Selected Topics of Theory and Experiment on the Space-Charge-Dominated Beam Physics

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Outline

- Part I: general concepts of space-charge-dominated beams.
- Part II: University of Maryland Electron Ring(UMER) and its components.
  - Diagnostics: BPM, energy analyzer …
- Part III: selected experimental and theoretical results
  - Experimental study of beam energy spread evolution in intense beam
  - Theoretical study of beam emittance of a gridded electron gun
  - Experimental study of Resistive wall instability

Beam Transport in a Uniform Focusing Channel

Beam envelope equation:

$$R'' + k_0^2 R - \frac{K}{R} - \frac{\varepsilon^2}{R^3} = 0$$

External focusing force

Space charge force

Emittance

$$K = \frac{I}{I_0}\frac{2}{\beta^3\gamma^3}(1-\gamma^2 f_e)$$

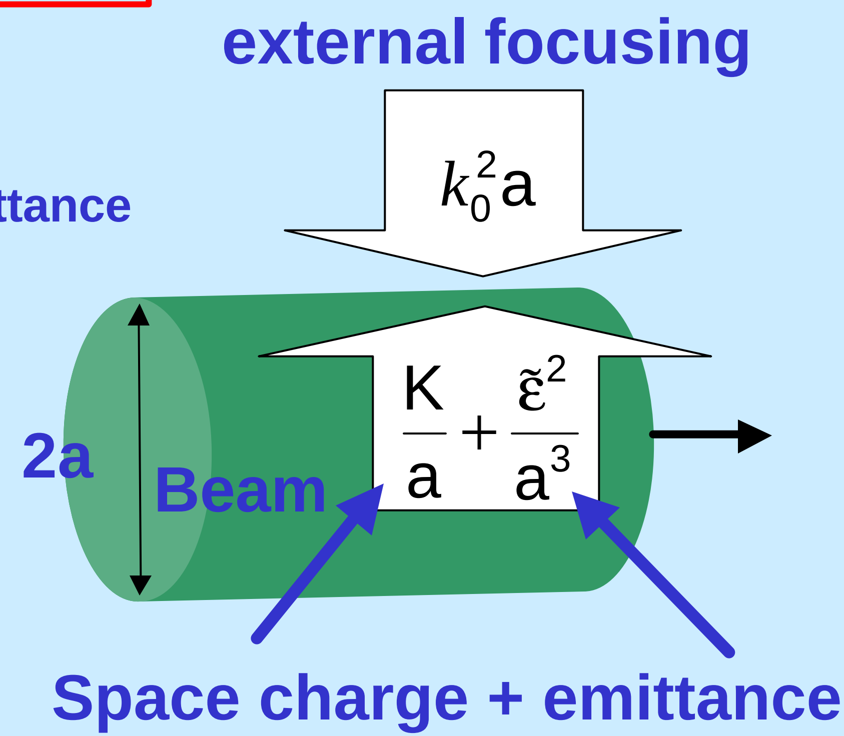
generalized perveance

Matched Beam:

$$k_o^2 a = \frac{K}{a} + \frac{\varepsilon^2}{a^3}$$

Define **intensity parameter (χ)**

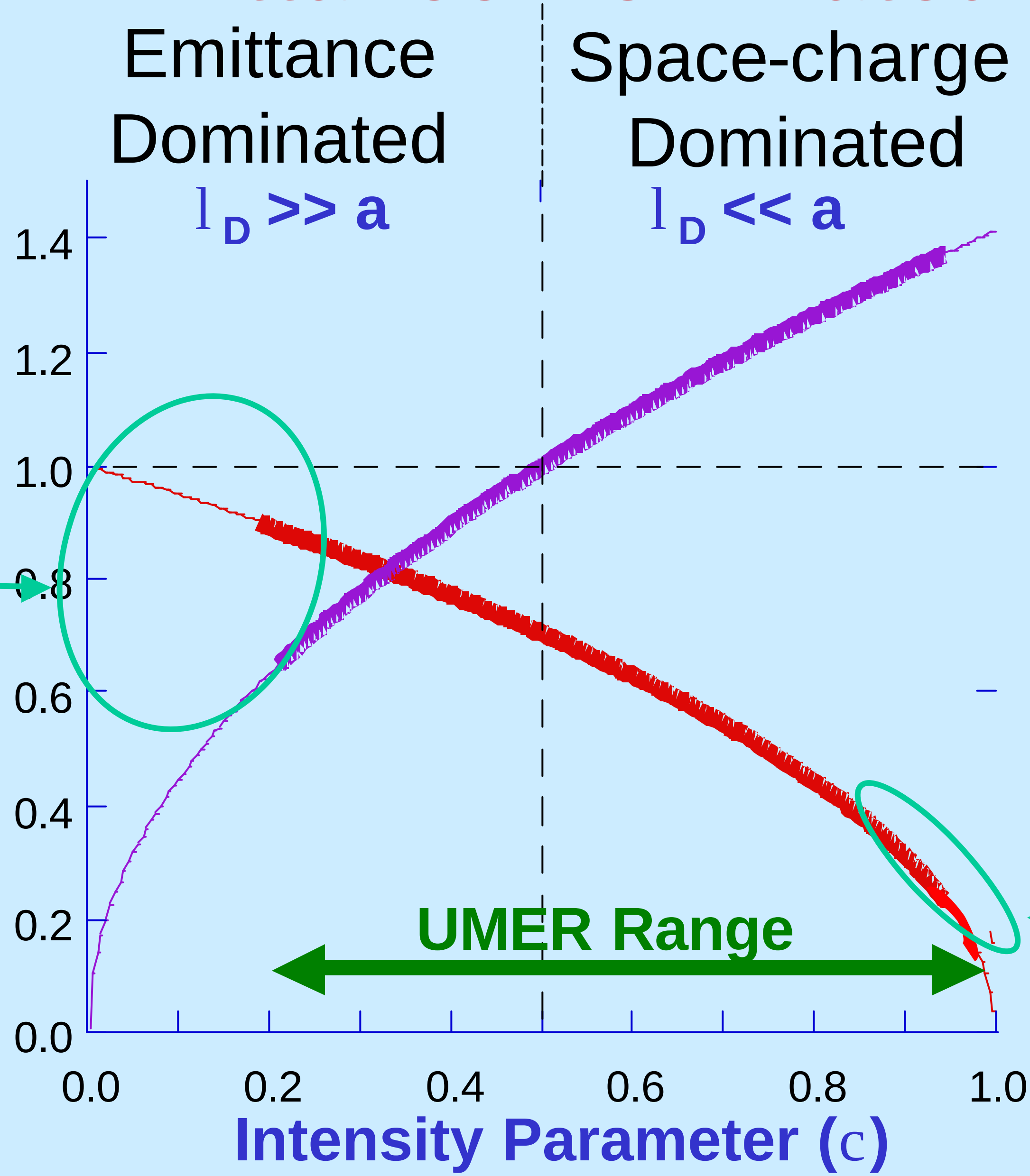
$$\chi = \frac{K}{k_0^2 a^2} = \frac{\text{space charge force}}{\text{external force}}$$



Betatron tune depression: $\frac{k}{k_0} = \frac{\nu}{\nu_0} = \sqrt{1-\chi}$

Plasma frequency $\frac{\omega_p}{\omega_0} = \sqrt{2\chi}$

Space-Charge Dominated vs Emittance Dominated



Plasma Oscillations Curve

$$\frac{\omega_P}{\omega_0} = \sqrt{2\chi}$$

Betatron Oscillations Curve

$$\frac{k}{k_0} = \sqrt{1-\chi}$$

Existing rings

HIF Drivers

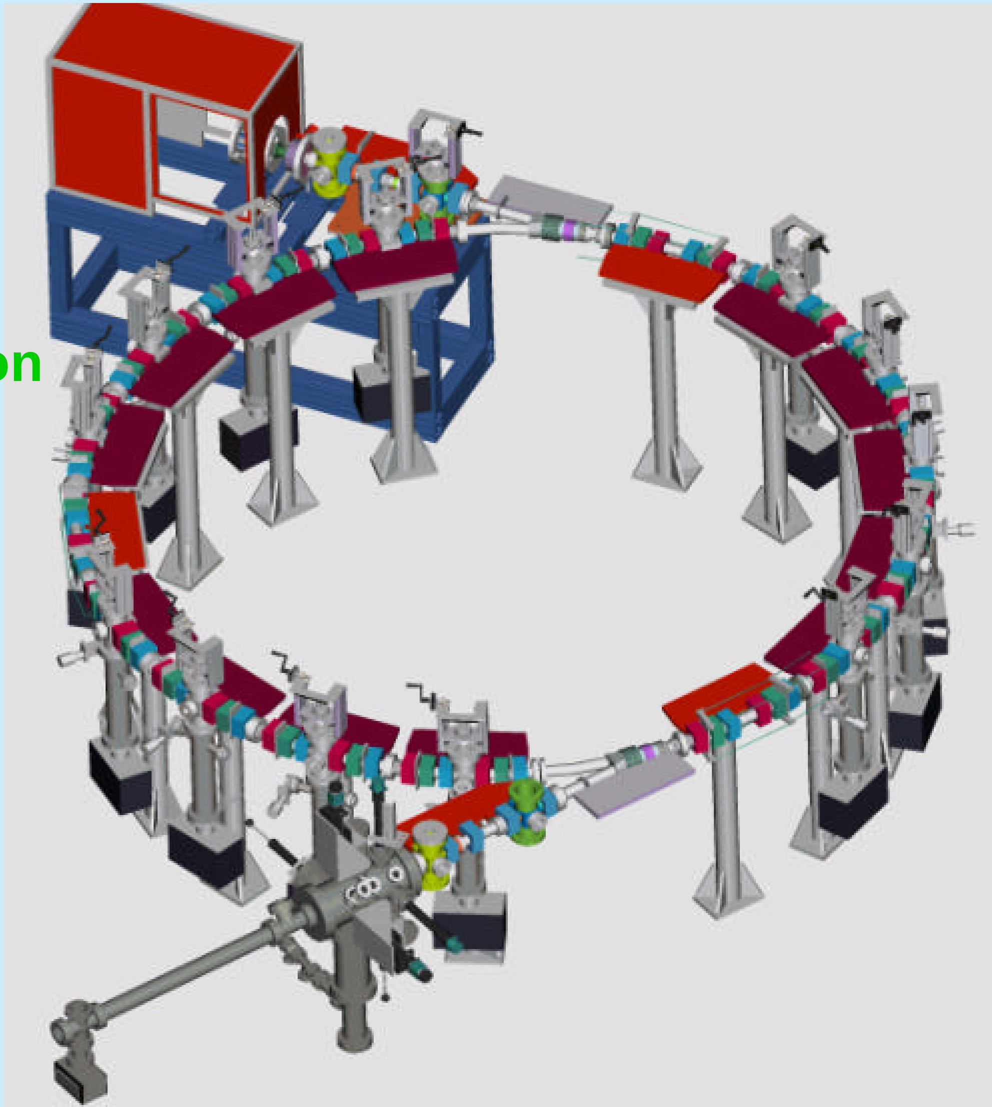
Intensity Parameter:

$$\chi = \frac{K}{k_o^2 a^2}$$

University of Maryland Electron Ring

UMER designed to serve as a **research platform** for intense beam physics

- **Beam Energy: 10 keV**
- **Beam current: 100 mA**
- **Generalized perveance 1.5 x 10^{-3}**
- **Emittance, 4x rms, norm 10 micron**
- **Pulse Length 50 - 100 ns**
- **Bunch charge 5 nC**
- **Circumference 11.52 m**
- **Lap time 197 ns**
- **Tune Depression (k/k_0) >0.15**

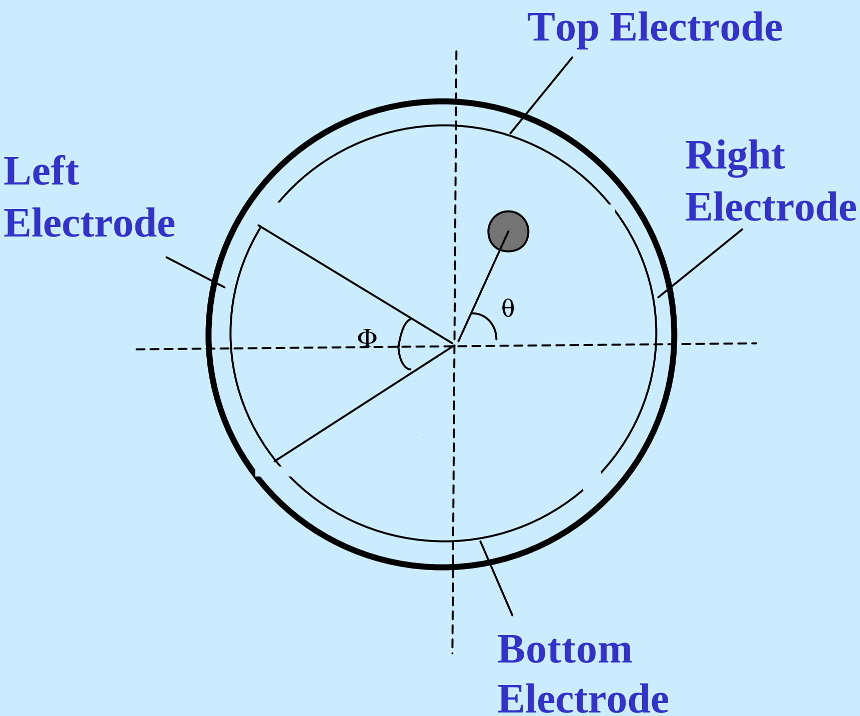


Diagnostics Available

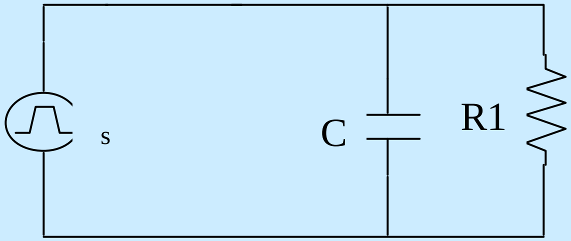
- Fast Current Monitors (2+) (rise time < 200 ps)
- **Beam Position Monitors** (17 BPMs)
- Phosphor Screens (18+ P-Screens)

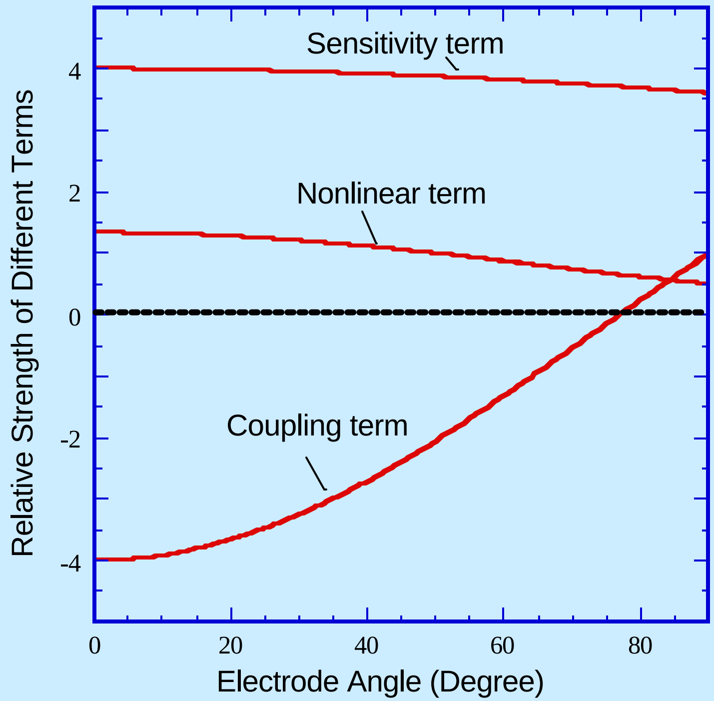
- End Diagnostic Chamber:
  - **Energy Analyzer**
  - Pepper-pot Emittance (Phase Space) Monitor
  - Slit-Wire Emittance (Phase Space) Monitor
  - Faraday Cup

UMER Diagnostics –BPM[1,2]



$$20Ln(V_R / V_L) = A\frac{x}{b} + B\left(\frac{x}{b}\right)^3 + C\frac{xy^2}{b^3}$$

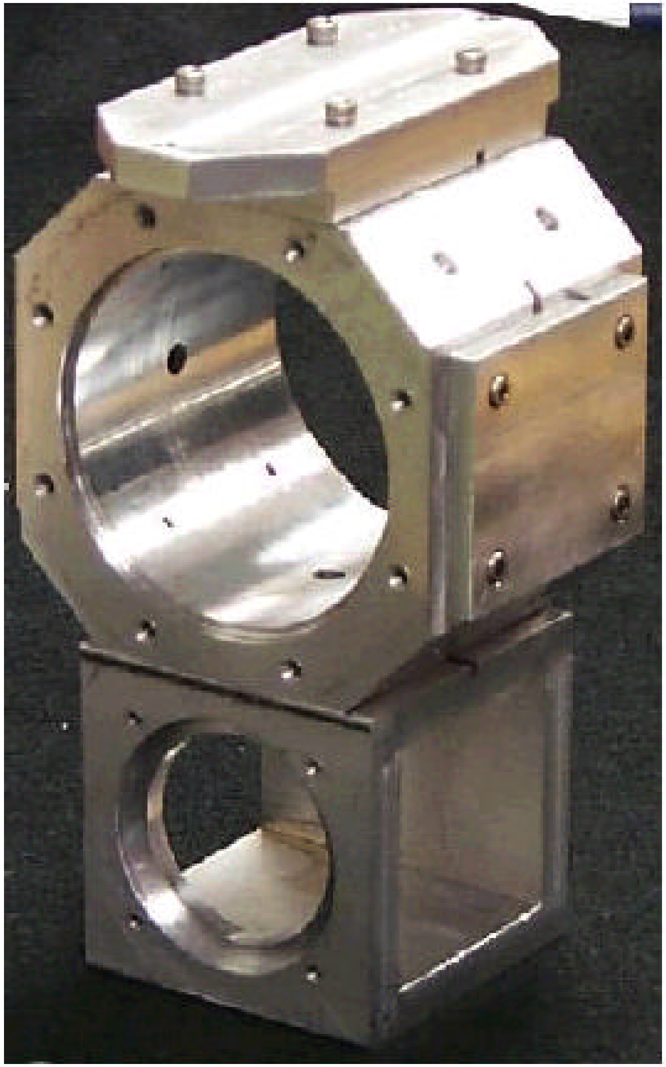


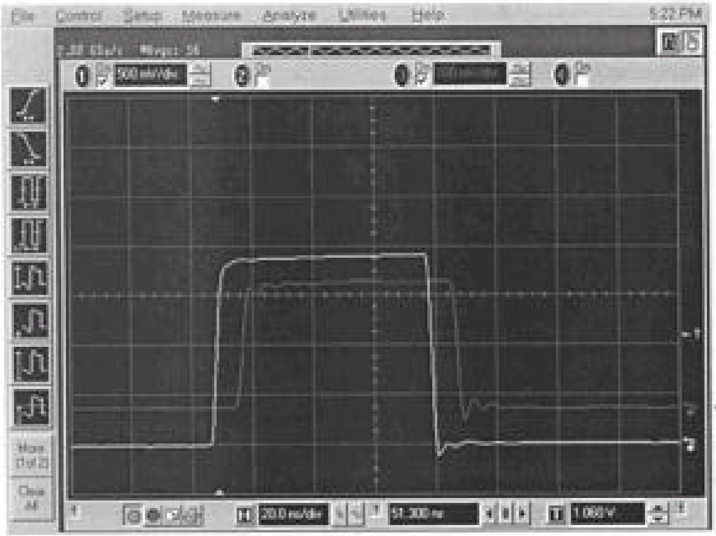


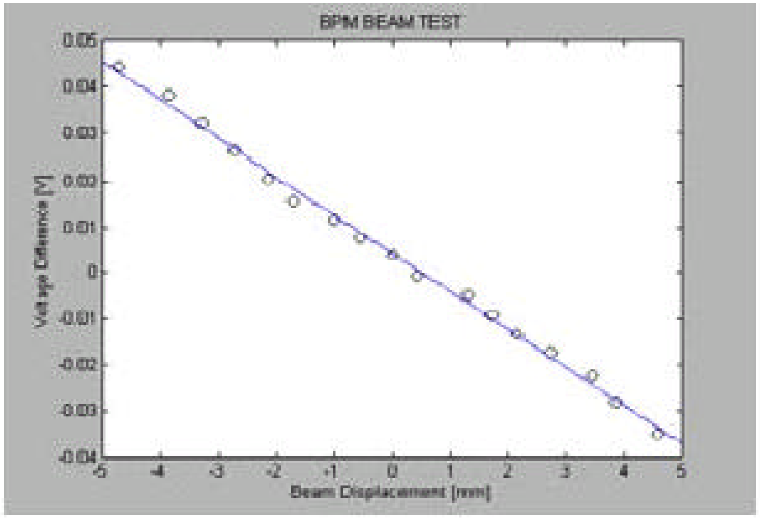
F is chosen to be 76.99^o to remove the coupling between X and Y direction [3].

[1] Y. Zou etal, PAC 1999
[2] B. Quinn etal, PAC 2003
[3] Y. Zou, Ph.D dissertation, UMD, 2000

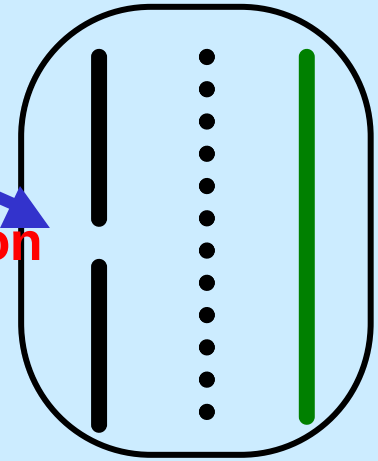
UMER Diagnostics -BPM

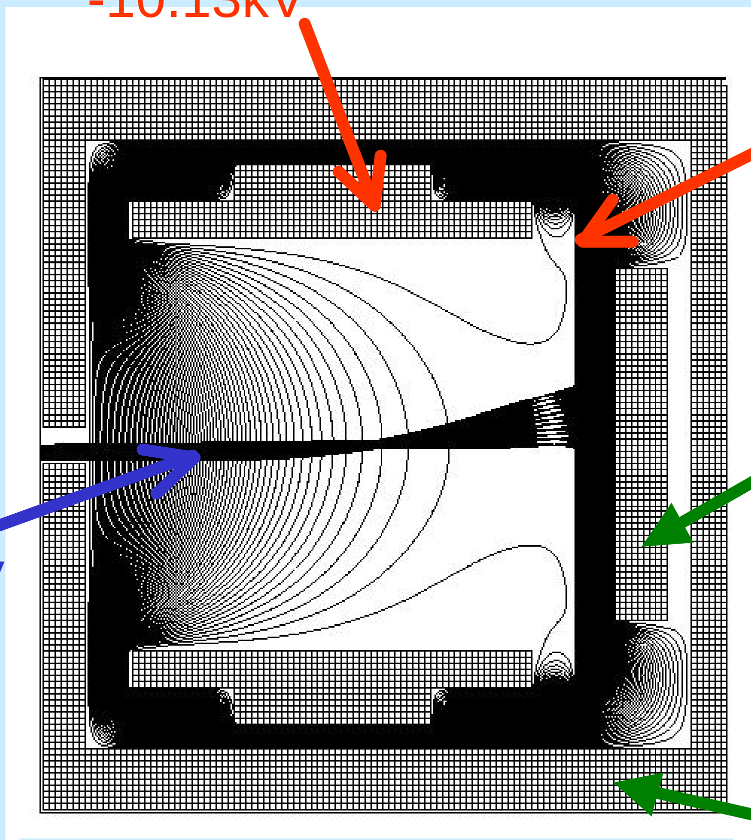




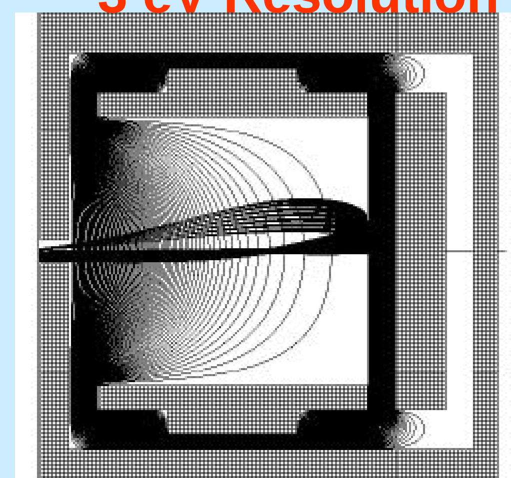


Design of Energy Analyzer[1,2]





3rd Generation: Res. < 1 eV

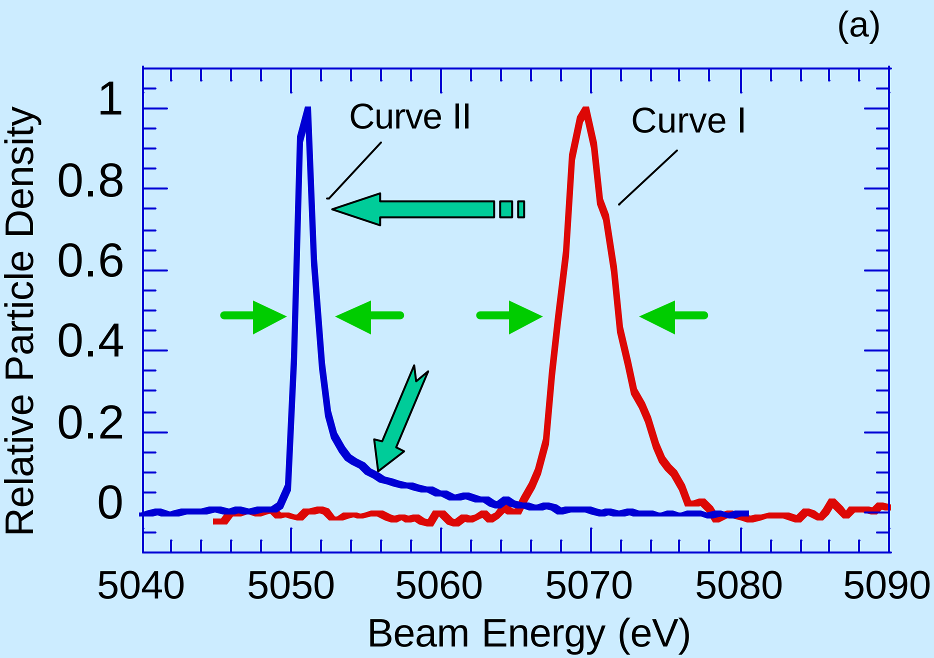


[1] Y. Zou, etal, Phys. Rev. ST Accel. Beams 5(7), 2002, p. 011502.
[2] Y. Cui, Y. Zou etal, to submit to RSI.

Longitudinal space-charge effect inside the Analyzer

Problems:

- Shift the measured mean energy towards low-energy side.
- Leave a large tail at the high-energy side.
- Make the FWHM of measured spectrum narrower than the true spectrum
- Measured rms energy spread are different



Parameters: 5 keV, 135 mA beam,

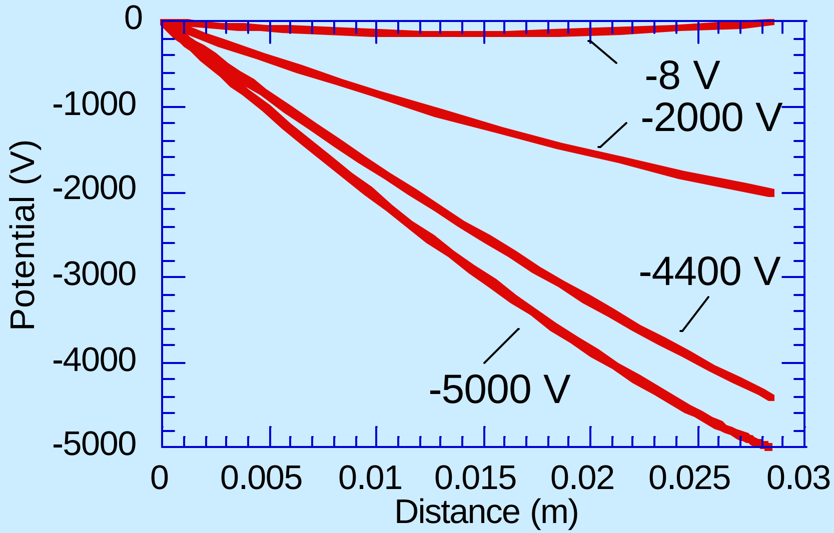
Curve 1: 0.2 mA beam current inside the device

Curve 2: 2.2 mA beam current inside the device

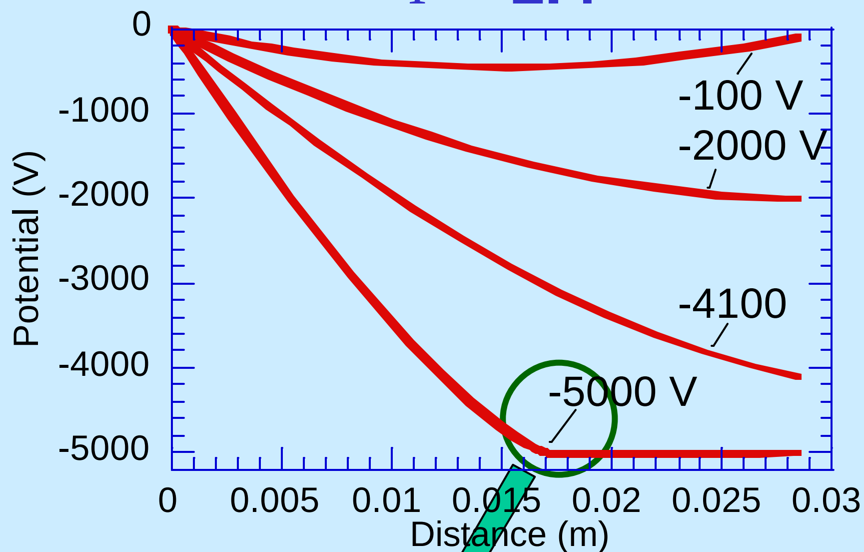
Potential solutions for thermal beam[1]

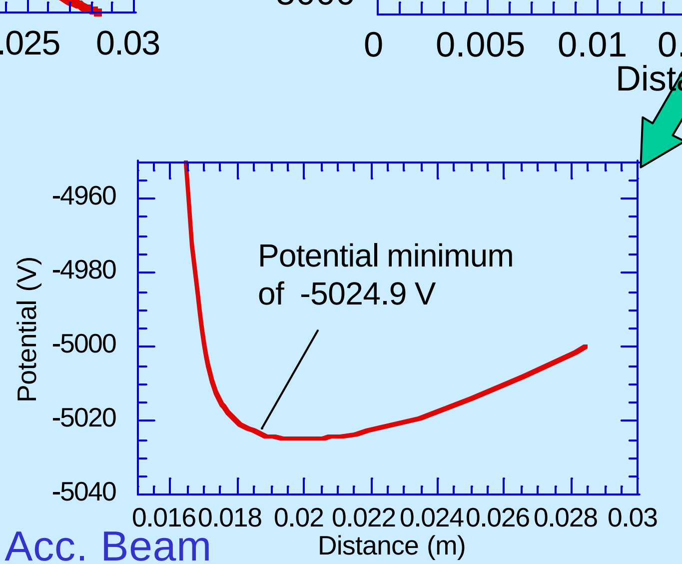
Beam energy: 5 keV,
Initial beam energy spread: 10 eV ($\lambda = J_{in}/J_{lim}$)

λ = 0.5



λ = 1.4





[1] Y. Zou etal, submitted Phys. Rev. ST Acc. Beam

Comparison of Simulation Results and Experiments

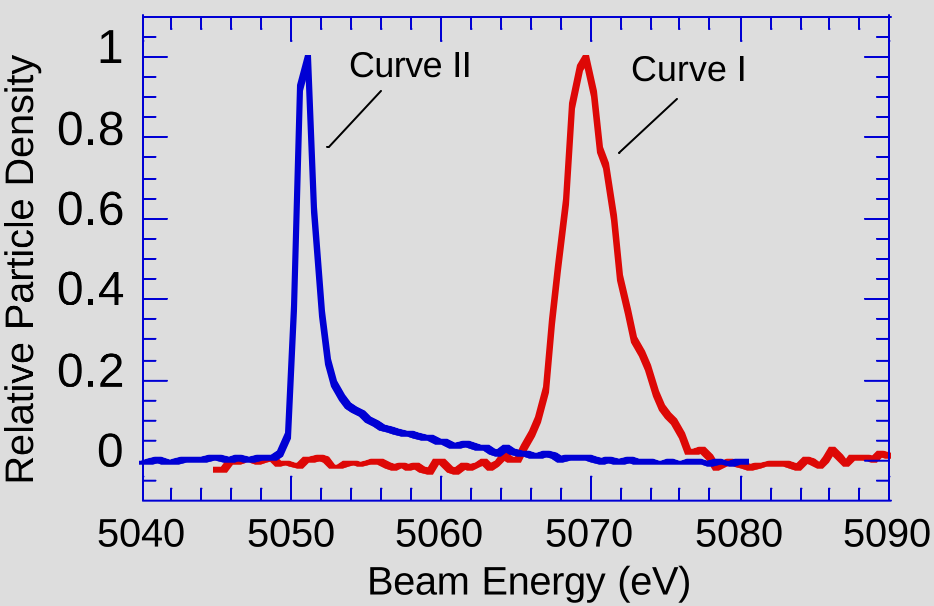
Nominal Energy : 5 keV, Current: 135 mA

Experiment

- Curve I: 0.2 mA inside the device (λ =0.062, estimated), E_{rms}= 2.2 eV, FWHM=3.4 eV

- Curve II: 2.2 mA inside the device (λ =0.8, estimated), E_{rms}= 3.2 eV, FWHM=1.1 eV

(a)

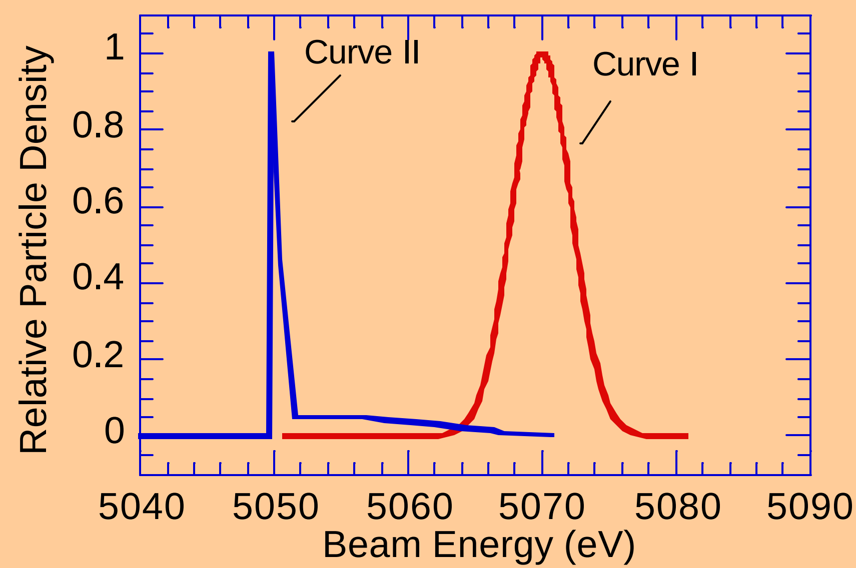


1D Theory and simulation plus 2D correction

- Curve I: λ = 0.062, E_{rms}= 2.2 eV, FWHM = 5.1eV

Curve II: λ = 1.2, E_{rms}= 5.1eV, FWHM = 0.49 eV

(c)



Part III: Selected Physics Topics

• Experimental study of beam energy spread evolution in intense beam
• Transverse beam emittance of a gridded electron gun
• Experimental study of Resistive wall instability

Experimental Study of Beam Energy Spread in Space-Charge-Dominated Electron Beam*

* Y. Zou etal, to submit to Phys. Rev. STAB

Energy Spread Growth in the Intense Electron Beam

- **Longitudinal-transverse relaxation (intra beam scattering)[1]**
  - **Long relaxation time**
- **Longitudinal-longitudinal relaxation[2]**
  - **Short relaxation time, ~ plasma period**

•Theoretical prediction for the longitudinal energy spread including both effects is given by:

$$\Delta E_{//,rms} = [(2qV_0 k_B T_{//})^2 + (C/pe_0) q n^{1/3} q V_0]^{1/2}$$

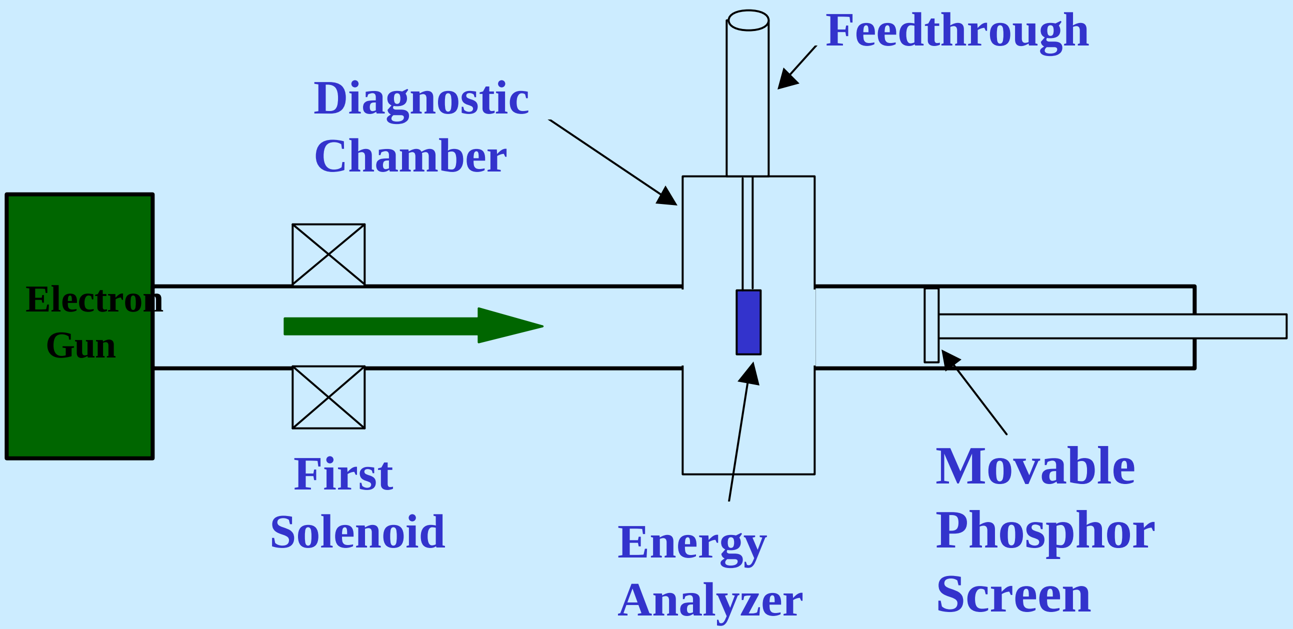
•Scaling law for the energy spread due to the L-T relaxation:

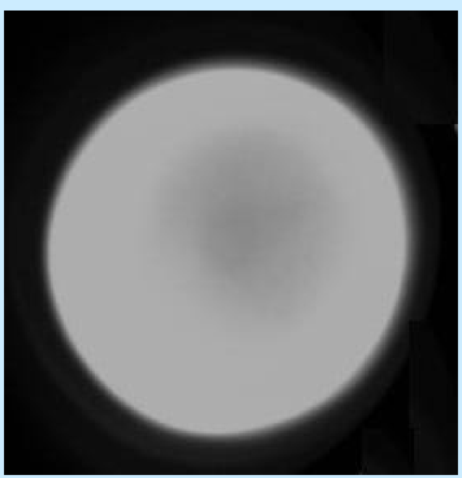
$$\Delta E_{rms} \sim (I * D / a)^{1/2} \sim (J * a * D)^{1/2}$$

[1] See the reviews in Chapters 5 and 6 of M. Reiser, “Theory and Design of Charged Particle Beams”, John Willey & Sons, 1994.

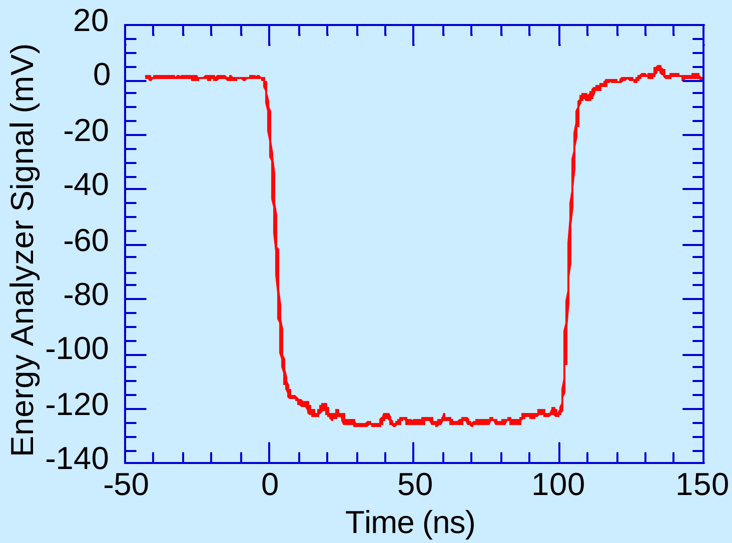
[2] See, for instance, A. V. Aleksandrov et al. Phys. Rev. A, 46, 6628 (1992)

Phase I Experimental Setup





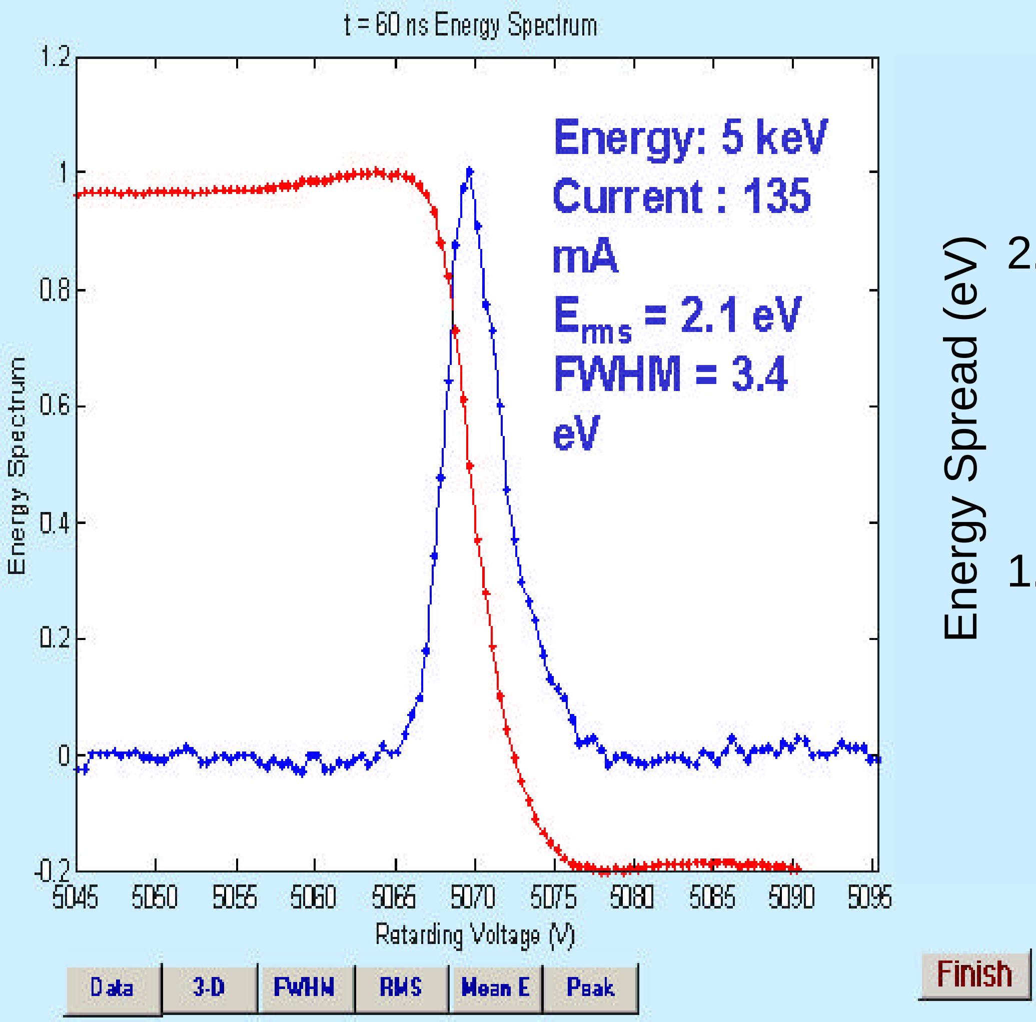
Phosphor screen image

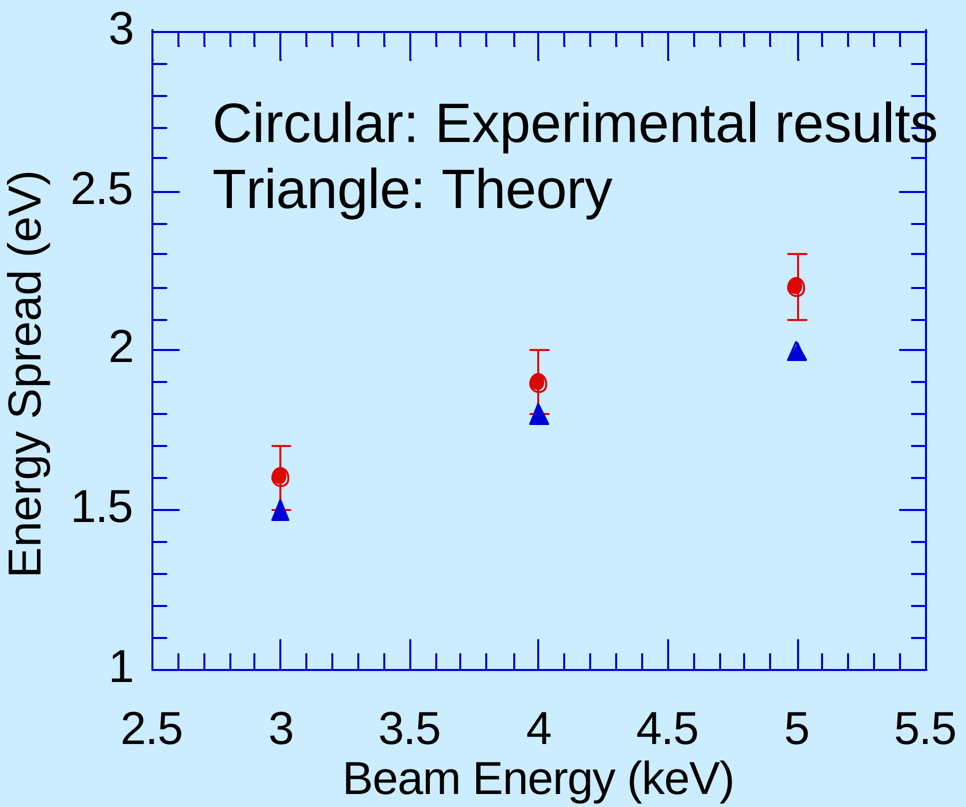


Typical EA Signal

UMER

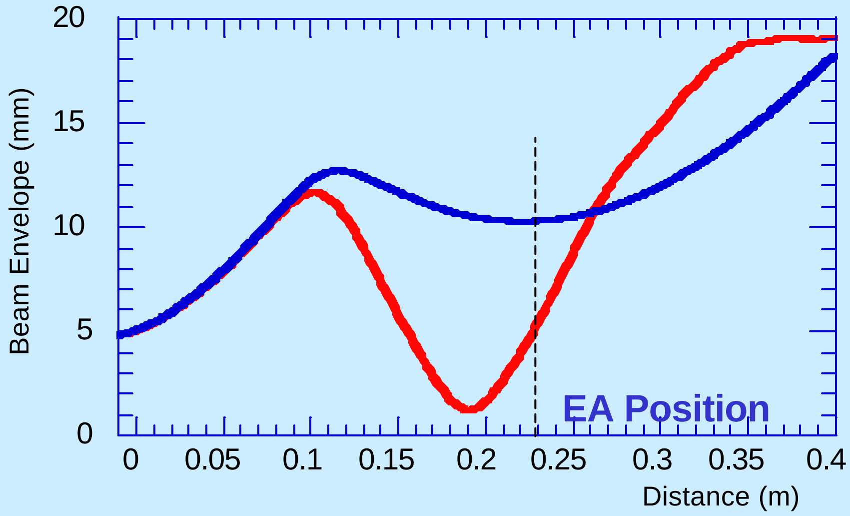
Typical Energy Spread Measurement Results



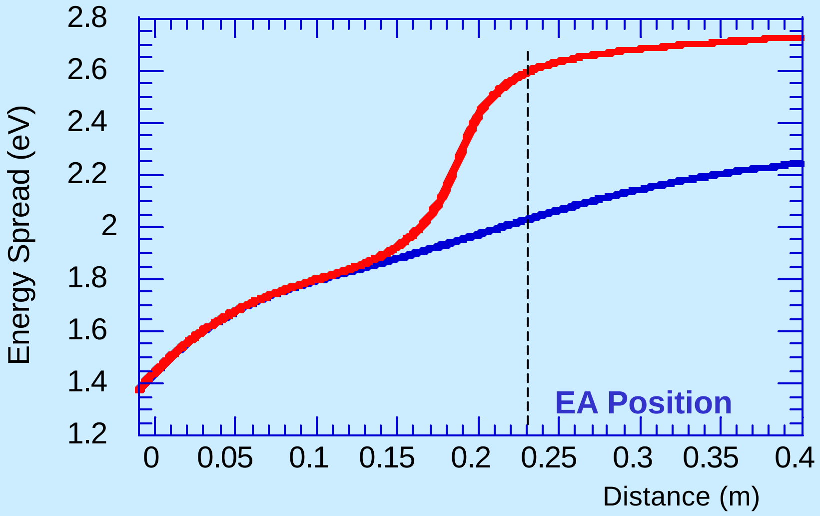


Energy Spread vs Beam Energy at Different Particle Densities

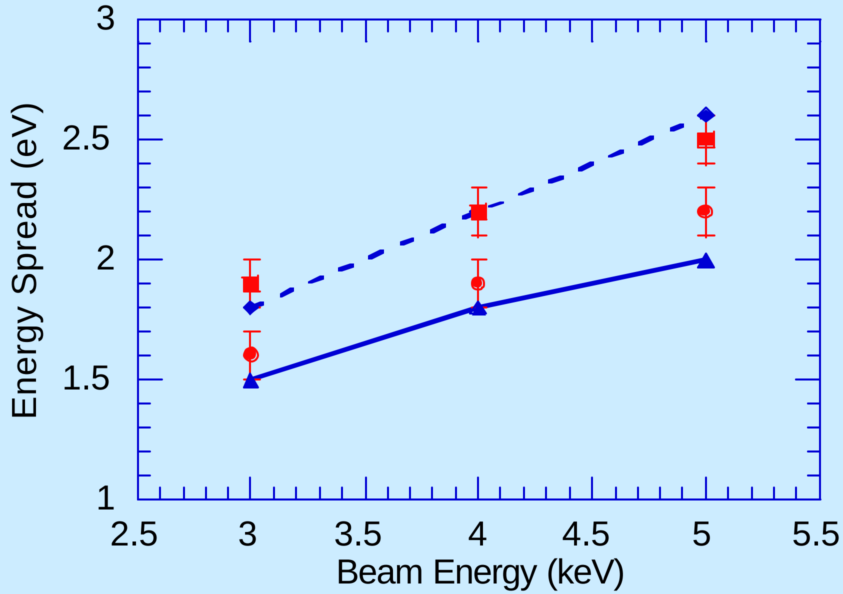
Beam envelope (5 keV)



Calculated energy spread



Comparison of experimental results and theory



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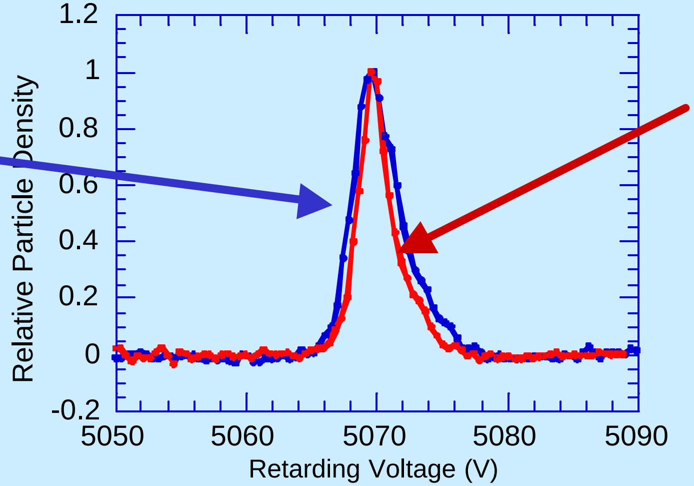
Energy Spread at Different Beam Currents

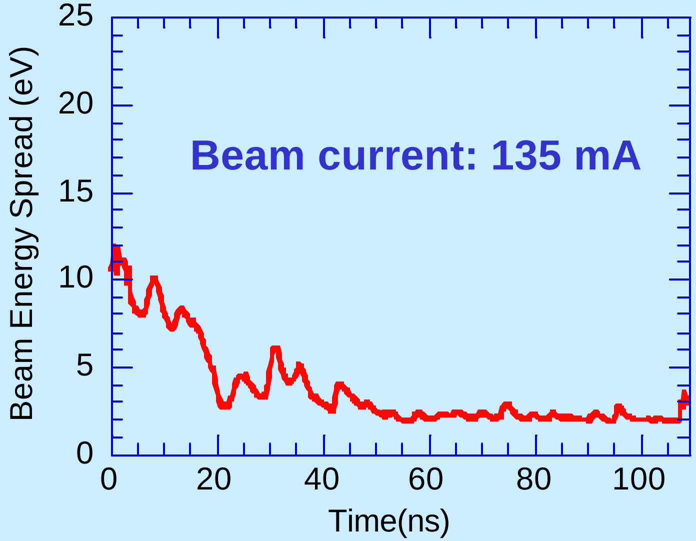
Beam Energy : 5 keV, Sampled position: 60 nS

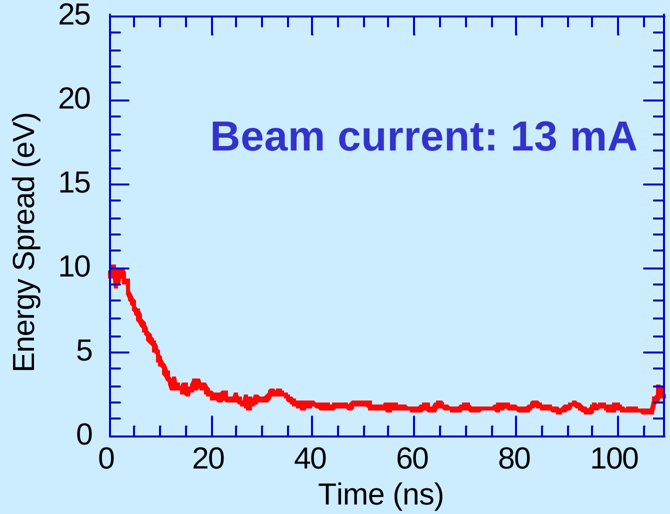
Beam current: 135 mA
Energy spread: 2.1 eV



Beam current: 13 mA
Energy spread: 1.7 eV

Energy spread along the pulse (time resolved)





Transverse Beam Emittance Growth in a Gridded Electron Gun[1]

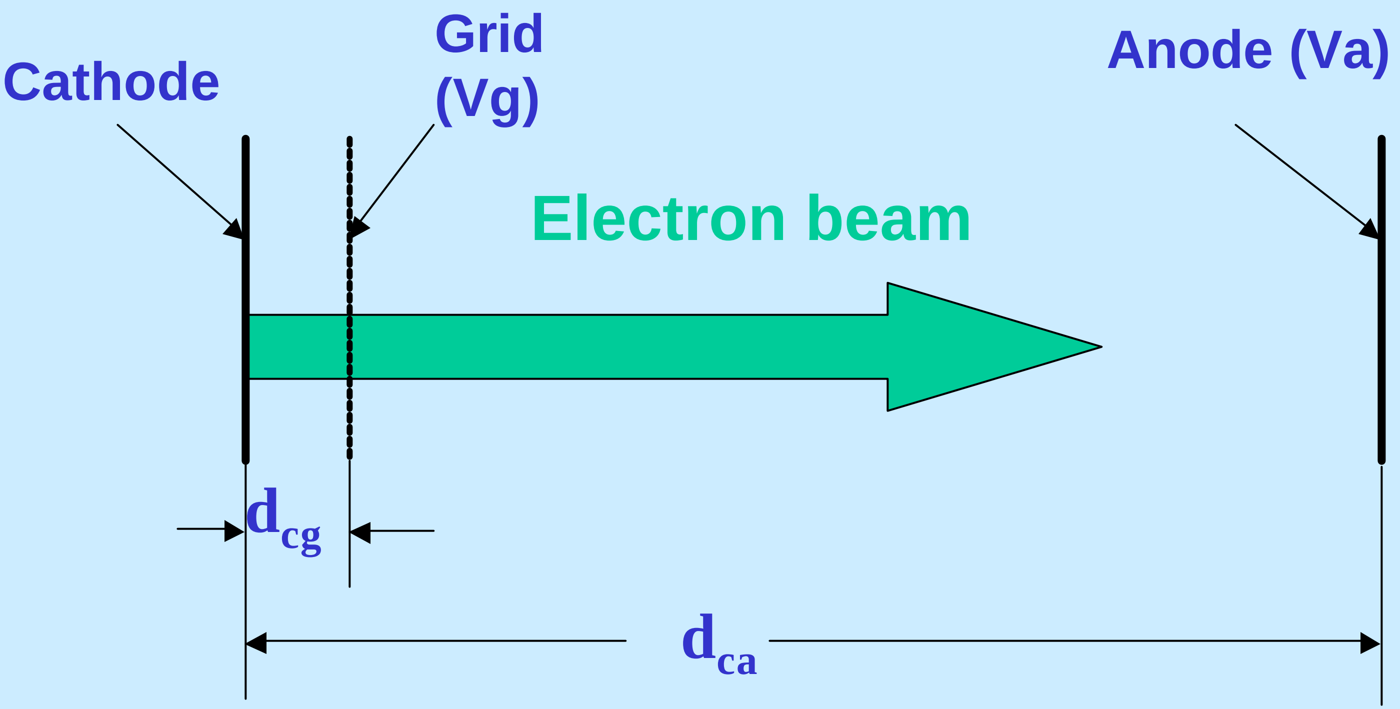


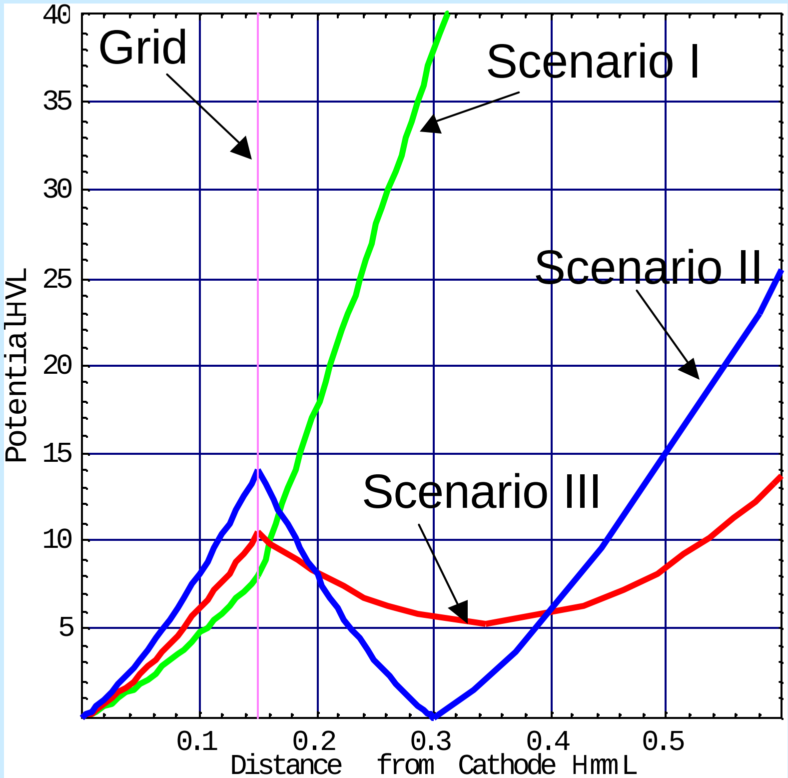
Figure: Schematic of a Grid Gun

[1] Y. Zou etal, to appear in NIM

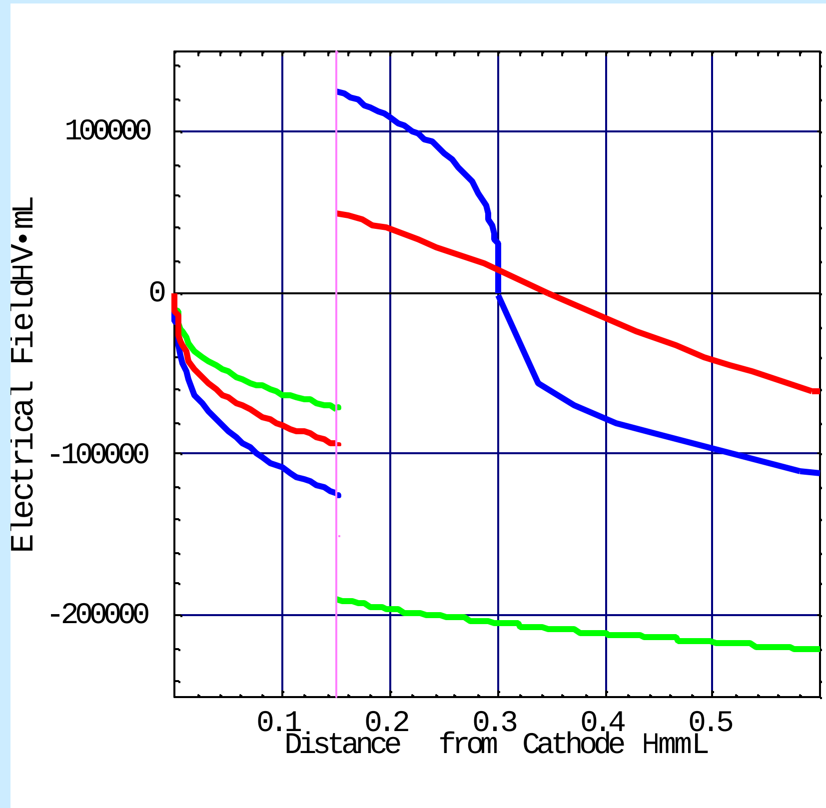
Potential Distribution at Different Grid Voltages

Cathode grid distance: d_{cg} = 0.15e-3 m, d_{ca} = 0.027 m, Va = 10000V

Potential distribution



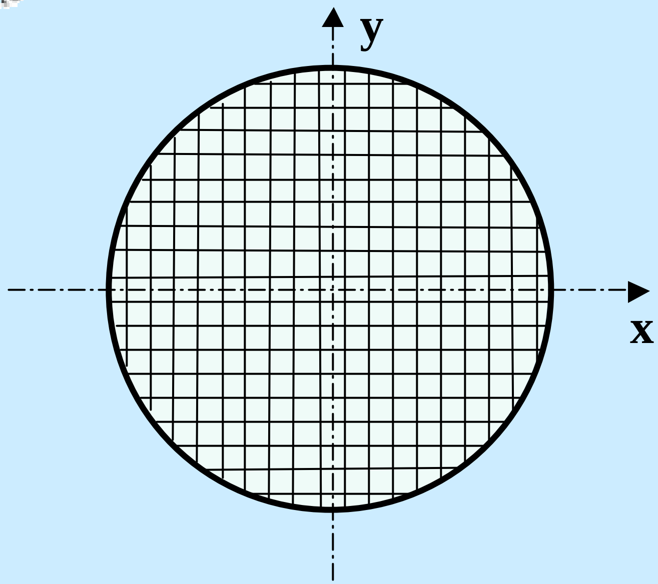
Electrical field



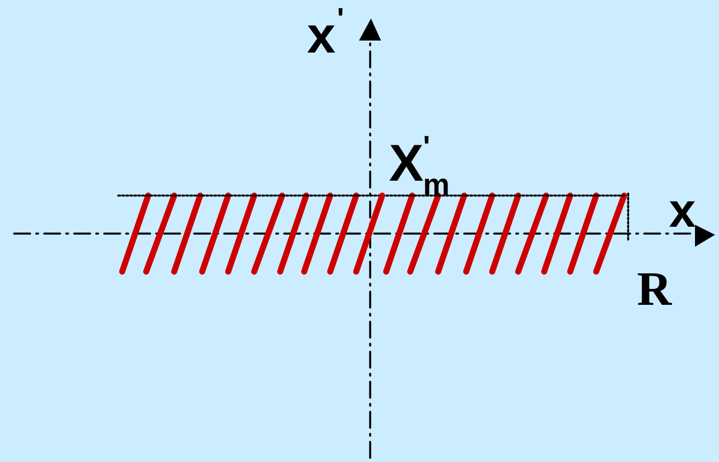
Field discontinuity due to the non-natural grid potential

$$\Delta E_z = c_1\left(V_g^{1/2} + c_2\right)^{1/2} + \frac{4}{3}\frac{V_g}{d_g}$$

Emittance of Multi Beam Systems



Configuration space



Beam trace space

Effective normalized emittance can be calculated as

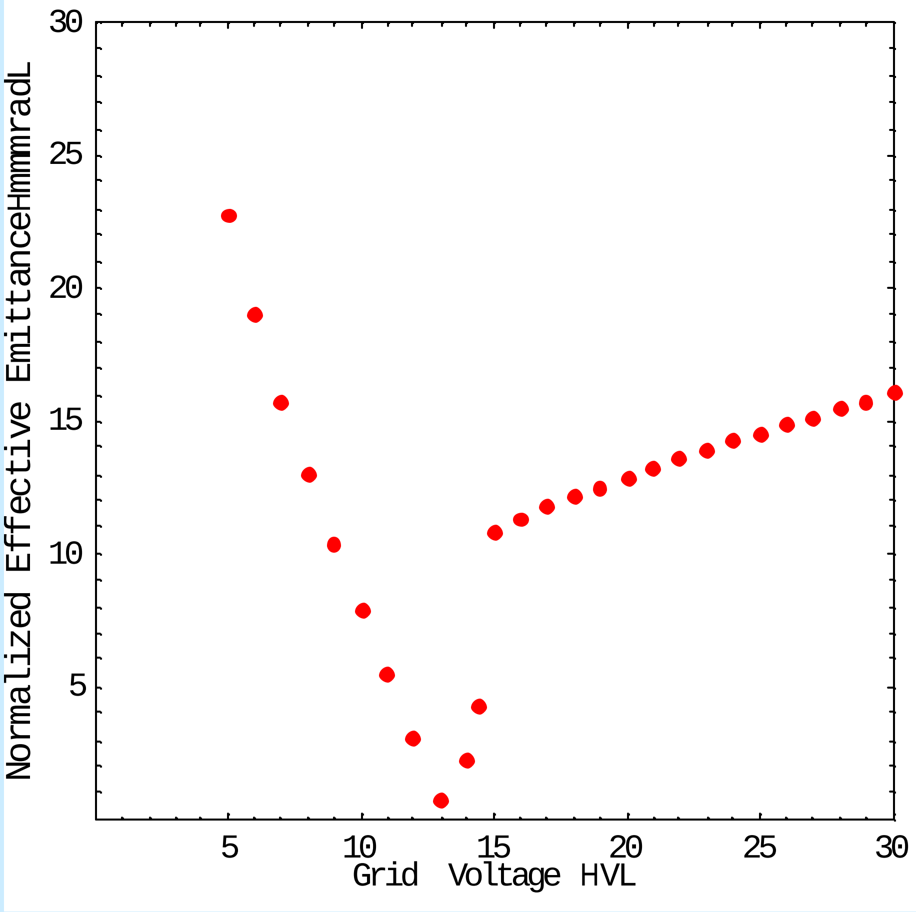
$$e_{n,g} = \frac{GRa}{4}\left(\frac{2eV_g}{mc^2}\right)^{1/2}\frac{|\Delta E_z|}{V_g}$$

Where $G = 16\left(\frac{l^3}{3p}\sum_{i=-N}^{N}\sqrt{1-4l^2i^2}\,i^2\right)^{1/2}$ **is geometry factor**

Calculated Emittance Growth Vs. Grid Voltage

Cathode grid distance: d_{cg} = 0.15e-3 m, d_{ca} = 0.0255 m, Beam radius: R = 4e-3 m

Half opening of mesh: a = 0.075e-3 m, Anode Voltage: Va = 10000 V



Normalized effective emittance vs. grid voltage

Compare with the Experimental Results

Cathode grid distance: d_{cg} = 0.15e-3 m, d_{ca} = 0.0255 m, Beam radius: R = 4e-3 m

Half opening of mesh: a = 0.075e-3 m, Anode Voltage: Va = 10000 V, Grid Voltage: ~25 V

Calculation results:

- emittance due to grid: $\varepsilon_{n,g}$ = 14 mm mrad
- emittance due to intrinsic thermal motion: $\varepsilon_{n,i}$ = 3.5 mm-mrad
- emittance due to non-ideal gun focusing structure[1]:

  $\varepsilon_{n,f}$ =6 mm mrad

- **Total calculated emittance**: ~ $\sqrt{e_{n,g}^2 + e_{n,i}^2 + e_{n,f}^2}$ =**15. 6 mm-mrad**

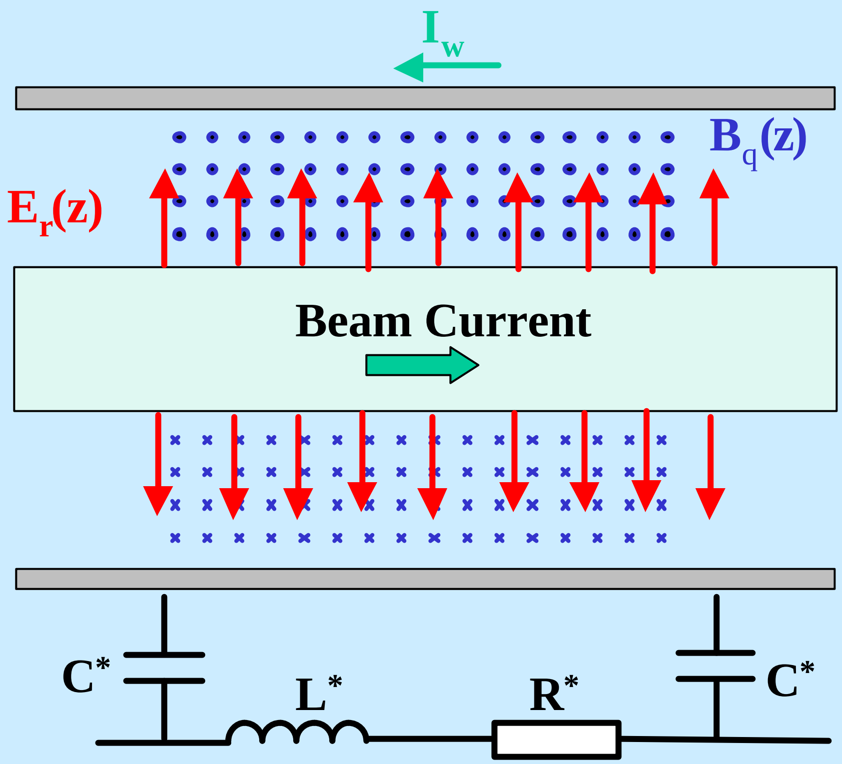
Experimental results[2]: 12~16 mm-mrad

3-D realistic PIC simulation with WARP? - I. Haber

[1] D. Kehne, 10 KV Electron Gun Manual

[2] S. Bernal, Internal report, IREAP, 2000

Experiments on Resistive-wall Instability in Space-Charge-Dominated Electron Beam [1,2]



[1] Y. Zou etal, Phys. Rev. Lett., Vol. 84, p 5138(2000)

[2] H. Suk, J.G. Wang, Y. Zou, etal J. Appl. Phys. Vol. 86, No. 3, p 1699 (1999)

Linear Theory

In one-dimensional theory, the resistive-wall instability is governed by following two linearized equations:

$$\frac{\partial \Lambda_1(z,t)}{\partial t} + \frac{\partial i_1(z,t)}{\partial z} = 0$$

and

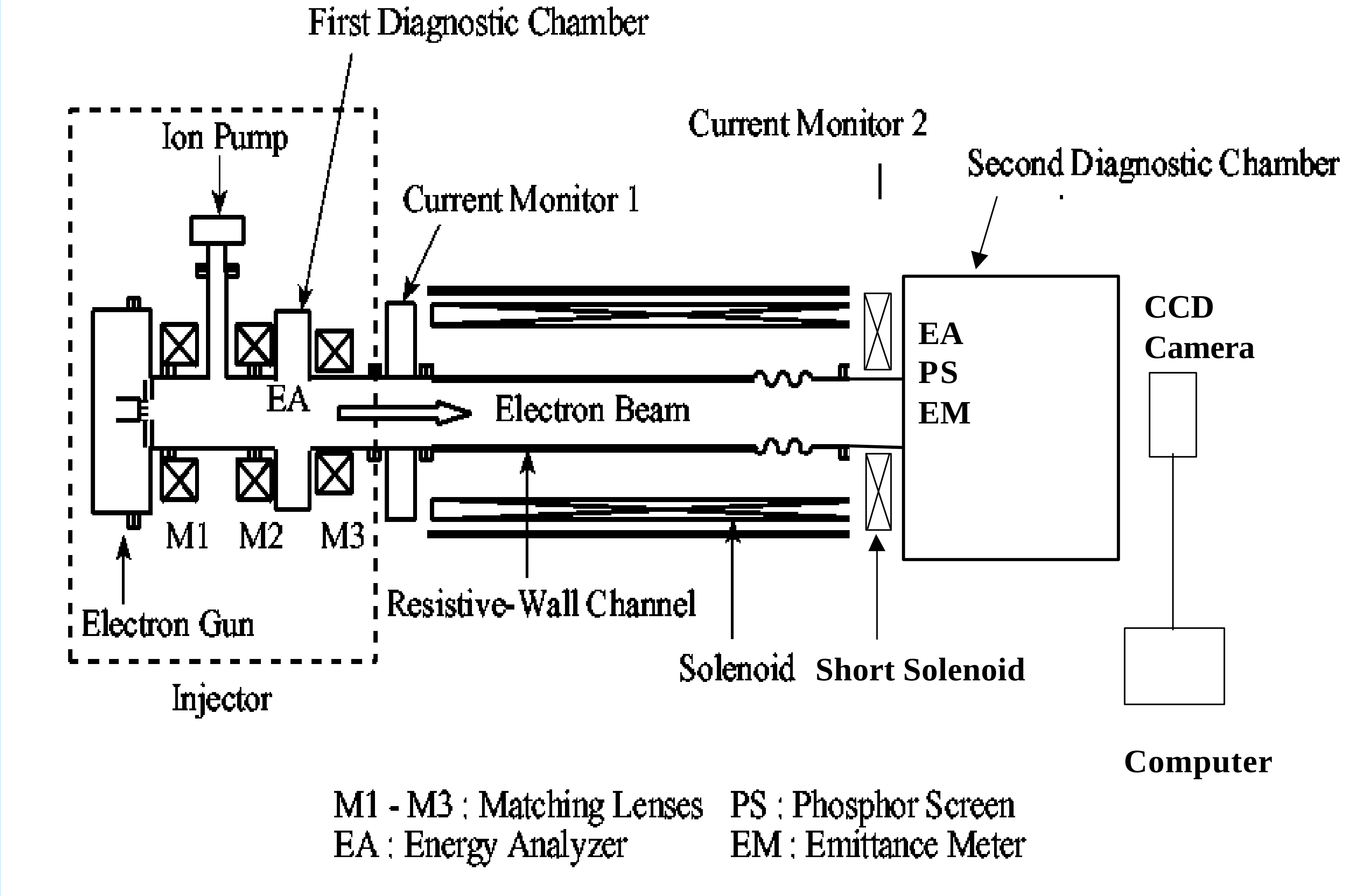
$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z} + qE_0 \frac{\partial}{\partial p} \right) f_1(z,p,t) = -qE_1(z,t)\frac{\partial f_0(p)}{\partial p}$$

In the long-wavelength range (l >>d), the resistive-wall instability solution can be expressed as:

$$e^{\pm k_i z} e^{i(wt \pm k_r z)}$$

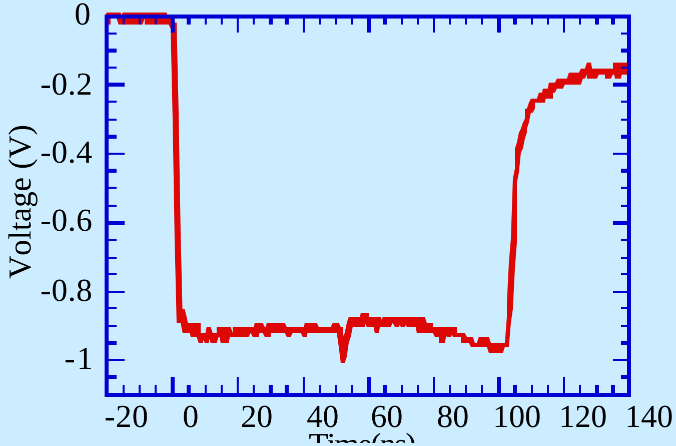
with $k_i = R_w^* \sqrt{\frac{pe_0 e\Lambda_0}{gmg}}$

Experimental Apparatus(1)

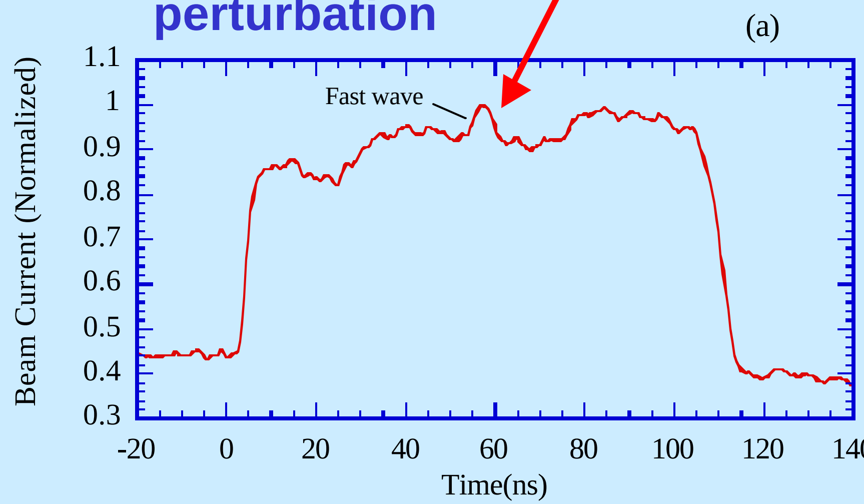


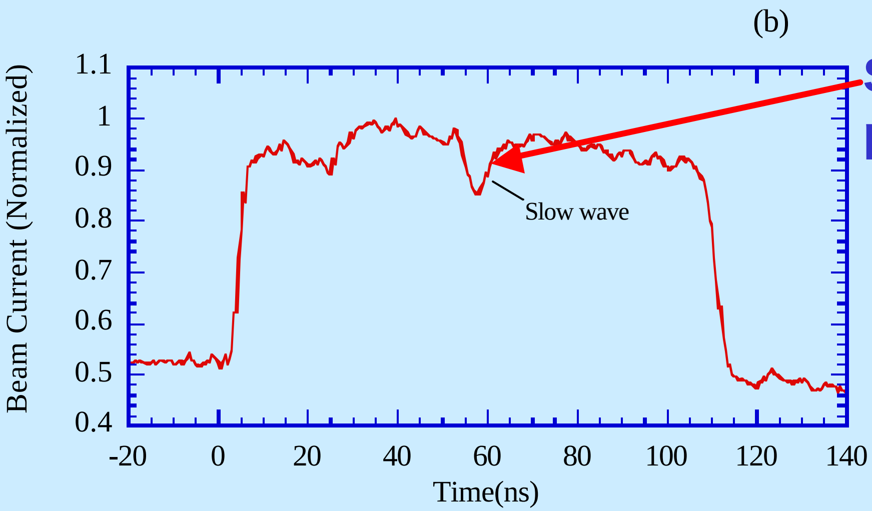
Generation of Space-Charge Waves[1]

Cathode-Grid pulse



Fast wave has positive current perturbation





Slow wave has negative current perturbation

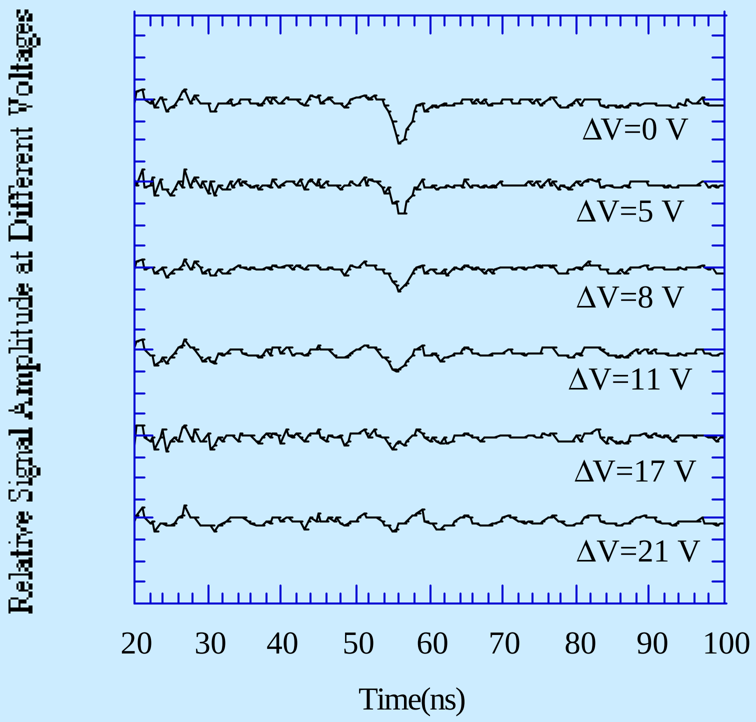
[1] J. G. Wang and M. Reiser, Rev. Sci. Instrum., 65(11), 3444 (1994).

Linear Results for Fast Waves

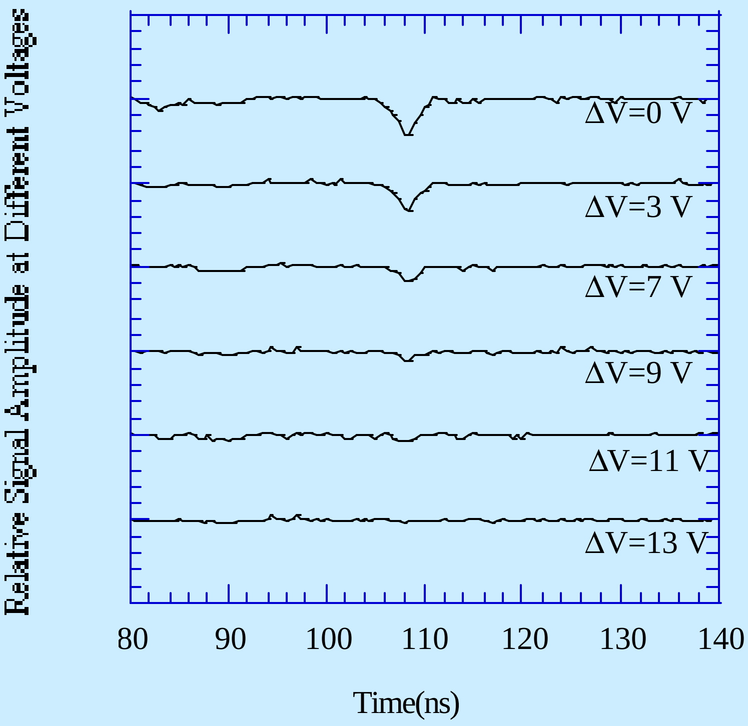
Fast wave decays in the resistive-wall pipe. DE_1=21eV, DE_2=13 eV, decay rate k_i= -0.48 /m

Beam energy: 3.5 keV, Beam current: 19.8 mA

Before the resistive wall



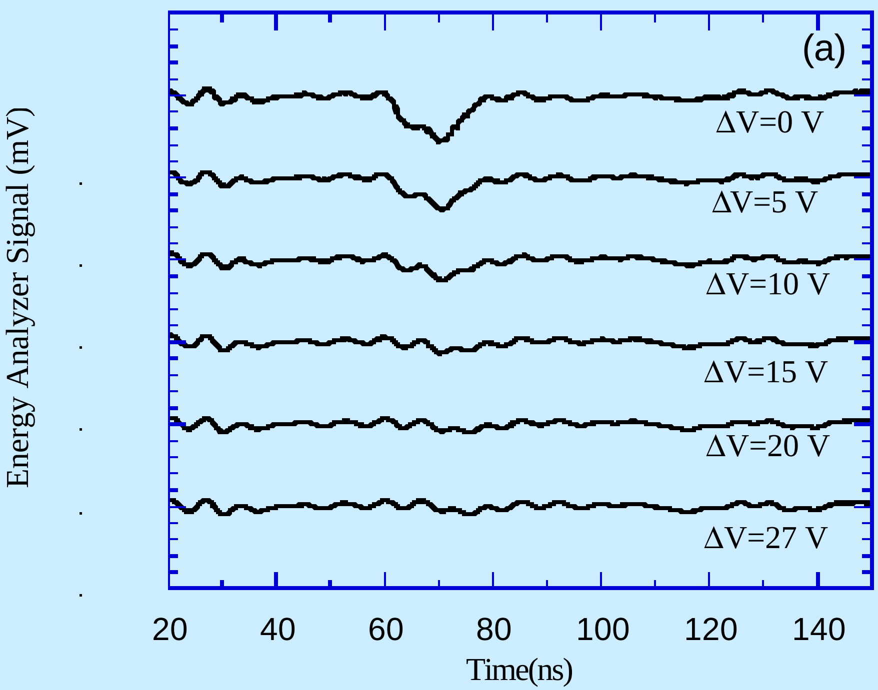
After the resistive wall



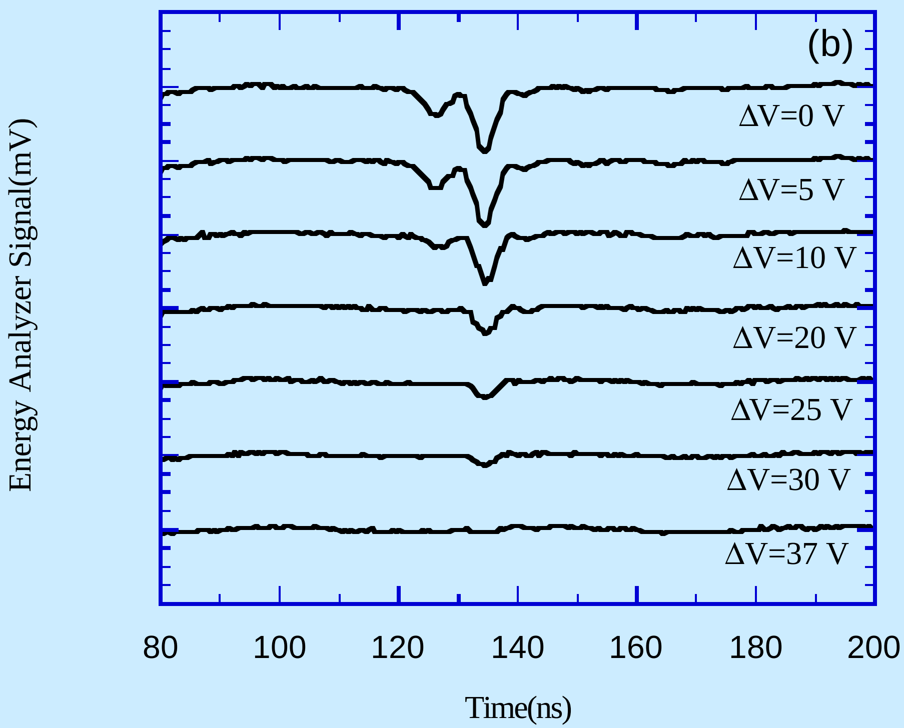
Linear Results for Slow Waves

Slow wave grows in the resistive-wall pipe.
DE_1=27eV, DE_2=37 eV, growth rate k_i= 0.32/m
Beam energy: 2.5 keV, Beam current: 30 mA

Before resistive wall



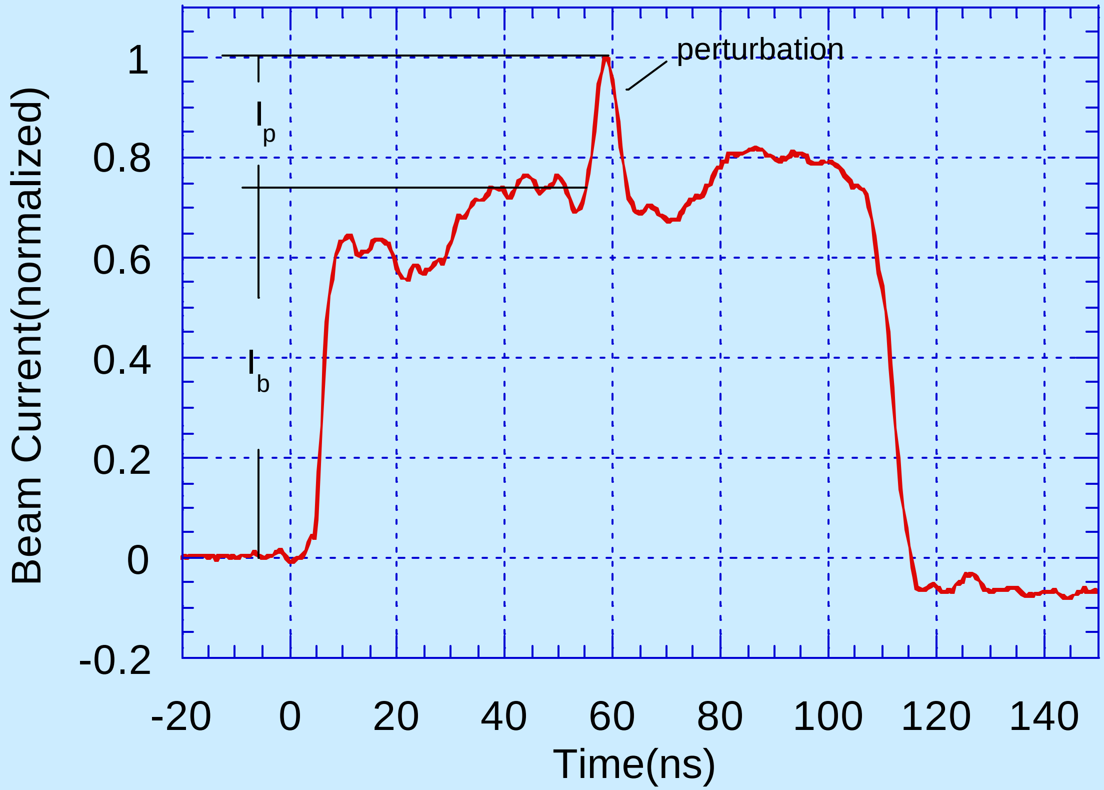
After resistive wall



Fast Wave Decay Rate at Different Beam Energies

Beam Energy (keV)	2.5	3.5	4
Beam Current (mA)	15.6	19.8	23.2
DE_1 (eV)	12 ± 1	21 ± 1	18 ± 1
DE_2 (eV)	7 ± 1	13 ± 1	16 ± 1
Experimental k_i (1/m)	-0.54 ± 0.2	-0.48 ± 0.12	-0.12 ± 0.12
Calculated k_i from Eq.(3.21) (1/m)	-0.41	-0.4	-0.39

Large Perturbation (Nonlinear Regime)



Example of a nonlinear initial current perturbation.

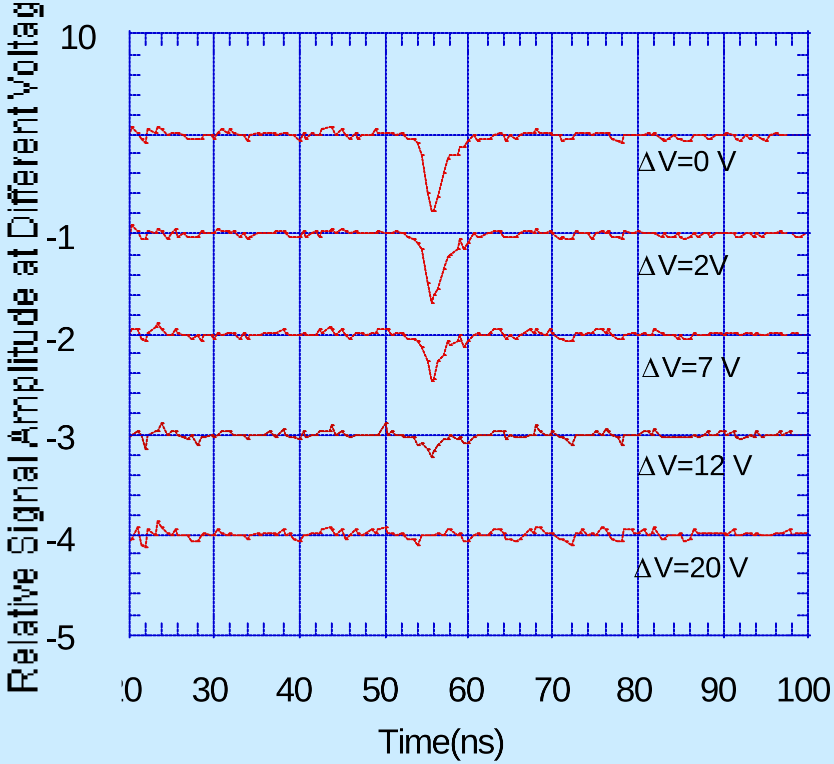
perturbation strength = I_p/I_b

Fast Wave in the Nonlinear Regime

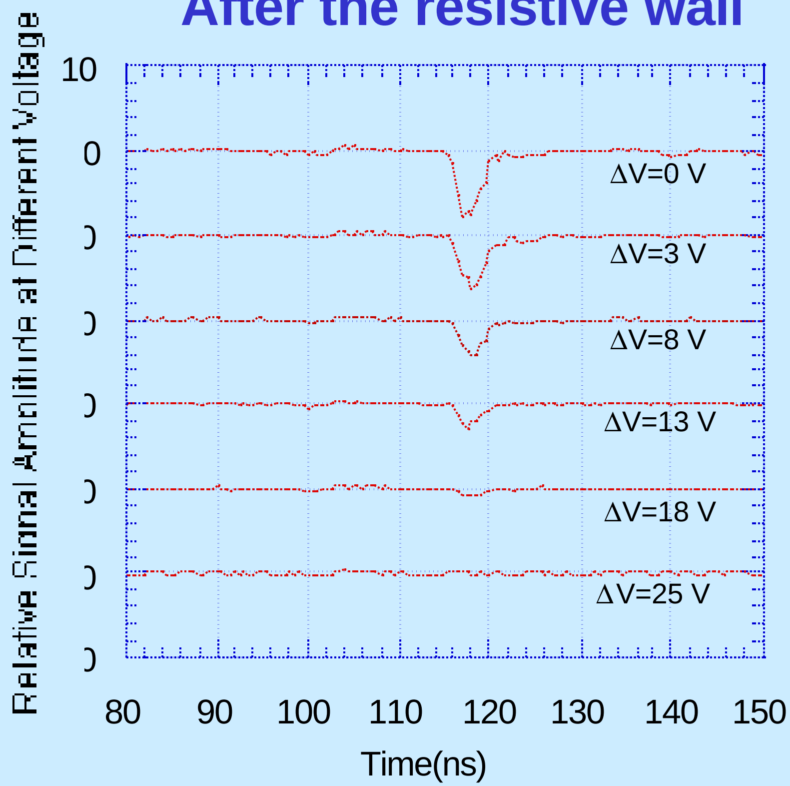
In the nonlinear regime, energy width of particles associated with fast wave increases. DE_1=20 eV, DE_2=25eV, Ki = 0.23 1/m

Beam energy 2.5 keV, Beam current 16 mA

Before the resistive wall

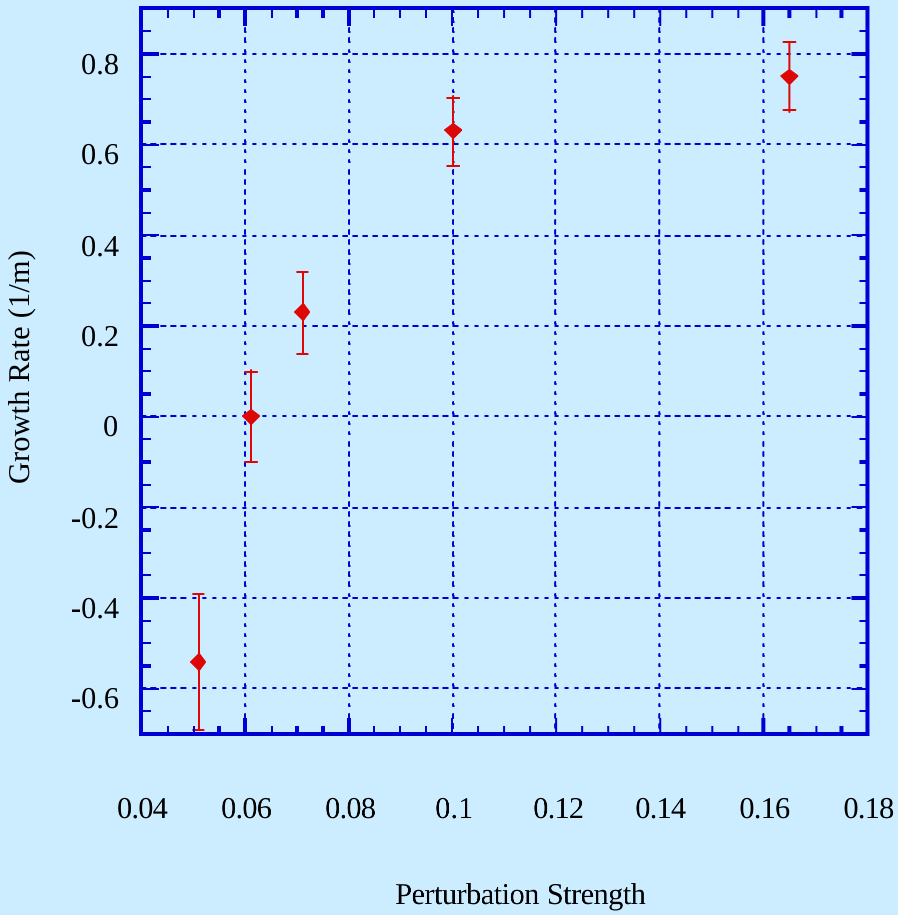


After the resistive wall



Fast Wave Growth Rate vs Initial Perturbation Strength

Beam energy 2.5 keV, Beam Current 16 mA.



Summary

- Overview of UMER and its design and diagnostics
  - Design of BPM and Energy analyzer
- Experimental study of beam energy spread in the intense electron beam
- Beam emittance growth in a gridded electron gun
- Experimental study of resistive wall instability