Selected Topics of Theory and Experiment on the Space-Charge-Dominated Beam Physics

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Outline

• Part I: general concepts of space-charge-dominated beams.

• Part II: University of Maryland Electron Ring (UMER) and its components.
  • Diagnostics: BPM, energy analyzer …

• Part III: selected experimental and theoretical results
  • Experimental study of beam energy spread evolution in intense beam
  • Theoretical study of beam emittance of a gridded electron gun
  • Experimental study of Resistive wall instability
Beam Transport in a Uniform Focusing Channel

Beam envelope equation:

\[ R'' + k^2_0 R - \frac{K}{R} - \frac{\varepsilon^2}{R^3} = 0 \]

External focusing force

**Matched Beam:**

\[ k_0^2 a = \frac{K}{a} + \frac{\varepsilon^2}{a^3} \]

Define **intensity parameter** \( \chi \)

\[ \chi = \frac{K}{k_0^2 a^2} = \frac{\text{space charge force}}{\text{external force}} \]

**Betatron tune depression:**

\[ \frac{k}{k_0} = \frac{v}{v_0} = \sqrt{1 - \chi} \]

**Plasma frequency**

\[ \frac{\omega_p}{\omega_0} = \sqrt{2\chi} \]
Space-Charge Dominated vs Emittance Dominated

Emittance Dominated
\[ \lambda_D \gg a \]

Space-charge Dominated
\[ \lambda_D \ll a \]

Intensity Parameter:
\[ \chi = \frac{K}{k_0^2a^2} \]

Plasma Oscillations Curve
\[ \frac{\omega_p}{\omega_0} = \sqrt{2\chi} \]

Betatron Oscillations Curve
\[ \frac{k}{k_0} = \sqrt{1 - \chi} \]

Existing rings

UMER Range

HIF Drivers

Intensity Parameter (\(\chi\))
University of Maryland Electron Ring
UMER designed to serve as a research platform for intense beam physics

• Beam Energy: 10 keV
• Beam current: 100 mA
• Generalized perveance $1.5 \times 10^{-3}$
• Emittance, 4x rms, norm 10 micron
• Pulse Length 50 - 100 ns
• Bunch charge 5 nC
• Circumference 11.52 m
• Lap time 197 ns
• Tune Depression $(k/k_0) > 0.15$
Diagnostics Available

• Fast Current Monitors (2+) (rise time < 200 ps)
• **Beam Position Monitors** (17 BPMs)
• Phosphor Screens (18+ P-Screens)

• End Diagnostic Chamber:
  – Energy Analyzer
  – Pepper-pot Emittance (Phase Space) Monitor
  – Slit-Wire Emittance (Phase Space) Monitor
  – Faraday Cup
UMER Diagnostics – BPM$^{[1,2]}$

\[
20 \ln \left( \frac{V_R}{V_L} \right) = A \frac{x}{b} + B \left( \frac{x}{b} \right)^3 + C \frac{xy^2}{b^3}
\]

F is chosen to be 76.99$^\circ$ to remove the coupling between X and Y direction $^{[3]}$.

[1] Y. Zou et al, PAC 1999
UMER Diagnostics - BPM
Design of Energy Analyzer\cite{1,2}

1\textsuperscript{st} Generation: Parallel-Plate Retarding EA
> 20 eV Resolution

2\textsuperscript{nd} Generation
~ 3 eV Resolution

3\textsuperscript{rd} Generation: Res. < 1 eV

Collimating Cylinder
-10.13\text{keV}

Retarding Mesh
-9999.5 V

Collector

Grounded Housing

10 keV Beam

[2] Y. Cui, Y. Zou et al, to submit to RSI.
Longitudinal space-charge effect inside the Analyzer

Problems:

- Shift the measured mean energy towards low-energy side.
- Leave a large tail at the high-energy side.
- Make the FWHM of measured spectrum narrower than the true spectrum
- Measured rms energy spread are different

Parameters: 5 keV, 135 mA beam,

Curve 1: 0.2 mA beam current inside the device

Curve 2: 2.2 mA beam current inside the device
Potential solutions for thermal beam[1]

Beam energy: 5 keV,
Initial beam energy spread: 10 eV \((\lambda = J_{in}/J_{lim})\)

\[ \lambda = 0.5 \]

\[ \lambda = 1.4 \]

Comparison of Simulation Results and Experiments

Nominal Energy: 5 keV, Current: 135 mA

**Experiment**
- Curve I: 0.2 mA inside the device ($\lambda = 0.062$, estimated), $E_{\text{rms}} = 2.2$ eV, FWHM = 3.4 eV
- Curve II: 2.2 mA inside the device ($\lambda = 0.8$, estimated), $E_{\text{rms}} = 3.2$ eV, FWHM = 1.1 eV

**1D Theory and simulation plus 2D correction**
- Curve I: $\lambda = 0.062$, $E_{\text{rms}} = 2.2$ eV, FWHM = 5.1 eV
- Curve II: $\lambda = 1.2$, $E_{\text{rms}} = 5.1$ eV, FWHM = 0.49 eV
Part III: Selected Physics Topics

- Experimental study of beam energy spread evolution in intense beam
- Transverse beam emittance of a gridded electron gun
- Experimental study of Resistive wall instability
Experimental Study of Beam Energy Spread in Space-Charge-Dominated Electron Beam*

* Y. Zou et al, to submit to Phys. Rev. STAB
Energy Spread Growth in the Intense Electron Beam

- **Longitudinal-transverse relaxation (intra beam scattering)**\[^1\]
  - Long relaxation time
- **Longitudinal-longitudinal relaxation**\[^2\]
  - Short relaxation time, \( \sim \) plasma period

**Theoretical prediction for the longitudinal energy spread including both effects is given by:**

\[
\Delta E_{\parallel,\text{rms}} = \left[ (2qV_0k_BT_{\parallel})^2 + \left( \frac{C}{\pi\varepsilon_0}qn^{1/3}qV_0 \right) \right]^{1/2}
\]

**Scaling law for the energy spread due to the L-T relaxation:**

\[
\Delta E_{\text{rms}} \sim \left( \frac{I \ast D}{a} \right)^{1/2} \sim \left( J \ast a \ast D \right)^{1/2}
\]

\[^1\] See the reviews in Chapters 5 and 6 of M. Reiser, “Theory and Design of Charged Particle Beams”, John Willey & Sons, 1994.

Phase I Experimental Setup

- Electron Gun
- First Solenoid
- Diagnostic Chamber
- Energy Analyzer
- Movable Phosphor Screen
- Feedthrough

Phosphor screen image

Typical EA Signal
Typical Energy Spread Measurement Results

Energy: 5 keV
Current: 135 mA
$E_{\text{rms}} = 2.1$ eV
FWHM = 3.4 eV

Circular: Experimental results
Triangle: Theory
Energy Spread vs Beam Energy at Different Particle Densities

Beam envelope (5 keV)

Calculated energy spread

Comparison of experimental results and theory
Energy Spread at Different Beam Currents

Beam Energy: 5 keV, Sampled position: 60 nS

Beam current: 135 mA
Energy spread: 2.1 eV

Beam current: 13 mA
Energy spread: 1.7 eV

Energy spread along the pulse (time resolved)
Transverse Beam Emittance Growth in a Gridded Electron Gun[1]

[1] Y. Zou et al., to appear in NIM
Potential Distribution at Different Grid Voltages

Cathode grid distance: $d_{cg} = 0.15 \times 10^{-3} \text{ m}$, $d_{ca} = 0.027 \text{ m}$, $V_a = 10000 \text{ V}$

Potential distribution

Electrical field

Field discontinuity due to the non-natural grid potential

$$\Delta E_z = c_1 \left( V_g^{1/2} + c_2 \right)^{1/2} + \frac{4 V_g}{3 d_g}$$
Emittance of Multi Beam Systems

Configuration space  
Beam trace space

Effective normalized emittance can be calculated as

\[ \varepsilon_{n,g} = \frac{GRA}{4} \left( \frac{2eV_g}{mc^2} \right)^{1/2} \frac{|\Delta E_z|}{V_g} \]

Where \( G = 16 \left( \frac{\lambda^3}{3\pi} \sum_{i=-N}^{N} \sqrt{1-4\lambda^2 i^2 i^2} \right)^{1/2} \) is geometry factor.
Calculated Emittance Growth Vs. Grid Voltage

Cathode grid distance: $d_{cg} = 0.15e^{-3}$ m, $d_{ca} = 0.0255$ m, Beam radius: $R = 4e^{-3}$ m

Half opening of mesh: $a = 0.075e^{-3}$ m, Anode Voltage: $V_a = 10000$ V

Normalized effective emittance vs. grid voltage
Compare with the Experimental Results

Cathode grid distance: $d_{cg} = 0.15 \times 10^{-3} \text{ m}$, $d_{ca} = 0.0255 \text{ m}$, Beam radius: $R = 4 \times 10^{-3} \text{ m}$

Half opening of mesh: $a = 0.075 \times 10^{-3} \text{ m}$, Anode Voltage: $V_a = 10000 \text{ V}$, Grid Voltage: $\sim 25 \text{ V}$

Calculation results:

- emittance due to grid: $\varepsilon_{n,g} = 14 \text{ mm mrad}$
- emittance due to intrinsic thermal motion: $\varepsilon_{n,i} = 3.5 \text{ mm-mrad}$
- emittance due to non-ideal gun focusing structure$^{[1]}$:
  \[ \varepsilon_{n,f} = 6 \text{ mm mrad} \]
- Total calculated emittance: $\sim \sqrt{\varepsilon_{n,g}^2 + \varepsilon_{n,i}^2 + \varepsilon_{n,f}^2} = 15.6 \text{ mm-mrad}$

Experimental results$^{[2]}$: $12\sim16 \text{ mm-mrad}$

3-D realistic PIC simulation with WARP? - I. Haber

$^{[1]}$ D. Kehne, 10 KV Electron Gun Manual
$^{[2]}$ S. Bernal, Internal report, IREAP, 2000
Experiments on Resistive-wall Instability in Space-Charge-Dominated Electron Beam \cite{1,2}

\[ I_w \]

\[ E_r(z) \]

\[ B_\theta(z) \]

\[ \text{Beam Current} \]

\[ C^* \quad L^* \quad R^* \quad C^* \]


Linear Theory

In one-dimensional theory, the resistive-wall instability is governed by following two linearized equations:

\[
\frac{\partial \Lambda_1(z,t)}{\partial t} + \frac{\partial i_1(z,t)}{\partial z} = 0
\]

and

\[
\left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial z} + qE_0 \frac{\partial}{\partial p} \right) f_1(z, p, t) = -qE_1(z, t) \frac{\partial f_0(p)}{\partial p}
\]

In the long-wavelength range (\(\lambda >> d\)), the resistive-wall instability solution can be expressed as:

\[
e^{\pm k_i z} e^{i(wt \pm k_r z)}
\]

with

\[
k_i = R_w^* \sqrt{\frac{\pi \varepsilon_0 e \Lambda_0}{g m \gamma}}
\]
Experimental Apparatus (1)

- Ion Pump
- First Diagnostic Chamber
- Current Monitor 1
- Electron Gun
- M1 - M3: Matching Lenses
- EA: Energy Analyzer
- Solenoid
- Short Solenoid
- Resistive-Wall Channel
- Second Diagnostic Chamber
- Current Monitor 2
- CCD Camera
- Computer
- PS: Phosphor Screen
- EM: Emittance Meter
Generation of Space-Charge Waves[1]

Cathode-Grid pulse

Fast wave has positive current perturbation

Slow wave has negative current perturbation

Linear Results for Fast Waves

Fast wave decays in the resistive-wall pipe. $\text{DE}_1=21\text{eV}$, $\text{DE}_2=13\text{eV}$, decay rate $k_i=-0.48\text{ /m}$

Beam energy: 3.5 keV, Beam current: 19.8 mA

Before the resistive wall

After the resistive wall
Linear Results for Slow Waves

Slow wave grows in the resistive-wall pipe. \(\Delta E_1 = 27\text{ eV}, \Delta E_2 = 37\text{ eV},\) growth rate \(k_i = 0.32/\text{m}\)
Beam energy: 2.5 keV, Beam current: 30 mA

Before resistive wall

After resistive wall

Energy Analyzer Signal (mV)

Time(ns)

Energy Analyzer Signal (mV)

Time(ns)
# Fast Wave Decay Rate at Different Beam Energies

<table>
<thead>
<tr>
<th>Beam Energy (keV)</th>
<th>Energy</th>
<th>2.5</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Current (mA)</td>
<td></td>
<td>15.6</td>
<td>19.8</td>
<td>23.2</td>
</tr>
<tr>
<td>ΔE₁ (eV)</td>
<td></td>
<td>12 ± 1</td>
<td>21 ± 1</td>
<td>18 ± 1</td>
</tr>
<tr>
<td>ΔE₂ (eV)</td>
<td></td>
<td>7 ± 1</td>
<td>13 ± 1</td>
<td>16 ± 1</td>
</tr>
<tr>
<td>Experimental kᵢ (1/m)</td>
<td></td>
<td>-0.54 ± 0.2</td>
<td>-0.48 ± 0.12</td>
<td>-0.12 ± 0.12</td>
</tr>
<tr>
<td>Calculated kᵢ from Eq.(3.21) (1/m)</td>
<td></td>
<td>-0.41</td>
<td>-0.4</td>
<td>-0.39</td>
</tr>
</tbody>
</table>
Large Perturbation (Nonlinear Regime)

Example of a nonlinear initial current perturbation.

perturbation strength = $I_p/I_b$
In the nonlinear regime, energy width of particles associated with fast wave increases. $D_{E_1} = 20\, \text{eV}$, $D_{E_2} = 25\, \text{eV}$, $K_i = 0.23\, \text{1/m}$

Beam energy 2.5 keV, Beam current 16 mA
Fast Wave Growth Rate vs Initial Perturbation Strength

Beam energy 2.5 keV, Beam Current 16 mA.
Summary

• Overview of UMER and its design and diagnostics
  • Design of BPM and Energy analyzer
• Experimental study of beam energy spread in the intense electron beam
• Beam emittance growth in a gridded electron gun
• Experimental study of resistive wall instability