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USPAS Course on
4^{th} Generation Light Sources II
ERLs and Thomson Scattering

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RF and SRF

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Thomas Jefferson National Accelerator Facility

Operated by the Southeastern Universities Research Association for the U. S. Department of Energy

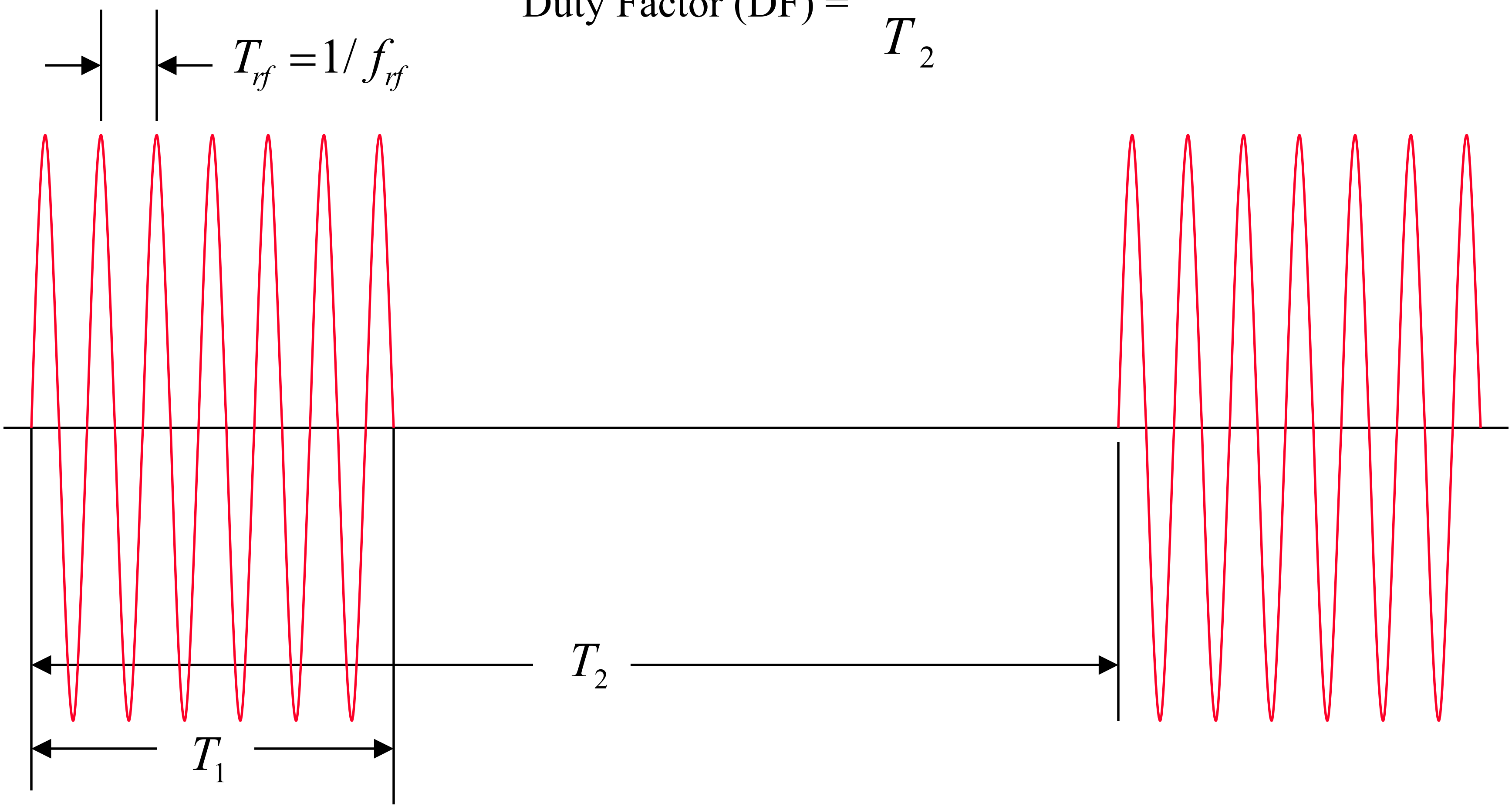
Outline*

- Duty Factor – CW Operation
  - Definitions
  - Superconducting Recirculating Linacs
    - Nuclear Physics
    - FEL Driver Accelerators
- Superconducting RF (SRF)
  - Historical Foundations of SRF
  - State of the Art in SRF in the 70's, 80's and 90's
  - RF Cavity Fundamental Concepts
  - SRF Performance Limitations
    - Multipacting
    - Thermal Breakdown
    - Field Emission
  - State of the Art in SRF in 2000
- Conclusions

*Most of this talk was compiled by Lia Merminga in preparation for a previous USPAS course

Duty Factor – CW Operation: Definitions

$$\text{Duty Factor (DF)} = \frac{T_1}{T_2}$$



Duty Factor – CW Operation: Definitions (cont'd)

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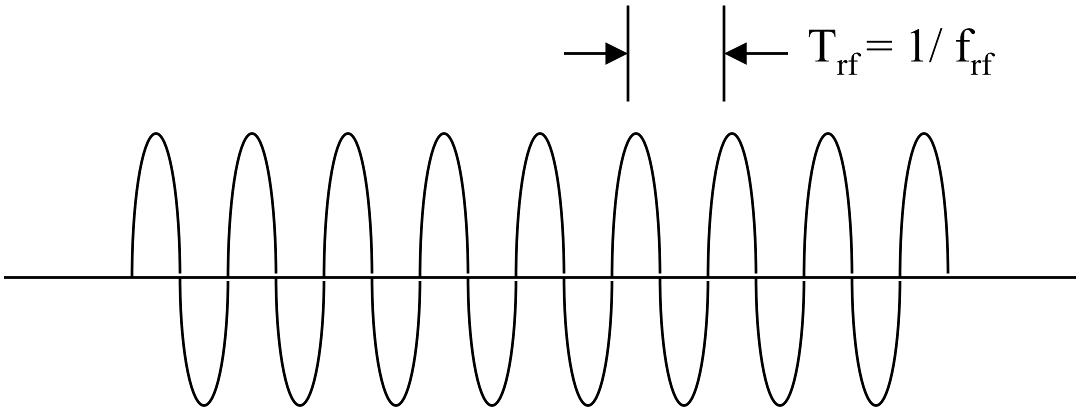

- CW RF: rf is continuously on
- CW Beam: Beam pulse is continuously on at the RF repetition rate or a subharmonic of it
  - Example 1. JLAB FEL: cw rf at 1497 MHz
    cw beam at 74.85 MHz, the 20th subharmonic of the rf wave
  - Example 2. CEBAF: cw rf at 1497 MHz
    cw beam at 499 MHz, the 3rd subharmonic of the rf wave
- High Duty Factor: Duty Factor > 10% with beam pulse lengths of several msecs or more.

The Need for High Duty Factor

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- High energy electrons have been used as nuclear probes: the electromagnetic interaction provides an ideal tool to measure the structure of nuclei.
- In 1977, Livingston Panel to examine: "The Role of Electron Accelerators in U.S. Medium Energy Nuclear Science."
- Livingston report:
  - Points out that almost all significant investigations to that date were "single-arm" experiments: only the scattered electron was detected. The reason: The best duty factors available from electron accelerators were only ~1%.
  - Recommends the next generation of electron accelerators to be capable of carrying out coincidence experiments: both the scattered electron and the associated ejected particle are detected.
- Conclusions of Livingston Panel are reinforced by sub-committee of the U.S. Nuclear Science Advisory Committee (Barnes et al. 1982).
- This committee recommends the construction of at least one cw electron accelerator with maximum energy of 4 GeV.

SRF Recirculating Linacs for Nuclear Physics

- The original motivation (Schwettman et al., 1967) for building a superconducting electron linac at Stanford-HEPL for Nuclear Physics research was to provide:
  - continuous operation
  - high accelerating gradients
  - exceptional stability
  - energy resolution of order 10^{-4}
  - beam currents up to 400 μA

- These characteristics continue to be desirable in modern electron accelerators for Nuclear Physics.

To date these objectives have all been met!

SRF Recirculating Linacs for Nuclear Physics (cont'd)

- For cw operation, power dissipated in the walls of a Cu structure is substantial. Not so for superconductors:
  - Power dissipated in cavity walls is: $P_{diss} = \frac{E_{acc}^2}{(r/Q)Q_0}$
    - (r/Q) is the shunt impedance in Ohms, depends on geometry (more later)
    - Microwave surface resistance of a superconductor is ~ 10^{-5} lower than Cu

$$\Rightarrow Q_0^{SC} \sim 10^5 \, Q_0^{Cu}$$

Option	Superconducting	Normal Conducting
Q_0	2×10^9	2×10^4
r/Q (ohm/m) @ 500 MHz	330	900
P_{diss}/L (Watt/m) @ E_{acc}=1MV/m	1.5	56000 !!!
AC Power (kW/m)	0.54	112

- Furthermore, the RF power to beam power efficiency is much higher in SC accelerators.

SRF Recirculating Linacs for Nuclear Physics (cont'd)

- For applications demanding high cw voltage, SC cavities have a clear advantage:
  - Recall: $P_{diss} \sim E_{acc}^2$
  - High fields (>50 MV/m) can be produced in Cu cavities, but only for μsecs, before either the rf power required becomes prohibitive or the cooling becomes impossible!
  - Operating gradient in NC cavities, in cw mode, is limited to <1.75 MV/m!

SRF Recirculating Linacs for Nuclear Physics (cont'd)

- Recall: $P_{diss} \sim 1/(r/Q)$
- In NC cavities, r/Q must be maximized by using a small beam aperture.
- In SC cavities, one can afford to make the beam aperture much larger than NC cavities:
  - The resulting drop in r/Q for the accelerating mode is not a concern, because of immensely larger Q_0 .
  - But, advantages of bigger beam aperture are:

    a) It reduces short-range wakes $\Rightarrow$ reduces emittance growth along the linac
    b) It reduces the impedance of dangerous Higher Order Modes (HOMs)
    c) It reduces beam loss and beam-loss-induced radioactivity

SRF Recirculating Linacs for Nuclear Physics (cont'd)

- cw srf linacs make highly stable operation possible

  ⇒ rf phase and amplitude can be controlled very precisely

  ⇒ very low energy spread (~ 10^{-5} at CEBAF)

- In cw operation (made possible by srf cavities) high average current can be achieved with low peak current.

  ⇒ interaction of beam with cavity and vacuum chamber is weak and small beam emittance can be preserved through the linac!

SRF Recirculating Linacs as FEL Driver Accelerators

- There are many candidate drivers for an FEL: dc electrostatic accelerators, storage rings, induction linacs, rf linacs

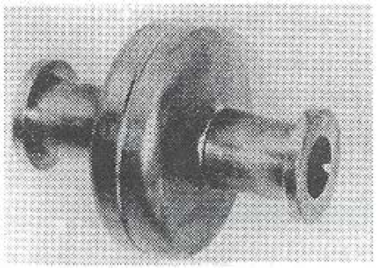
- RF linacs are suitable for a variety of FELs because they
  - Offer high extraction efficiency $\Rightarrow$ higher P_{FEL}
  - Offer good beam quality: low energy spread and low emittance, necessary for adequate overlap between laser and electron beam

- SRF linacs are ideal as FEL drivers because:
  - excellent beam quality (as outlined earlier)
  - allow both high average power (in cw mode) and high efficiency!

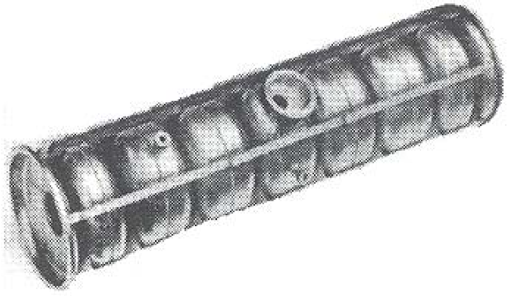
- In most high-power lasers, most of input energy is rejected as waste heat
  $\Rightarrow$ ability to remove heat sets power limitation (e.g. CO_2 lasers)
- In an FEL, the waste heat is in the form of electron beam power
  $\Rightarrow$ electron beam not converted to laser power is largely recoverable!

Historical Foundations of SRF

- HEPL at Stanford University was the pioneer laboratory in exploration of srf for accelerator applications.
- Exploration of srf for particle accelerators began at Stanford University in 1965 with acceleration of electrons in a lead-plated resonator.
- In 1977 HELP completed first SCA providing 50 MV in 27m linac at 1.3 GHz.



(a)



(b)

(a) Single-cell HEPL Nb cavity, resonant frequency 1.3 GHz
(b) HEPL 7-cell subsection

Historical Foundations of SRF (cont'd)

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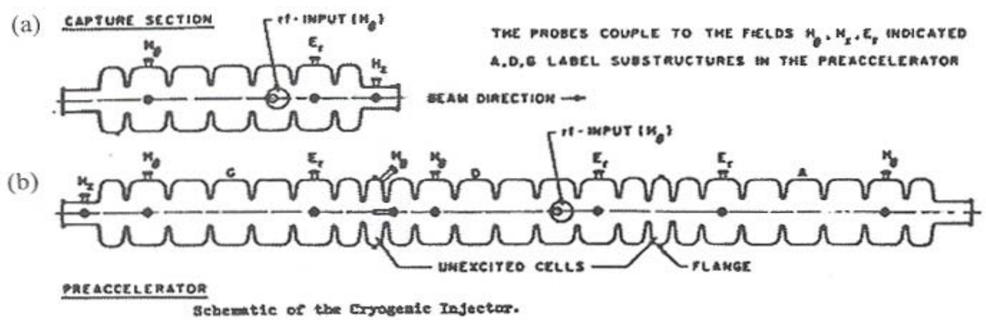
- In 1977 the U. of Illinois completes construction of a microtron with SCA sections that provided 13 MV.
- Both SCA and MUSL-I aimed to provide cw high current (~ several 10 μA) for precision Nuclear Physics research.
- Both operated at accelerating gradient of ~ 2MV/m
- Multipacting limited the achievable gradient.



(a) 7-cell subsection of the HEPL structure, with 5 cells and 2 half end-cells

(b) Multicell subsection of the HEPL SCA. Several subsections are joined together at the unexcited cells

Historical Foundations of SRF (cont'd)

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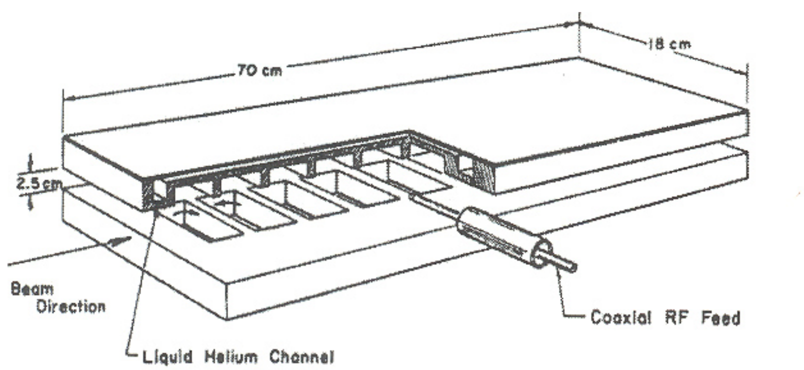
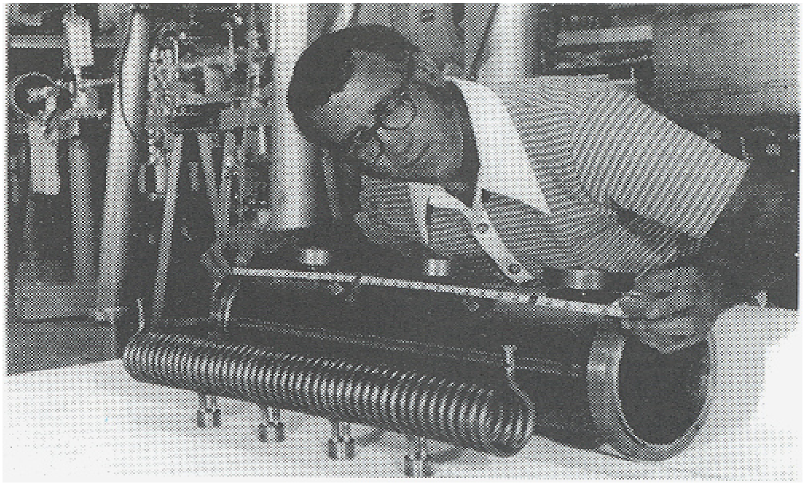
- At the same time Cornell wants to increase the energy of the 12 GeV CESR storage ring using high gradient srf structures.
- In aiming for high gradient, an important idea was to use higher rf frequency (2.86 GHz).
- It would push the fields at which multipacting would occur to higher values!
- In 1975 the muffin-tin structure was developed. It accelerated electron beam of 100 μA to 4 GeV.
- It operated at 4 MV/m, at $Q_0 \sim 10^9$.
- It was limited by thermal breakdown.



Historical Foundations of SRF (cont'd)

- In parallel srf activities are going on at the protons and heavy-ions front:
- In late 1960's, KFK began exploring an srf linac for protons using helically loaded Nb cavities.
- Similar exploration started at ANL.



Helical Nb resonator developed at ANL for a heavy-ion linac.

- The structure exhibits poor mechanical stability: vibrations make control of rf phase difficult.

Historical Foundations of SRF (cont'd)

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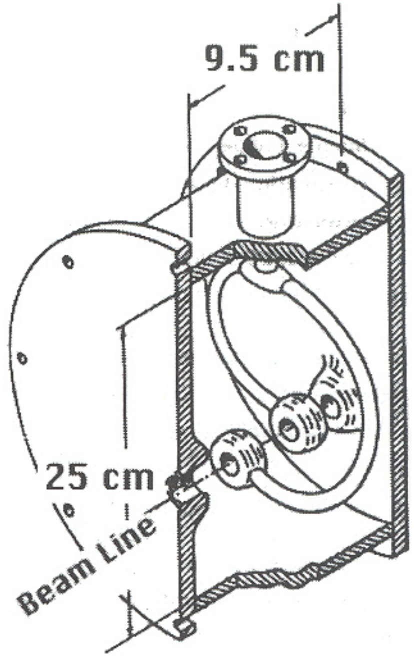
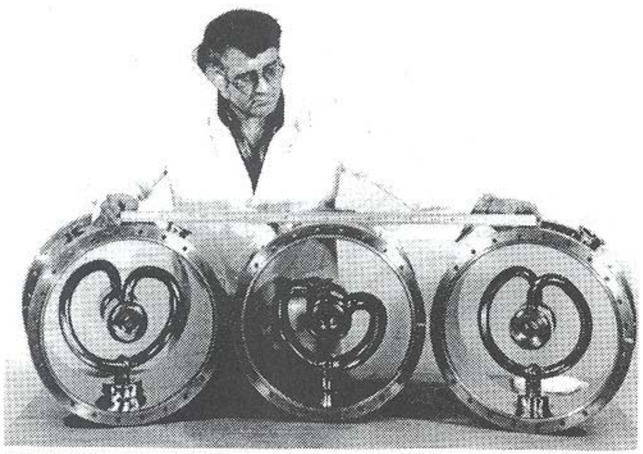
- In 1974 Caltech develops a split-ring cavity geometry and successfully tackles the stability problem.
- It was fabricated from a thin film of lead electroplated on a Cu structure.
- It became the basis of the booster accelerator at SUNY.
- Typical operating gradient ~ 2.5 MV/m.



The three-gap split-loop, lead-on-copper resonator originated at Caltech. It accelerated charged particles with β=0.1, rf frequency is 150 MHz.

Historical Foundations of SRF (cont'd)

- In 1975 ANL starts development of Nb split-ring structure. It became the basis of ATLAS.



Nb split-ring resonators used in ATLAS. The Nb components are hollow and filled with liquid He at 4.2 K. Outer housing is made from Nb-Cu composite.

- ATLAS was commissioned in 1978.
- Holds world record for longest running srf accelerator.
- Operating gradient for the split-ring Nb structures is ~ 2.5-3.5 MV/m.

State of the Art in SRF in the 70's, 80's and 90's

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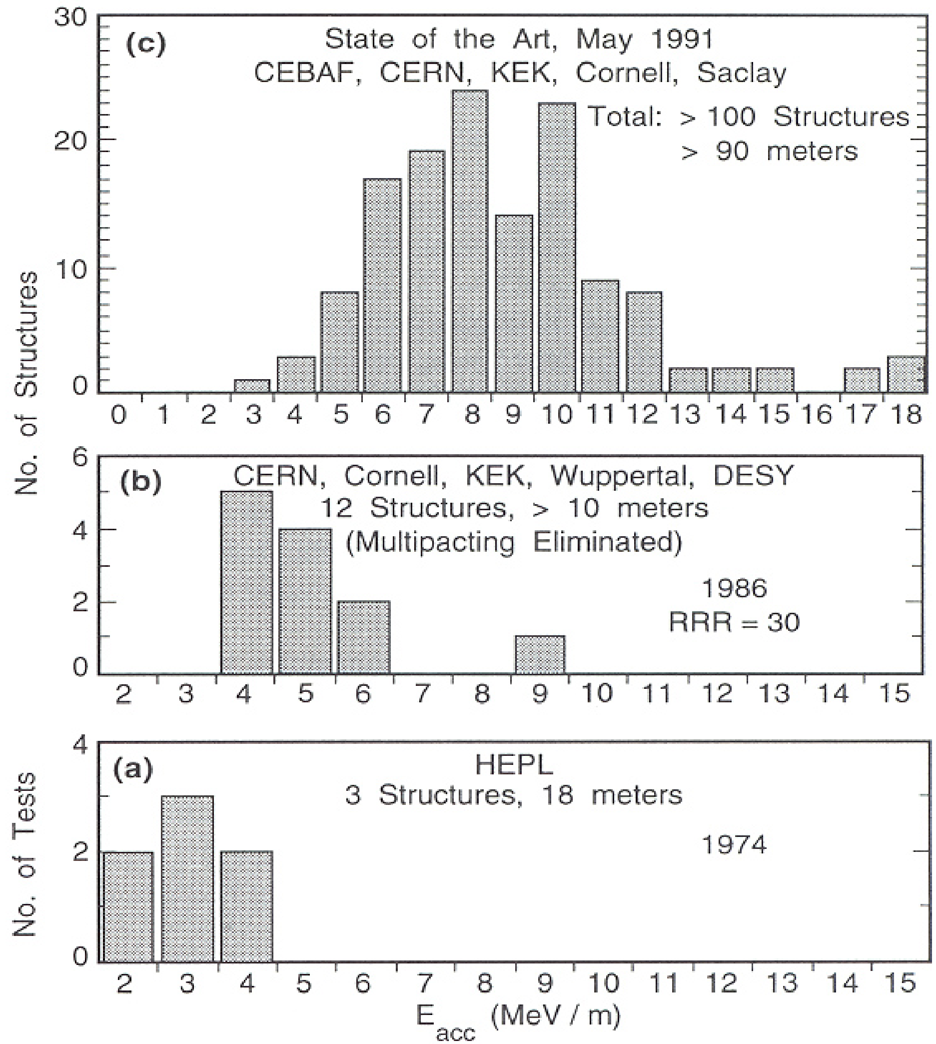
(a) Gradients limited by multipacting
(b) Multipacting is solved but gradients are limited by thermal breakdown.
(c) Thermal breakdown is alleviated by using high-purity, high-thermal-conductivity Nb. Now gradients are predominantly limited by field emission, although thermal breakdown is also encountered.
(Courtesy of H. Padamsee.)



State of the Art in SRF in the 70's, 80's and 90's

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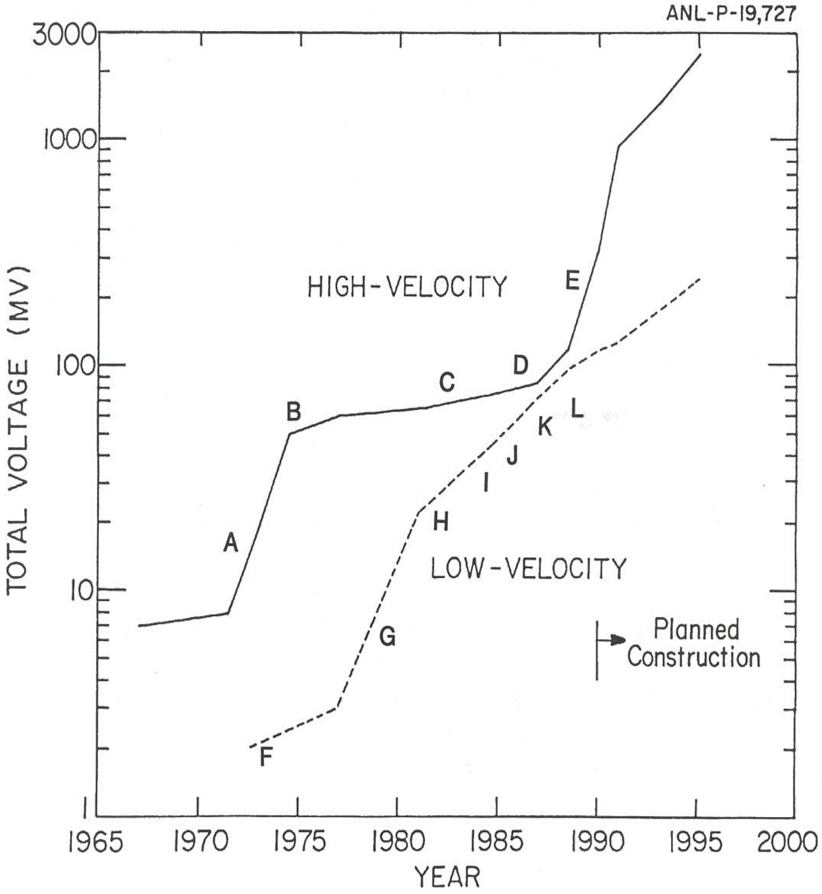

Total installed voltage capability with srf cavities for electron and heavy-ion accelerators. The growth for electron accelerators levels out between 1974 and 1984 because of problems with multipacting and thermal breakdown.

(Courtesy of Jean Delayen)

RF Cavity Fundamental Concepts

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- Quality Factor Q_0

$$Q_0 = \frac{\omega_0 U}{P_c} = \frac{\text{Energy stored in cavity}}{\text{Energy lost in one rf period}}$$

- Q_0 measures the number of oscillations a resonator will go through before dissipating its stored energy.
- Q_0 is frequently written as: $Q_0 = \frac{G}{R_s}$
- G is the *geometry constant* and R_s is the surface resistance:

$$R_s = R_{BCS}(T) + R_0$$

- A convenient expression and a good fit for the BCS term is,

$$R_{BCS}(\text{ohm}) = 2\times 10^{-4}\frac{1}{T}\left(\frac{f}{1.5}\right)^2 \exp\left(-\frac{17.67}{T}\right)$$

- A well-prepared niobium surface can reach a residual resistance R_0 of 10-20 nΩ. Record values are near 1 nΩ.

RF Cavity Fundamental Concepts (cont'd)

- An important quantity used to characterize losses in a cavity is the shunt impedance R_a (ohms per cell) defined as

$$R_a = \frac{E_{acc}^2}{P_{diss}}$$

- Ideally one wants R_a to be large for the accelerating mode so that dissipated power is minimized. Particularly important for Cu cavities!
- Note that R_a/Q_0 is:
  - independent of the surface resistance
  - independent of cavity size, $R_a/Q_0 \sim 100\ \Omega$/cell
- R_a/Q_0 is used to determine the level of mode excitation by charges passing through the cavity.
- In NC cavities R_a/Q_0 is maximized by using small beam aperture
  - But R_a/Q_0 of HOMs tend to increase also
    $\Rightarrow$ beam interacts more strongly with the HOMs
    $\Rightarrow$ beam quality degrades $\Rightarrow$ bunch charge is limited.

SRF Performance Limitations: Multipacting

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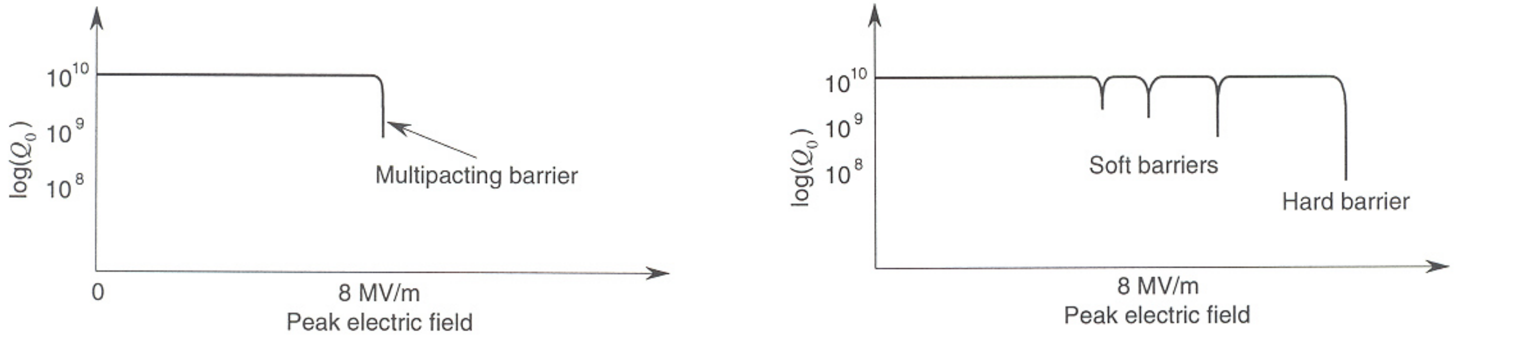
- Multipacting was a major performance limitation of srf cavities in the past.
- Multipacting in rf structures is a resonant process.
- A large number of electrons build up and multipacting, absorbing rf power so cavity fields can not increase by raising incident power.
- Electrons collide with structure walls, leading to large temperature rise and, in srf cavities, thermal breakdown.
- Multipacting can be avoided in β=1 cavities by selecting proper cavity shape.

Multipacting (cont'd)

- Onset of multipacting (MP) is usually recognized when the field level in the cavity remains fixed, as more rf power is supplied. In effect, Q_0 abruptly reduces at the MP threshold.
- Often a MP barrier can be overcome by processing.
- Barriers that can be processed are *soft* barriers, ones that persist are *hard* barriers.
- A processed soft barrier may reappear after the cavity is exposed to air: MP is strongly dependent on the condition of the first few monolayers of rf surface.



Q_0 vs. E_{pk} curves for srf cavity when one (a) or several (b) MP barriers are encountered.

Multipacting (cont'd)

Multipacting Mechanism

- An electron is emitted from one of the structure's surfaces. The emitted electron is accelerated by the rf fields and eventually impacts a wall again.
- Secondary electrons (SE's) are produced.
- The number of SE's depends on the surface characteristics and impact energy of the primary.
- The SE's are accelerated, impact and produce another generation of electrons, etc…
- If the number of emitted electrons exceeds the number of impacting, then the electron current will increase exponentially.

$$N_e = N_0 \prod_{m=1}^{k} \delta(K_m)$$

$\delta(K_m)$: the secondary emission coefficient (SEC), material-dependent, function of impact energy. If $\delta(K_m)>1$ for all impact sites, MP will occur.

- Electron current will only be limited by available power or space charge effects.

Multipacting (cont'd)

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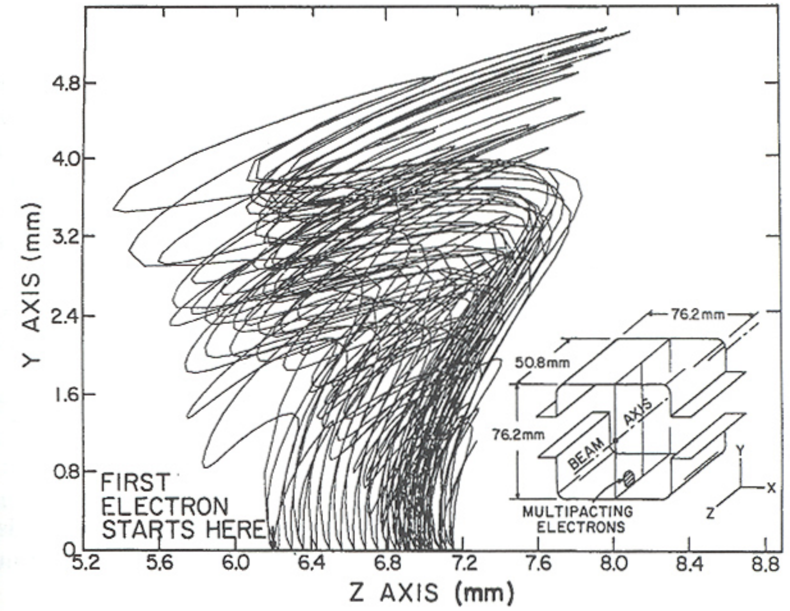
A Common Multipacting Scenario

- Most frequent type of MP in β=1 cavities: charges impact the cavity wall at, or near, the emission site itself.
- Emitted electrons are accelerated by $E_\perp$ to the surface while the surface magnetic field force electrons in quasi-circular orbits, so they return to their point of origin.



Multipacting (cont'd)

Several solutions were explored:

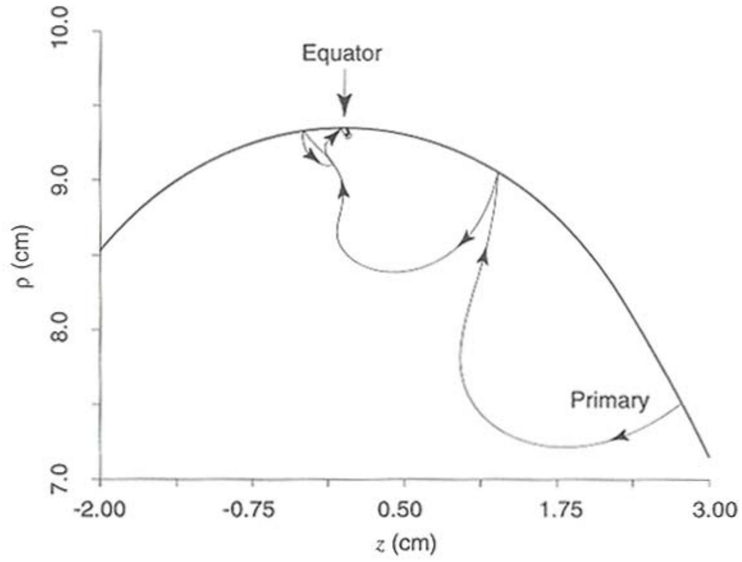
- Reducing the E field by making subtle alterations to the shape of the cavity.
  Was done on the HEPL cavities. Method was successful for the muffin-tin cavity.

- Placing grooves in the surface where MP occurs. $E_\perp$ is strongly attenuated at the bottom of the grooves, and secondaries remain trapped.

- The most successful solution to MP problem was to make a spherical cavity. The magnetic field varies along the entire cavity wall , so there are no stable trajectories, as the electrons drift to the equator. At the equator, $E_\perp$ vanishes, so secondaries do not gain any energy.

- Elliptical is better than spherical from mechanical stability point of view. Elliptical cavity shapes are now universally adopted for β=1 SRF cavities and MP is no longer a serious problem.

Multipacting (cont'd)



Electron trajectories in an elliptical cavity. The charges drift to the equator where MP is not possible.

Multipacting (cont'd)

Avoiding Multipacting

- Localize MP orbits by diagnostic tools. Track electrons and implement geometric modifications to shift barriers or to change resonant conditions.
- Use materials with low SEC.
- Cleanliness is imperative.
- Destabilize MP trajectories at the operating fields by applying a dc electric or magnetic field, such that it alters the MP trajectories.

SRF Performance Limitations: Thermal Breakdown

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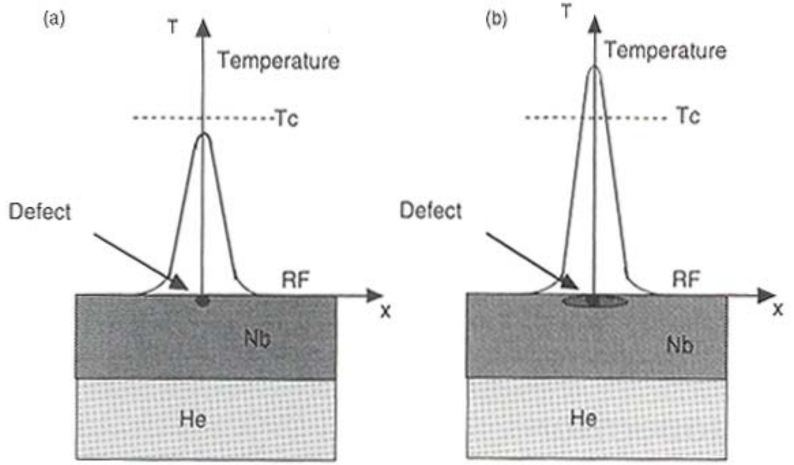
- Phenomena that limit the achievable magnetic field: thermal breakdown of superconductivity and field emission.
- Thermal breakdown (or "quench") originates at sub-mm-size regions, "defects," that have rf losses substantially higher than the surface resistance of an ideal superconductor.
- In dc case, supercurrents flow around the defects.
- At rf frequencies, the reactive part of the impedance causes the rf current to flow through the defect, producing ohmic heating.
- When T at the outside edge of the defect exceeds T_C, the superconducting region surrounding the defect becomes normal -> power dissipation is increased.
- As NC region grows, power dissipation increases and results in thermal instability.



Thermal Breakdown (cont'd)

- Examples of defects:
  - 50 μm Cu particle attached to Nb surface
  - Chemical or drying stain 440 μm
  - 50 μm crystal containing S, Ca, Cl, K
- There are many opportunities for such defects to enter an srf cavity during the various stages of production and preparation.
- Statistically, number of defects increases with cavity area ⇒ larger cavities break down at lower fields.

Thermal Breakdown (cont'd)

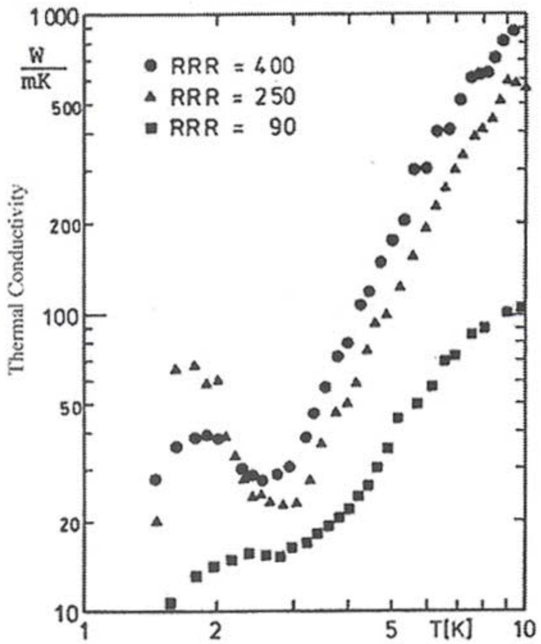
- Solutions to thermal breakdown

a) Guided Repair

One or two gross defects can be located by thermometry and removed by mechanical grinding. Example: 350 MHz single-cell Nb cavity E_{acc} was increased from 5 MV/m ->10 MV/m.
Not easy for smaller defects.

b) Raising Thermal Conductivity of Niobium

$H_{max} \propto \sqrt{\kappa} \Rightarrow$
If raise κ, H_{max} will increase.
Defects will be able to tolerate more power before driving neighboring superconductor into normal state.
Approximately, $\kappa = 0.25$ (W/m-K) x RRR



Thermal conductivity of Nb w

Thermal Breakdown (cont'd)

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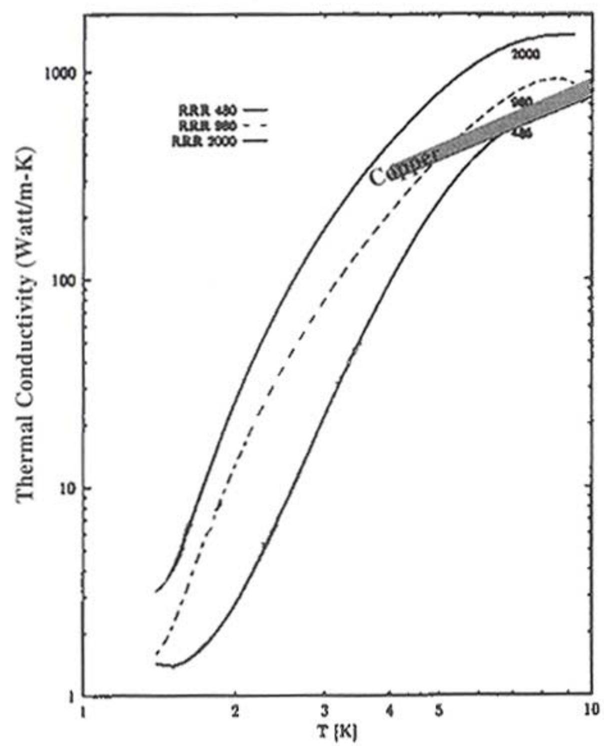
c) Thin Films of Niobium on Copper

- Use µm-thick film of Nb on a thermally stabilizing copper substrate.
- Thermal conductivity of Cu is much greater than of Nb.



Thermal conductivity of high-purity Cu samples compared to low-temperature thermal conductivity of Nb samples of various RRR. Note that at RRR~1000, the thermal conductivity of Nb begins to approach that of Cu.

State of the Art in SRF in 2000

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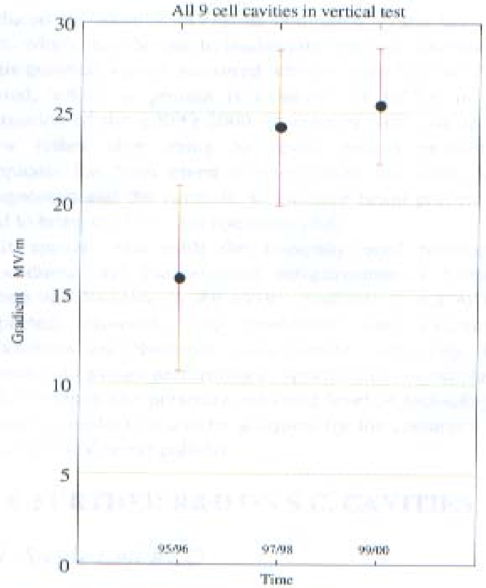


Figure 3: Average gradient of all 9-cell cavities measured in vertical tests during the past 5 years.

(a)

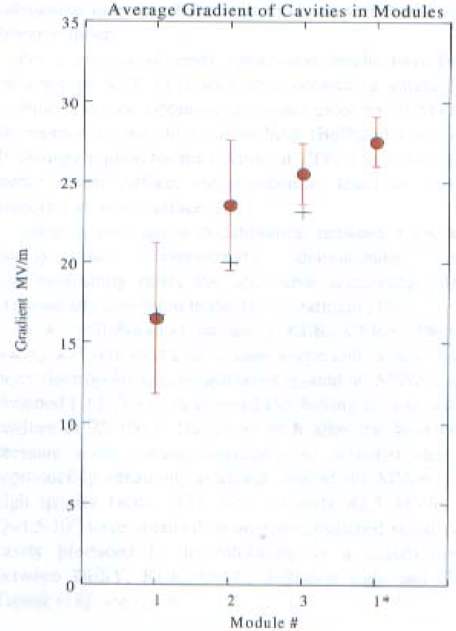


Figure 4: Average gradient, as measured in vertical tests, of the 9-cell cavities assembled into accelerator modules.

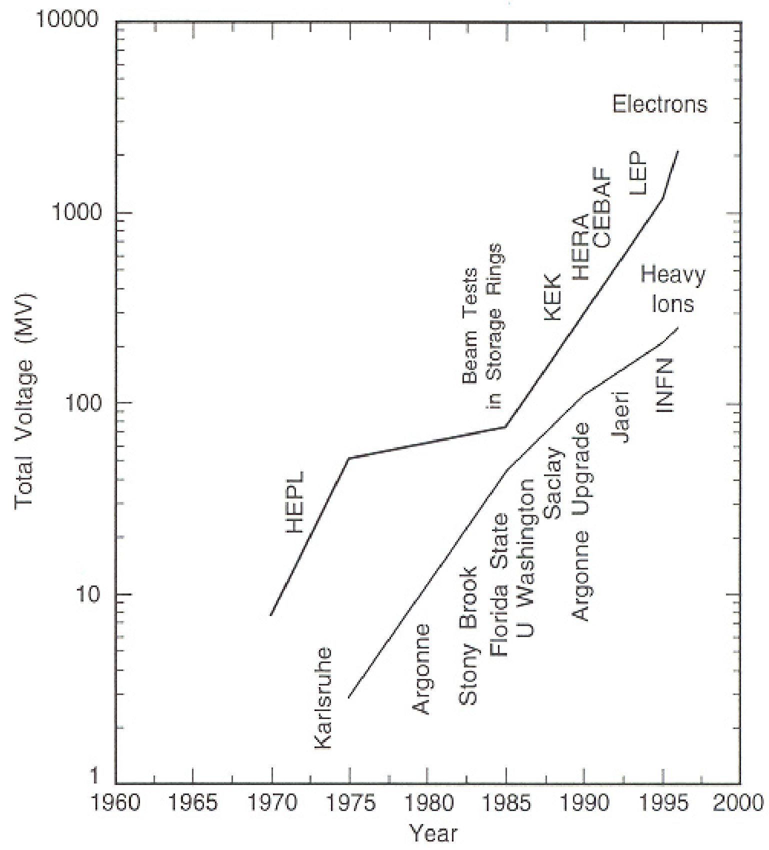
(b)

(a) Average gradient in all 9-cell TESLA cavities measured in vertical tests during the past 5 years.

(b) Average gradient as measured in vertical tests, of the TESLA 9-cell cavities assembled into accelerator modules.

State of the Art in SRF in 2000

Total installed voltage capability with srf cavities for electron and heavy-ion accelerators.



Conclusions

- Recirculating linacs used for Nuclear Physics research or as driver accelerators for FELs and synchrotron radiation sources benefit from cw of high duty factor operation.
- CW or high duty factor operation at higher gradients/high currents practically necessitates the use of superconducting cavities.
- RF superconductivity applied to particle accelerators has made tremendous progress in the last ~ 30 years.
- Major srf limitations due to multipacting, thermal breakdown, and field emission have been understood and successfully combated to a large extent.
- CW operation at higher gradients (> 20MV/m) and higher average currents yet (~100 mA), is the new challenge rf superconductivity will soon have to face!

Outline

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- Introduction
- Cavity Fundamental Parameters
- RF Cavity as a Parallel LCR Circuit
- Coupling of Cavity to an rf Generator
- Equivalent Circuit for a Cavity with Beam Loading
  - On Crest and on Resonance Operation
  - Off Crest and off Resonance Operation
    - Optimum Tuning
    - Optimum Coupling
- Q-external Optimization under Beam Loading and Microphonics
- RF Modeling
- Conclusions

Introduction

- Goal: Ability to predict rf cavity's steady-state response and develop a differential equation for the transient response
- We will construct an equivalent circuit and analyze it
- We will write the quantities that characterize an rf cavity and relate them to the circuit parameters, for
  - a) a cavity
  - b) a cavity coupled to an rf generator
  - c) a cavity with beam

RF Cavity Fundamental Quantities

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- Quality Factor Q_0:

$$Q_0 \equiv \frac{\omega_0 W}{P_{diss}} = \frac{\text{Energy stored in cavity}}{\text{Energy dissipated in cavity walls per radian}}$$

- Shunt impedance R_a:

$$R_a \equiv \frac{V_a^2}{P_{diss}} \quad \text{in ohms per cell}$$

(accelerator definition); V_a = accelerating voltage

- Note: Voltages and currents will be represented as complex quantities, denoted by a tilde. For example:

$$\tilde{V}_c = V_c e^{i\phi(t)}$$

where $V = |\tilde{V}|$ is the magnitude of $\tilde{V}$

Equivalent Circuit for an rf Cavity

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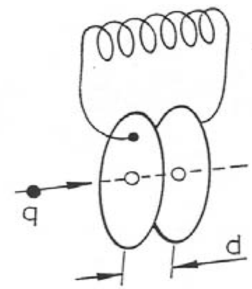
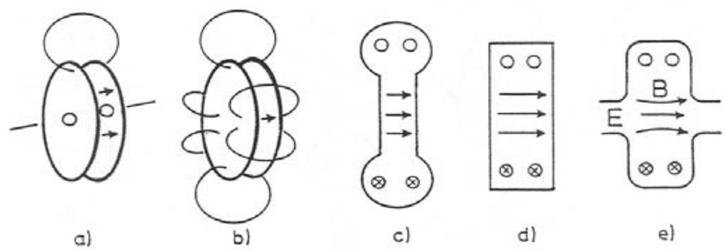
Simple LC circuit representing an accelerating resonator.



Simple lumped L-C circuit repesenting an accelerating resonator.
$\omega_o^2 = 1/LC$

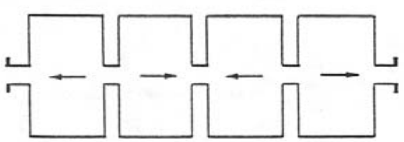
Metamorphosis of the LC circuit into an accelerating cavity.



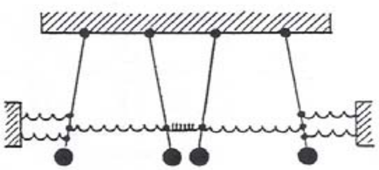
Metamorphosis of the L-C circuit of Fig. 1 into an accelerating cavity (after R.P.Feynman[33]). Fig. 5d shows the cylindrical "pillbox cavity" and Fig. 5e a slightly modified pillbox cavity with beam holes (typical β between 0.5 and 1.0). Fig. 5c resembles a low β version of the pillbox variety ($0.2<\beta<0.5$).

Chain of weakly coupled pillbox cavities representing an accelerating cavity.

Chain of coupled pendula as its mechanical analogue.



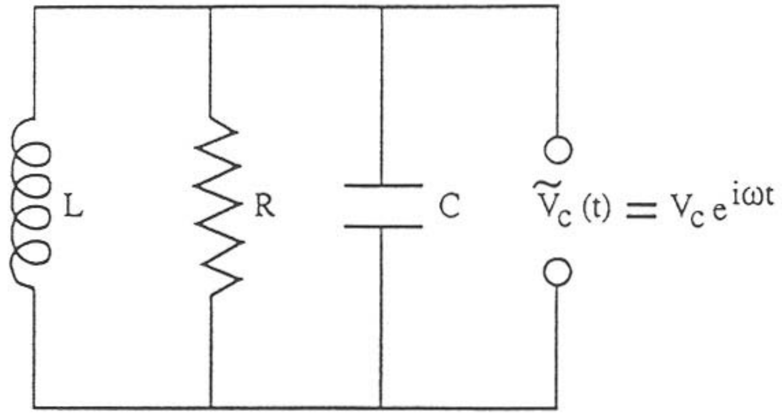
Chain of weakly-coupled pillbox cavities representing an accelerating module



Chain of coupled pendula as a mechanical analogue to Fig. 6a

Equivalent Circuit for an rf Cavity (cont'd)

- An rf cavity can be represented by a parallel LCR circuit:



- Impedance Z of the equivalent circuit: $$\tilde{Z} = \left[\frac{1}{R} + \frac{1}{iL\omega} + iC\omega\right]^{-1}$$

- Resonant frequency of the circuit: $\omega_0 = 1/\sqrt{LC}$

- Stored energy W: $W = \frac{1}{2}CV_c^2$

Equivalent Circuit for an rf Cavity (cont'd)

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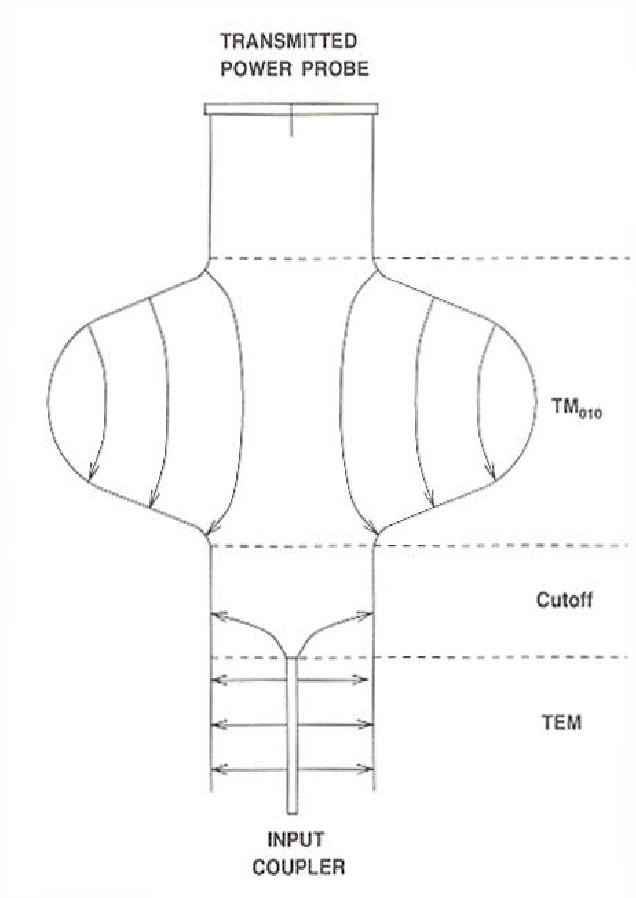
- Power dissipated in resistor R: $$P_{diss} = \frac{1}{2}\frac{V_c^2}{R}$$
- From definition of shunt impedance $$R_a \equiv \frac{V_a^2}{P_{diss}} \qquad \therefore\; R_a = 2R$$
- Quality factor of resonator: $$Q_0 \equiv \frac{\omega_0 W}{P_{diss}} = \omega_0 CR$$
- Note: $$\tilde{Z} = R\left[1 + iQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]^{-1}$$ For $\omega \approx \omega_0$, $$\tilde{Z} \approx R\left[1 + 2iQ_0\left(\frac{\omega - \omega_0}{\omega_0}\right)\right]^{-1}$$

Cavity with External Coupling

- Consider a cavity connected to an rf source
- A coaxial cable carries power from an rf source to the cavity
- The strength of the input coupler is adjusted by changing the penetration of the center conductor
- There is a fixed output coupler, the *transmitted power probe*, which picks up power transmitted through the cavity



Cavity with External Coupling (cont'd)

Consider the rf cavity after the rf is turned off.

Stored energy W satisfies the equation: $$\frac{dW}{dt} = -P_{tot}$$

Total power being lost, P_{tot}, is: $$P_{tot} = P_{diss} + P_e + P_t$$

P_e is the power leaking back out the input coupler. P_t is the power coming out the transmitted power coupler. Typically P_t is very small $\Rightarrow P_{tot} \approx P_{diss} + P_e$

Recall $$Q_0 \equiv \frac{\omega_0 W}{P_{diss}}$$

Similarly define a "loaded" quality factor Q_L: $$Q_L \equiv \frac{\omega_0 W}{P_{tot}}$$

Now $$\frac{dW}{dt} = -\frac{\omega_0 W}{Q_L} \quad \Rightarrow \quad W = W_0 e^{-\frac{\omega_0 t}{Q_L}}$$

$\therefore$ energy in the cavity decays exponentially with time constant: $$\tau_L = \frac{Q_L}{\omega_0}$$

Cavity with External Coupling (cont'd)

Equation

$$\frac{P_{tot}}{\omega_0 W} = \frac{P_{diss} + P_e}{\omega_0 W}$$

suggests that we can assign a quality factor to each loss mechanism, such that

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e}$$

where, by definition,

$$Q_e \equiv \frac{\omega_0 W}{P_e}$$

Typical values for CEBAF 7-cell cavities: $Q_0=1x10^{10}$, $Q_e \approx Q_L=2x10^{7.}$

Cavity with External Coupling (cont'd)

- Define "coupling parameter":

$$\beta \equiv \frac{Q_0}{Q_e}$$

therefore

$$\frac{1}{Q_L} = \frac{(1+\beta)}{Q_0}$$

- β is equal to:

$$\beta = \frac{P_e}{P_{diss}}$$

It tells us how strongly the couplers interact with the cavity. Large β implies that the power leaking out of the coupler is large compared to the power dissipated in the cavity walls.

Equivalent Circuit of a Cavity Coupled to an rf Source

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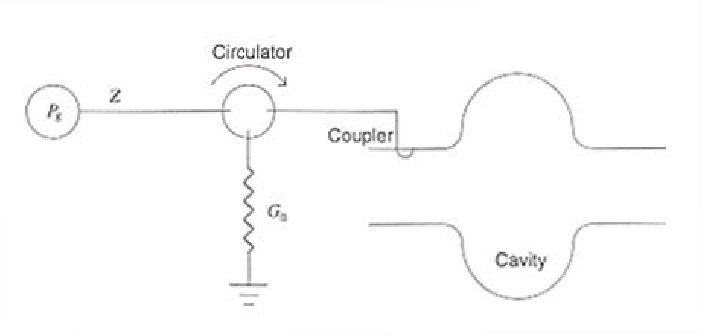
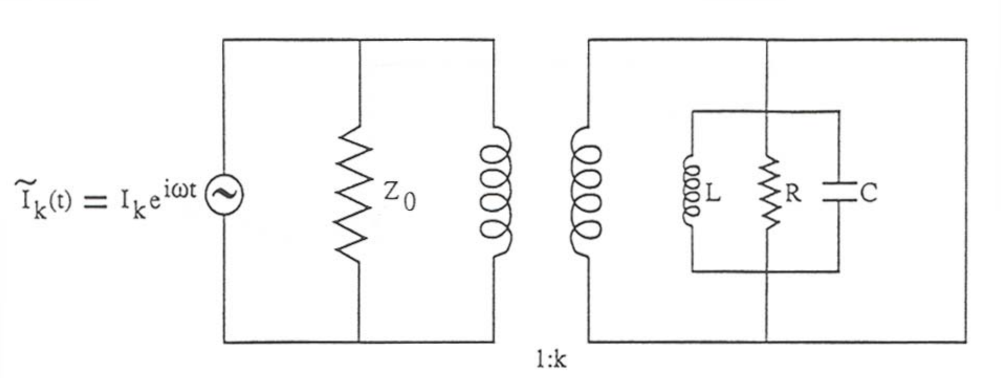
- The system we want to model:



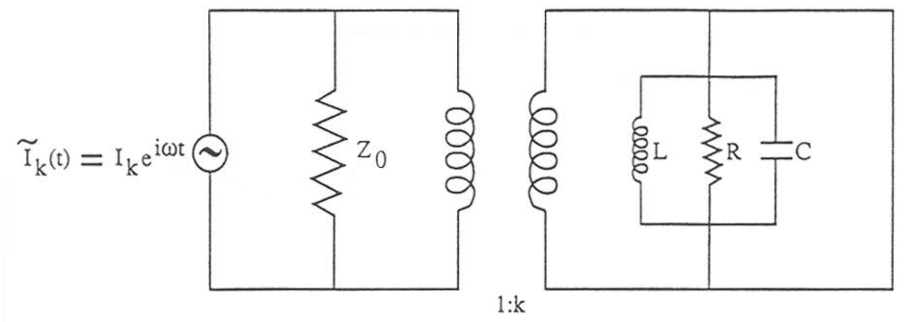
- Between the rf generator and the cavity is an isolator – a circulator connected to a load. Circulator ensures that signals coming from the cavity are terminated in a matched load.
- Equivalent circuit:



RF Generator + Circulator Coupler Cavity

- Coupling is represented by an ideal transformer of turn ratio 1:k

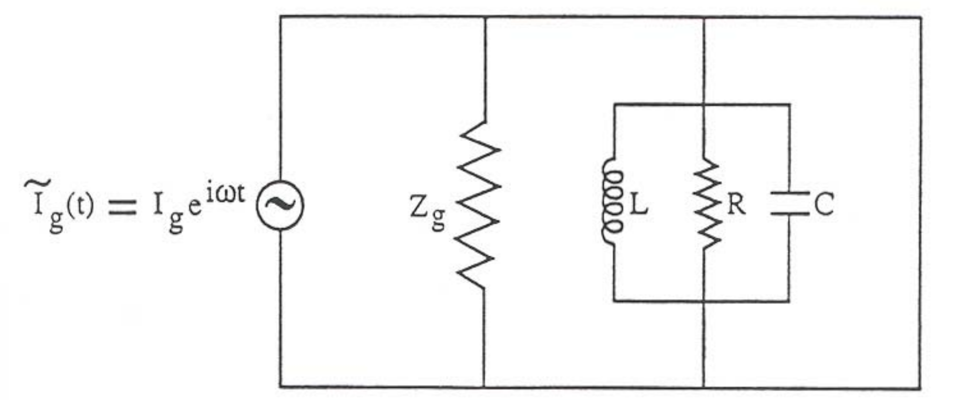
Equivalent Circuit of a Cavity



$\Downarrow$

$$I_g = \frac{I_k}{k}$$

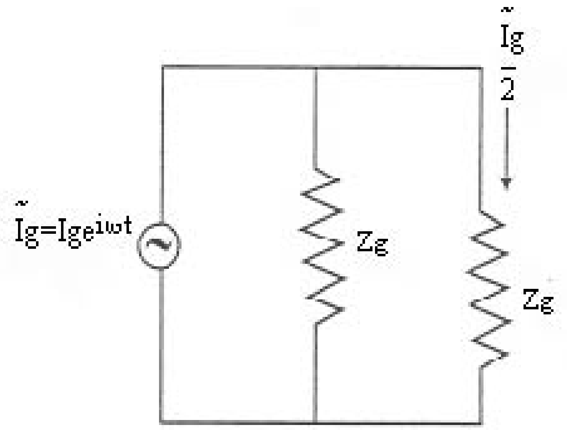
$$Z_g = k^2 Z_0$$



By definition, $$\beta \equiv \frac{R}{Z_g} = \frac{R}{k^2 Z_0} \quad \therefore \quad Z_g = \frac{R}{\beta}$$

Generator Power

- When the cavity is matched to the input circuit, the power dissipation in the cavity is maximized.



$$P_{diss}^{max} = \frac{1}{2} Z_g \left(\frac{I_g}{2} \right)^2 \quad \text{or} \quad P_{diss}^{max} = \frac{1}{16\beta} R_a I_g^2 \equiv P_g$$

- We define the available generator power P_g at a given generator current $\tilde{I}_g$ to be equal to P_{diss}^{max} .

Some Useful Expressions

- We derive expressions for W, P_{diss}, P_{refl}, in terms of cavity parameters

$$\frac{W}{P_g} = \frac{\frac{Q_0}{\omega_0} P_{diss}}{\frac{1}{16\beta} R_a I_g^2} = \frac{\frac{Q_0}{\omega_0} \frac{V_c^2}{R_a}}{\frac{1}{16\beta} R_a I_g^2} = \frac{16\beta}{R_a^2} \frac{Q_0}{\omega_0} \frac{V_c^2}{I_g^2}$$

$$V_c = I_g Z_{TOT}$$

$$Z_{TOT} = \left[\frac{1}{Z_g} + \frac{1}{Z} \right]^{-1}$$

$$Z_{TOT} = \frac{R_a}{2} \left[(1+\beta) + iQ_0 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right]^{-1}$$

$$\therefore \quad W = 4\beta \frac{Q_0}{\omega_0} \frac{1}{(1+\beta)^2 + Q_0^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2} P_g$$

$$\text{For} \quad \omega \approx \omega_0 \quad \Rightarrow$$

$$W \approx \frac{4\beta}{(1+\beta)^2} \frac{Q_0}{\omega_0} \frac{1}{1 + \left[2 \frac{Q_0}{(1+\beta)} \frac{\omega - \omega_0}{\omega_0} \right]^2} P_g$$

Some Useful Expressions (cont'd)

$$W \;\square\; \frac{4\beta}{(1+\beta)^2}\frac{Q_0}{\omega_0}\frac{1}{1+\left[2\dfrac{Q_0}{(1+\beta)}\dfrac{\omega-\omega_0}{\omega_0}\right]^2}P_g$$

- Define "Tuning angle" Ψ:

$$\tan\Psi \equiv -Q_L\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) \approx -2Q_L\frac{\omega-\omega_0}{\omega_0} \quad \text{for } \omega\approx\omega_0$$

$$\therefore$$

$$\mathrm{W}=\frac{4\beta}{(1+\beta)^2}\frac{\mathrm{Q}_0}{\omega_0}\frac{1}{1+\tan^2\Psi}\mathrm{P}_g$$

- Recall:

$$P_{diss} = \frac{\omega_0 W}{Q_0}$$

$$\therefore$$

$$P_{diss} = \frac{4\beta}{(1+\beta)^2}\frac{1}{1+\tan^2\Psi}P_g$$

Some Useful Expressions (cont'd)

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- Reflected power is calculated from energy conservation,

$$P_{refl} = P_g - P_{diss}$$

$$P_{refl} = P_g \left[1 - \frac{4\beta}{(1+\beta)^2} \frac{1}{1+\tan^2\Psi} \right]$$

- On resonance:

$$W = \frac{4\beta}{(1+\beta)^2} \frac{Q_0}{\omega_0} P_g$$

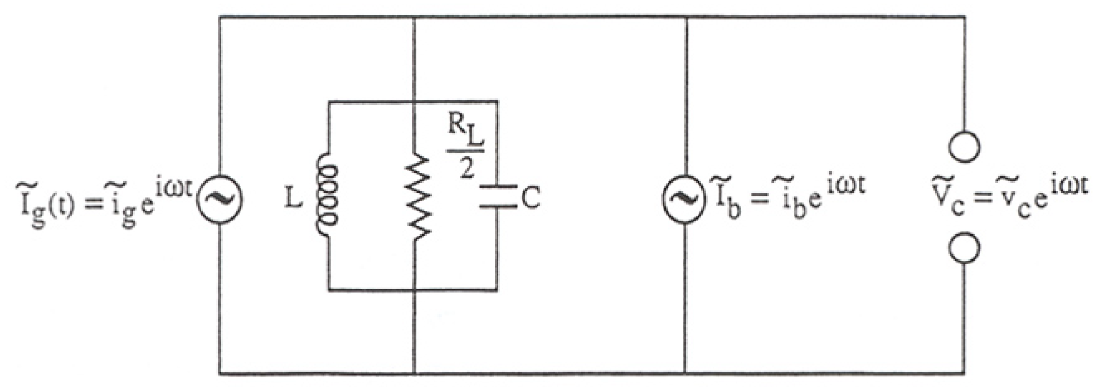
$$P_{diss} = \frac{4\beta}{(1+\beta)^2} P_g$$

$$P_{refl} = \left(\frac{1-\beta}{1+\beta} \right)^2 P_g$$

- Example: For V_a=20MV/m, L_{cav}=0.7m, P_g=3.65 kW, $Q_0=1\text{x}10^{10}$, $\omega_0=2\pi\text{x}1497\text{x}10^6$ rad/sec, β=500, on resonance W=31 Joules, P_{diss}=29 W, P_{refl}=3.62 kW.

Equivalent Circuit for a Cavity with Beam

- Beam in the rf cavity is represented by a current generator.
- Equivalent circuit:



- Differential equation that describes the dynamics of the system:

$$i_C = C\frac{dv_C}{dt}, \quad i_R = \frac{v_C}{R_L/2}, \quad v_C = L\frac{di_L}{dt}$$

- R_L is the loaded impedance defined as: $R_L = \frac{R_a}{(1+\beta)}$

Equivalent Circuit for a Cavity with Beam (cont'd)

- Kirchoff's law: $$\tilde{i}_L + \tilde{i}_R + \tilde{i}_C = \tilde{i}_g - \tilde{i}_b$$
- Total current is a superposition of generator current and beam current and beam current opposes the generator current.

$$\frac{d^2\tilde{v}_c}{dt^2} + \frac{\omega_0}{Q_L}\frac{d\tilde{v}_c}{dt} + \omega_0^2\tilde{v}_c = \frac{\omega_0 R_L}{2Q_L}\frac{d}{dt}\left(\tilde{i}_g - \tilde{i}_b\right)$$

- Assume that $\tilde{v}_c, \tilde{i}_g, \tilde{i}_b$ have a fast (rf) time-varying component and a slow varying component:

$$\tilde{v}_c = \tilde{V}_c e^{i\omega t}$$
$$\tilde{i}_g = \tilde{I}_g e^{i\omega t}$$
$$\tilde{i}_b = \tilde{I}_b e^{i\omega t}$$

where ω is the generator angular frequency and $\tilde{V}_c, \tilde{I}_g, \tilde{I}_b$ are complex quantities.

Equivalent Circuit for a Cavity with Beam (cont'd)

- Neglecting terms of order $\frac{d^2\tilde{V}_c}{dt^2}, \frac{d\tilde{I}}{dt}, \frac{1}{Q_L}\frac{d\tilde{V}_c}{dt}$ we arrive at:

$$\frac{d\tilde{V}_c}{dt} + \frac{\omega_0}{2Q_L}(1 - i\tan\Psi)\tilde{V}_c = \frac{\omega_0 R_L}{4Q_L}(\tilde{I}_g - \tilde{I}_b)$$

where Ψ is the tuning angle.

- For short bunches: $|\tilde{I}_b| \approx 2I_0$ where I_0 is the average beam current.

Equivalent Circuit for a Cavity with Beam (cont'd)

$$\frac{d\tilde{V}_c}{dt} + \frac{\omega_0}{2Q_L}(1 - i\tan\Psi)\tilde{V}_c = \frac{\omega_0 R_L}{4Q_L}(\tilde{I}_g - \tilde{I}_b)$$

- At steady-state: $$\tilde{V}_c = \frac{R_L/2}{(1 - i\tan\Psi)}\tilde{I}_g - \frac{R_L/2}{(1 - i\tan\Psi)}\tilde{I}_b$$

or $$\tilde{V}_c = \frac{R_L}{2}\tilde{I}_g \cos\Psi e^{i\Psi} - \frac{R_L}{2}\tilde{I}_b \cos\Psi e^{i\Psi}$$

or $$\tilde{V}_c = \boxed{\tilde{V}_{gr}\cos\Psi e^{i\Psi}} + \boxed{\tilde{V}_{br}\cos\Psi e^{i\Psi}}$$

or $$\tilde{V}_c = \tilde{V}_g + \tilde{V}_b$$

$$\left\{ \begin{array}{l} \tilde{V}_{gr} = \dfrac{R_L}{2}\tilde{I}_g \\ \tilde{V}_{br} = -\dfrac{R_L}{2}\tilde{I}_b \end{array} \right\}$$ are the generator and beam-loading voltages on resonance

and $\left\{ \begin{array}{l} \tilde{V}_g \\ \tilde{V}_b \end{array} \right\}$ are the generator and beam-loading voltages.

Equivalent Circuit for a Cavity with Beam (cont'd)

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- Note that:

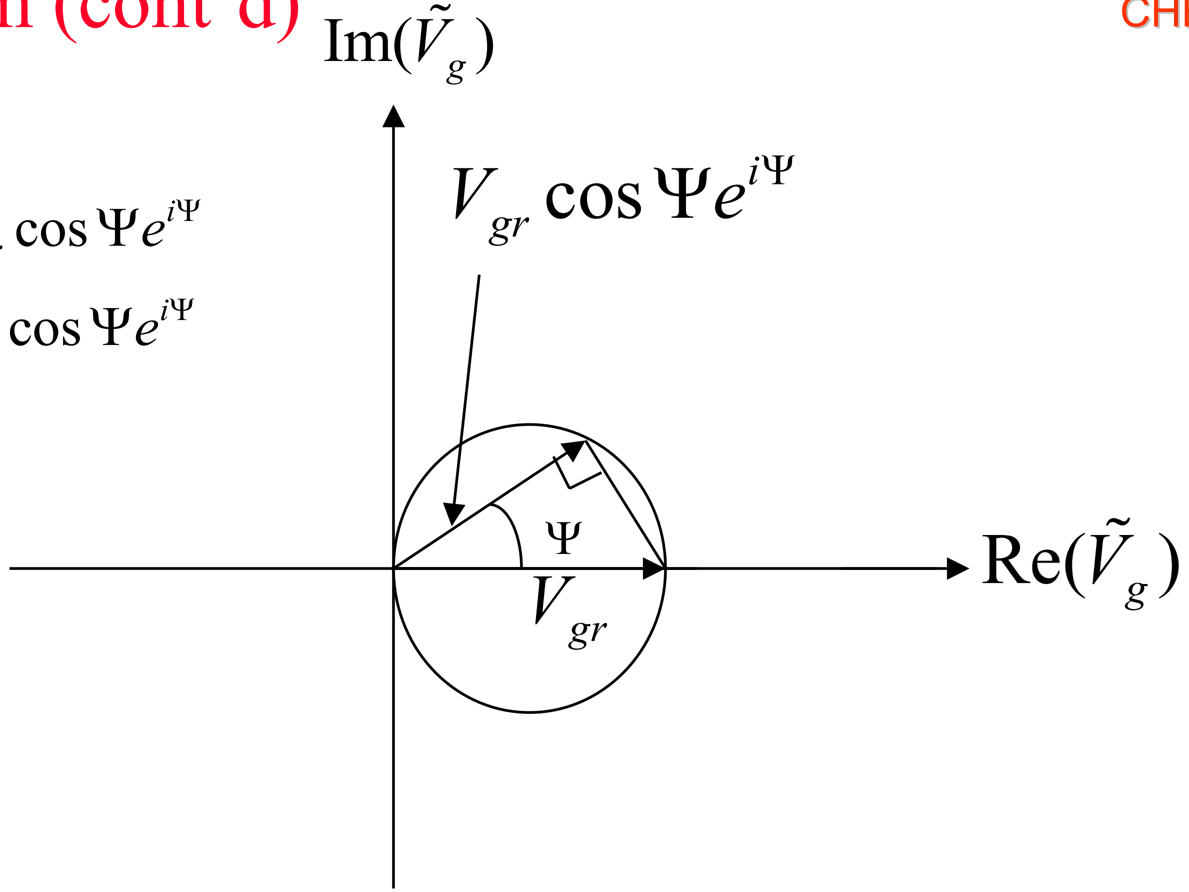
$$|\tilde{V}_{gr}| = \frac{2\sqrt{\beta}}{\sqrt{1+\beta}}\sqrt{P_g R_L} \approx 2\sqrt{P_g R_L} \quad \text{for large } \beta$$

$$|\tilde{V}_{br}| = R_L I_0$$

Equivalent Circuit for a Cavity with Beam (cont'd)

$$\tilde{V}_g = \tilde{V}_{gr} \cos\Psi e^{i\Psi}$$

$$\tilde{V}_b = \tilde{V}_{br} \cos\Psi e^{i\Psi}$$



As Ψ increases the magnitude of both V_g and V_b decreases while their phases rotate by Ψ.

Equivalent Circuit for a Cavity with Beam (cont'd)

$$\tilde{V}_c = \tilde{V}_g + \tilde{V}_b$$

- Cavity voltage is the superposition of the generator and beam-loading voltage.
- This is the basis for the vector diagram analysis.

Example of a Phasor Diagram

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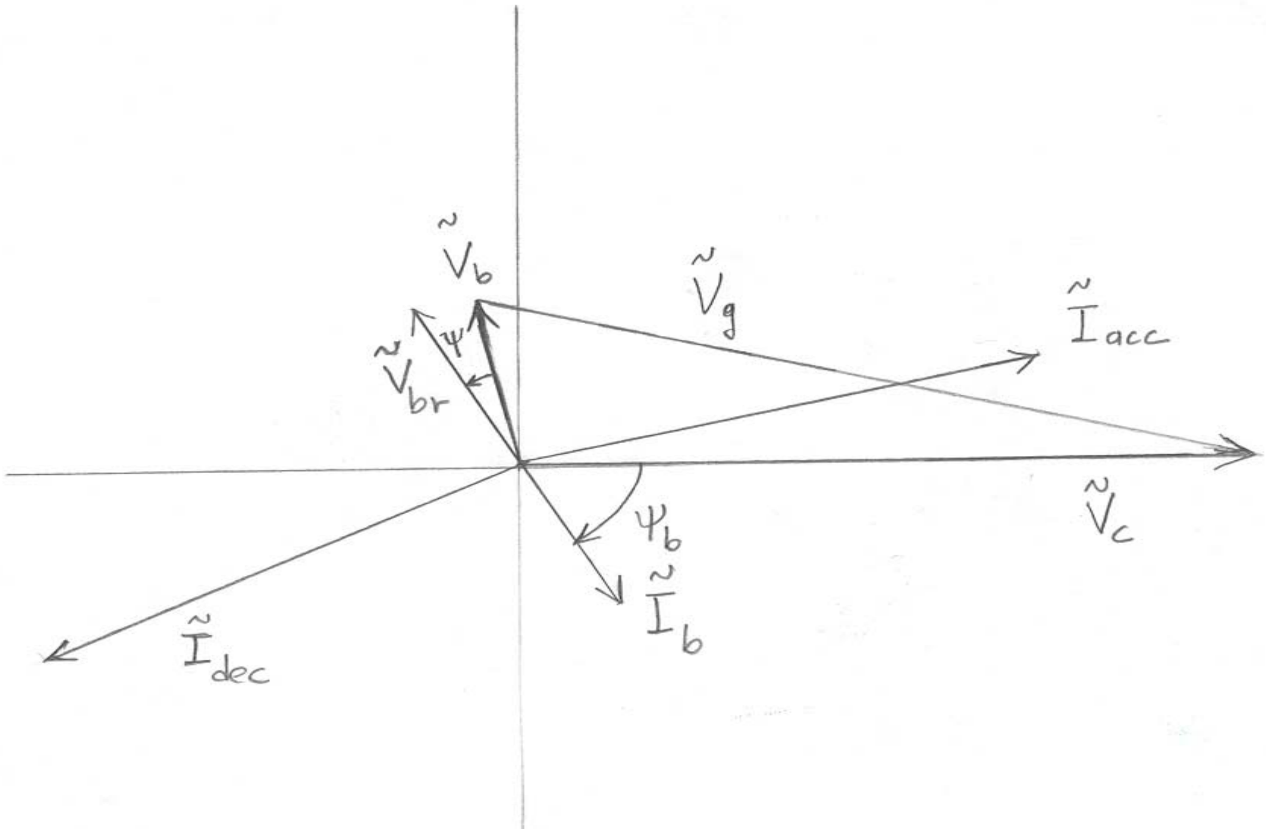
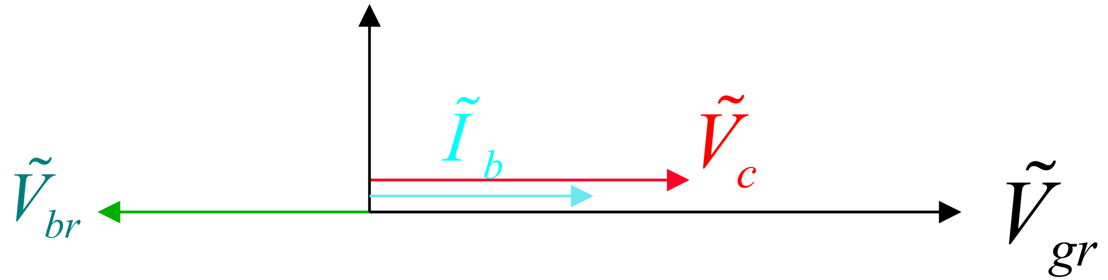


On Crest and On Resonance Operation

- Typically linacs operate on resonance and on crest in order to receive maximum acceleration.
- On crest and on resonance



$$\Rightarrow \qquad V_a = V_{gr} - V_{br}$$

where V_a is the accelerating voltage.

More Useful Equations

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- We derive expressions for W, V_a, P_{diss}, P_{refl} in terms of β and the loading parameter K, defined by: $K=I_0/2\sqrt{R_a/P_g}$

From:

$$|\tilde{V}_{gr}| = \frac{2\sqrt{\beta}}{\sqrt{1+\beta}}\sqrt{P_g R_L}$$

$$|\tilde{V}_{br}| = R_L I_0$$

$$V_a = V_{gr} - V_{br}$$

$$\Rightarrow$$

$$V_a = \sqrt{P_g R_a}\left\{\frac{2\sqrt{\beta}}{\sqrt{1+\beta}}\left(1-\frac{K}{\sqrt{\beta}}\right)\right\}$$

$$W = \frac{4\beta}{(1+\beta)^2}\frac{Q_0}{\omega_0}\left(1-\frac{K}{\sqrt{\beta}}\right)^2 P_g$$

$$P_{diss} = \frac{4\beta}{(1+\beta)^2}\left(1-\frac{K}{\sqrt{\beta}}\right)^2 P_g$$

$$I_0 V_a = I_0\sqrt{R_a P_{diss}}$$

$$\eta \equiv \frac{I_0 V_a}{P_g} = \frac{2\sqrt{\beta}}{\sqrt{1+\beta}}2K\left(1-\frac{K}{\sqrt{\beta}}\right)$$

$$P_{refl} = P_g - P_{diss} - I_0 V_a \;\Rightarrow\; P_{refl} = \frac{\left[(\beta-1)-2K\sqrt{\beta}\right]^2}{(\beta+1)^2}P_g$$

More Useful Equations (cont'd)

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- For β large,

$$P_g \approx \frac{1}{4R_L}(V_a + I_0 R_L)^2$$

$$P_{refl} \approx \frac{1}{4R_L}(V_a - I_0 R_L)^2$$

- For $P_{refl}=0$ (condition for matching) $\Rightarrow$

$$R_L = \frac{V_a^M}{I_0^M}$$

and

$$P_g \approx \frac{I_0^M V_a^M}{4}\left(\frac{V_a}{V_a^M} + \frac{I_0}{I_0^M}\right)^2$$

Example

- For V_a=20 MV/m, L=0.7 m, $Q_L=2\text{x}10^7$, $Q_0=1\text{x}10^{10}$:

Power	$I_0 = 0$	$I_0 = 100\ \mu A$	$I_0 = 1$ mA
P_g	3.65 kW	4.38 kW	14.033 kW
P_{diss}	29 W	29 W	29 W
I_0V_a	0 W	1.4 kW	14 kW
P_{refl}	3.62 kW	2.951 kW	~ 4.4 W

Off Crest and Off Resonance Operation

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- Typically electron storage rings operate off crest in order to ensure stability against phase oscillations.
- As a consequence, the rf cavities must be detuned off resonance in order to minimize the reflected power and the required generator power.
- Longitudinal gymnastics may also impose off crest operation operation in recirculating linacs.
- We write the beam current and the cavity voltage as

$$\tilde{I}_b = 2I_0 e^{i\psi_b}$$

$$\tilde{V}_c = V_c e^{i\psi_c} \quad \text{and set } \psi_c = 0$$

- The generator power can then be expressed as:

$$P_g = \frac{V_c^2}{R_L}\frac{(1+\beta)}{4\beta}\left\{\left[1+\frac{I_0 R_L}{V_c}\cos\psi_b\right]^2 + \left[\tan\Psi - \frac{I_0 R_L}{V_c}\sin\psi_b\right]^2\right\}$$

Off Crest and Off Resonance Operation (cont'd)

- Condition for optimum tuning:

$$\tan\Psi = \frac{I_0 R_L}{V_c}\sin\psi_b$$

- Condition for optimum coupling:

$$\beta_0 = 1 + \frac{I_0 R_a}{V_c}\cos\psi_b$$

- Minimum generator power:

$$P_{g,\min} = \frac{V_c^2 \beta_0}{R_a}$$

Q_{ext} Optimization under Beam Loading and Microphonics

- Beam loading and microphonics require careful optimization of the external Q of cavities.
- Derive expressions for the optimum setting of cavity parameters when operating under

  a) heavy beam loading

  b) little or no beam loading, as is the case in energy recovery linac cavities

and in the presence of microphonics.

Q_{ext} Optimization (cont'd)

$$P_g = \frac{V_c^2}{R_L}\frac{(1+\beta)}{4\beta}\left\{\left[1+\frac{I_{tot}R_L}{V_c}\cos\psi_{tot}\right]^2+\left[\tan\Psi-\frac{I_{tot}R_L}{V_c}\sin\psi_{tot}\right]^2\right\}$$

$$\tan\Psi = -2Q_L\frac{\delta f}{f_0}$$

where δf is the total amount of cavity detuning in Hz, including static detuning and microphonics.

- Optimization of the generator power with respect to coupling gives:

$$\beta_{opt} = \sqrt{(b+1)^2+\left[2Q_0\frac{\delta f}{f_0}+b\tan\psi_{tot}\right]^2}$$

$$\text{where} \quad b \equiv \frac{I_{tot}R_a}{V_c}\cos\psi_{tot}$$

where I_{tot} is the magnitude of the resultant beam current vector in the cavity and ψ_{tot} is the phase of the resultant beam vector with respect to the cavity voltage.

Q_{ext} Optimization (cont'd)

$$P_g = \frac{V_c^2}{R_L}\frac{(1+\beta)}{4\beta}\left\{\left[1+\frac{I_{tot}R_L}{V_c}\cos\psi_{tot}\right]^2 + \left[\tan\Psi - \frac{I_{tot}R_L}{V_c}\sin\psi_{tot}\right]^2\right\}$$

- Write: $$\tan\Psi = -2Q_L\frac{\delta f_0 + \delta f_m}{f_0}$$

  where δf_0 is the static detuning
  and δf_m is the microphonic detuning

- To minimize generator power with respect to tuning:

$$\delta f_0 = -\frac{f_0}{2Q_0}b\tan\Psi$$

independent of β!

$$\Rightarrow \qquad P_g = \frac{V_c^2}{R_L}\frac{(1+\beta)}{4\beta}\left\{(1+b+\beta)^2 + \left[2Q_0\frac{\delta f_m}{f_0}\right]^2\right\}$$

Q_{ext} Optimization (cont'd)

- Condition for optimum coupling:

$$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta f_m}{f_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_a}\left[|b+1| + \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta f_m}{f_0}\right)^2}\right]$$

- In the absence of beam (b=0):

$$\beta_{opt} = \sqrt{1 + \left(2Q_0 \frac{\delta f_m}{f_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_a}\left[1 + \sqrt{1 + \left(2Q_0 \frac{\delta f_m}{f_0}\right)^2}\right]$$

Example

- ERL Injector and Linac:

  δf_m=25 Hz, Q_0=1x10^{10} , f_0=1300 MHz, I_0=100 mA, V_c=20 MV/m, L=1.04 m, R_a/Q_0=1036 ohms per cavity

- ERL linac: Resultant beam current, I_{tot} = 0 mA (energy recovery)

  and β_{opt}=385 $\Rightarrow$ Q_L=2.6x10^7 $\Rightarrow$ P_g = 4 kW per cavity.

- ERL Injector: I_0=100 mA and β_{opt}= 5x10^4 ! $\Rightarrow$ Q_L= 2x10^5 $\Rightarrow$ P_g = 2.08 MW per cavity!

  Note: $I_0 V_a$ = 2.08 MW $\Rightarrow$ optimization is entirely dominated by beam loading.

RF System Modeling

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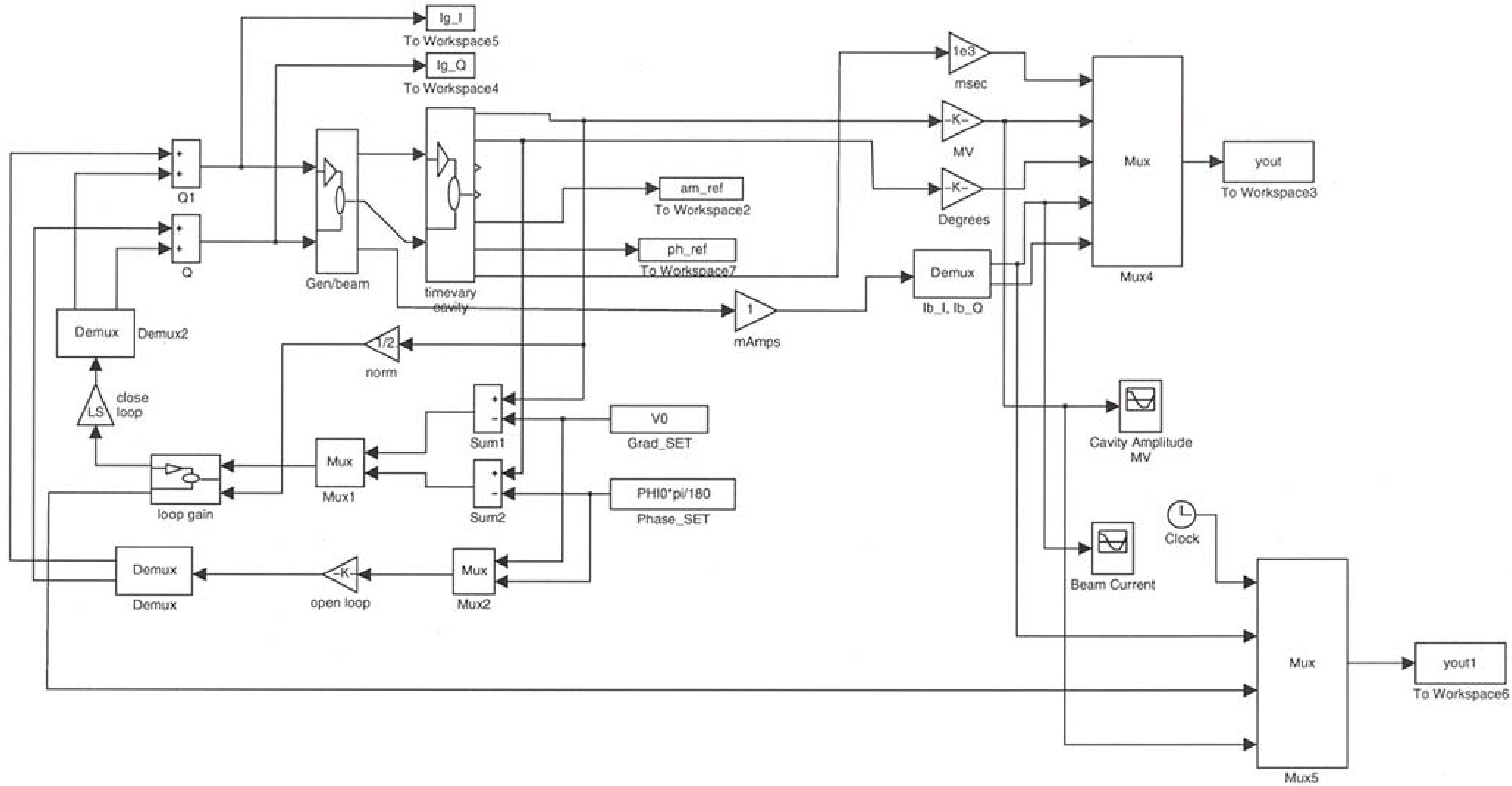
- To include amplitude and phase feedback, nonlinear effects from the klystron and be able to analyze transient response of the system, response to large parameter variations or beam current fluctuations
    - we developed a model of the cavity and low level controls using SIMULINK, a MATLAB-based program for simulating dynamic systems.
- Model describes the beam-cavity interaction, includes a realistic representation of low level controls, klystron characteristics, microphonic noise, Lorentz force detuning and coupling and excitation of mechanical resonances

RF System Model



RF Modeling: Simulations vs. Experimental Data

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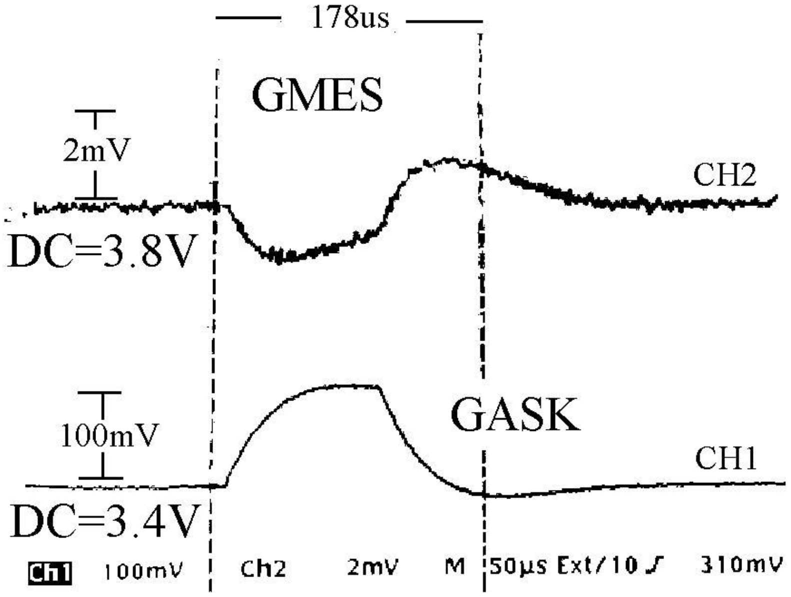

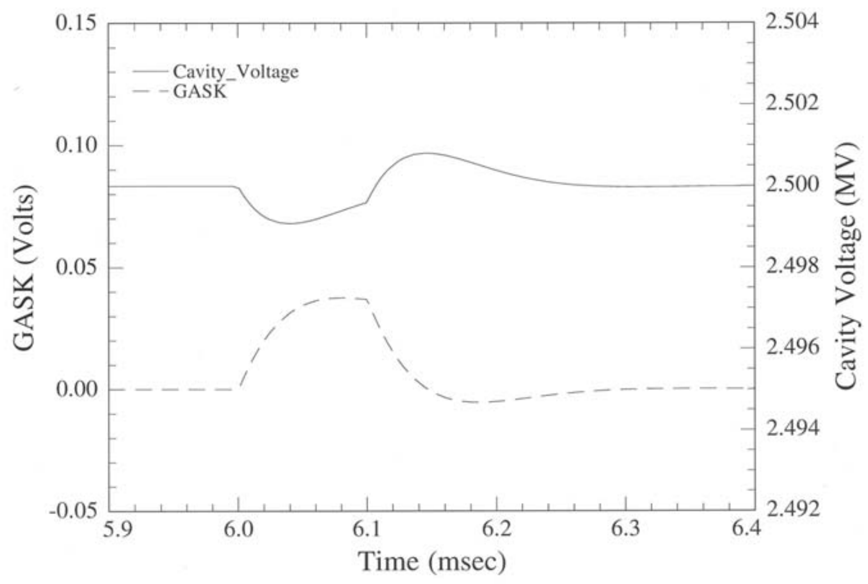

Measured and simulated cavity voltage and amplified gradient error signal (GASK) in one of CEBAF's cavities, when a 65 μA, 100 μsec beam pulse enters the cavity.

Conclusions

- We derived a differential equation that describes to a very good approximation the rf cavity and its interaction with beam.
- We derived useful relations among cavity's parameters and used phasor diagrams to analyze steady-state situations.
- We presented formula for the optimization of Q_{ext} under beam loading and microphonics.
- We showed an example of a Simulink model of the rf control system which can be useful when nonlinearities can not be ignored.