Collective Beam-Beam Effects In Hadron Colliders

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Thanks: DOE

OUTLINE

- Linear Coherent Beam-Beam Effect
- Nonlinear Coherent Beam-Beam Effect
- Beam-Beam Simulation for Hadron Beams
- Final Comments

Equation for Beam Particle Distributions

Consider one IP in a collider. Neglecting intra-beam collisions, transverse Hamiltonian for beam particles can be written as

$$H^{(i)}(\vec{I},\vec{\phi},t) = \vec{\nu}\cdot\vec{I} + U_{err}(\vec{r},t) + U_{bb}^{(j)}(\vec{r})\sum_n \delta(t-t_n)$$

$$U_{bb}^{(j)}(\vec{r}) \sim \int \frac{1}{|\vec{r}-\vec{r'}|} f_j(\vec{r'},\vec{p'},t)\, d\vec{r'}d\vec{p'}$$

where $i, j = 1$ or 2, and $i \neq j$. For single-particle distributions,

$$\frac{\partial f_i}{\partial t} + \vec{\nu}\cdot\frac{\partial f_i}{\partial\vec{\phi}} = \frac{\partial U_{err}}{\partial\vec{r}}\cdot\frac{\partial f_i}{\partial\vec{p}} + \frac{\partial U_{bb}^{(j)}}{\partial\vec{r}}\cdot\frac{\partial f_i}{\partial\vec{p}}\sum_n \delta(t-t_n)$$

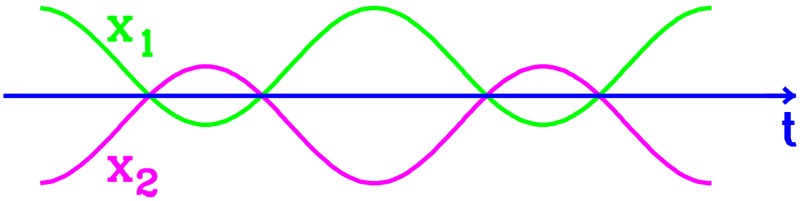
Comments on Beam-Beam Effects of Hadron Beams

Damping time scale is comparable to storage time. The motion of particles is a Hamiltonian dynamics. With nonlinear perturbations, the distributions may not reach any stationary state within a fraction of the storage time.

The problem of coherent beam-beam effects can be divided into near-linear (near-integrable) and nonlinear (non-integrable) regime based on the validity of linearized (or perturbative) Vlasov equation.

Linear Coherent Beam-Beam Effect

Stable Coherent Beam-Beam Oscillation:



In two-beams center-of-mass coordinate:

$$X_{CM} = \frac{M_1 X_1 + M_2 X_2}{M_1 + M_2}$$

$$\delta X = \frac{M_1 X_1 - M_2 X_2}{M_1 + M_2}$$

σ-mode:

X_{CM} oscillates with betatron tunes if $\nu_1 = \nu_2$.

π-mode, From Linear Theory:

δX oscillates with betatron tunes plus a coherent beam-beam tune shift.

- For symmetrical (round to flat) beams,
  $\Delta\nu \sim 1.21 - 1.33\,\xi$ (Yokoya factor).

- For unsymmetrical beams, when $\xi_1/\xi_2 > 0.55$ or $|\nu_1 - \nu_2| > \xi$, the π-mode would be damped.

$\Longrightarrow$ LINEAR THEORY: Coherent beam-beam effect is not important in the unsymmetrical or strong-weak cases.

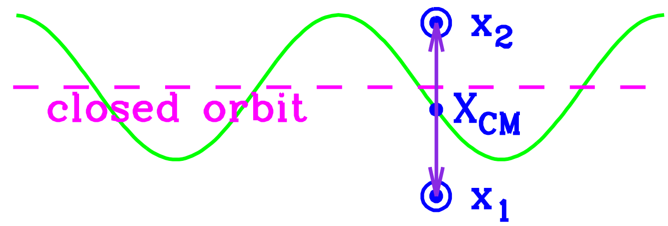
This is not right in the nonlinear regime of beam-beam interactions of hadron beams!

Stable Coherent Beam-Beam Oscillation

Consider Linear lattices with one IP. For beam centroids,

$$\ddot{X}_i + \nu_{0i}^2 \, X_i = \lambda_i \, F_x \, \delta_n(t - t_c)$$

$$i = 1, 2$$



$$\vec{F} = \int \rho_1(\vec{r}, t)\rho_2(\vec{r'}, t)\frac{\vec{r} - \vec{r'}}{|\vec{r} - \vec{r'}|^2} d\vec{r}\, d\vec{r'}$$

In the two-beams center-of-mass coordinate,

$$\begin{cases} \ddot{X}_{CM} + \frac{1}{2}\left(\nu_{01}^2 + \nu_{02}^2\right) X_{CM} + \frac{1}{2}\left(\nu_{01}^2 - \nu_{02}^2\right) \delta X & = 0 \\ \delta \ddot{X} + \frac{1}{2}\left(\nu_{01}^2 + \nu_{02}^2\right) \delta X + \frac{1}{2}\left(\nu_{01}^2 - \nu_{02}^2\right) X_{CM} & = \lambda \, F_x \end{cases}$$

Rigid-Beam Approximation:

Except the centers of the distributions, the shapes of the distributions do not change with time during the beam oscillation. To the lowest-order of X_i,

$$F_x \sim \left[\left(1 + \frac{\xi_1}{\xi_2}\right) \delta X + \left(1 - \frac{\xi_1}{\xi_2}\right) X_{CM}\right] + O(\text{high-order-moments})$$

Neglect time-dependences of high-order moments, for matched beams

If $|\nu_{01} - \nu_{02}| > \xi_1, \, \xi_2,$ $\quad \Delta\nu_i = \frac{1}{2}\xi_i$

If $\nu_{01} = \nu_{02},$ $\quad \Delta\nu_i = \frac{1}{2}(\xi_1 + \xi_2)$

Coherent Beam-Beam Tune Shift From Linearized Vlasov Equation

— K. Yokoya

Vlasov equation for beam-beam interaction in one dimension:

$$\frac{\partial f_1}{\partial t} + \nu_{01}\frac{\partial f_1}{\partial \phi} = \epsilon\,\frac{\partial f_1}{\partial p}\int \frac{f_2(x',p',t)}{x-x'}dx'dp'$$

$$x = \sqrt{2I}\cos\phi, \qquad p = \sqrt{2I}\sin\phi.$$

Linearization:

Let $\quad f_j(x,p,t) = f_{0j}(x,p) + \Psi_j(x,p,t), \quad j = 1,2$

$$\frac{\partial \Psi_1}{\partial t} + \nu_{01}\frac{\partial \Psi_1}{\partial \phi} = \epsilon\left[\frac{\partial \Psi_1}{\partial p}\int \frac{f_{02}(x',p')}{x-x'}dx'dp' + \frac{\partial f_{01}}{\partial p}\int \frac{\Psi_2(x',p',t)}{x-x'}dx'dp'\right]$$

Single-mode (dipole) approximation:

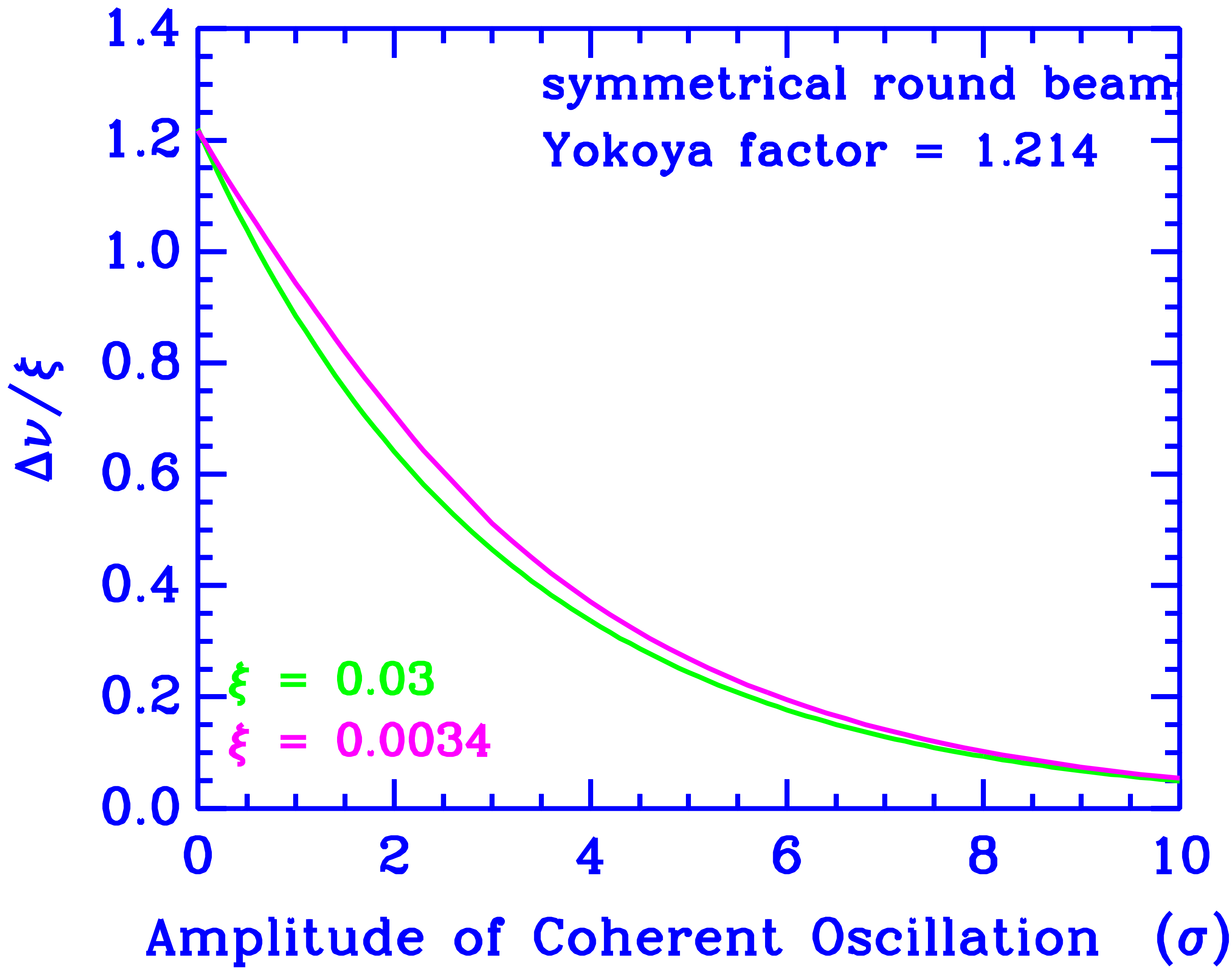
Let $\quad \Psi_j(I,\phi,t) = \sum_m \int F_{jm}(I,\nu)\, e^{-i\nu t}\, e^{im\phi}\, d\nu$

Keep only the $m = 1$ (dipole) mode,

$$\Longrightarrow \quad \Delta\nu = Y(\xi_1 + \xi_2)/2 \quad \text{when } \nu_{01} = \nu_{02}$$

Yokoya factor: $\quad Y = 1.21$ — 1.33

Coherent Beam-Beam Tune Shift From Beam-Beam Simulation



HERA Accelerator Study 2000

——— G. Hoffstaetter, *et. al.*

The experiment studied the beam-beam effect in HERA at a very large beam-beam tune shift.

$\xi_{e,y}$ was increased by increasing $\beta_{e,y}$

$$\xi_{e,y} = \frac{r_e N_p}{2\pi\gamma_e} \frac{\beta^*_{e,y}}{\sigma^*_{p,y}(\sigma^*_{p,x} + \sigma^*_{p,y})}$$

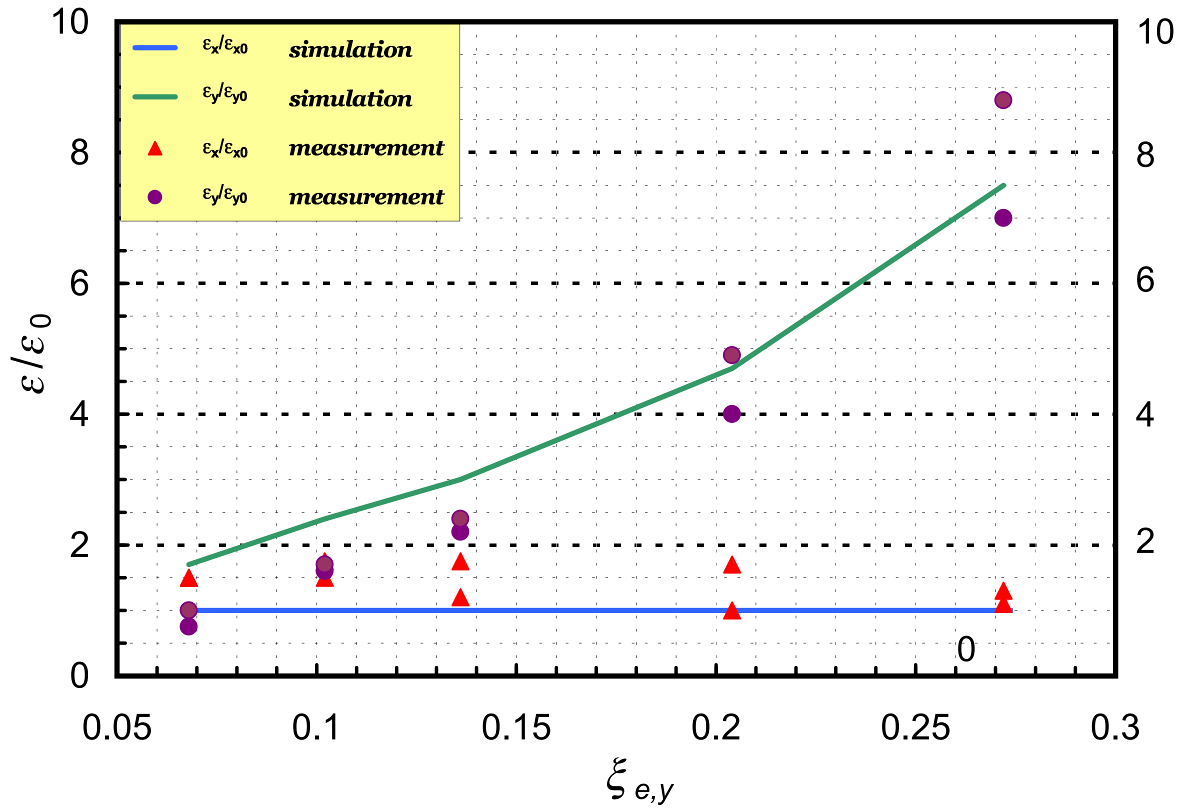
$\beta^*_{e,y}(m)$	I_{e^+} (mA)	$\sigma^*_{e,y}(\mu m)$	$\xi_{e,x}/\xi_{e,y}$	$\xi_{p,x}/\xi_{p,y}$ (10^{-4})
1.0	19	35.8	0.041/0.068	2.54/1.40
1.5	18	43.8	0.041/0.102	2.35/1.06
2.0	17	50.6	0.041/0.136	2.18/0.85
3.0	3.5	62.0	0.041/0.204	0.43/0.14
4.0	2.6	72.0	0.041/0.272	0.31/0.09

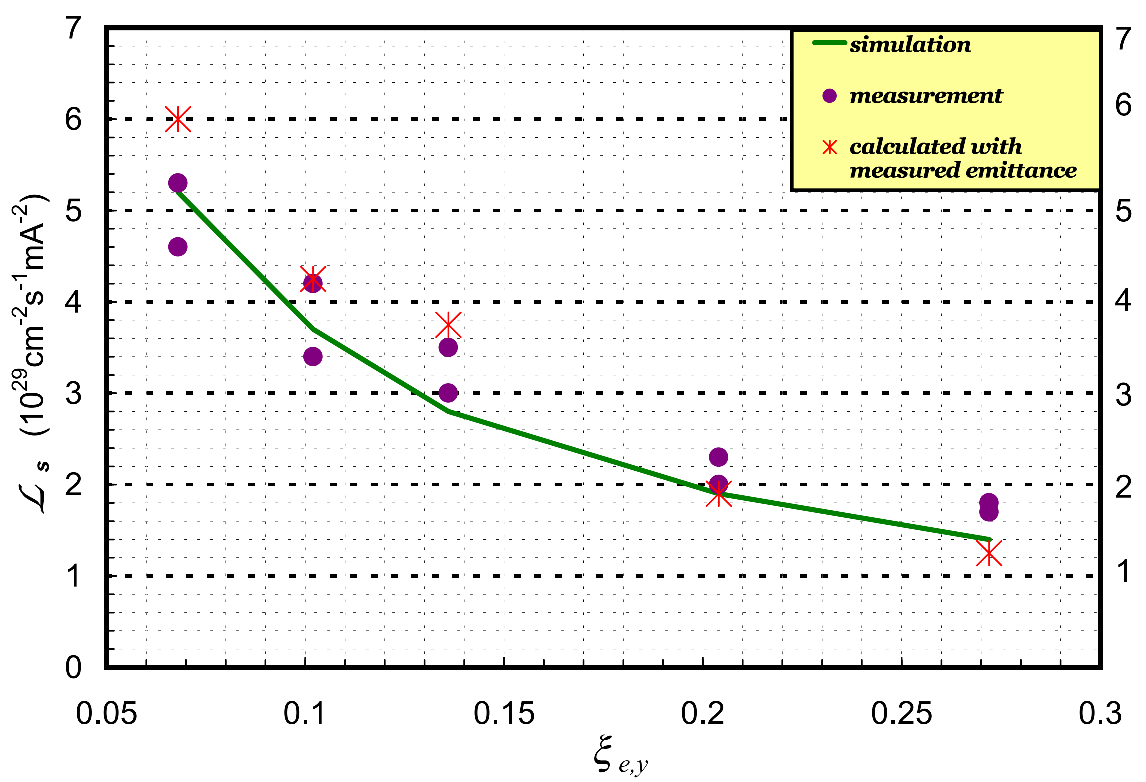
Some of Other Parameters in HERA 2000 Study

	Positron	Proton
E (GeV)	27.5	920
N_{tot}/N_{col}	189/174	180/174
I (mA)	I_{e^+}	90
$\beta^*_x/\beta^*_y(m)$	$2.5/\beta^*_{e,y}$	7/0.5
$\sigma^*_x/\sigma^*_y(\mu m)$	$283/\sigma^*_{e,y}$	164/39.9
$\epsilon_y/\epsilon_x(nm)$	32/1.28	3.82/3.18
ν_x/ν_y	0.169/0.246	0.291/0.297

HERA 2000 STUDY: Experiment and Simulation

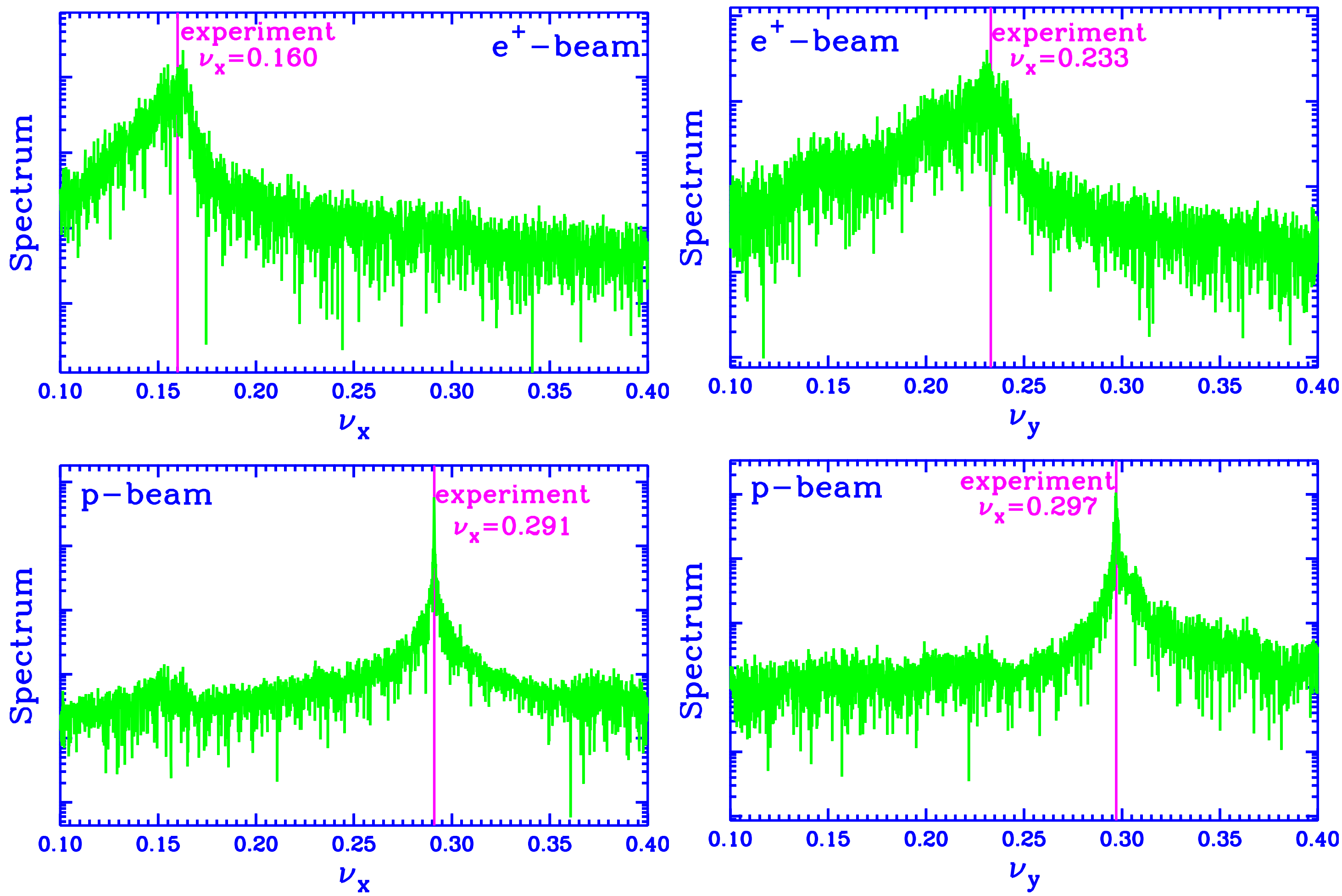
Emittance of the e^+ Beam and Specific Luminosity *vs.* $\beta^*_{e,y}$





COMPARISON OF HERA 2000 STUDY & SIMULATION

Coherent Modes When $\vec{\xi}_e = (0.041,\ 0.272)$

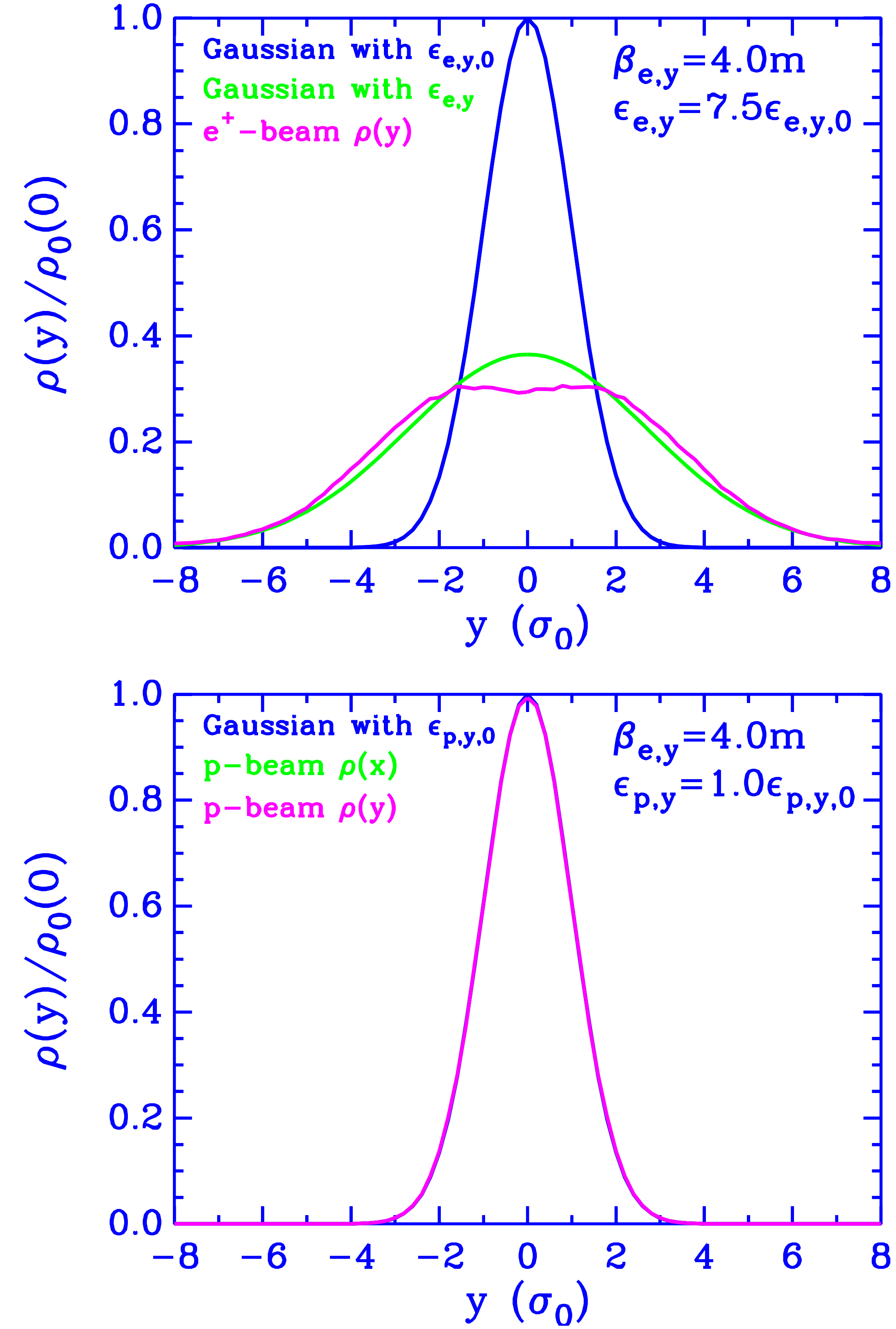


	$\nu_{e,x}$	$\nu_{e,y}$	$\Delta\nu_{e,x}/(2\xi_{e,x})$	$\Delta\nu_{e,y}/(2\xi_{e,y})$
Experiment	52.160	52.233	0.110	0.024
Simulation	52.162	52.232	0.085	0.026
Rigid Beam	52.156	52.227	0.159	0.035
Lattice tune	52.169	52.246		

"Rigid Beam": calculated by using the derived formula based on rigid Gaussian beams.

Simulation for HERA 2000 Study

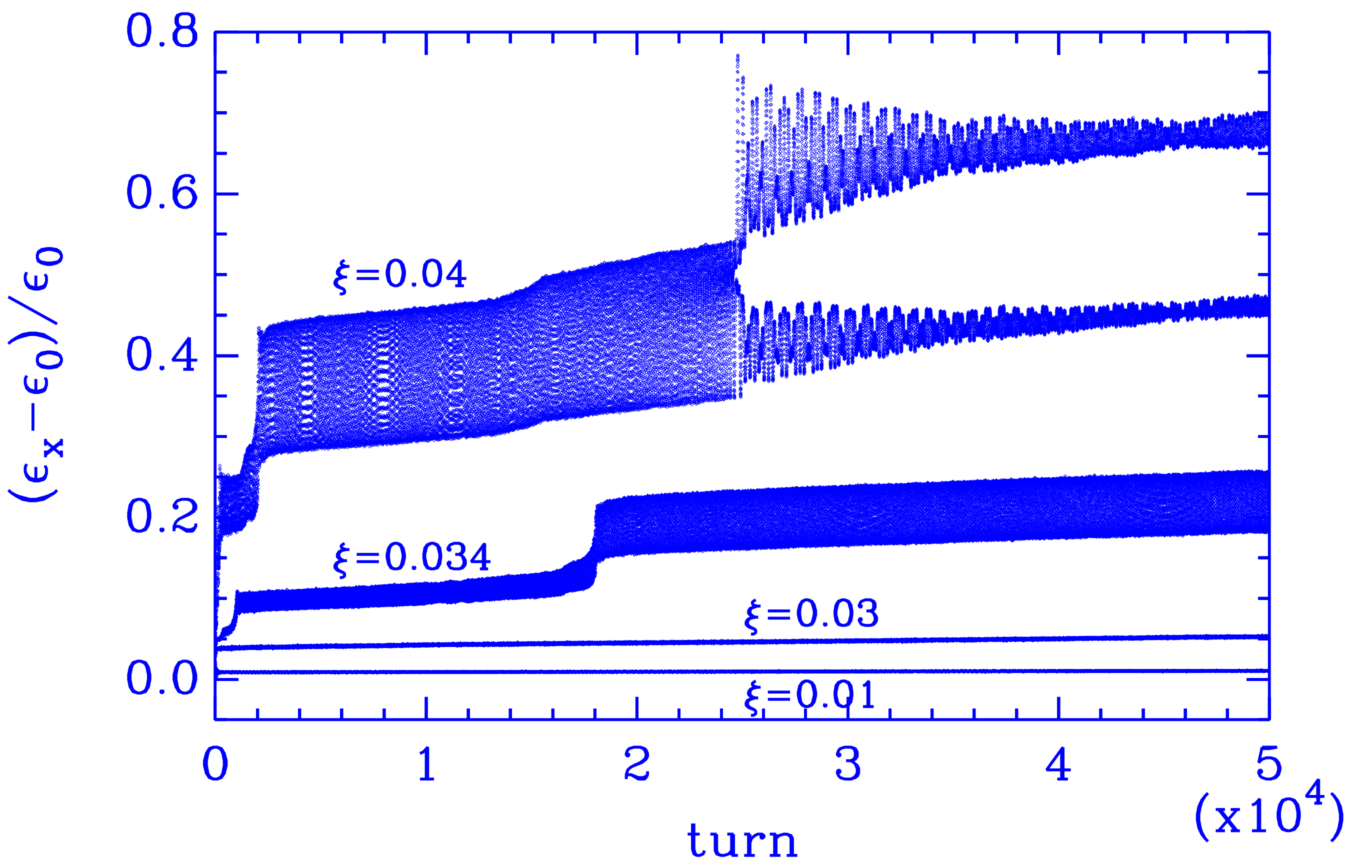
Beam Particle Distributions at $\beta^*_{e,y} = 4.0$ m



Nonlinear Coherent Beam-Beam Effects
(Coherent Beam-Beam Instability)

EMITTANCE GROWTH FOR LHC WITH ν_x=0.31, ν_y=0.32

The beam−beam instability occurs when $\xi > \xi_c \approx 0.03$.



Coherent Beam-Beam Instability of Hadron Beams

Stable Coherent Oscillation When $\xi < \xi_c$

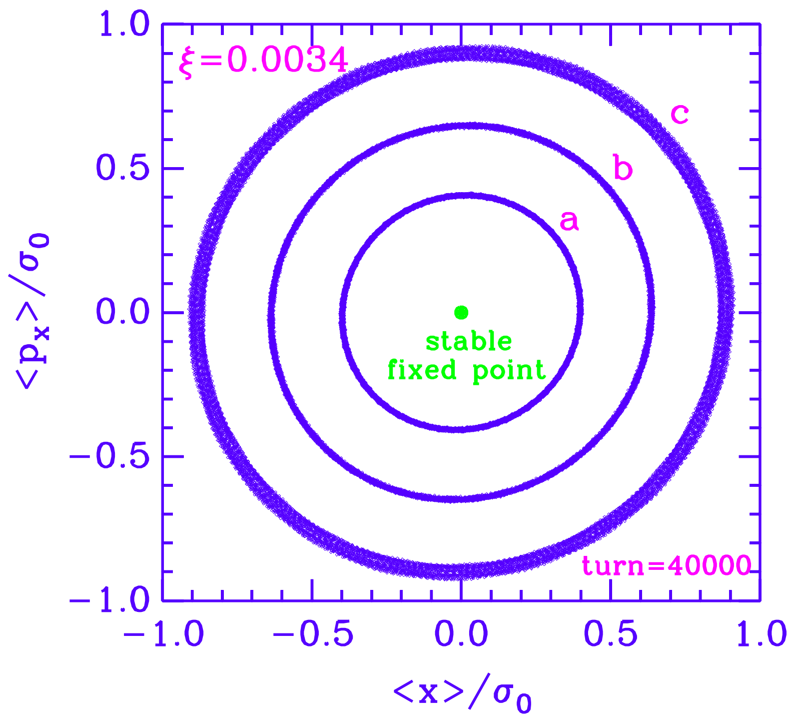
The origin of phase space is stable for coherent oscillation.

Symmetrical Beams:

- Coherent oscillations are stable.

Unsymmetrical Beams:

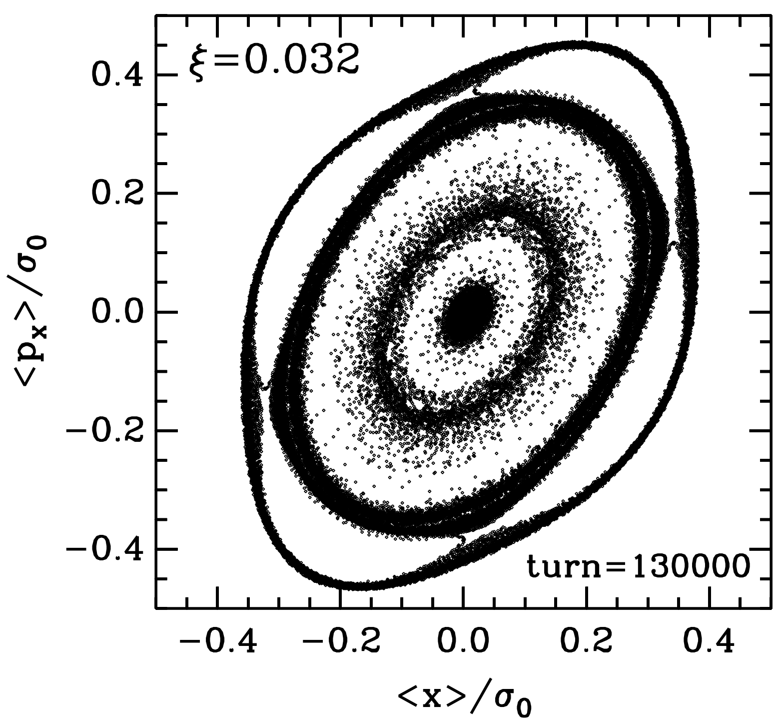
- Landau damping could suppress coherent motions but result in a emittance increase.



Chaotic Coherent Oscillation When $\xi > \xi_c$

The origin of phase space is unstable for coherent motion.

- Coherent oscillations are chaotic.
- Onset of collective beam-beam instability could occur with both strong-strong and strong-weak beam-beam interactions.



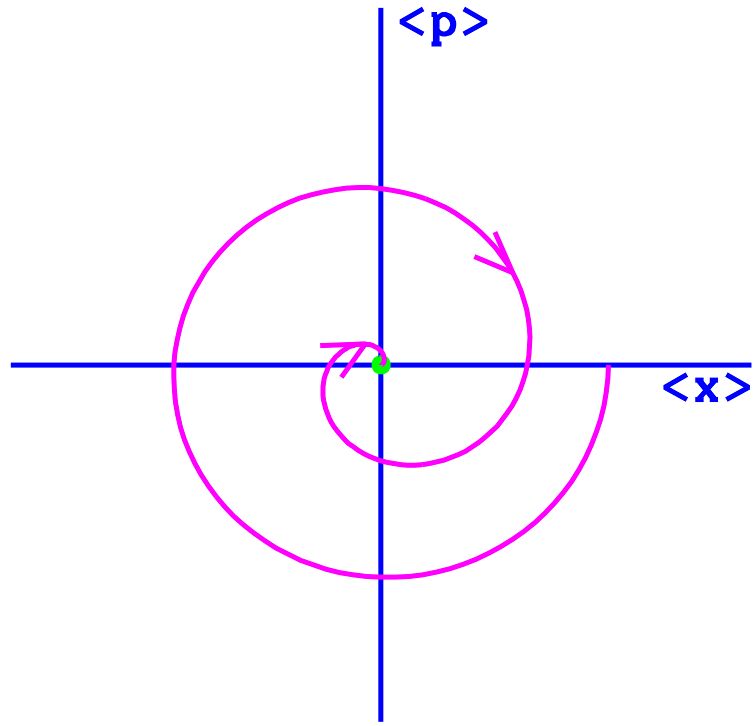
Coherent Beam-Beam Instability of Lepton Beams

——— **A.W. Chao and R.D. Ruth**

$\xi < \xi_c$,

- **The origin of the phase space for beam-centroid motion is a stable fixed point.**

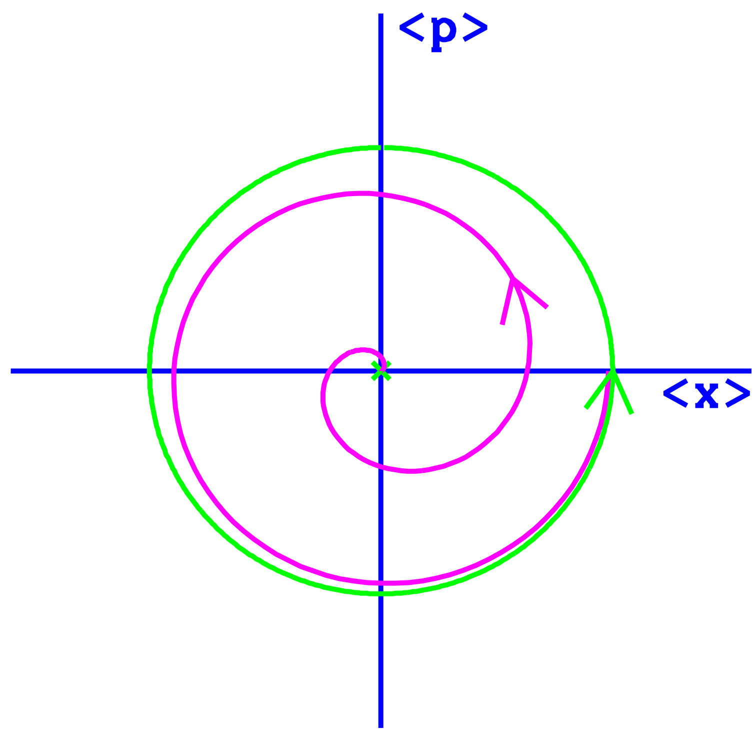
- **Damped coherent oscillation due to radiation damping.**



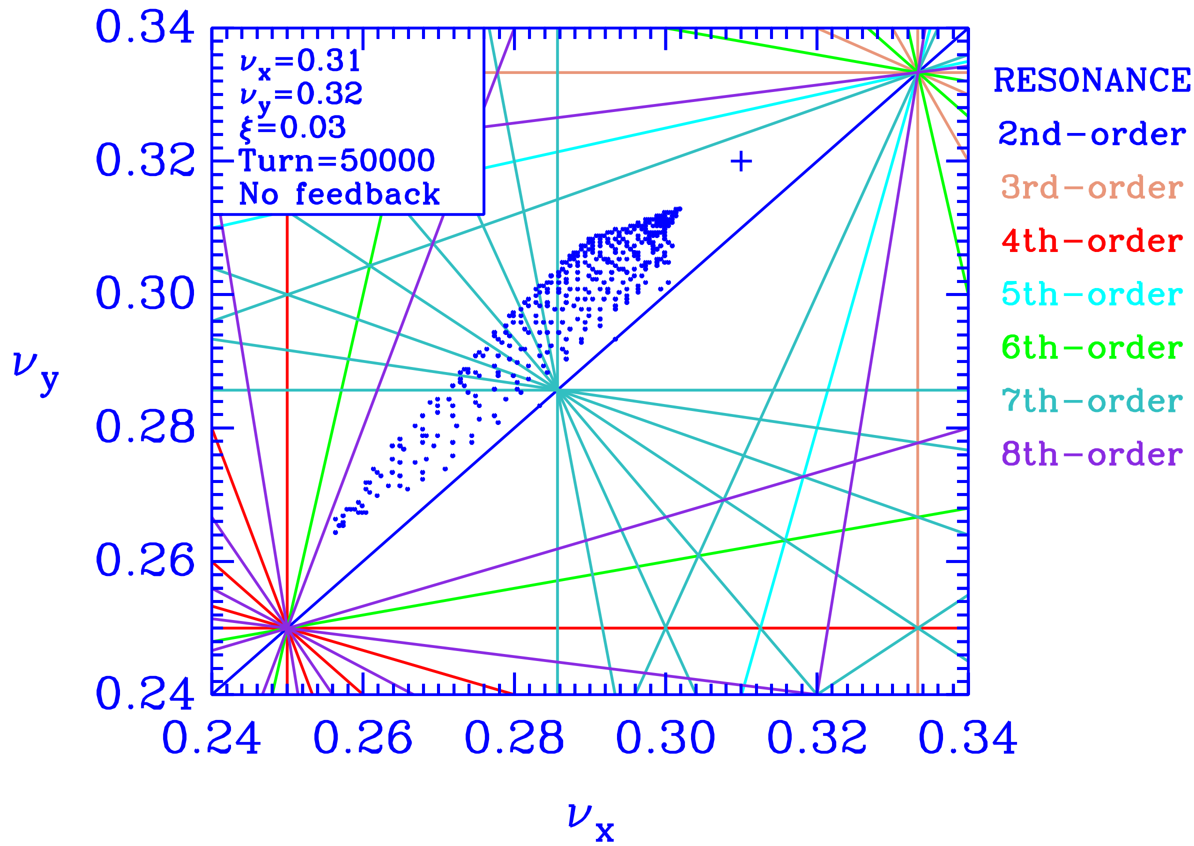
$\xi > \xi_c$,

- **The origin of the phase space for beam-centroid motion is an unstable fixed point.**

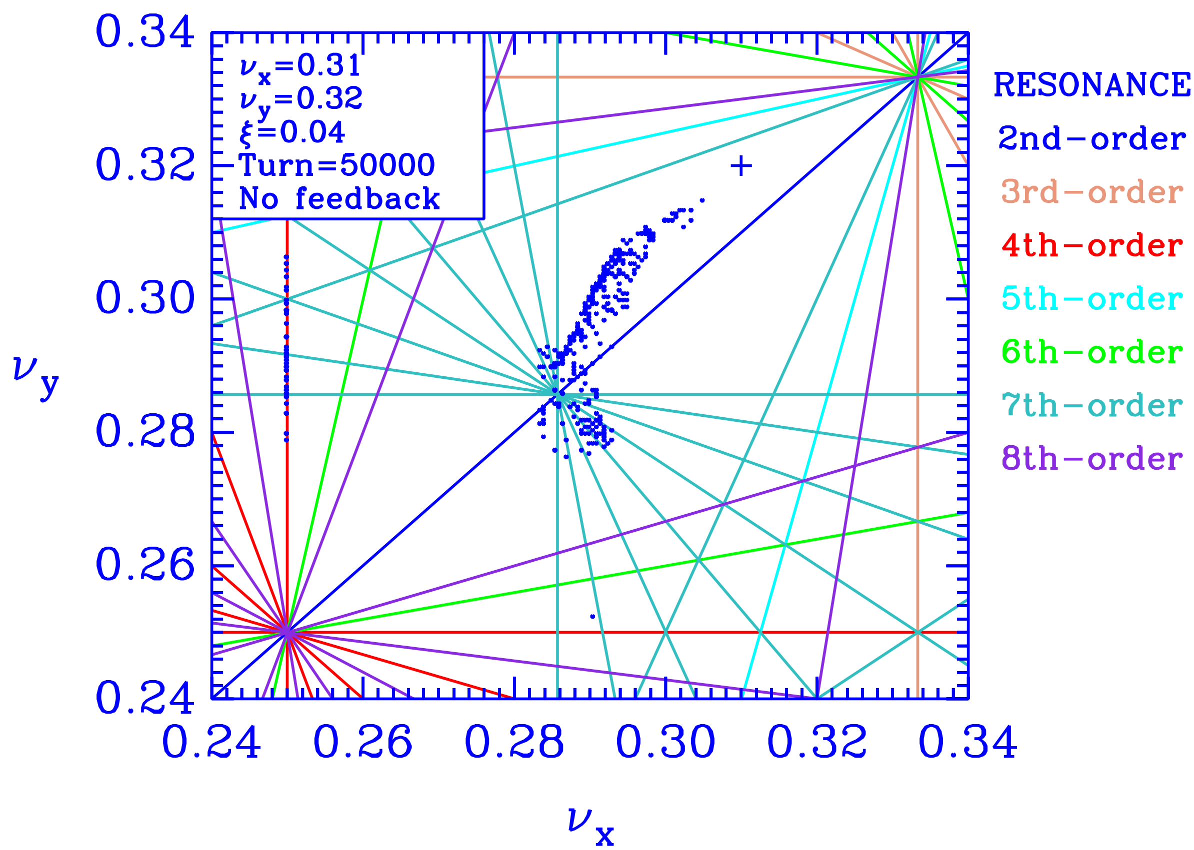
- **The competition between the instability and the damping could result in stable π or high-order modes.**



BEAM TUNE SPREAD OF LHC LATTICE WHEN $\xi < \xi_c$



BEAM TUNE SPREAD OF LHC LATTICE WHEN $\xi > \xi_c$



Incoherent Resonances vs. Beam-Beam Instability

$\underline{\nu_x = 0.310,\ \nu_y = 0.320:}$ $\xi_c \simeq 0.03$

7th-order resonance at $\xi \sim 0.012$
4th-order resonance at $\xi \sim 0.03$

$\underline{\nu_x = 0.270,\ \nu_y = 0.280:}$ $\xi_c \simeq 0.01$

4th-order resonance at $\xi \sim 0.01$

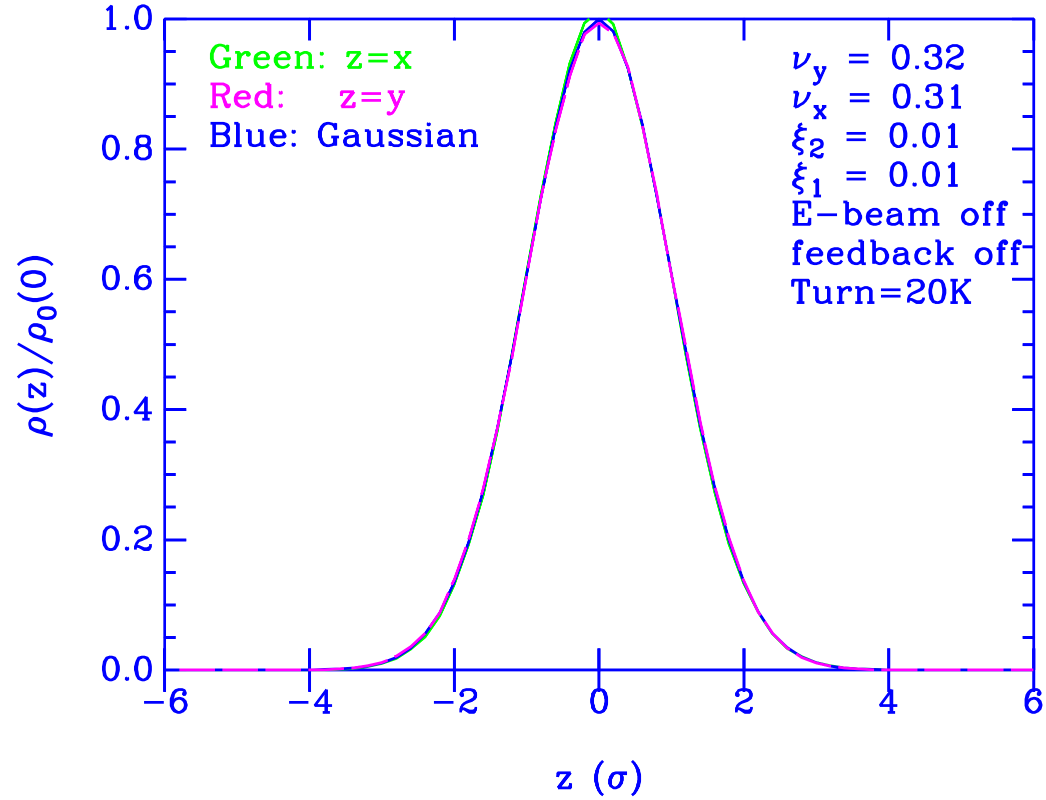
$\underline{\nu_x = 0.232,\ \nu_y = 0.242:}$ $\xi_c \simeq 0.04$

5th-order resonance at $\xi \sim 0.016$
6th-order resonance at $\xi \sim 0.033$

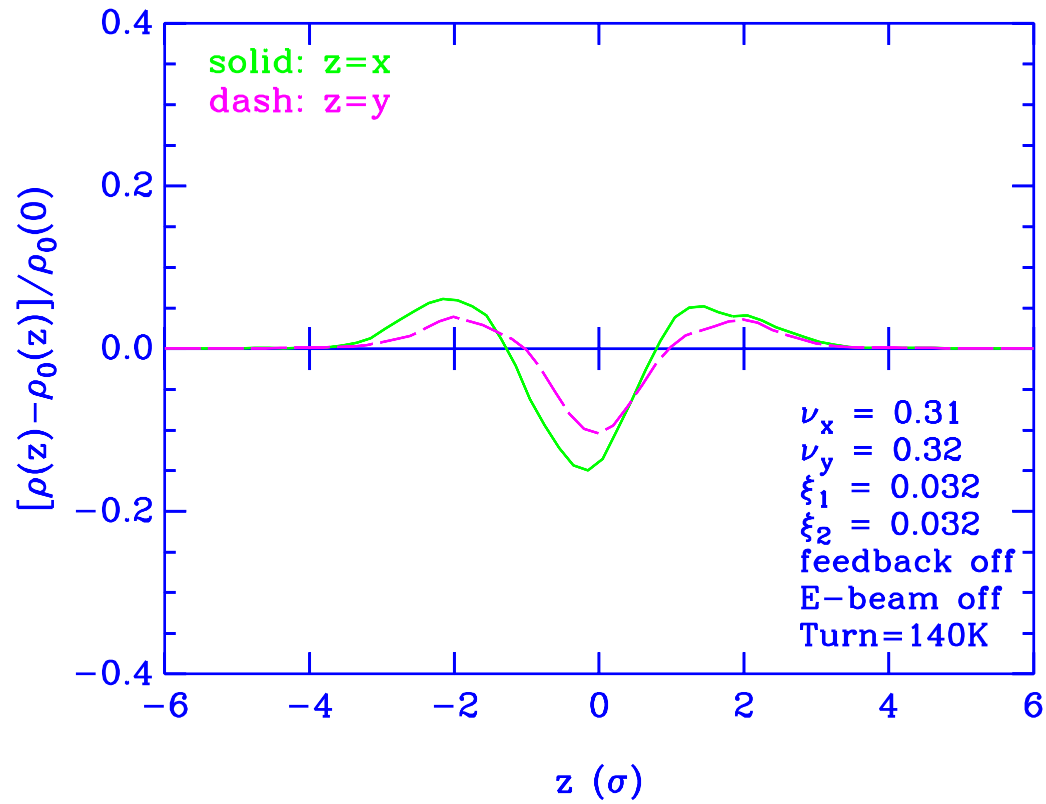
$\underline{\nu_x = 0.385,\ \nu_y = 0.395:}$ $\xi_c \simeq 0.02$

8th-order resonance at $\xi \sim 0.005$
3rd-order resonance at $\xi \sim 0.026$

Maintaining a Gaussian Beam When $\xi < \xi_c$



Beam Halo Due to Beam-Beam Instability When $\xi > \xi_c$



Collective Beam-Beam Instability of Hadron Beams

When the beam-beam parameter (ξ) exceeds a threshold (ξ_c), a chaotic coherent beam-beam instability occurs with the following characteristics:

- **Chaotic Coherent Oscillation**

  The phase-space region nearby the closed orbit could be unstable for beam centroids.

  $\Longrightarrow$ Spontaneous Chaotic Coherent Oscillation

- **Emittance Growth**

  An enhanced emittance growth is due to the dynamics of the counter-rotating beam.

- **Formation of Beam Halo**

  Beam distributions could significantly deviate from a Gaussian due to beam halo. The formation of the beam halo is a result of chaotic transport of particles from beam cores to beam tails.

[Ref.: J. Shi & D. Yao, PRE 62, 1258 (2000)]

Parameters of HERA Upgrade and HERA 2003 High-Luminosity Study

Parameter	Upgrade		2003 Experiment	
	HERA-e	HERA-p	HERA-e	HERA-p
$E(GeV)$	27.5	920	27.5	920
$I(mA)/N_{tot}$	0.3069	0.7778	0.2742	0.3917
$\beta_x^*/\beta_y^*(m)$	0.63/0.26	2.45/0.18	0.63/0.26	2.45/0.18
$\epsilon_x(nm)$	20	5.1	20	5.61
ϵ_y/ϵ_x	17%	1	17%	1
$\sigma_x^*/\sigma_y^*(\mu m)$	112/30	112/30	112/30	117/32
ν_x/ν_y	0.14/0.21	0.294/0.298	0.215/0.296	0.294/0.298
ξ_x	0.034	0.00155	0.01556	0.00138
ξ_y	0.0515	0.00045	0.02359	0.00040

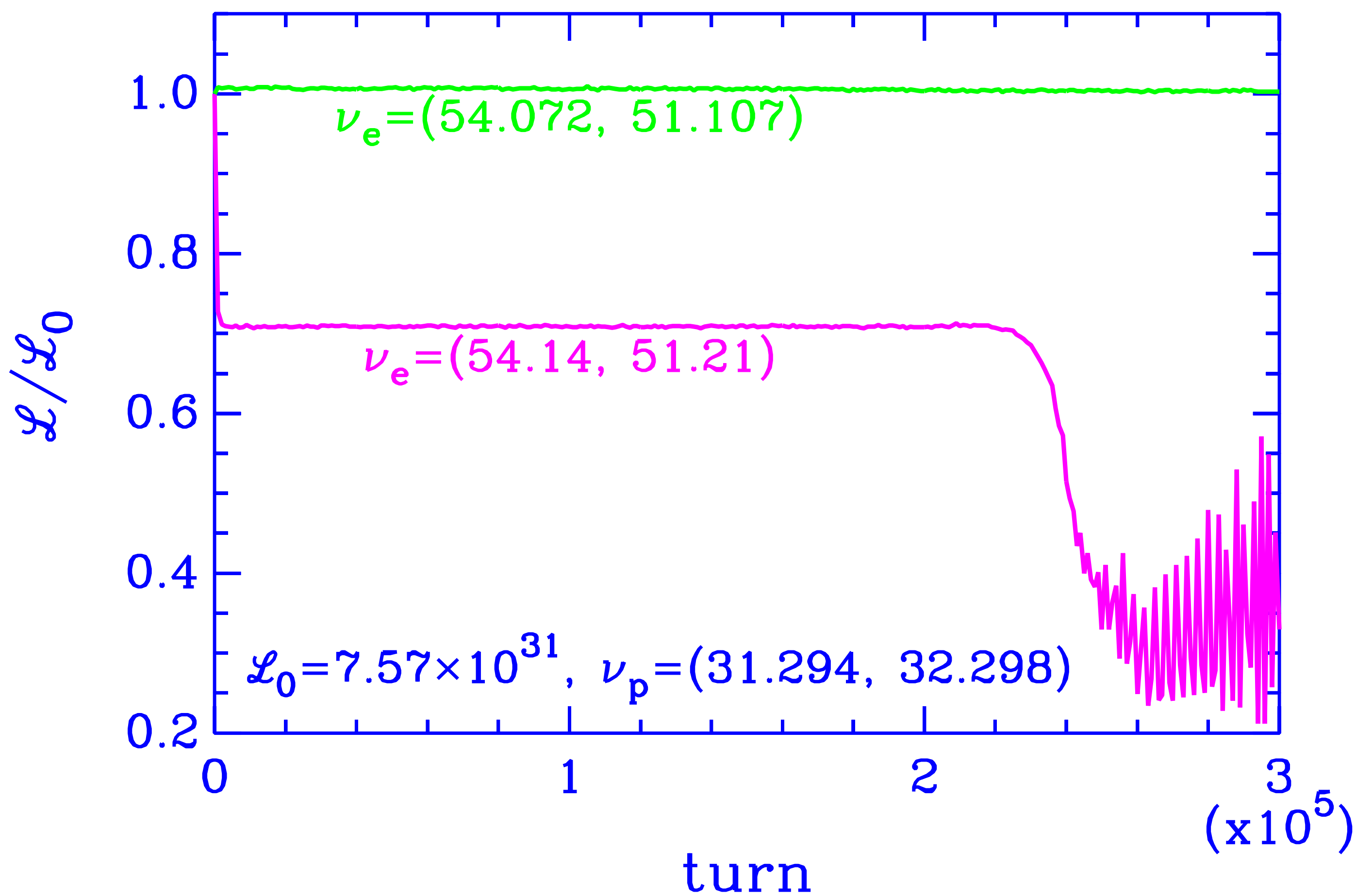
In 2003 experiment, ν_x/ν_y is the tune with collision.

Note: $\xi_{e,x}/\xi_{p,x} > 10, \quad \xi_{e,y}/\xi_{p,y} > 50$

—— This is a typical case of strong-weak beam-beam interaction!

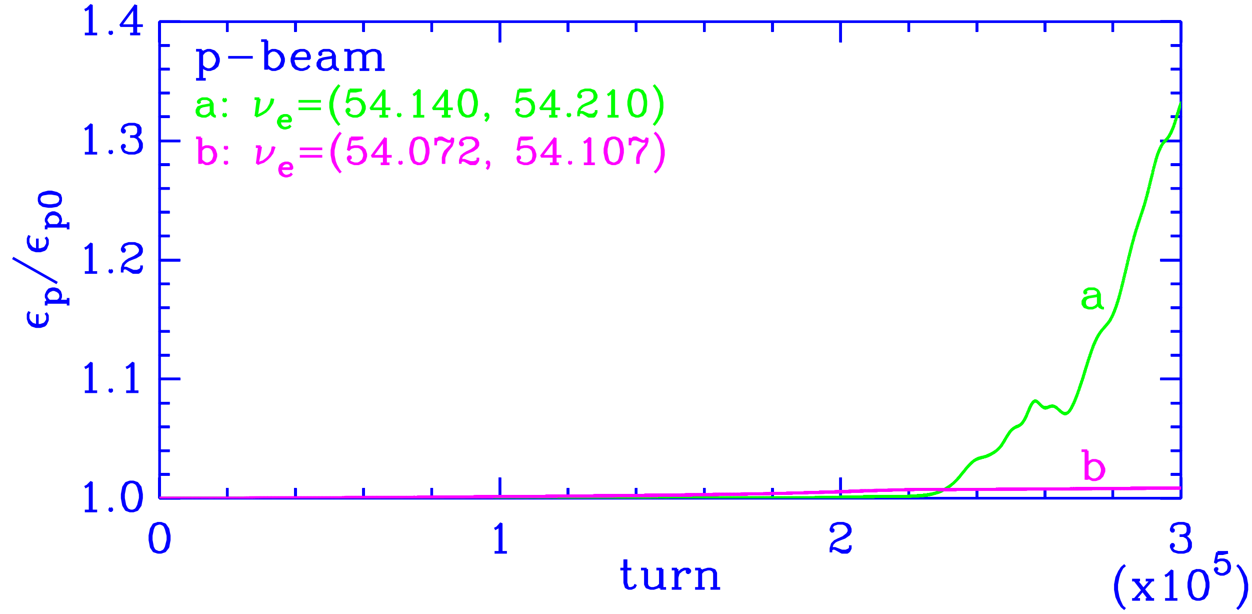
Luminosity in HERA Upgrade with/without the Chaotic Coherent Beam-Beam Instability

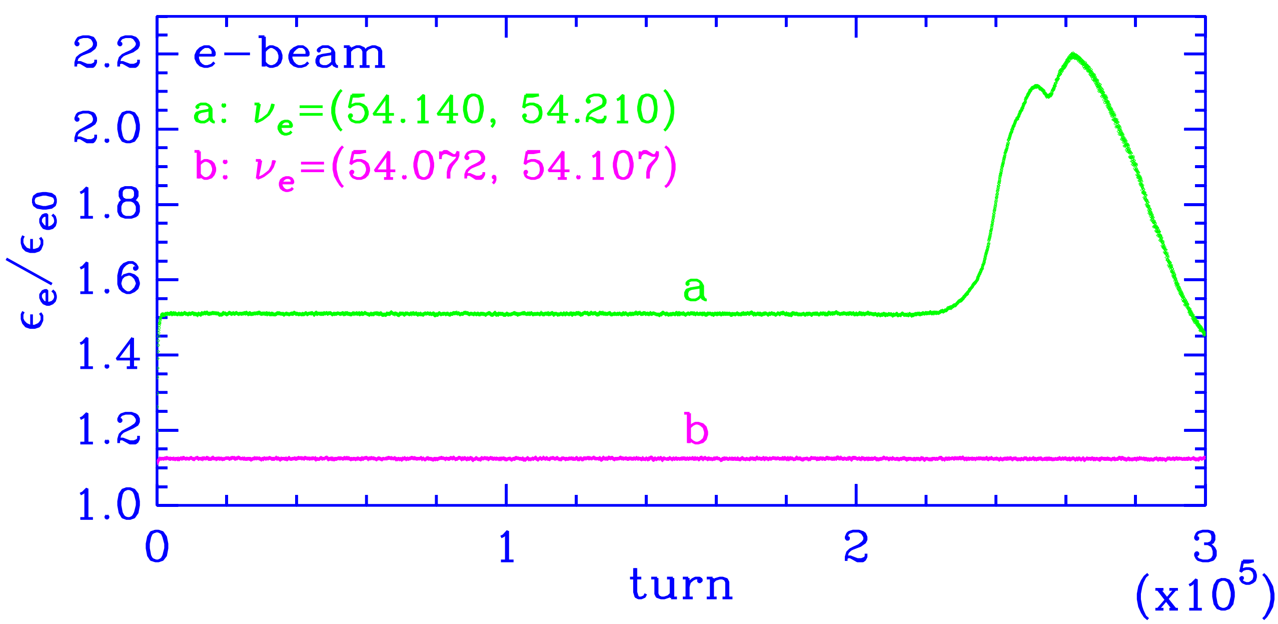
(Two *e*-*p* Collisions)



HERA Upgrade With Two *e*-*p* Collisions

Emittance Growth due to Coherent Beam-Beam Instability



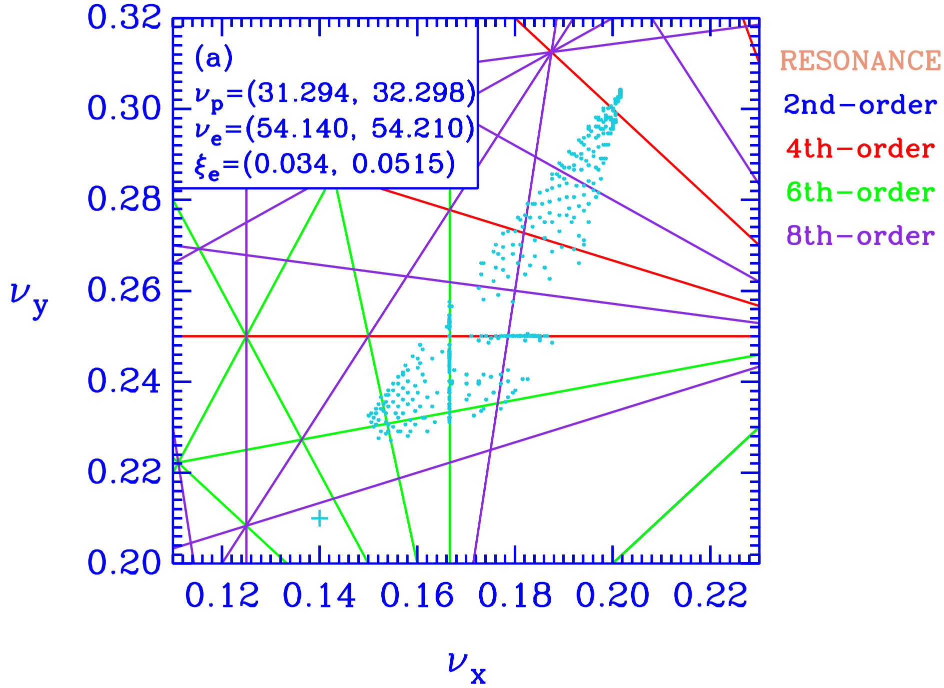


$$\vec{\xi}_p = (0.00155,\ 0.00045), \quad \vec{\nu}_p = (31.294,\ 32.298)$$

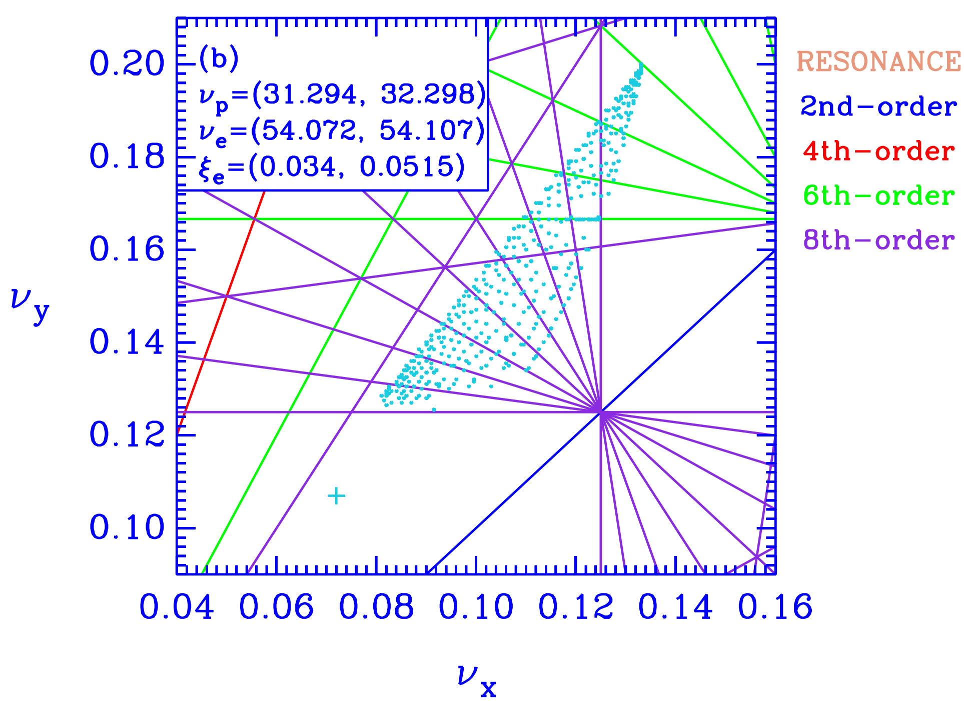
$$\vec{\xi}_e = (0.03400,\ 0.05150)$$

Tune Spread of e-Beam In HERA Upgrade (2 IP)

$\vec{\nu}_e = (54.140,\ 51.210)$:

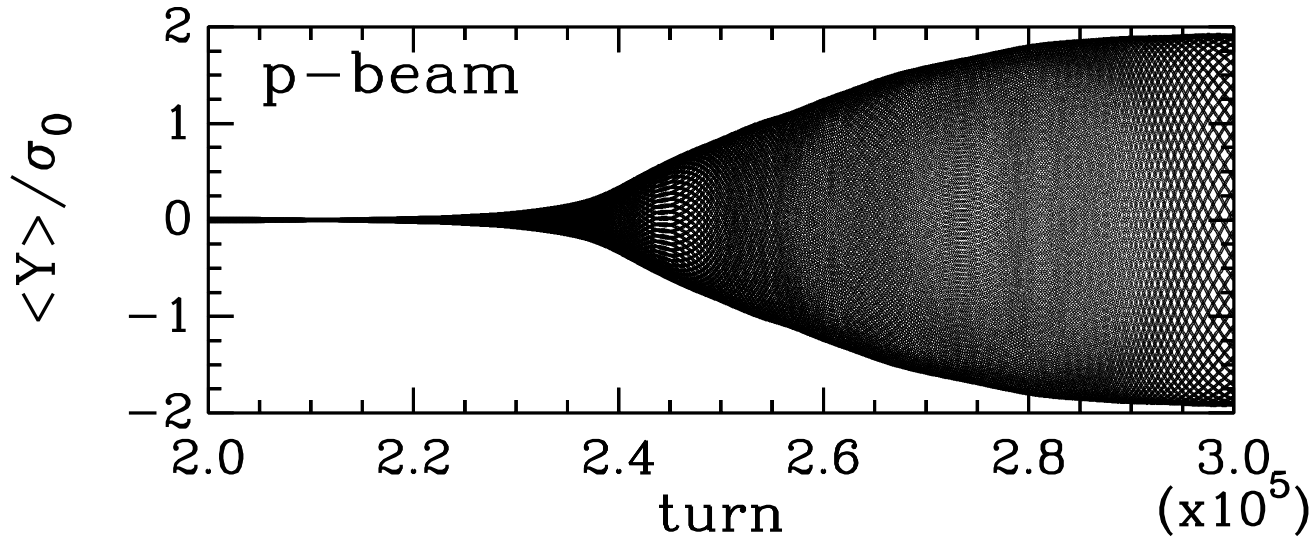


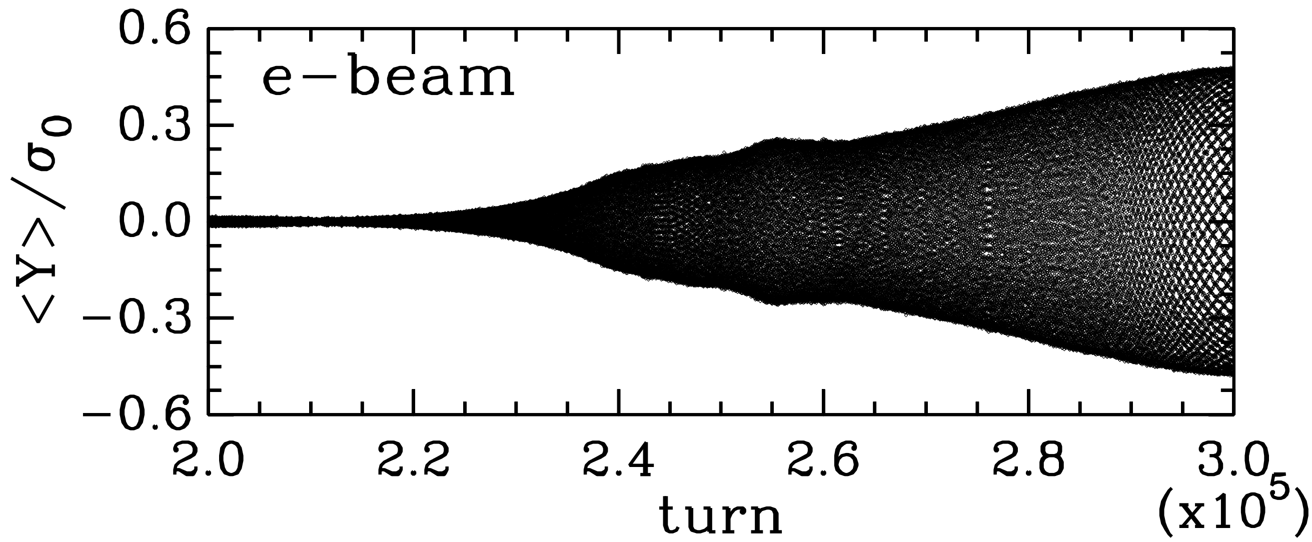
$\vec{\nu}_e = (54.140,\ 51.210) - 2\vec{\xi}_e = (54.072,\ 51.107)$:



$$\vec{\xi}_p = (0.00155,\ 0.00045), \quad \vec{\xi}_e = (0.034,\ 0.0515)$$

Chaotic Coherent Beam-Beam Instability in HERA Upgrade (2 IPs) When $\vec{\nu}_e = (54.14,\ 51.21)$

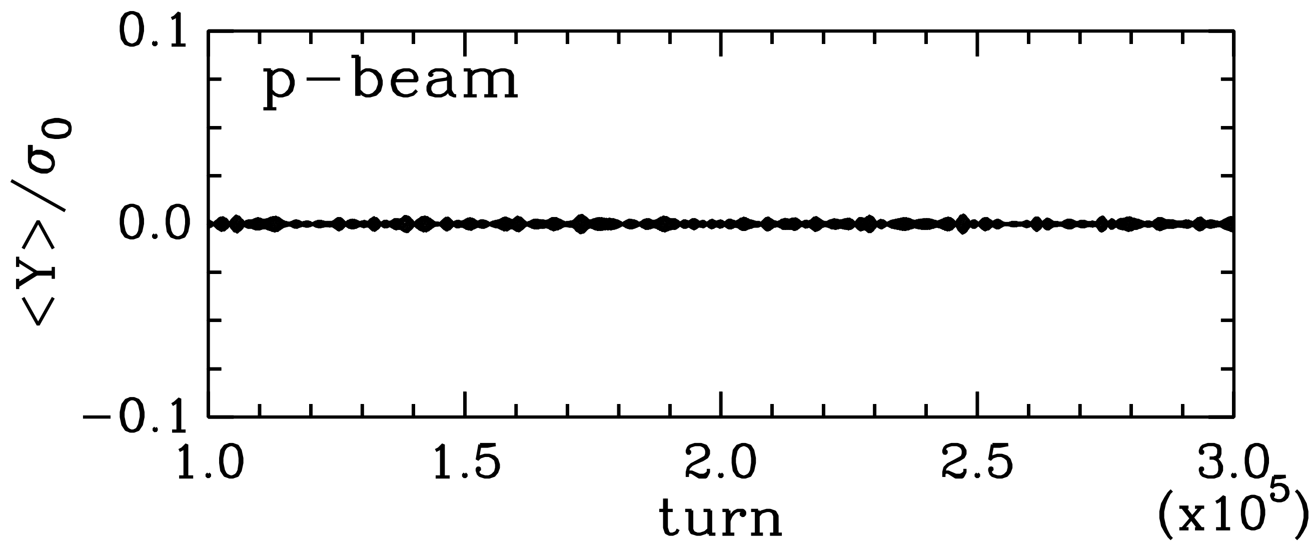


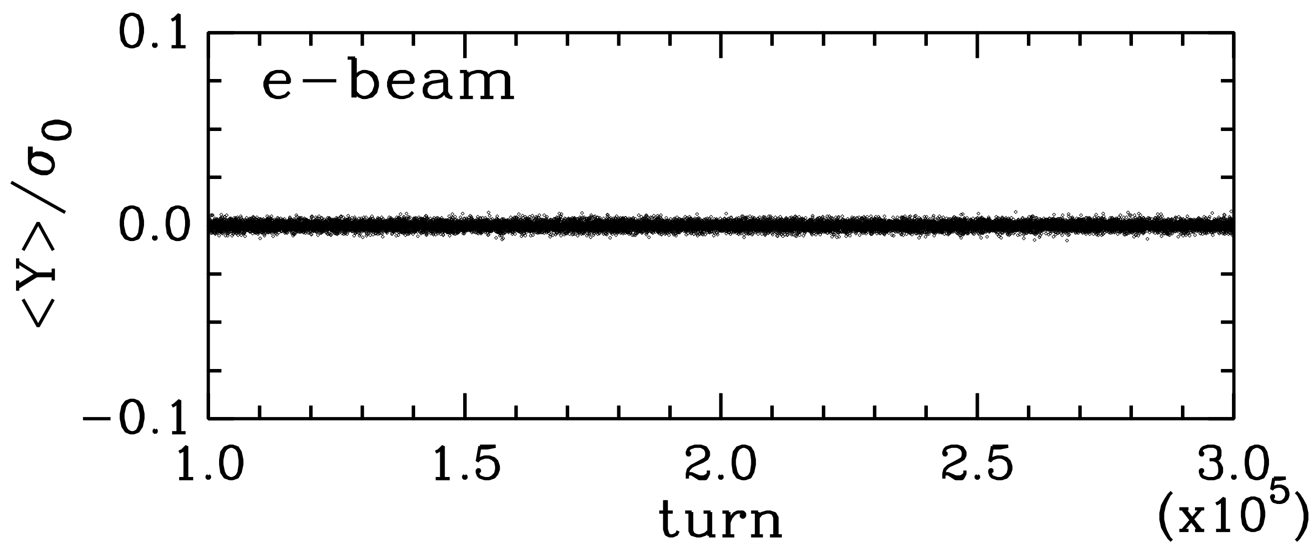


$\vec{\xi}_p = (0.00155,\ 0.00045), \quad \vec{\nu}_p = (31.294,\ 32.298)$

$\vec{\xi}_e = (0.03400,\ 0.05150)$

Stable Beam-Centroid Motion In HERA Upgrade (2 IPs) When $\vec{\nu}_e = (54.072,\ 51.107)$



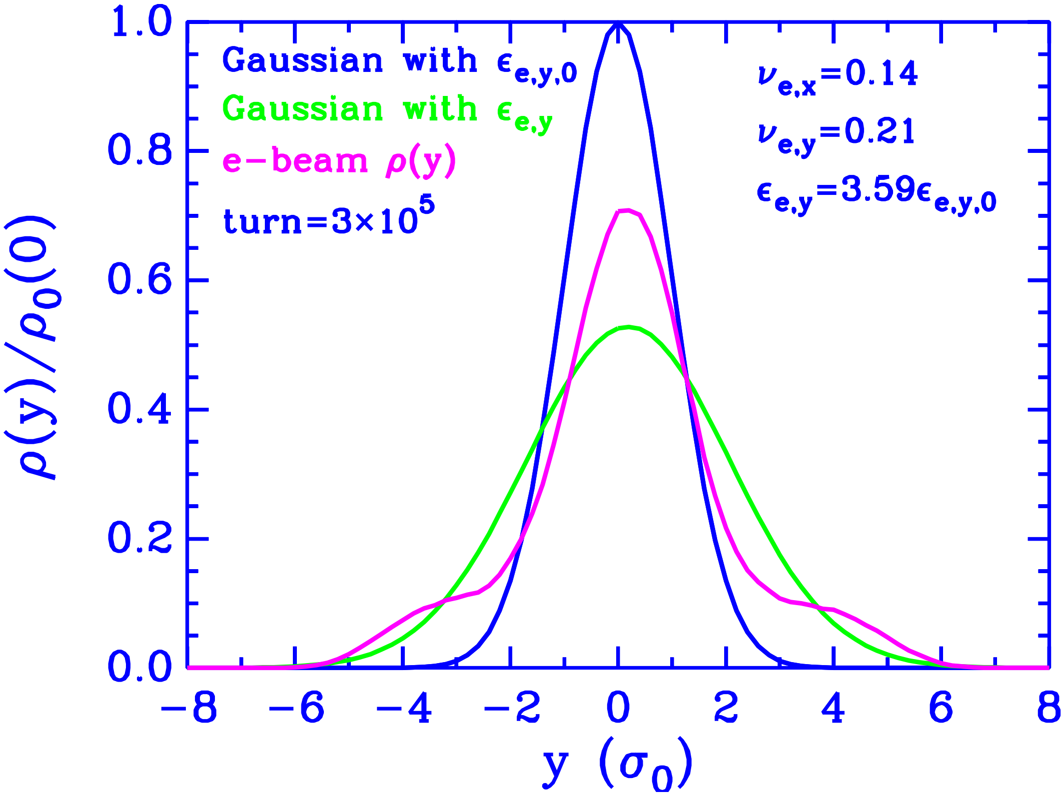


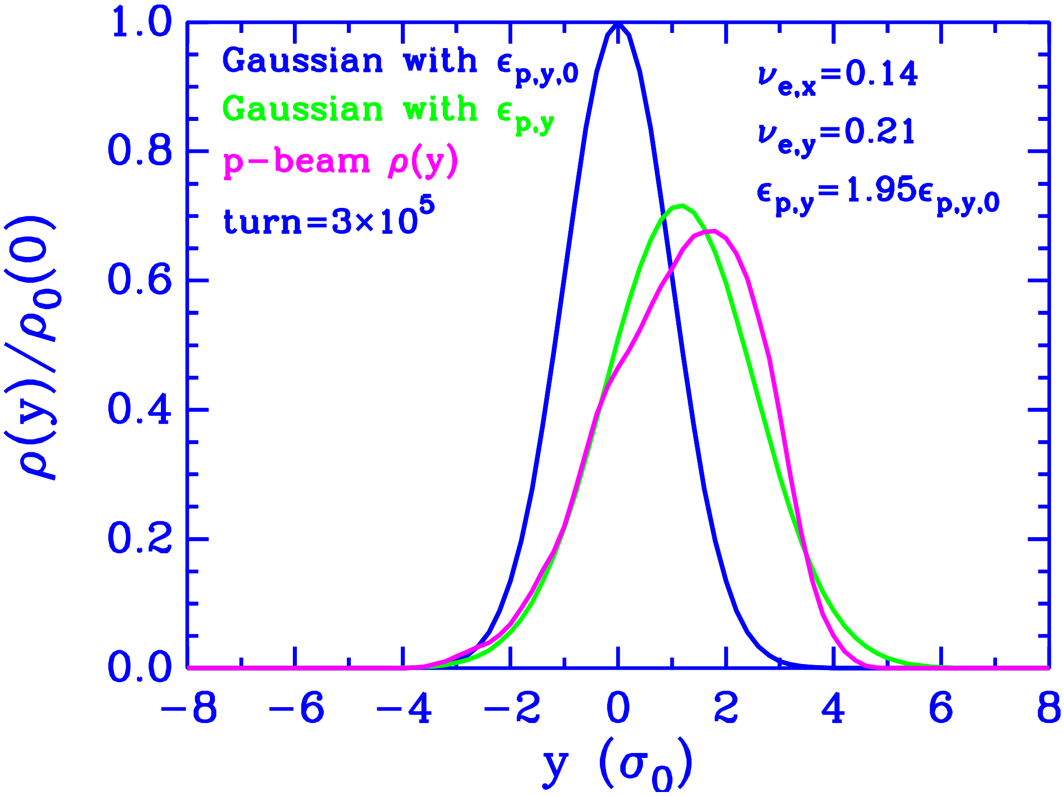
$\vec{\xi}_p = (0.00155,\ 0.00045), \quad \vec{\nu}_p = (31.294,\ 32.298)$

$\vec{\xi}_e = (0.03400,\ 0.05150)$

Beam Distributions in HERA After the Onset of the Chaotic Coherent Beam-Beam Instability

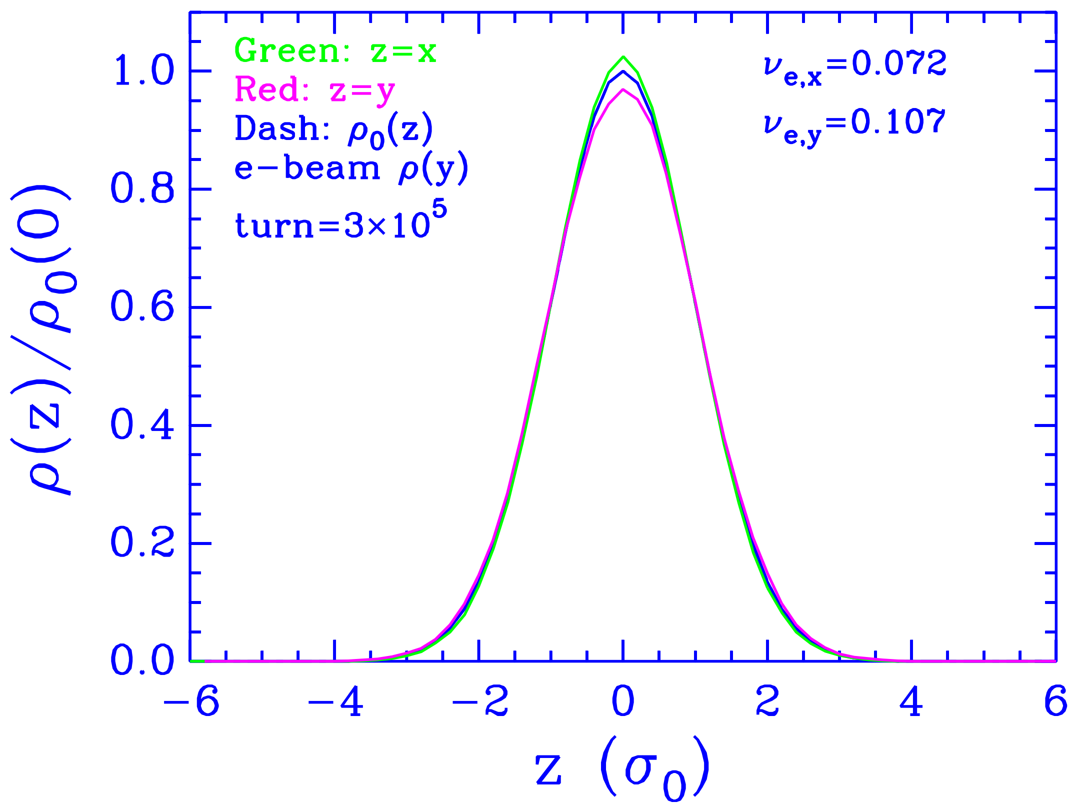
Two e-p Collisions, $\vec{\nu}_e = (54.140,\ 51.210)$

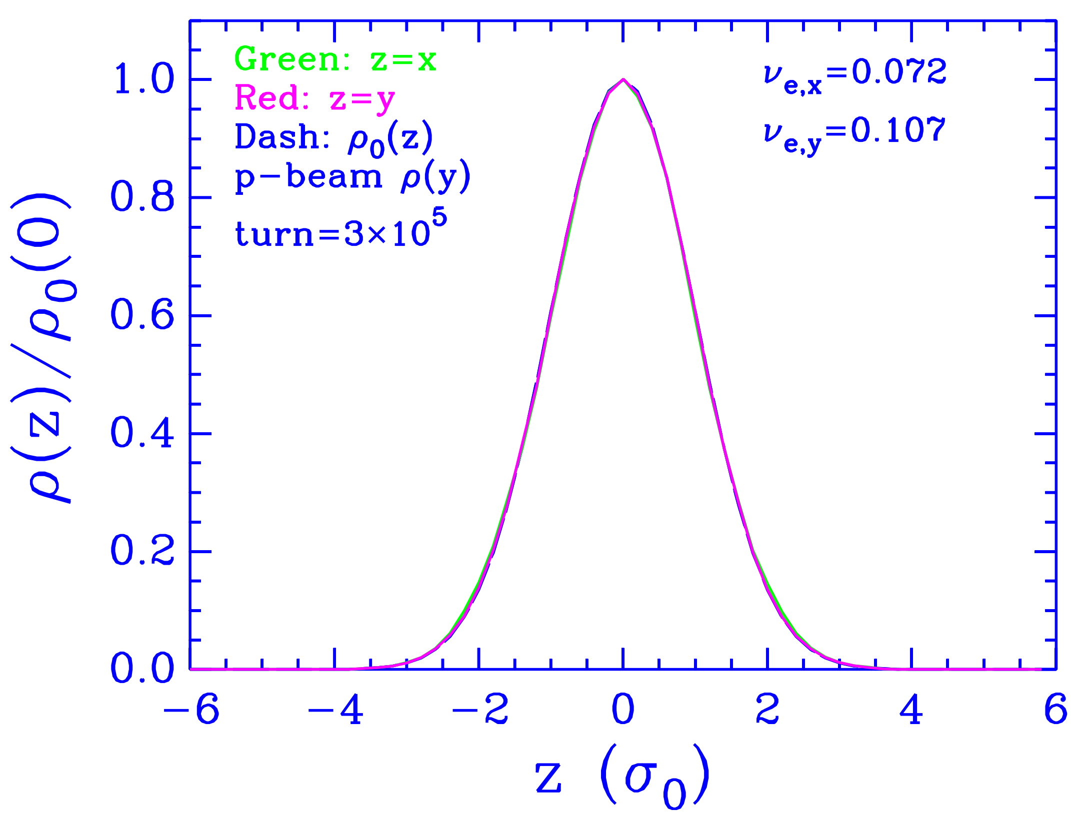




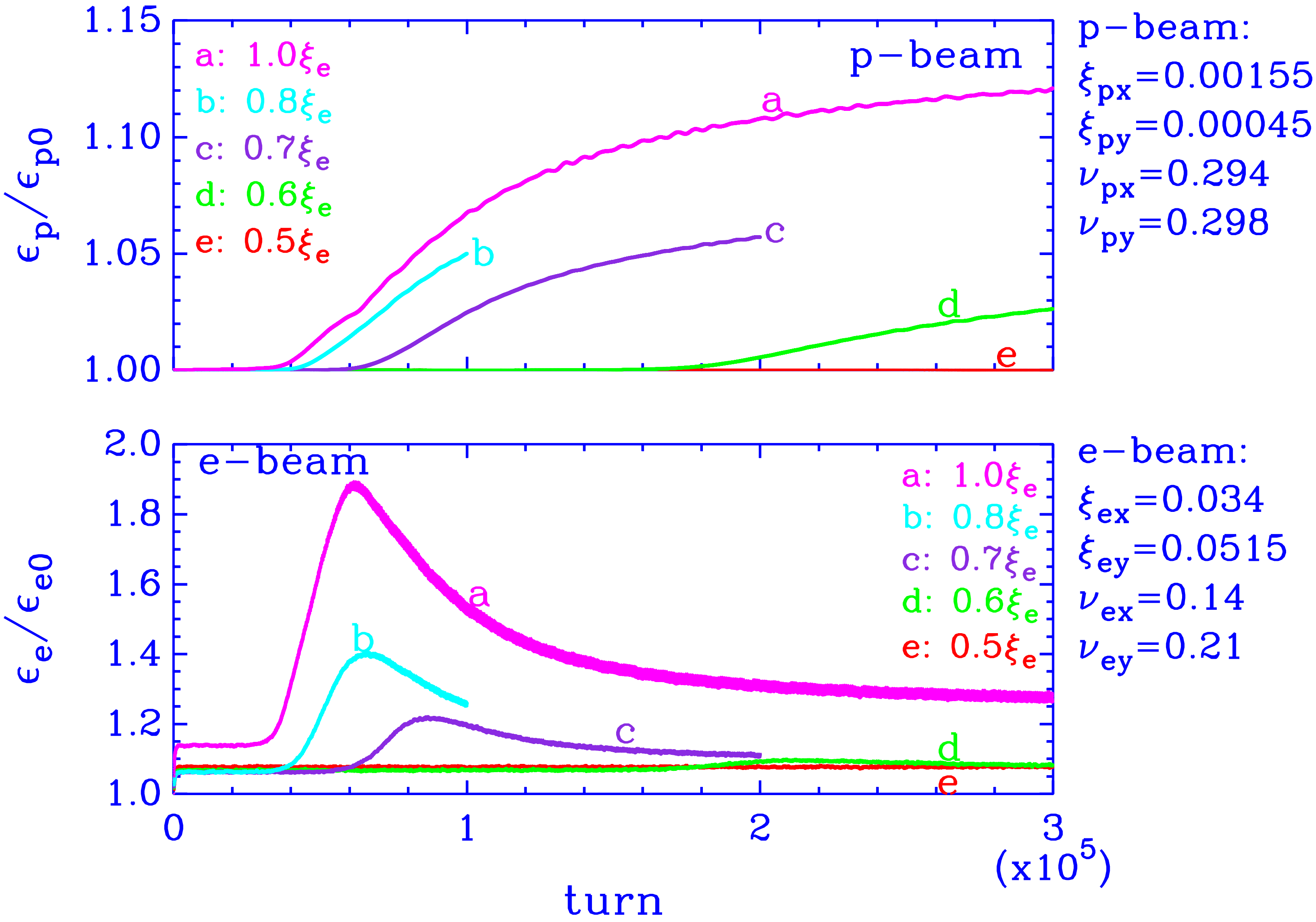
Stable Beam Distributions in HERA Upgrade

Two e-p Collisions, $\vec{\nu}_e = (54.072,\ 51.107)$





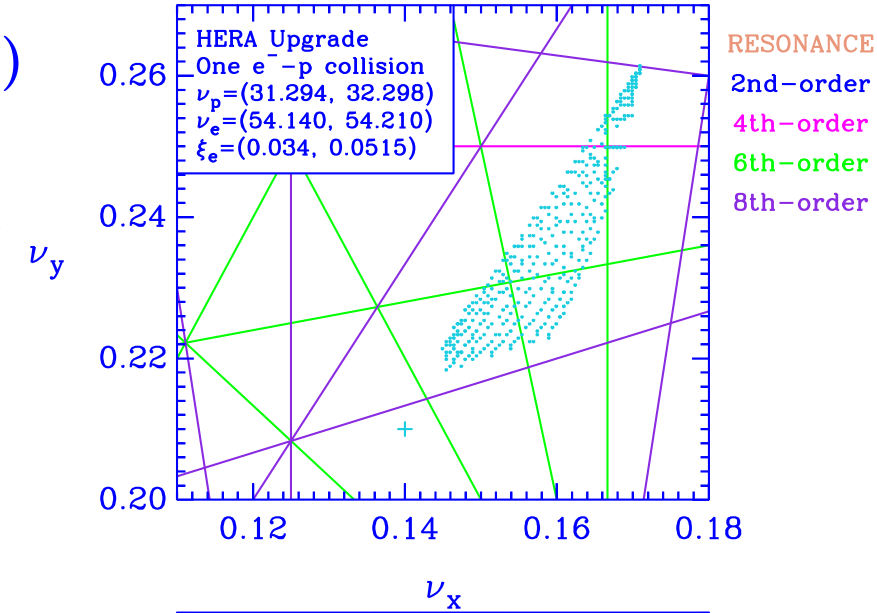
Threshold of Chaotic Coherent Beam−Beam Instability
HERA Upgrade with One p−e Collision



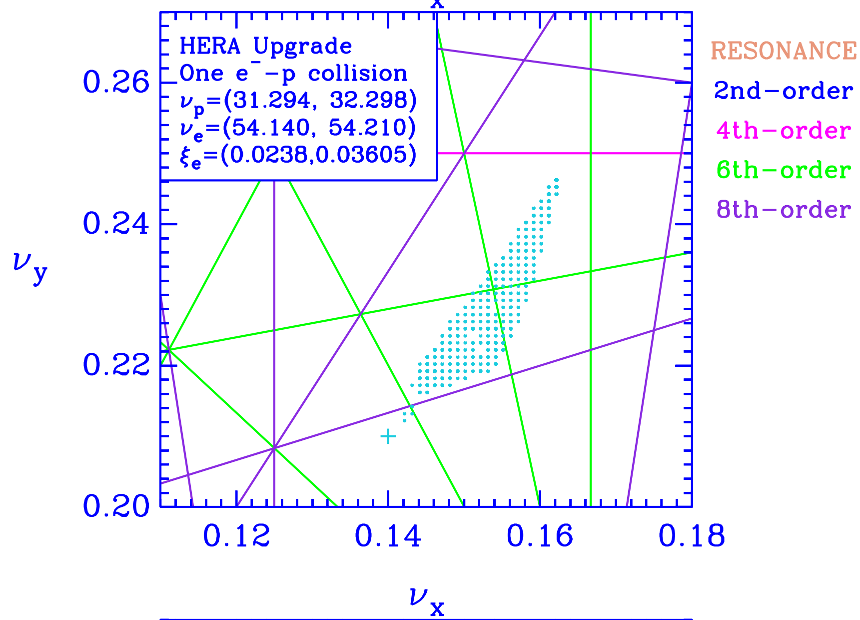
Tune Spread of e-Beam in HERA Upgrade (1 IP)

$\vec{\nu}_e = (0.14,\ 0.21)$

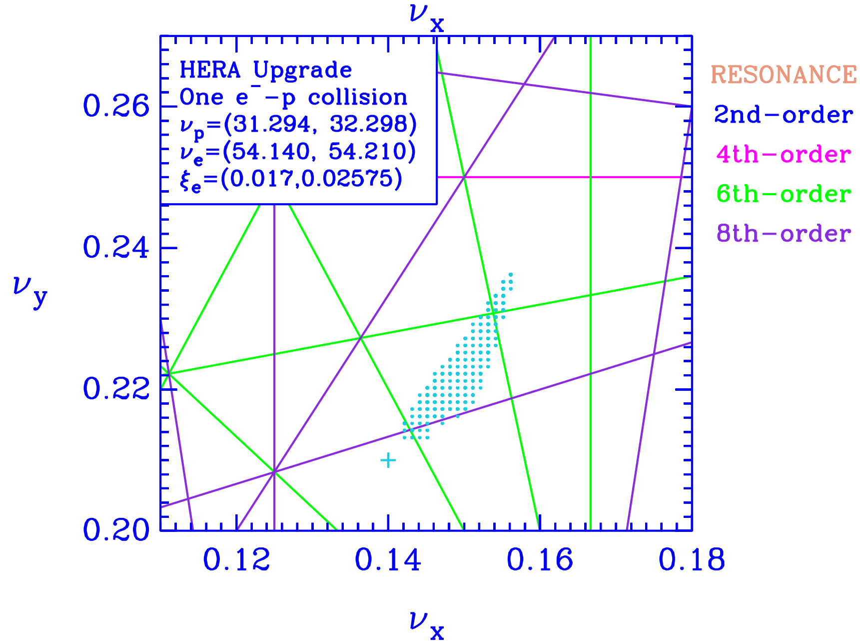
$I_p = 1.0\ I_{design}$



$I_p = 0.7\ I_{design}$

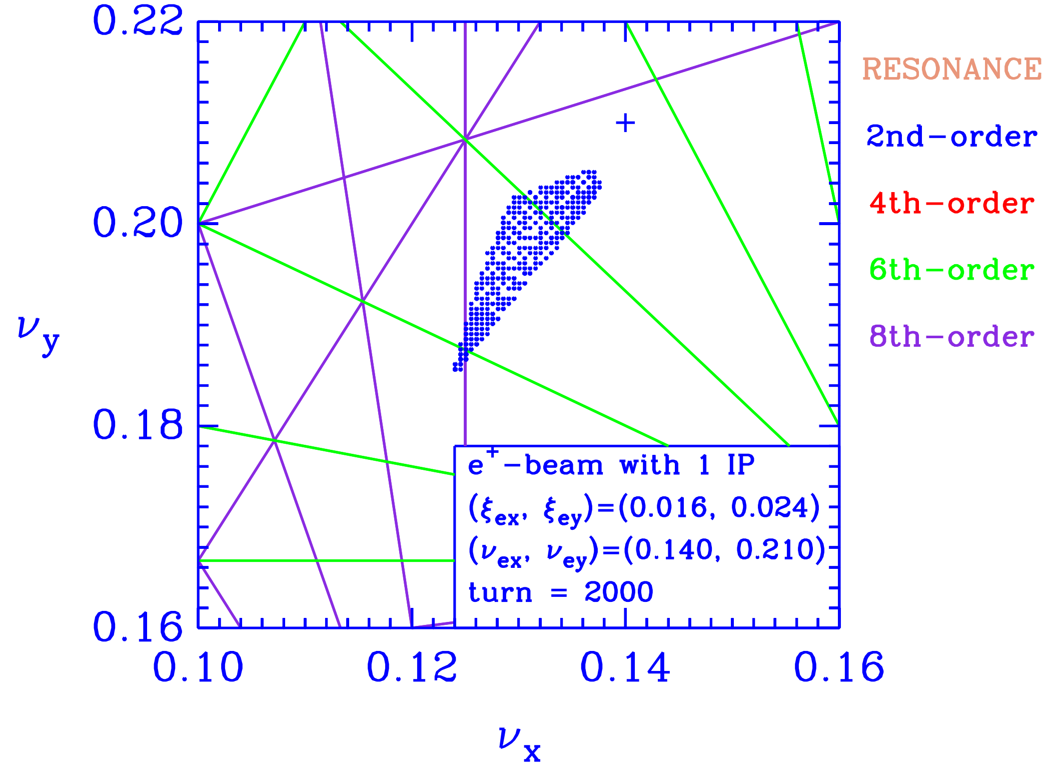


$I_p = 0.5\ I_{design}$

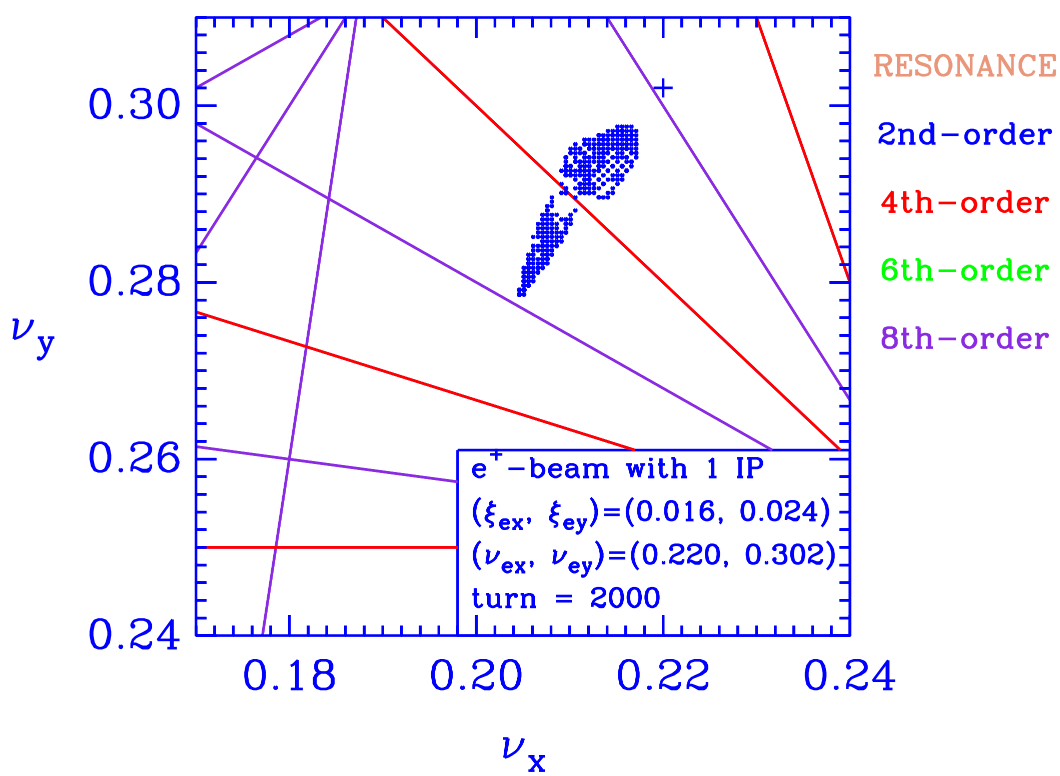


HERA 2003 STUDY: Tune Spread of e^+ Beam (1 IP)

The e^+ beam is at nominal working point :

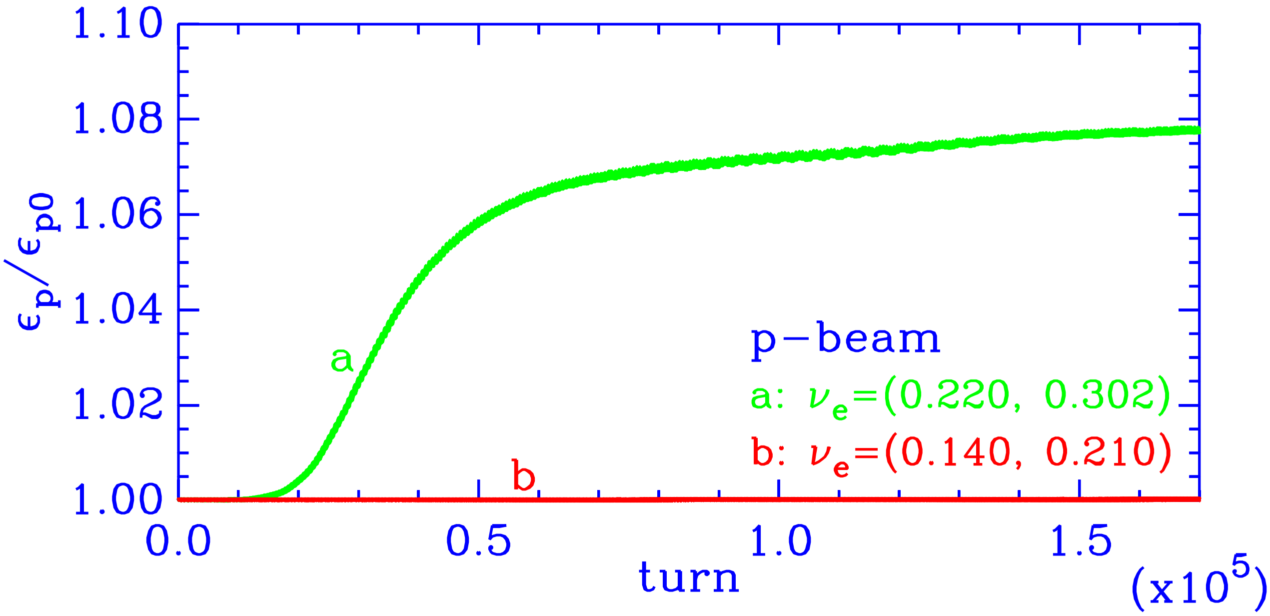


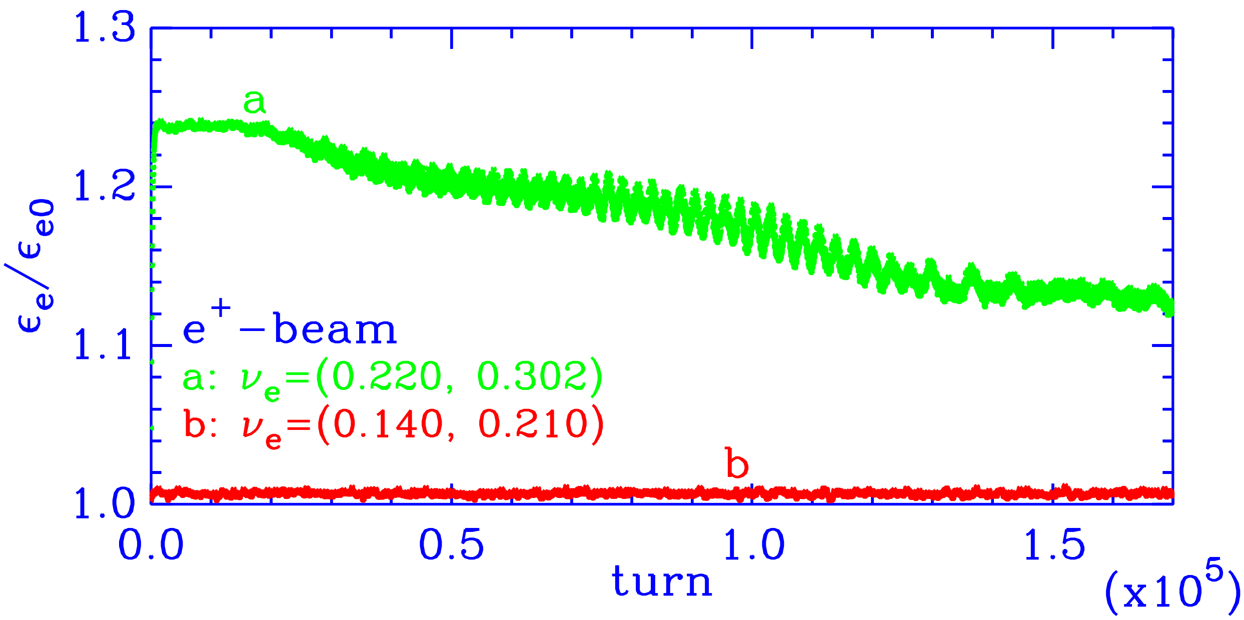
The e^+ beam crosses $2\nu_x + 2\nu_y$=1 :



HERA 2003 High-Luminosity Study With One IP

Emittance Growth due to Coherent Beam-Beam Instability





HERA 2003 Experimental Result:
In case a, the proton beam emittance increases ∼30% while in case b, no emittance increase was observed.

Collective Beam-Beam Instability in Strong-Weak Case of Beam-Beam Interactions

- **Traditionally, coherent (collective) beam-beam effects are considered to be not important in a strong-weak (or highly un-symmetrical) situation of beam-beam interactions.**

- **Computer simulation, however, showed that beam-beam interactions in HERA could induce a chaotic coherent beam-beam instability and result in a significant emittance growth in both weak and strong beams.**

- **Recently, such the collective beam-beam instability has been observed in HERA.**

- **For high-intensity beams, the collective beam-beam effect is therefore important in both situations of strong-strong and strong-weak beam-beam interactions.**

Methods of Numerical Simulation for Nonlinear Beam-Beam Effects

Near-Linear (Near-Integrable) Regime of Coherent Beam-Beam Effects

- **Quasi-stationary states of Vlasov equation may exist, especially when $\xi \longrightarrow 0$.**

- **Methods of perturbation could be employed.**

- **The system is forgiving on methods of numerical simulation.**

- **In principle, beams are stable in the consideration of beam-beam interactions and emittance growth is not important (or significant) after initial beam filementations.**

Nonlinear (Nonintegrable) Regime of Beam-Beam Interactions

- Stationary state of Vlasov equation may not be reachable.

  $\Longrightarrow$ We have to work with transient states of a nonlinear PDE — a very tough problem mathematically.

- Methods of perturbation such as various canonical perturbation expansions, the truncation of moment expansions, or the linearized Vlasov equation are no longer valid. The use of those approximation methods could distort the dynamics.

  $\Longrightarrow$ Only validated method we currently know is a correct numerical simulation.

- Fine Hamiltonian structure in phase space is important.

  $\Longrightarrow$ For a correct beam-beam simulation:

  Need to calculate a "smooth" and "undistorted" beam-beam force;

  In order to sample enough detail of phase-space structure for the time scale of interest, a large number of macro-particles are necessary.

- Be careful in using classical diffusion models for emittance growth or beam-particle loss. They are only valid mathematically in a fully chaotic region, otherwise the stickiness of resonances results in non-$\delta(\tau)$ correlations —— the problem of long-term tails.

Methods of Beam-Beam Simulation

1. **Soft Gaussian approximation: Assume Gaussian beams with varying width and center.**

   — Fast [$O(N_p)$]; but not right in the nonlinear regime of beam-beam interactions in which the distribution could deviate from the Gaussian; may be o.k. for incoherent beam-beam effects.

2. **Direct multi-particle tracking: the beam-beam force is calculated with particles-to-particle individually.**

   — Precise if N_p is large, but very slow [$\mathrm{O}(N_p^2)$], typical: $N_p \leq 10^4 \Longrightarrow$ wrong physics in the nonlinear regime.

3. **Particle-In-Cell (PIC): evaluate beam-beam force on a mesh.**

   — Precise, but very slow for separated beams.

   Variations:

   a. Calculate Beam-Beam Potential Without Boundary

   b. Calculate The Potential With Approximated Boundary

   c. Directly Calculate Beam-Beam Force on the Mesh

   d. With Weighted Functions

4. **Hybrid Fast Multipole Method (HFMM)**

   — Fast, better for separated beams.

5. **Canonical perturbations for solving Vlasov equation**

   — Only valid for $\xi \longrightarrow 0$.

Field Computation With PIC Method

1. Solve Beam-Beam Potential on the Mesh

Poisson eq. for potential $\Phi(x,y)$ with charge density $\rho(x,y)$,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi(x,y)=-2\pi\rho(x,y)$$

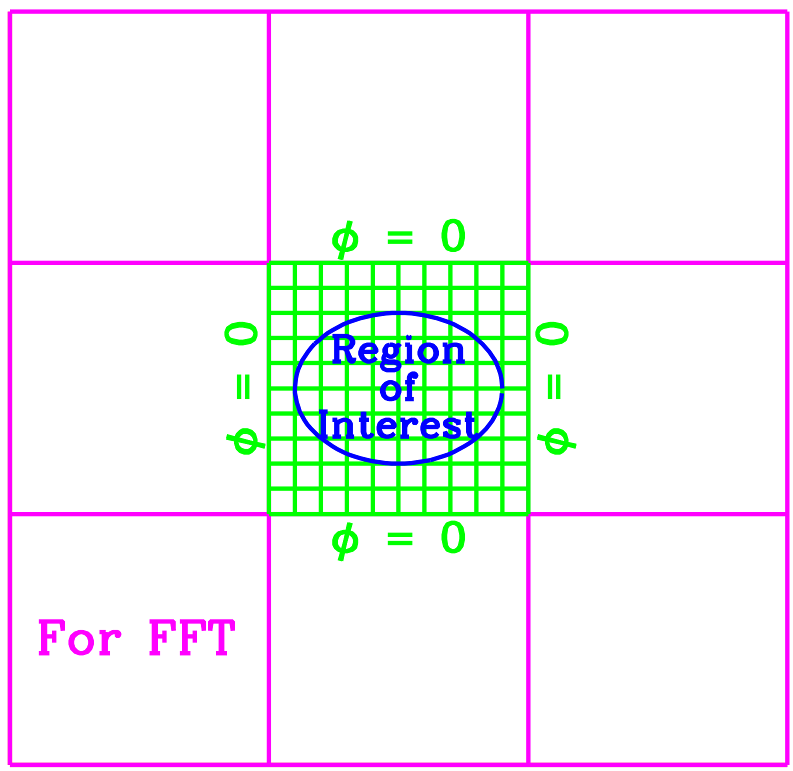
With Green's function,

$$\Phi(x,y)=\int G(x-x',y-y')\rho(x,y)dx'dy'$$

For open boundary,

$$G(x,y)=-\frac{1}{2}\ln\left(x^2+y^2\right)$$

- **FFT is usually used for solving $\Phi(x,y)$ on the mesh.**
- **The field is then computed with numerical derivatives.**



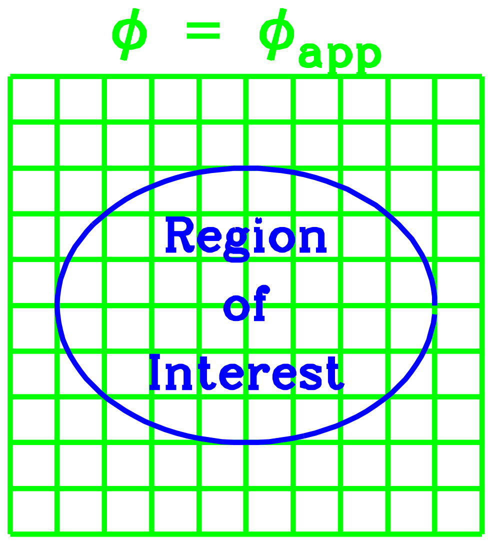
Comment:

- **Fast — Computation cost $\sim N_p N_m \ln N_m$.**
- **But the mesh has to be big to minimize errors from boundary. Many empty cells are wasted.**

Field Computation With PIC Method

2. PIC With Reduced Region of Mesh

— Y. Cai, A. Chao, S. Tzenov, T. Tejima



3. Direct Calculation of Beam-Beam Field on the Mesh

The field is calculated with

$$\vec{K}(\vec{r}) = \int d\vec{r'} \rho(\vec{r'}) \vec{G}_k(\vec{r} - \vec{r'})$$

where Green's function is

$$\vec{G}_k(\vec{r} - \vec{r'}) = \frac{(\vec{r} - \vec{r'})}{(x - x')^2 + (y - y')^2}$$

Comment:

- Accurate — Exact boundary condition
  No errors due to numerical derivatives.
- Only a small number of empty cells when using adaptive mesh.
- Slow when a large mesh has to be used (mis-matched beams) — Computation cost $\sim N_p N_m^2$.

Weighted Macro-Particles (WMPT)

—— M. Vogt, J.A. Ellison, T. Sen, R.L. Warnock

For any function in phase space $A(\vec{z})$,

$$\langle A \rangle_t = \int A(\vec{z}) f(\vec{z}, t) d^4 z$$

where $f(\vec{z}, t)$ is the beam distribution in phase space. Because of the symplecticity,

$$\langle A \rangle_t = \int A(\vec{z}(t)) f(\vec{z}(0), 0) d^4 z(0)$$

On grid points with weighted function w_i,

$$\langle A \rangle_t = \sum_i A(\vec{z}_i(t)) f(\vec{z}_i(0), 0) w_i$$

Advantage: better sampling beam tails.

Hybrid Fast Multipole Method (HFMM)

—— W. Herr, M.P. Zorzano, and F. Jones

The field is calculated on a mesh:

- Macro-particles inside the grid are assigned to grid points;
- Multipole expansions of the field are computed on every grid points.

Computing cost: Between $O(N_m)$ and $O(N_m \log N_m)$.

A better way to treat long-range beam-beam interactions.

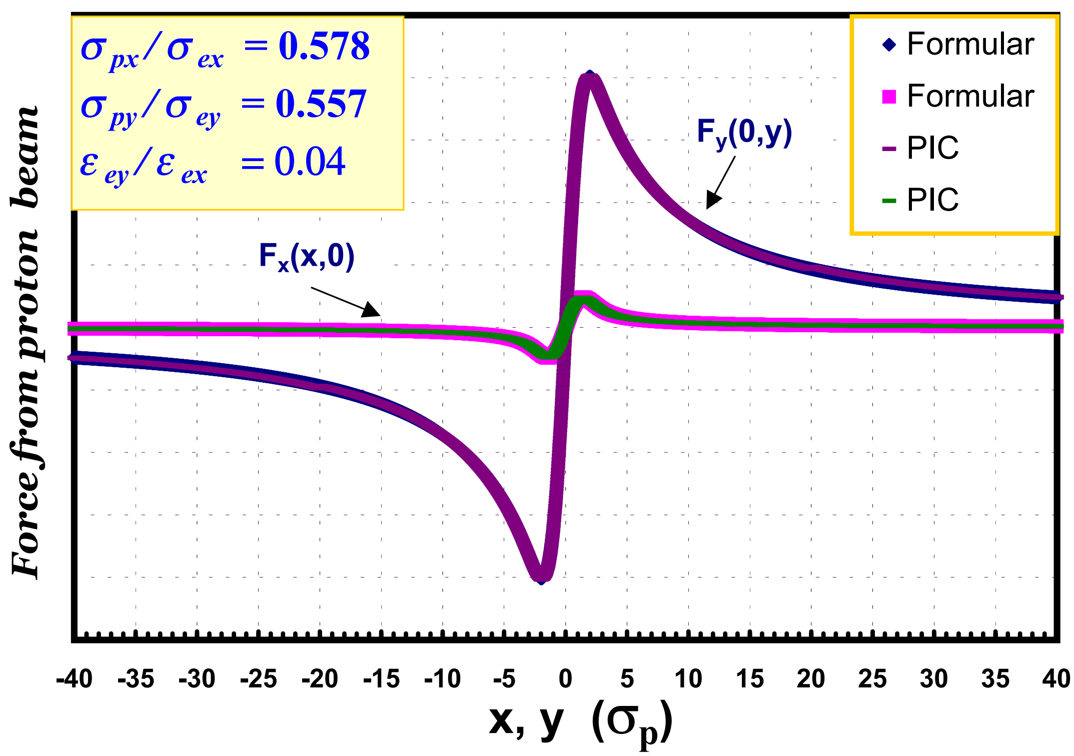
The Correct Way of Beam-Beam Simulation

All computational parameters in a numerical model should be tested for the computational convergence for the system in the worst possible situation (maximal beam-beam parameter, worst working point, ...).

—— A code should never be made as a "one-size-fits-all".

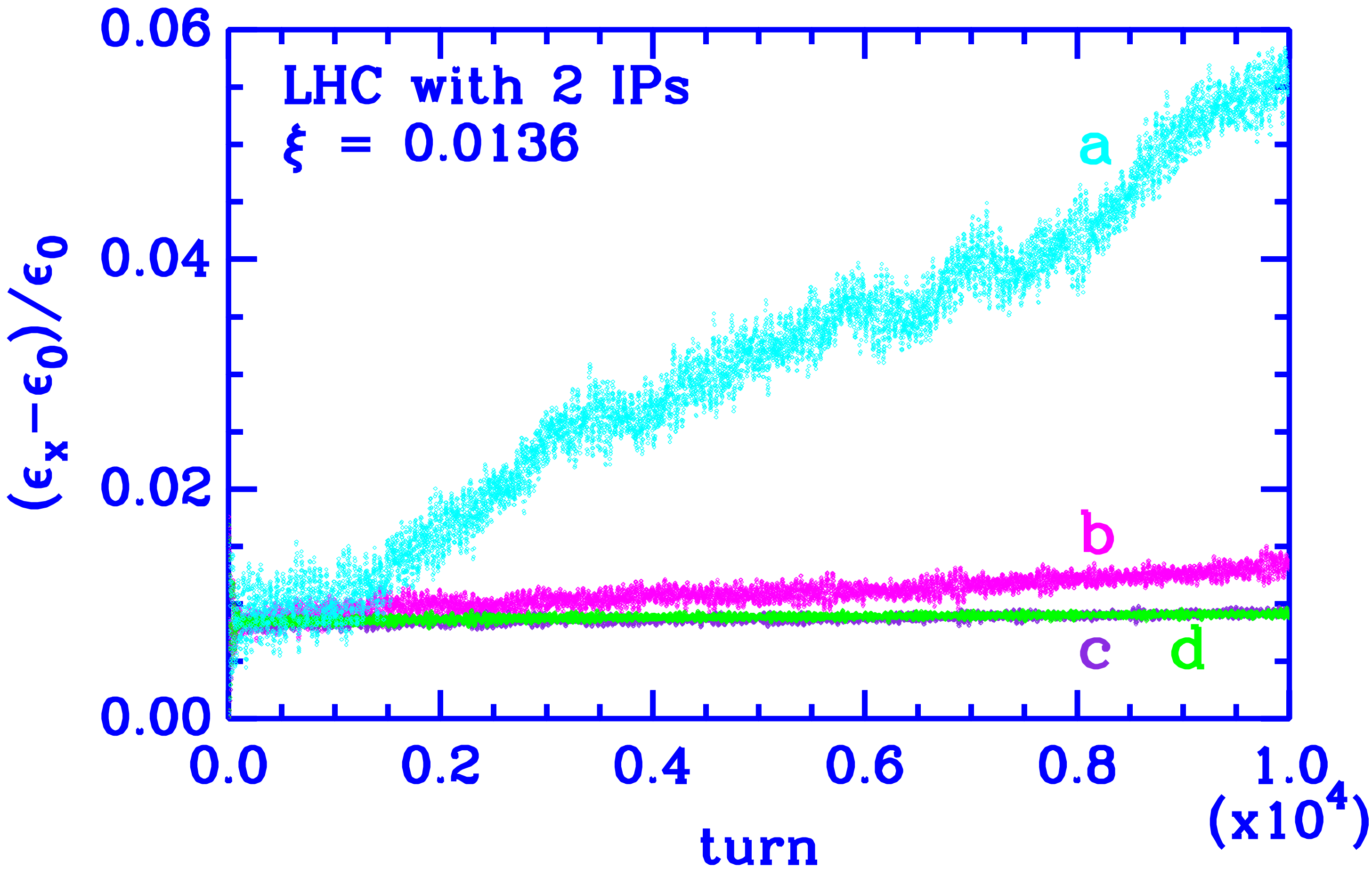
Importance of Computational Convergence

Traditionally, the "beauty" of the initial field has been used to show how "good" a simulation is,



This is far from enough especially in the nonlinear regime of beam-beam interactions.

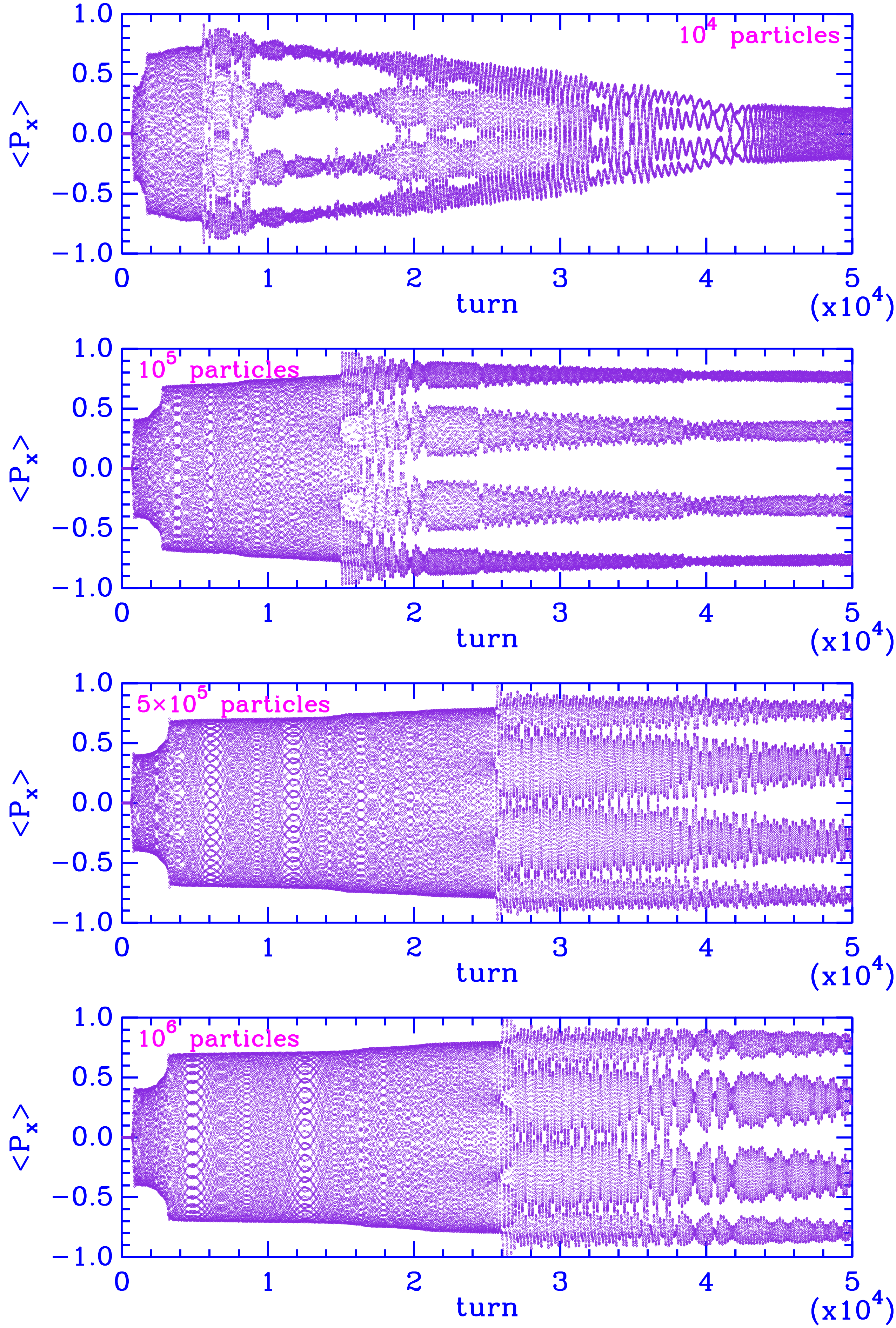
Comparison Between Different Numbers of Macro-Particles



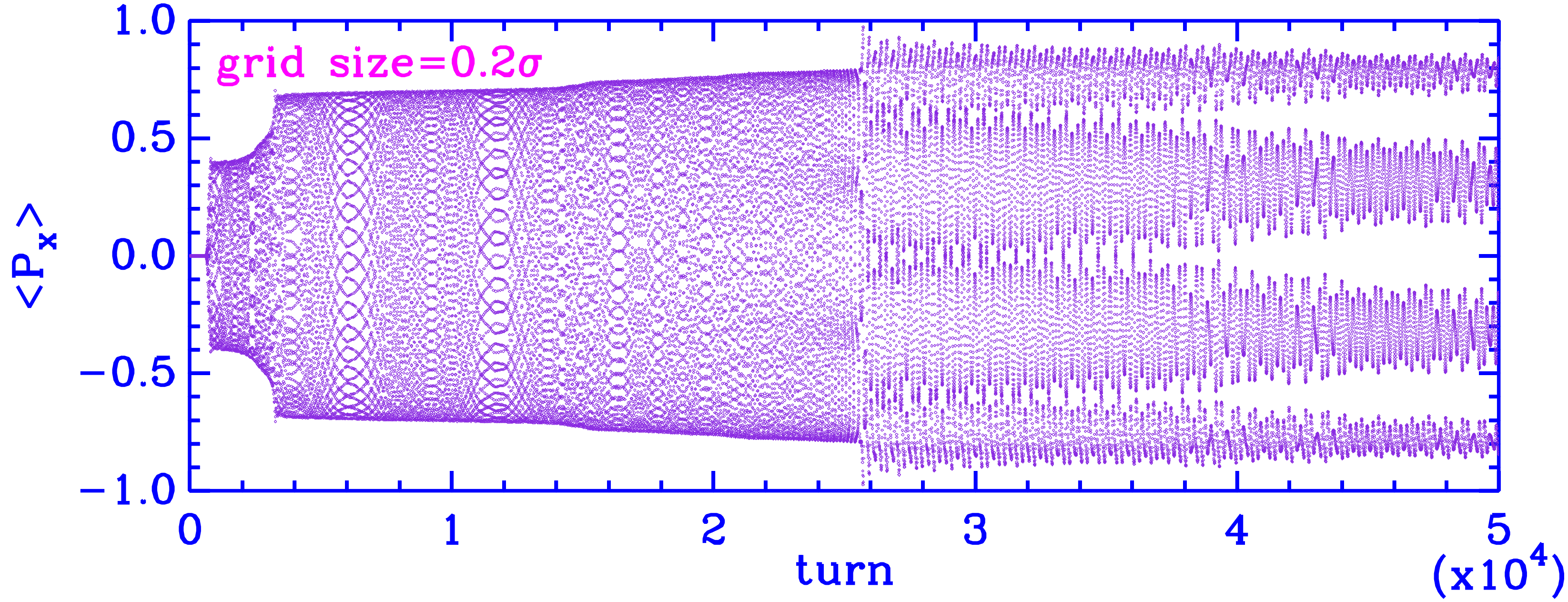
(a) 10^4 particles; (b) 10^5 particles;
(c) 5×10^5 particles; (d) 10^6 particles.

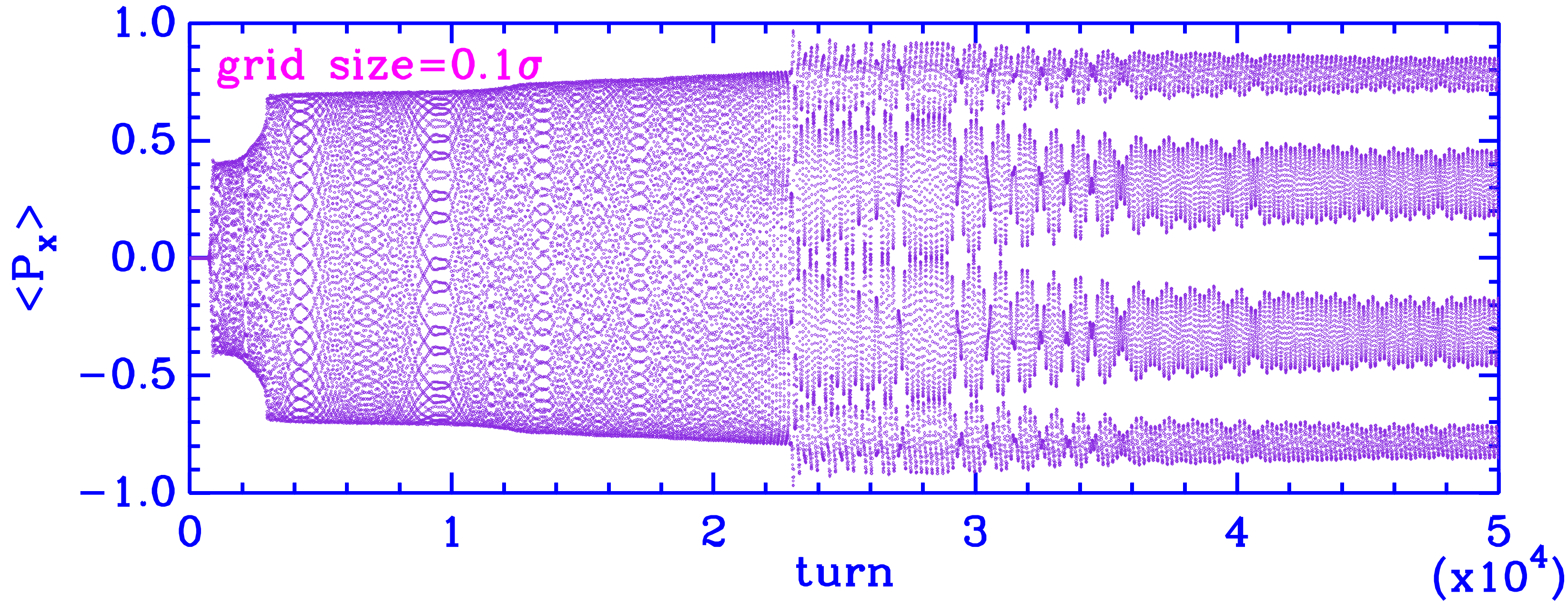
$$\epsilon_x = \int \frac{1}{2}(x^2 + p_x^2) f(\vec{r}, \vec{p}, t) d\vec{r} d\vec{p}$$

Comparison Between Different Numbers of Macro-Particles



COMPARISON BETWEEN DIFFERENT GRID CONSTANTS





Final Comments

- **In pushing the frontier of luminosity, hadron colliders could be more likely operated in the nonlinear regime of beam-beam interactions. An understanding of beam-beam effects in that regime is necessary.**

- **To study the beam-beam effects, especially in the nonlinear regime, we have to respect the Hamiltonian nature of hadron beams, and we have to recognize that the traditional mode analysis based on the linearized Vlasov equation, which is a very useful tool in lepton colliders, is invalid for hadron beams mathematically.**

- **In the nonlinear regime of beam-beam interactions, the traditional boundary between strong-strong and strong-weak beam-beam interactions is blurred and the beam-beam effect has to be studied (or at least checked) self-consistently in all situations. In this regime, only validated method for the study of nonlinear beam-beam effect is numerical simulation.**

- **What We Can Do Computationally**
  Understanding of Short-term beam-beam effects:
    Fast emittance growth (within $\sim 10^6$ tunes)
    Onset of beam-beam instabilities

    ...

- **What We Don't Have Confident Computationally**
  Understanding of Long-term beam-beam effects:
    Slow emittance growth, slow particle loss, and
    Slow diffusion due to nonlinearities
    Beam lift time ?