c(r_{i'})$$

$$\boxed{G_i(r,r') = \oint G_s(r,r')\,dr'}$$

E_y



x (sigma)

Spectral-finite difference solution of Poisson's equation scale as N²logN (cont'd)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\phi\right)+\frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2}\phi\right)=-\frac{\rho}{\varepsilon_0}$$

$$\phi(r,\theta)=\sum\phi_m(r)e^{-im\theta}$$

$$\rho(r,\theta)=\sum\rho_m(r)e^{-im\theta}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\phi_m\right)-\frac{m^2}{r^2}\phi_m=-\frac{\rho_m}{\varepsilon_0}\quad for\quad r\le a$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\phi_m\right)-\frac{m^2}{r^2}\phi_m=0\quad for\quad r>a$$

Spectral-finite difference solution of Poisson's equation

$$For \quad r \le a:$$

$$\left(\frac{1}{h^2}+\frac{1}{hr}\right)\phi_m^{n+1}-\left(\frac{2}{h^2}+\frac{m^2}{r^2}\right)\phi_m^{n}+\left(\frac{1}{h^2}-\frac{1}{hr}\right)\phi_m^{n-1}=-\frac{\rho_m}{\varepsilon_0};$$

$$\frac{\partial}{\partial r}\phi_m = 0 \quad for \quad r=0 \quad and \quad m=0$$

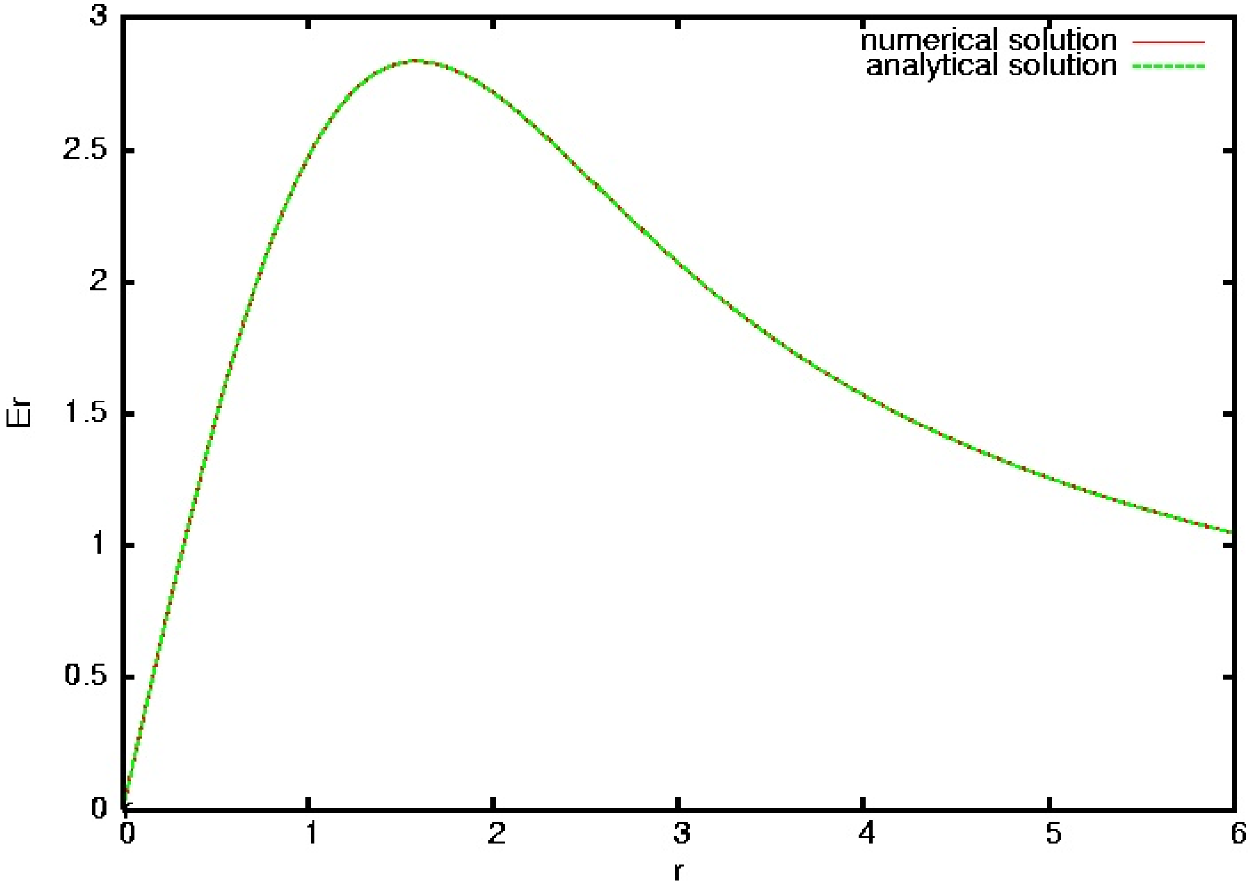
$$\phi_m = 0 \quad for \quad r=0 \quad and \quad m>0$$

$$For \quad r \ge a:$$

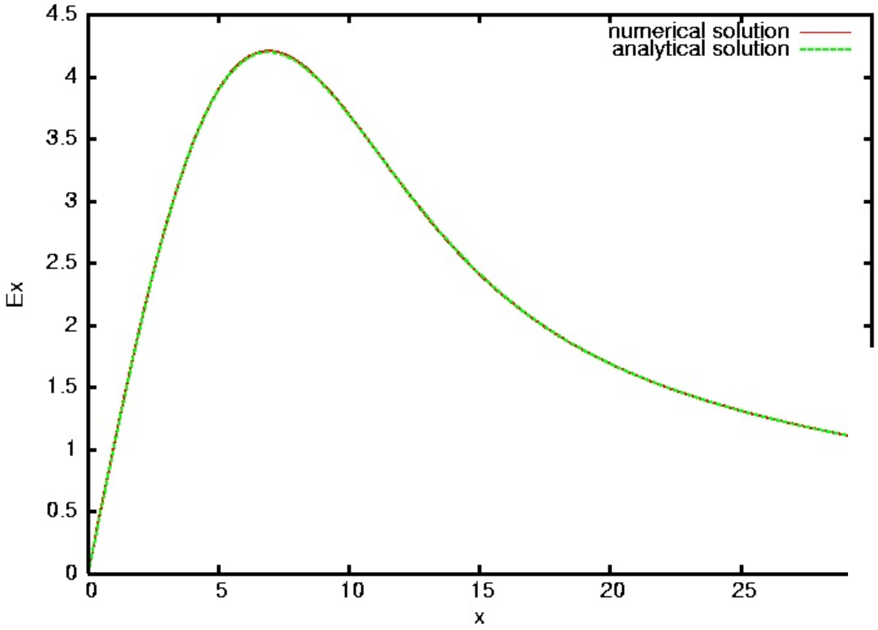
$$\phi = c\, r^{-m} \qquad m>0$$

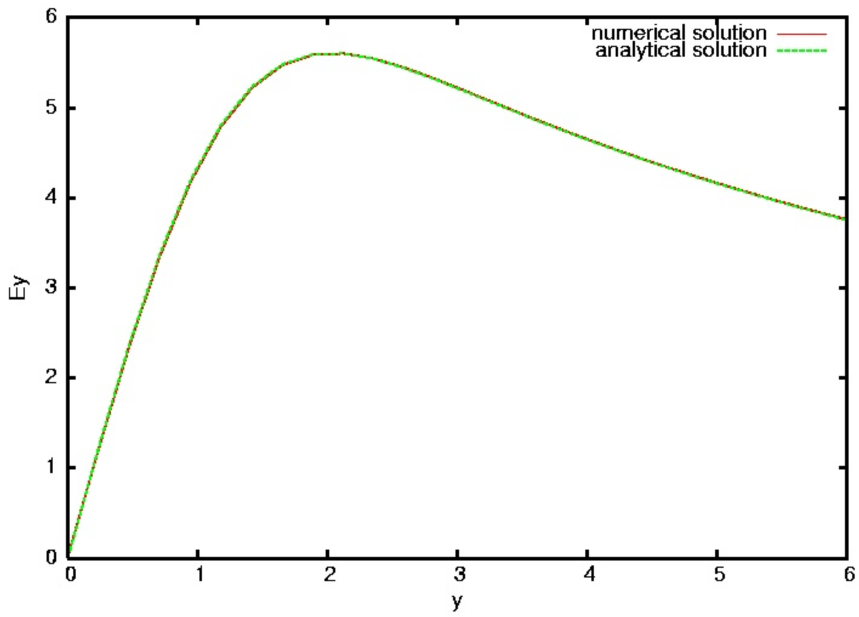
$$\phi = c \ln(r) \qquad m=0$$

Gaussian density distribution with aspect ratio of 1



Gaussian density distribution with aspect ratio of 5

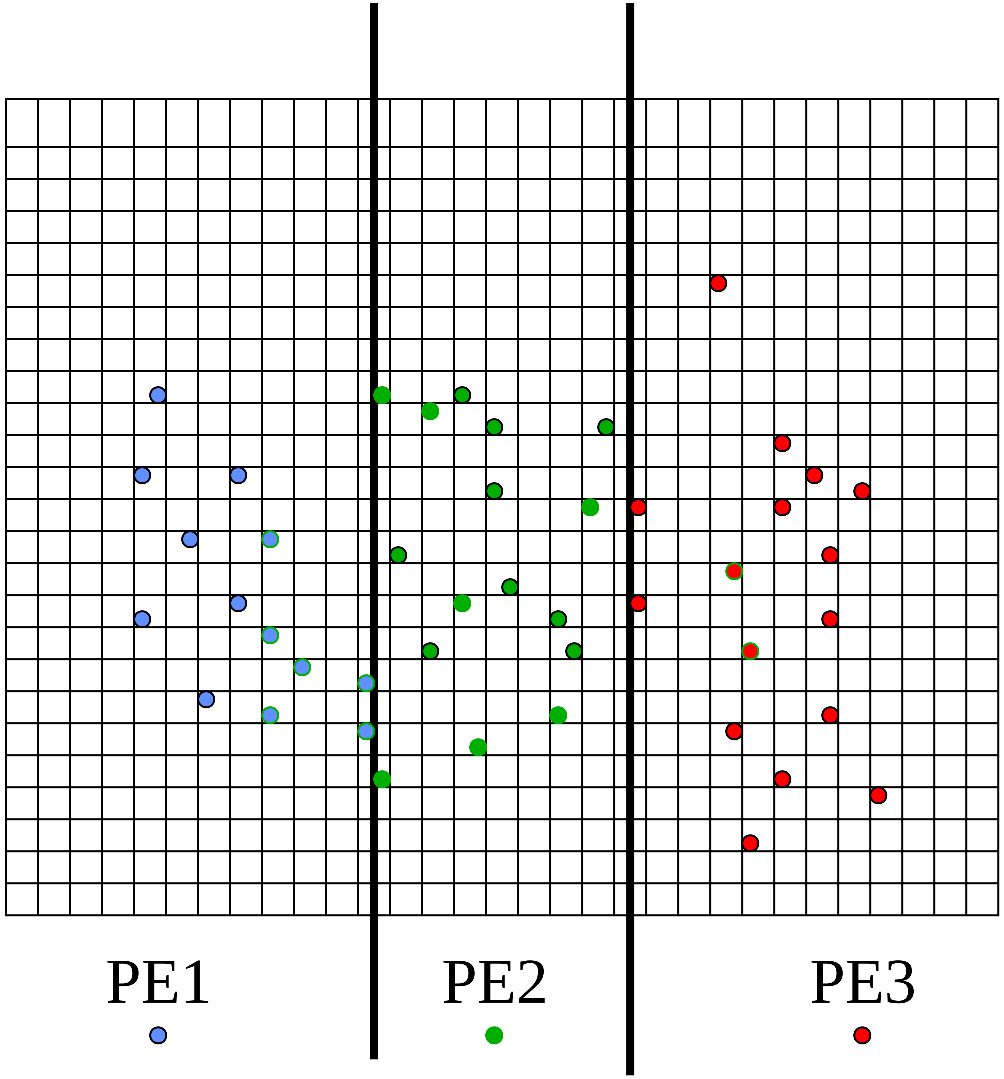




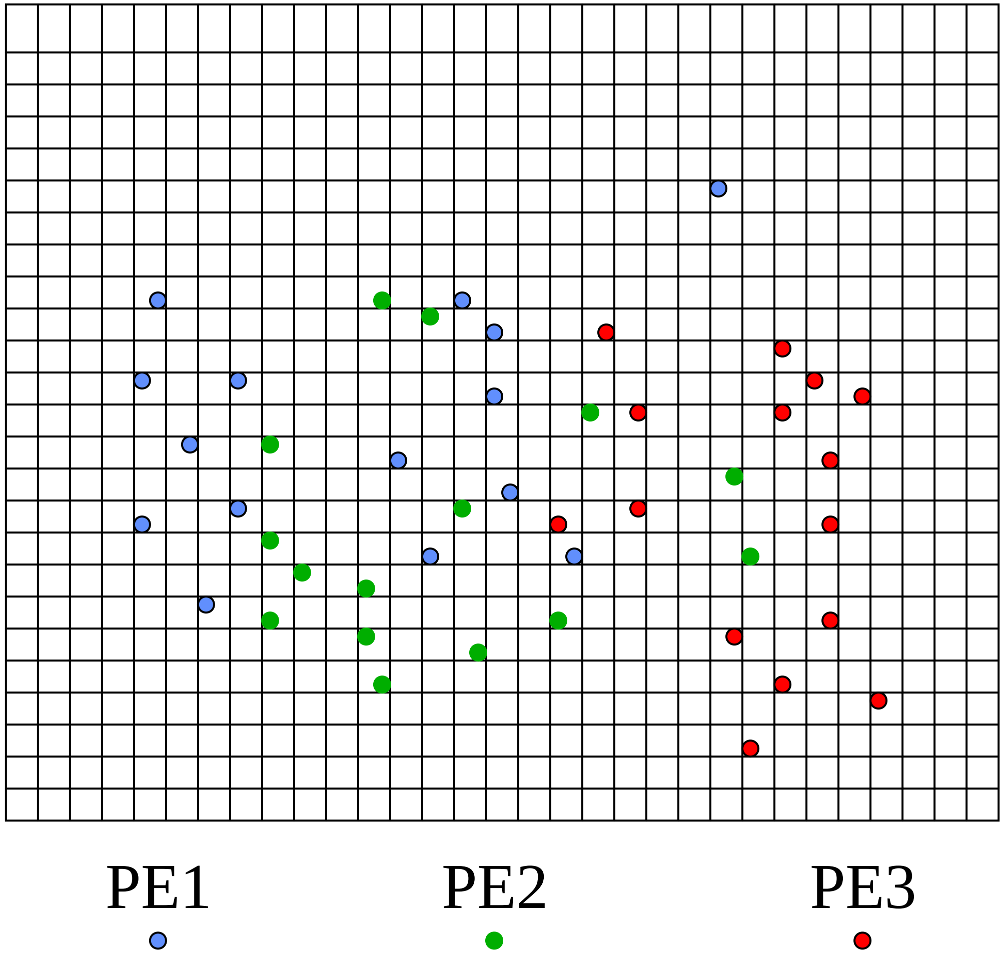
Parallel Implementation

- Uniformly distribute particles among processors
- Uniformly distribute the field domain among processors
- Exchange the local charge density among processors
- Solve the Poisson equation in parallel
- Collect the potential from the other processors

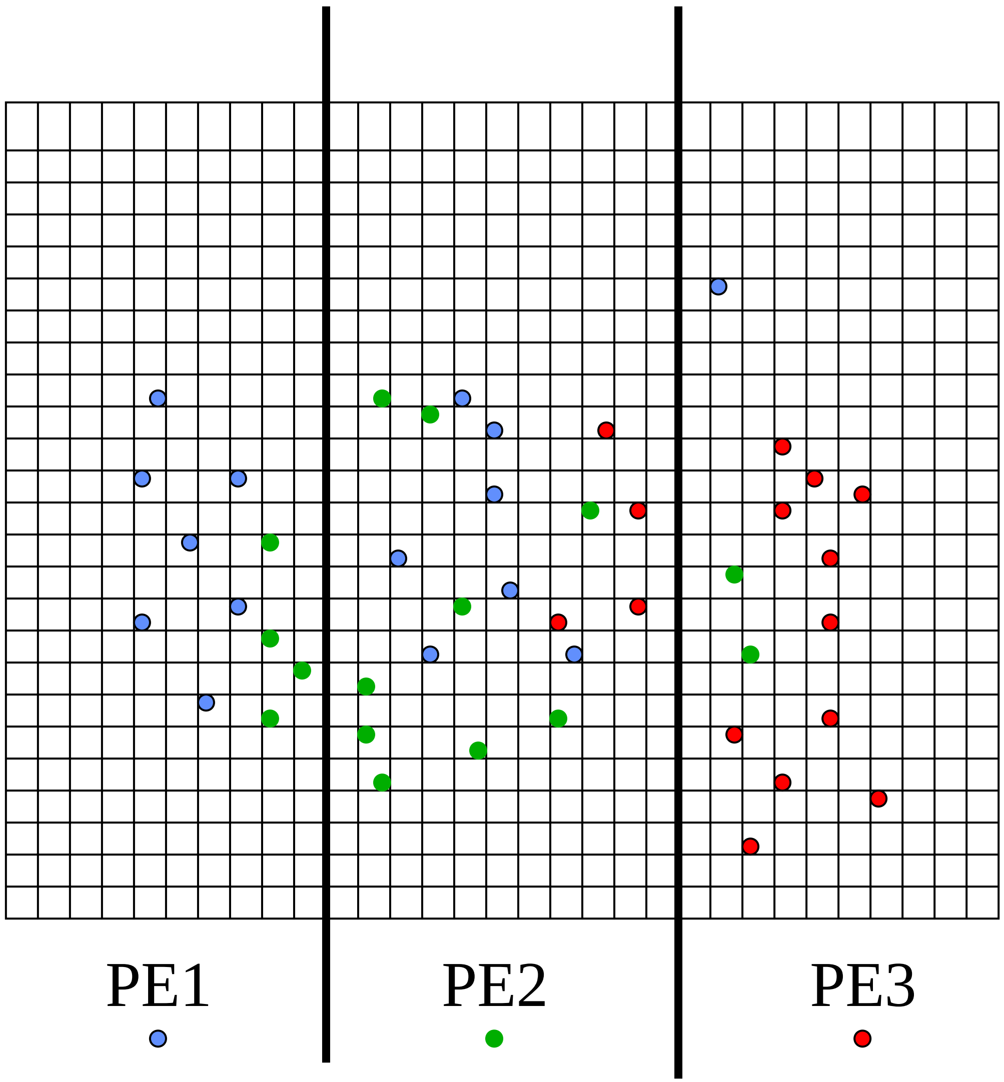
Domain Decomposition



Particle Decomposition



Particle and Field Decomposition

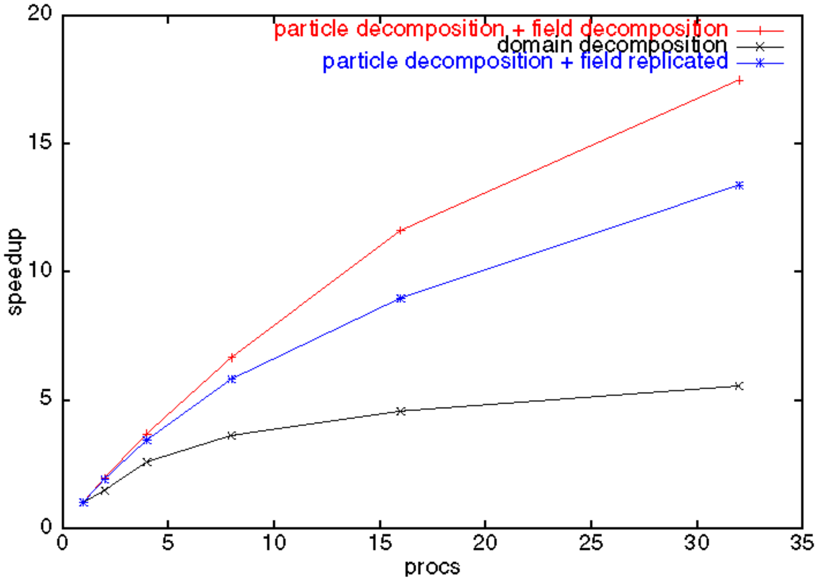


Parallel Implementation Issues: Performance Counts!

- **Example: Scaling of BeamBeam3D**

# of processors	execution time (sec)
128	1612
256	858
512	477
1024	303
2048	212

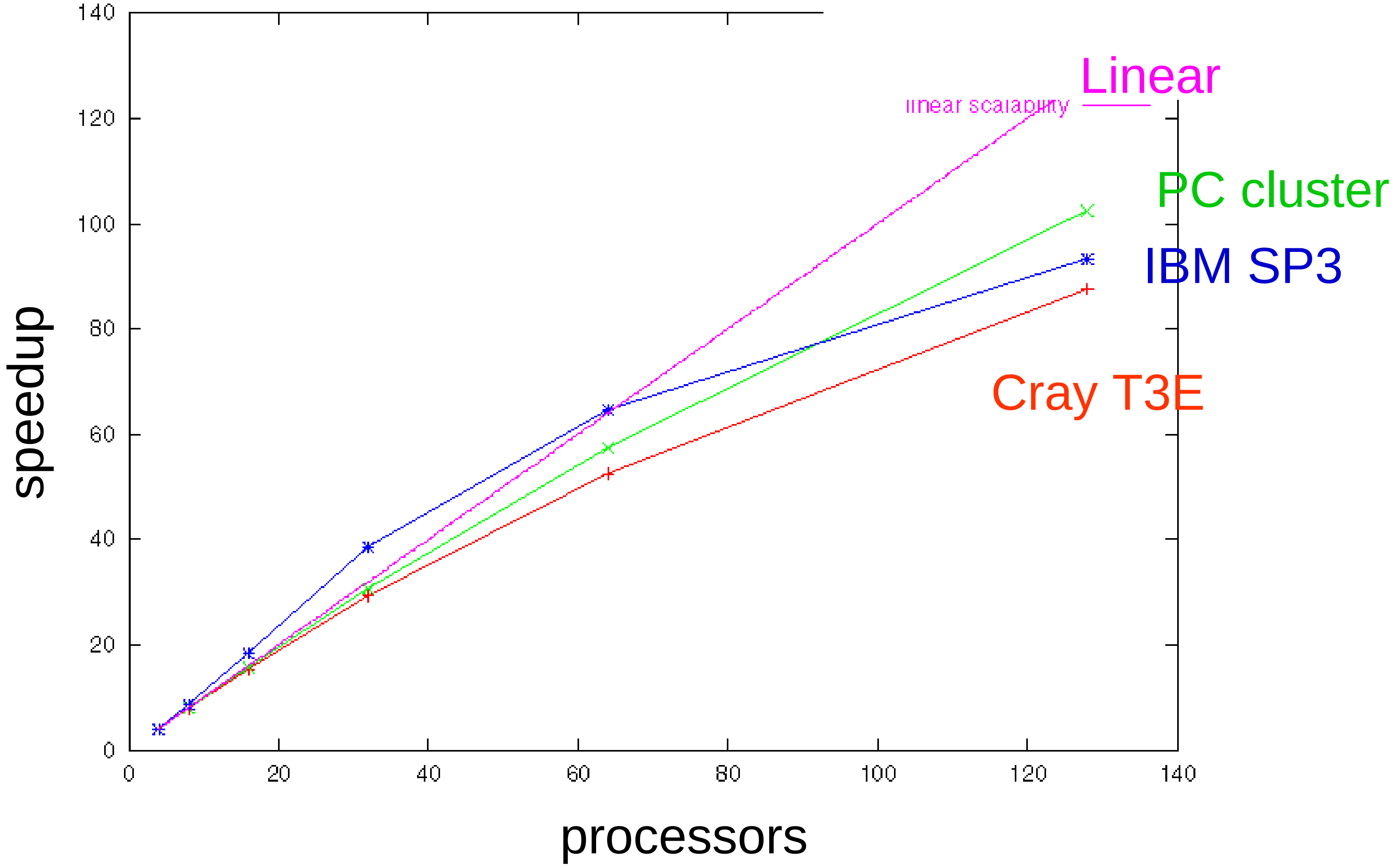
Scaling using weak-strong option



Performance of different parallelization techniques in strong-strong case

Strong-strong beam-beam will be crucial to LHC Optimization

Parallel Performance on IBM SP3, Cray T3E, and PC Cluster



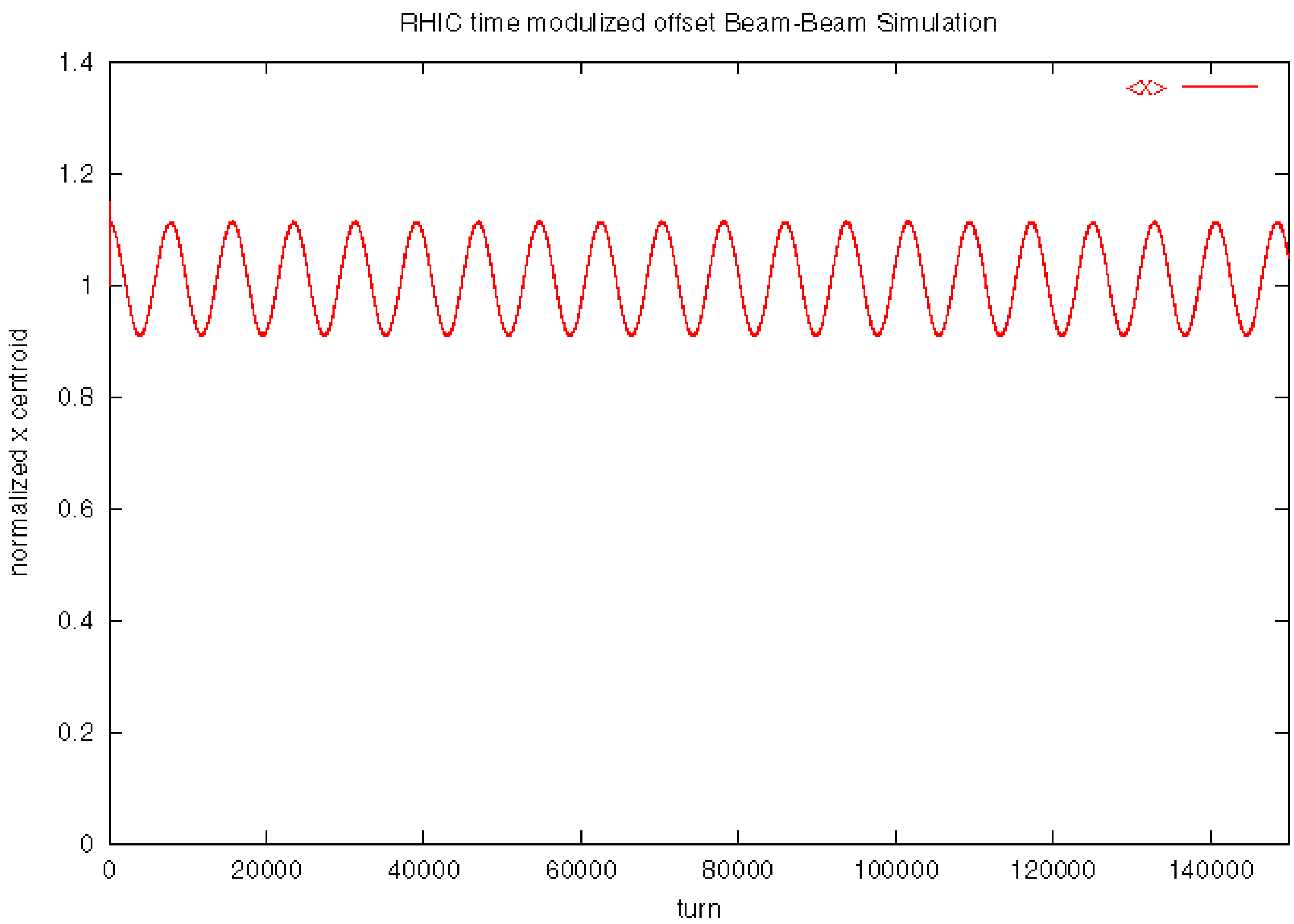
BeamBeam3D:
Parallel Strong-Strong / Strong-Weak Simulation Code

- Multiple physics models:
  - strong-strong (S-S); weak-strong (W-S)
- Multiple-slice model for finite bunch length effects
- New algorithm -- shifted Green function -- efficiently models long-range parasitic collisions
- Parallel particle-based decomposition to achieve perfect load balance
- Lorentz boost to handle crossing angle collisions
- W-S options: multi-IP collisions, varying phase adv,…
- Arbitrary closed-orbit separation (static or time-dep)
- Independent beam parameters for the 2 beams

RHIC Physical Parameters for the Beam-Beam Simulations

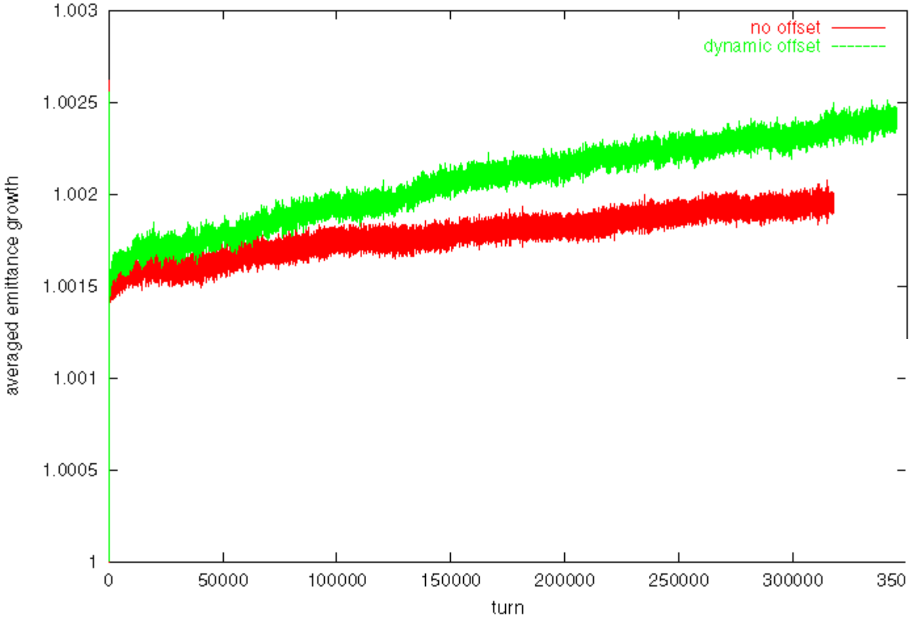
Beam energy (GeV)	23.4
Protons per bunch	8.4e10
Beta (m)	3
Rms spot size (mm)	0.629
Betatron tunes	(0.22,0.23)
Rms bunch length (m)	3.6
Synchrotron tune	3.7e-4
Momentum spread	1.6e-3
Offset	1 sigma
Oscillation frequency	10 Hz

Horizontal Centroid Oscillation

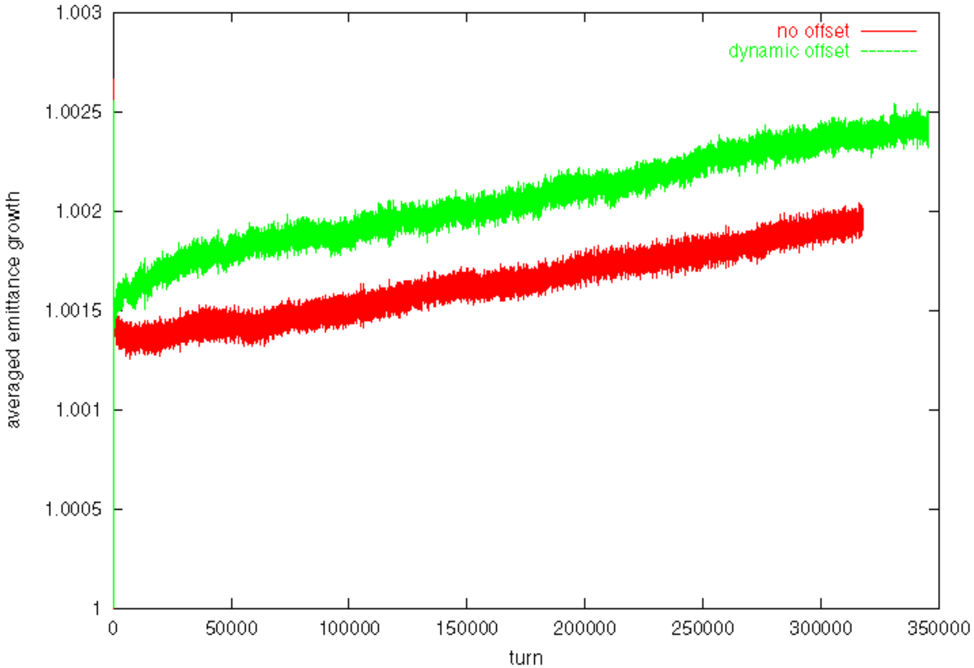


Averaged emittance growth

Beam 1



Beam 2

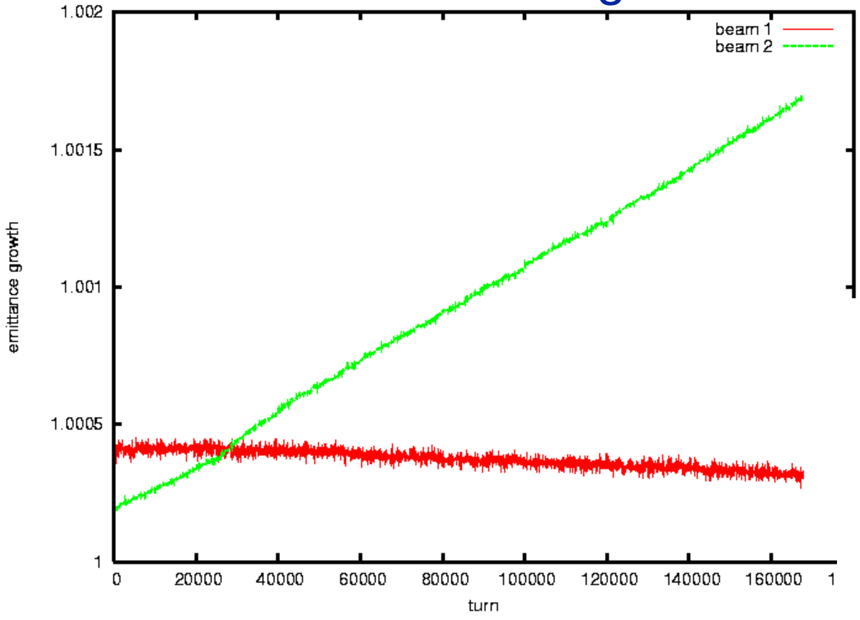


Nominal LHC Physical Parameters

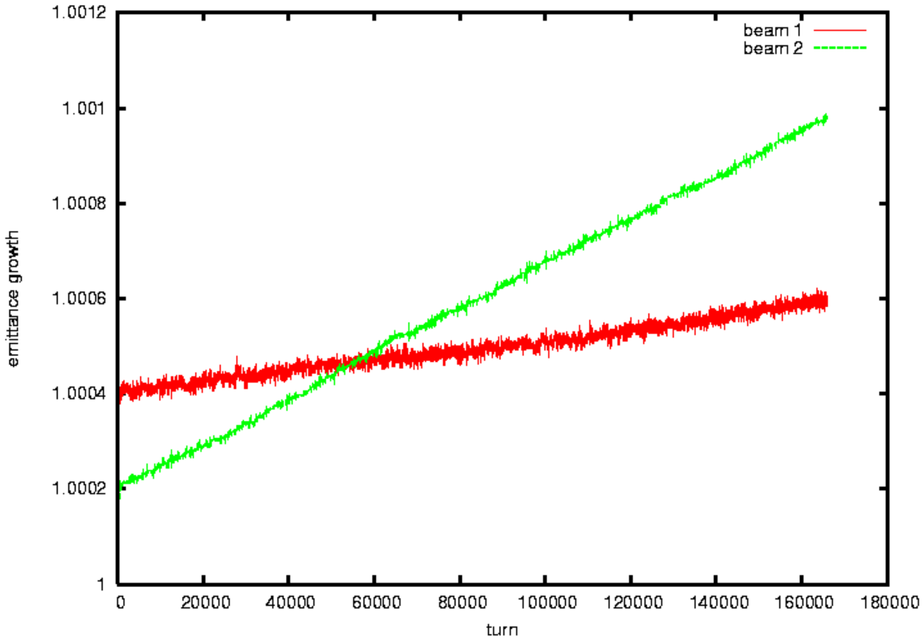
Beam energy (TeV)	7
Protons per bunch	1.05e11
Beta (m)	0.5
Rms spot size (um)	15.9
Betatron tunes	(0.31,0.32)
Rms bunch length (m)	0.077
Synchrotron tune	0.0021

Emittance Growth with Mismatched Beam-Beam Collisions at LHC

without detuning

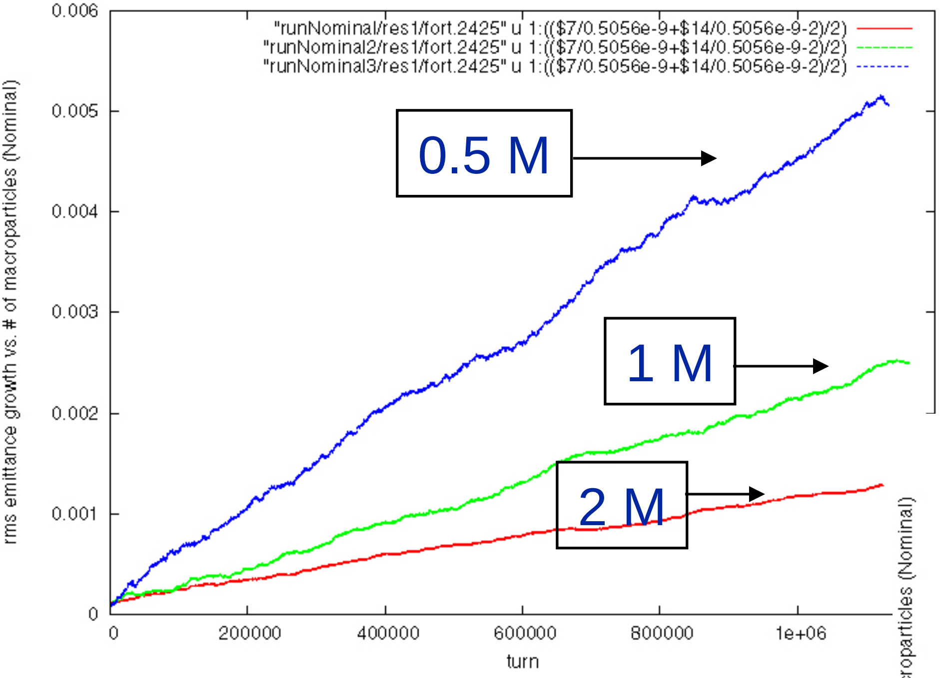


with detuning



Averaged X and Y rms emittance growth vs. # of macropaticles– nominal case

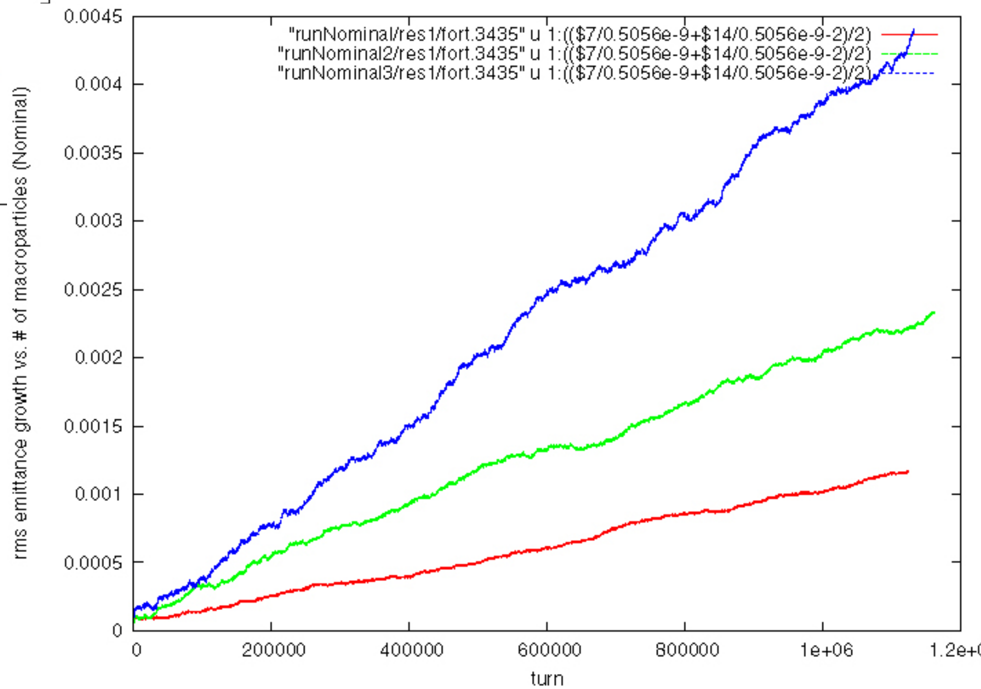
Beam 1



estimated emittance growth

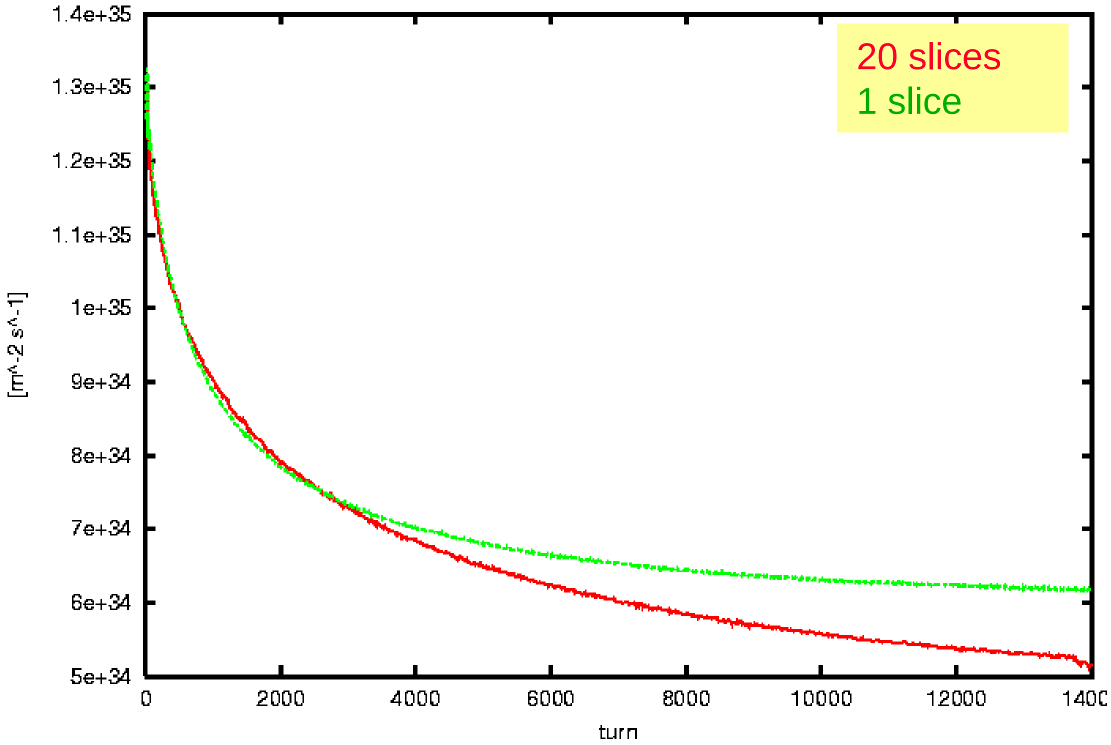
$$\varepsilon / \varepsilon_0 = 0.0015 + 0.0003 / N^{0.87} T$$

Beam 2



Beam-Beam Studies of PEP-II

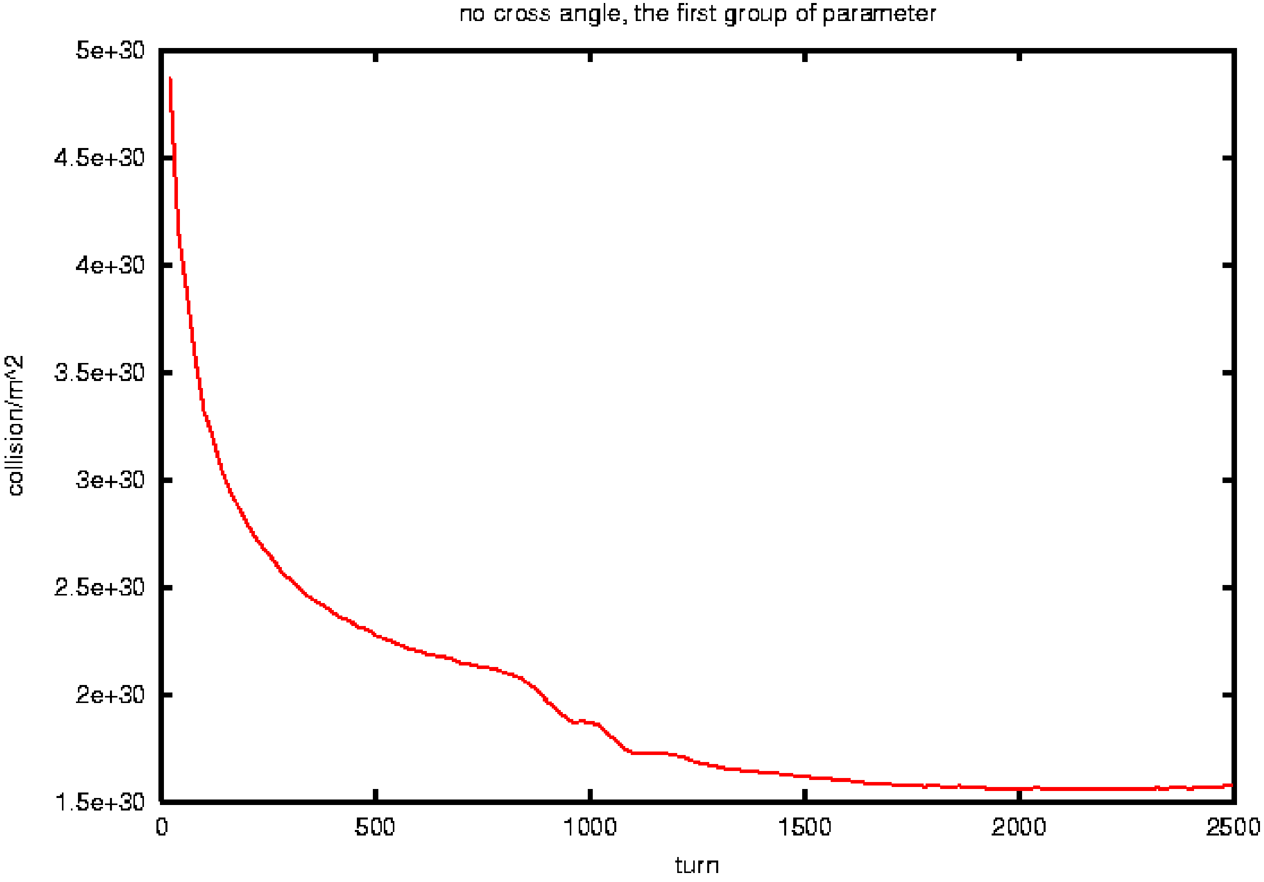
- **Collaborative study/comparison of beam-beam codes (J. Qiang/LBNL, Y. Cai/SLAC, K. Ohmi/KEK)**
- **Predicted luminosity sensitive to # of slices used in simulation**



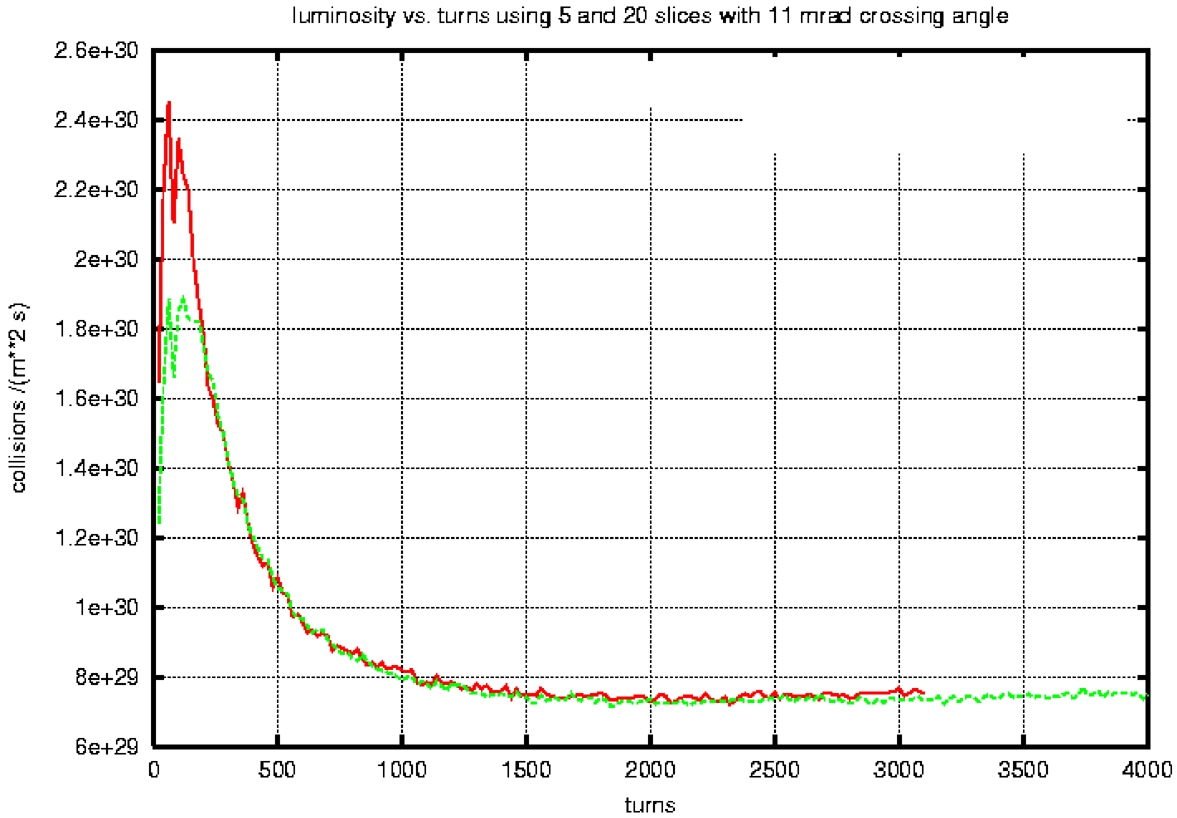
KEKB Physical Parameters

Beam energy (GeV)	8.0/3.5
Particles per bunch	$4.375e^{10}/10.0e^{10}$
Beta (m)	0.6/0.007/10.0
Emittance (m-rad)	$1.8e^{-18}/1.8e^{-18}/4.8e^{-6}$
Betatron tunes	(0.5151,0.5801)
Synchrotron tune	0.016
Damping time (/turn)	$2.5e^{-4}/2.5e^{-4}/5.0e^{-4}$

Single Collision Luminosity vs. Turn (head-on collision)



Single Collision Luminosity vs. Turn (11mrad crossing angle)



Future work

- Optimize the multiple slice model
- Include the nonlinear realistic lattice
- Studies of long range effects/wire compensation at RHIC
- Studies of the emittance growth and halo formation at LHC

Acknowledgements

- M. Furman, R. Ryne, W. Turner - LBNL
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