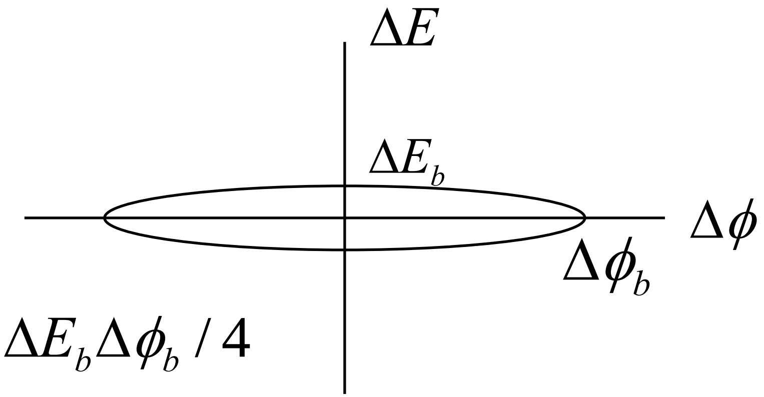
USPAS Course on
Recirculated and Energy Recovered Linear Accelerators

G. A. Krafft and L. Merminga
Jefferson Lab
I. Bazarov
Cornell

Lecture 9

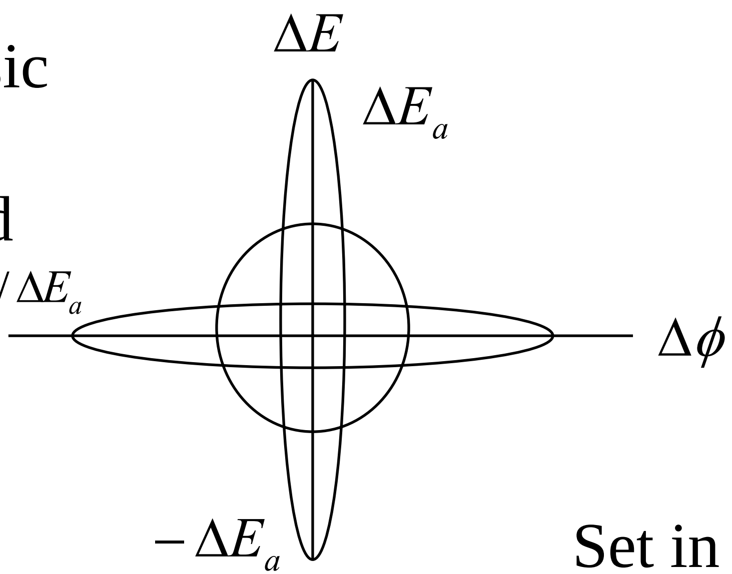
Energy Spread, Intrinsic



$$\varepsilon_L \approx \Delta E_b \Delta\phi_b / 4$$

Intrinsic Phase Spread

$$\sigma_\phi \approx 2\varepsilon_L / \Delta E_a$$



Intrinsic Energy Spread

$$\sigma_{E,inj} \approx 2\varepsilon_L / \Delta\phi_b$$

Set in injector/initial bunching

Longitudinal Optimization

Energy spread from perfectly phased linac

$$\frac{\sigma_E}{E} = \sqrt{\sigma_{E,inj}^2 / E^2 + \sigma_\phi^4 / 2}$$

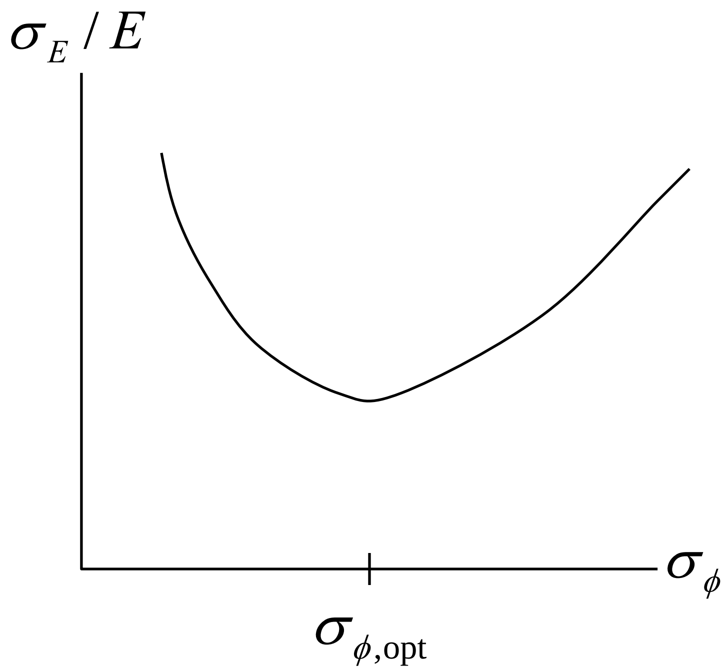
Using definition of longitudinal emittance, derive an optimum

$$\sigma_{\phi,opt} = \left(\frac{\varepsilon_L}{E}\right)^{1/3}$$

And minimum energy spread out the end

$$\frac{\sigma_E}{E} = \sqrt{\frac{3}{2}}\left(\frac{\varepsilon_L}{E}\right)^{2/3}$$

Longitudinal Optimization



Measurements of longitudinal emittance

Injector	45 MeV	<6.7 keV deg
Nlinac exit	500 MeV	<12.5 keV deg

Optimal spread using larger number 2E-5 @ 1 GeV (5E-6 @ 4 GeV)

RF Focussing

In any RF cavity that accelerates longitudinally, because of Maxwell Equations there must be additional transverse electromagnetic fields. These fields will act to focus the beam and must be accounted properly in the beam optics, especially in the low energy regions of the accelerator. We will discuss this problem in greater depth in injector lectures. Let **A**(x,y,z) be the vector potential describing the longitudinal mode (Lorenz gauge)

$$\nabla \cdot \vec{A} = -\frac{1}{c}\frac{\partial \phi}{\partial t}$$

$$\nabla^2 \vec{A} = -\frac{\omega^2}{c^2}\vec{A} \qquad \nabla^2 \phi = -\frac{\omega^2}{c^2}\phi$$

For cylindrically symmetrical accelerating mode, functional form can only depend on *r* and *z*

$$A_z(r,z) = A_{z0}(z) + A_{z1}(z)r^2 + \ldots$$

$$\phi(r,z) = \phi_0(z) + \phi_1(z)r^2 + \ldots$$

Maxwell's Equations give recurrence formulas for succeeding approximations

$$(2n)^2 A_{zn} + \frac{d^2 A_{z,n-1}}{dz^2} = -\frac{\omega^2}{c^2} A_{z,n-1}$$

$$(2n)^2 \phi_n + \frac{d^2 \phi_{n-1}}{dz^2} = -\frac{\omega^2}{c^2} \phi_{n-1}$$

Gauge condition satisfied when

$$\frac{dA_{zn}}{dz} = -\frac{i\omega}{c}\phi_n$$

in the particular case $n = 0$

$$\frac{dA_{z0}}{dz} = -\frac{i\omega}{c}\phi_0$$

Electric field is

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \overleftarrow{A}}{\partial t}$$

And the potential and vector potential must satisfy

$$E_z(0,z) = -\frac{d\phi_0}{dz} - \frac{i\omega}{c} A_{z0}$$

$$\therefore \quad \frac{i\omega}{c} E_z(0,z) = \frac{d^2 A_{z0}}{dz^2} + \frac{\omega^2}{c^2} A_{z0} = -4A_{z1}$$

So the magnetic field off axis may be expressed directly in terms of the electric field on axis

$$\therefore \quad B_\theta \approx -2rA_{z1} = \frac{i}{2} \frac{\omega r}{c} E_z(0,z)$$

And likewise for the radial electric field (see also $\nabla \cdot \vec{E} = 0$)

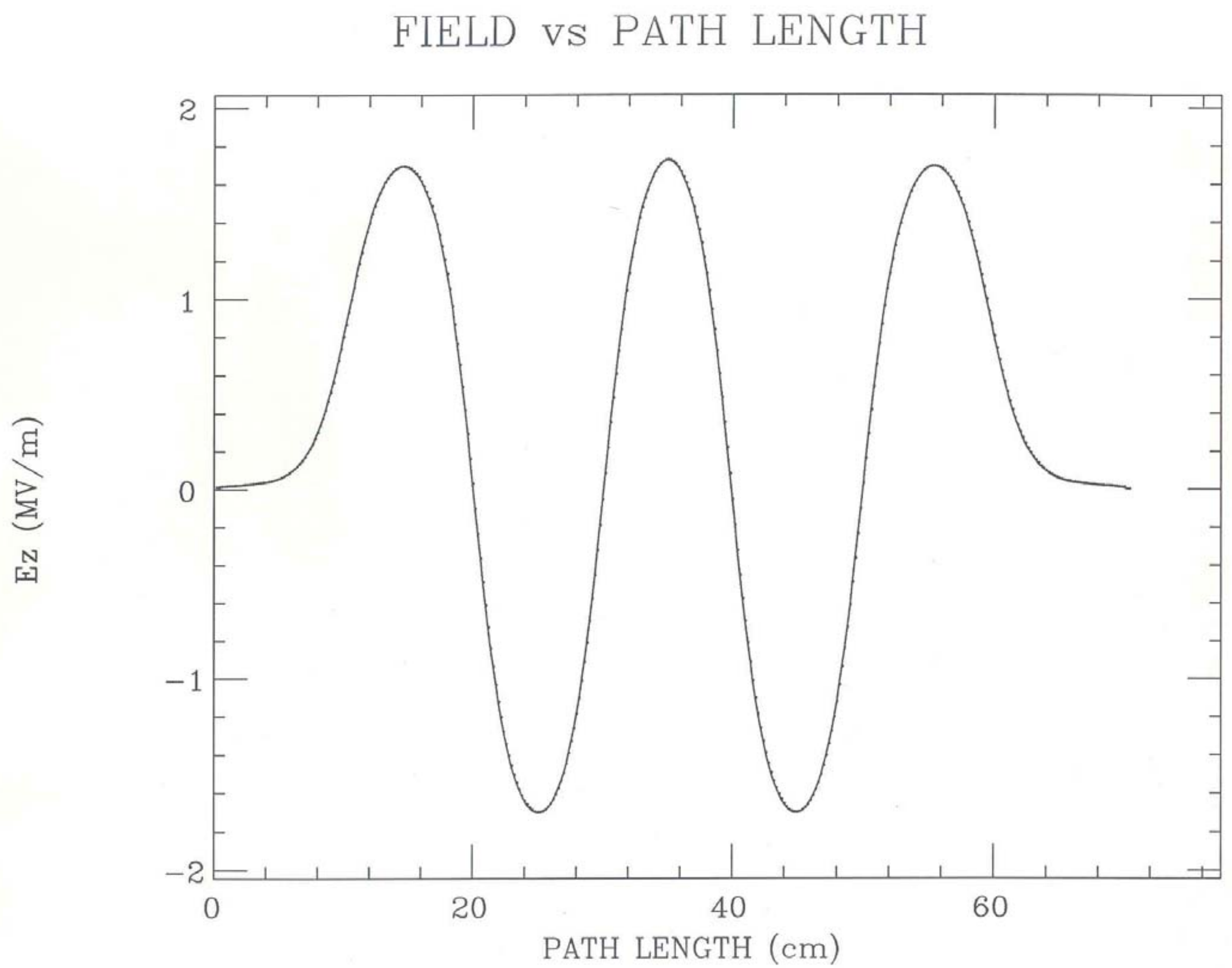
$$\therefore \quad E_r \approx -2r\phi_1(z) = -\frac{r}{2}\frac{dE_z(0,z)}{dz}$$

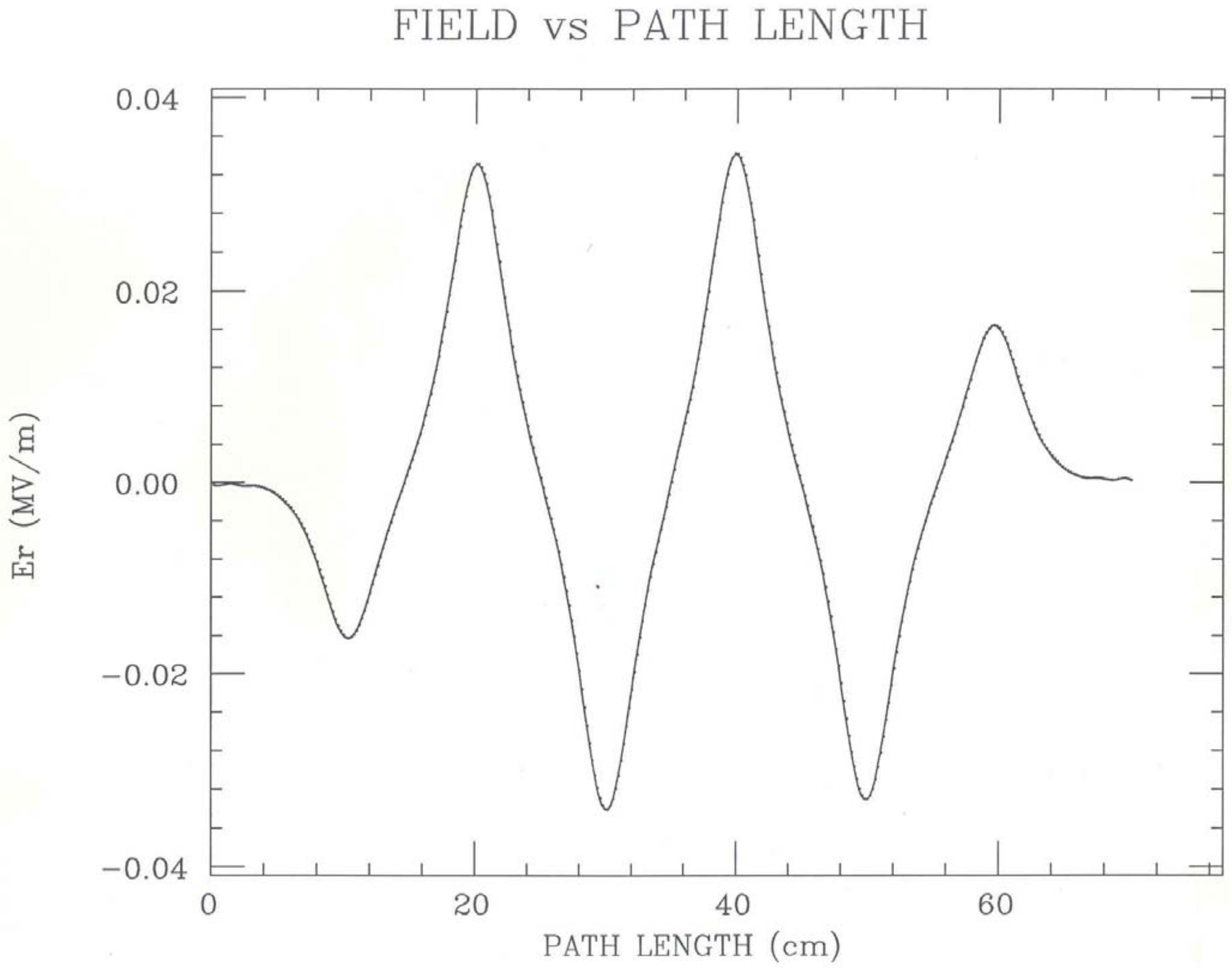
Explicitly, for the time dependence $\cos(\omega t + \delta)$

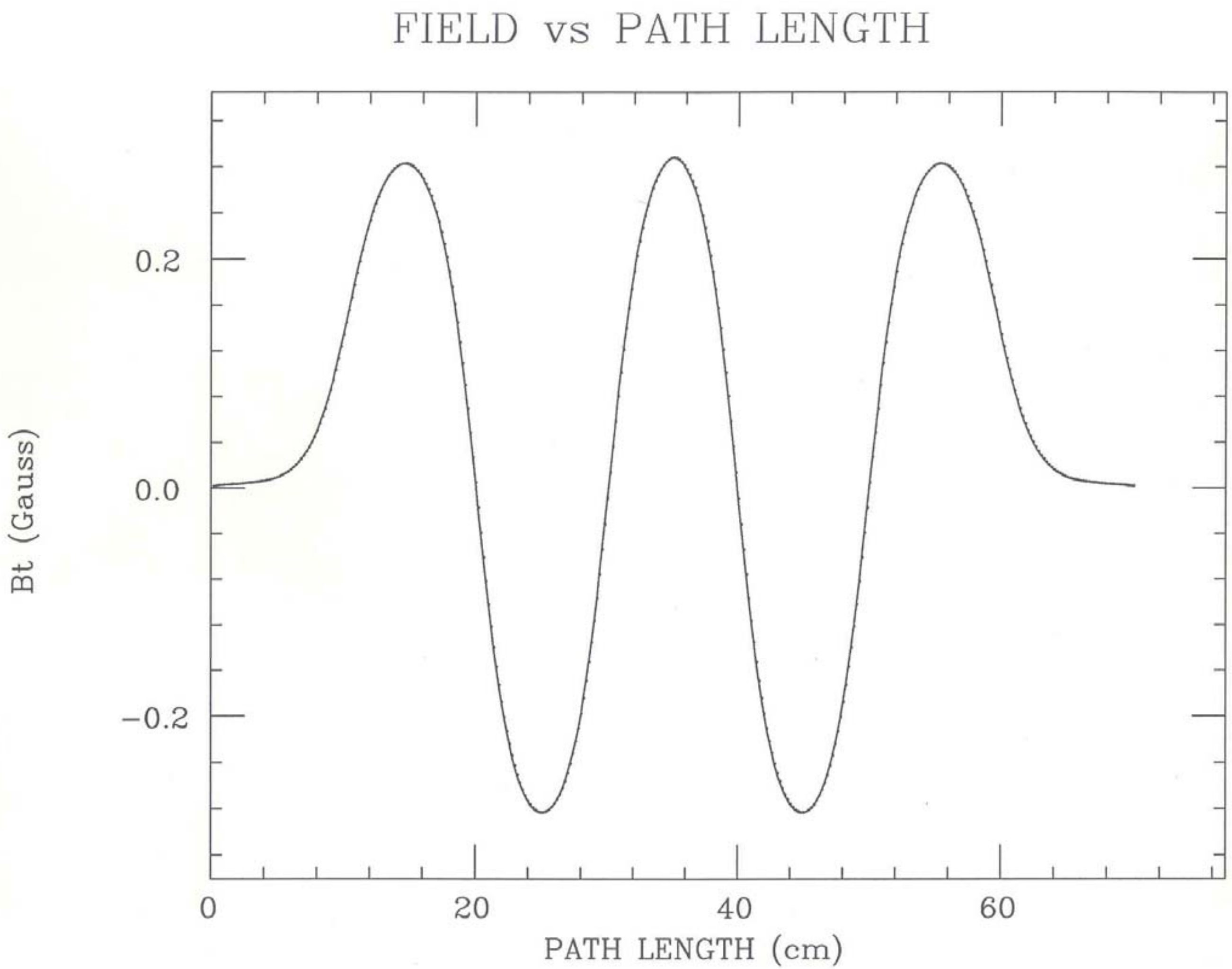
$$E_z(r,z,t) \approx E_z(0,z)\cos(\omega t + \delta)$$

$$E_r(r,z,t) \approx -\frac{r}{2}\frac{dE_z(0,z)}{dz}\cos(\omega t + \delta)$$

$$B_\theta(r,z,t) \approx -\frac{\omega r}{2c}E_z(0,z)\sin(\omega t + \delta)$$


FIELD vs PATH LENGTH
Ez (MV/m)
2
1
0
−1
−2
0
20
40
60
PATH LENGTH (cm)


FIELD vs PATH LENGTH
Er (MV/m)
0.04
0.02
0.00
−0.02
−0.04
0
20
40
60
PATH LENGTH (cm)


FIELD vs PATH LENGTH
0.2
0.0
−0.2
Bt (Gauss)
0
20
40
60
PATH LENGTH (cm)

Motion of a particle in this EM field

$$\frac{d\left(\gamma m\vec{\mathrm{V}}\right)}{dt} = -e\left(\vec{E} + \frac{\vec{V}}{c}\times\vec{B}\right)$$

$$\gamma(z)\beta_x(z) = \gamma(-\infty)\beta_x(-\infty) + \int_{-\infty}^{z}\left[\begin{array}{l} -\frac{x(z')}{2}\frac{dG(z')}{dz'}\cos(\omega t(z')+\delta) \\ +\frac{\omega\beta_z(z')x(z')}{2c}G(z')\sin(\omega t(z')+\delta)\end{array}\right]\frac{dz'}{\beta_z(z')}$$

The normalized gradient is

$$G(z) = \frac{eE_z(z,0)}{mc^2}$$

and the other quantities are calculated with the integral equations

$$\gamma(z) = \gamma(-\infty) + \int_{-\infty}^{z} G(z')\cos(\omega t(z') + \delta)dz'$$

$$\gamma(z)\beta_z(z) = \gamma(-\infty)\beta_z(-\infty) + \int_{-\infty}^{z} \frac{G(z')}{\beta_z(z')}\cos(\omega t(z') + \delta)dz'$$

$$t(z) = \lim_{z_0 \to -\infty} \frac{z_0}{\beta_z(-\infty)c} + \int_{-\infty}^{z} \frac{dz'}{\beta_z(z')c}$$

These equations may be integrated numerically using the cylindrically symmetric CEBAF field model to form the Douglas model of the cavity focussing. In the high energy limit the expressions simplify.

$$x(z) = x(a) + \int_a^z \frac{\gamma(z')\beta_x(z')}{\gamma(z')\beta_z(z')} dz'$$

$$\approx x(a) + \frac{\beta_x(-\infty)}{\beta_z(-\infty)}(z-a) - \int_a^z \frac{x(z')}{2} \frac{G(z')}{\gamma(z')\beta_z^2(z')} \cos(\omega t(z') + \delta) dz'$$

$$\gamma(z)\beta_z(z) = \gamma(-\infty)\beta_z(-\infty)\left[1 + \frac{E_G}{2E}\right]$$

$$- \int_{-\infty}^{\infty} \frac{x(z')}{4} \frac{G^2(z')}{\gamma(z')\beta_z^2(z')} \cos^2(\omega t(z') + \delta) dz'$$

$$E_G = mc^2 \cos(\delta) \int_{-\infty}^{\infty} G(z) \cos(\omega z / c) dz$$

Transfer Matrix

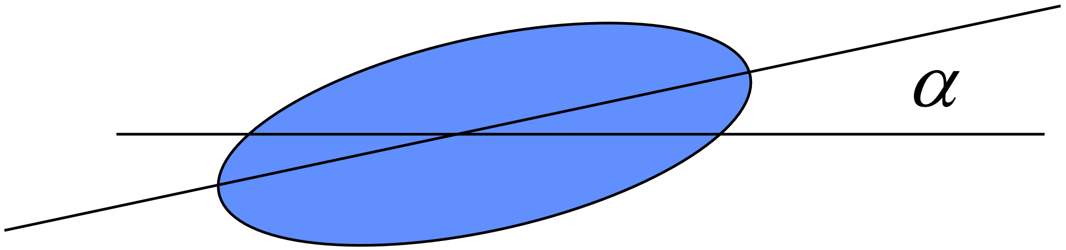
For position-momentum transfer matrix

$$T = \begin{pmatrix} 1 - \frac{E_G}{2E} & \frac{L}{\gamma} \\ -\frac{I}{4\gamma} & 1 + \frac{E_G}{2E} \end{pmatrix}$$

$$I = \cos^2(\delta) \int_{-\infty}^{\infty} G^2(z) \cos^2(\omega z / c) dz$$

$$+ \sin^2(\delta) \int_{-\infty}^{\infty} G^2(z) \sin^2(\omega z / c) dz$$

Kick Generated by mis-alignment



$$\Delta\gamma\beta = \frac{E_G \alpha}{2E}$$

Damping and Antidamping

By symmetry, if electron traverses the cavity exactly on axis, there is no transverse deflection of the particle, but there is an energy increase. By conservation of transverse momentum, there must be a decrease of the phase space area. For linacs NEVER use the word "adiabatic"

$$\frac{d\left(\gamma m \vec{\mathrm{V}}_{\mathrm{transverse}}\right)}{dt} = 0$$

$$\gamma(z)\,\beta_x(z) = \gamma(-\infty)\,\beta_x(-\infty)$$

Conservation law applied to angles

$$\beta_x, \beta_y << \beta_z \approx 1$$

$$\theta_x = \beta_x / \beta_z \sim \beta_x \quad \theta_y = \beta_y / \beta_z \sim \beta_y$$

$$\theta_x(z) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)}\theta_x(-\infty)$$

$$\theta_y(z) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)}\theta_y(-\infty)$$

Phase space area transformation

$$dx \wedge d\theta_x(z) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)} dx \wedge d\theta_x(-\infty)$$

$$dy \wedge d\theta_y(z) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)} dy \wedge d\theta_y(-\infty)$$

Therefore, if the beam is accelerating, the phase space area after the cavity is less than that before the cavity and if the beam is decelerating the phase space area is greater than the area before the cavity. The determinate of the transformation carrying the phase space through the cavity has determinate equal to

$$\mathrm{Det}\left(M_{cavity}\right) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)}$$

By concatenation of the transfer matrices of all the accelerating or decelerating cavities in the recirculated linac, and by the fact that the determinate of the product of two matrices is the product of the determinates, the phase space area at each location in the linac is

$$dx \wedge d\theta_x(z) = \frac{\gamma(0)\beta_z(0)}{\gamma(z)\beta_z(z)} dx \wedge d\theta_x(0)$$

$$dy \wedge d\theta_y(z) = \frac{\gamma(0)\beta_z(0)}{\gamma(z)\beta_z(z)} dy \wedge d\theta_y(0)$$

Same type of argument shows that things like orbit fluctuations are damped/amplified by acceleration/deceleration.

Summary of last three lectures

We have shown how proper manipulation of the longitudinal phase space can lead to accelerators with superior beam characteristics.

We have shown how phase space tends to be degraded by generation of "curvatures" in longitudinal phase space, and a means to quantify such effects.

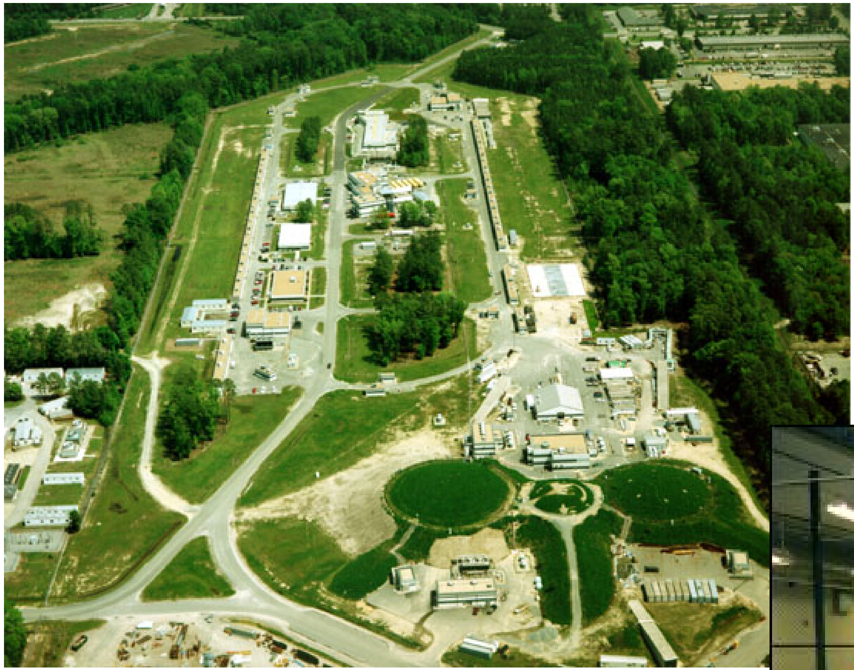
In this lecture and the preceding one, we've discussed some of the ways that people have combated this effect through (1) proper RF phase choices, (2) adding sextupoles in recirculation optics, and (3) RF linearization cavities.

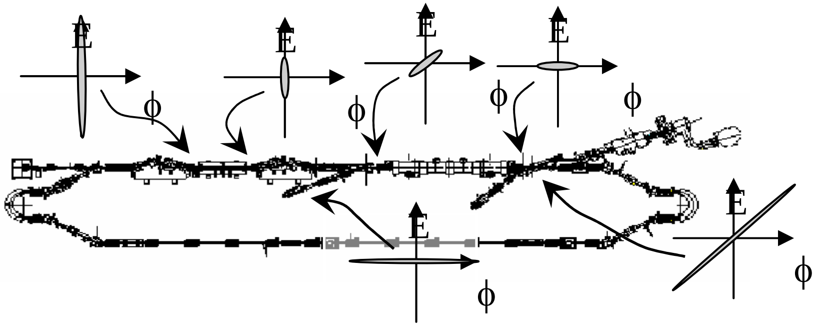
We've demonstrated in more detail estimated the energy spread generated by the effect of RF amplitude and phase fluctuations.

We’ve shown a method to estimate the RF focussing from the accelerating modes of the linear accelerator. This method has been used to determine some of the “standard” transfer matrices of cavities.

We’ve calculated the effect of damping and antidamping of betatron oscillations in a linac accelerating or decelerating.

Optics Issues for Recirculating Linacs

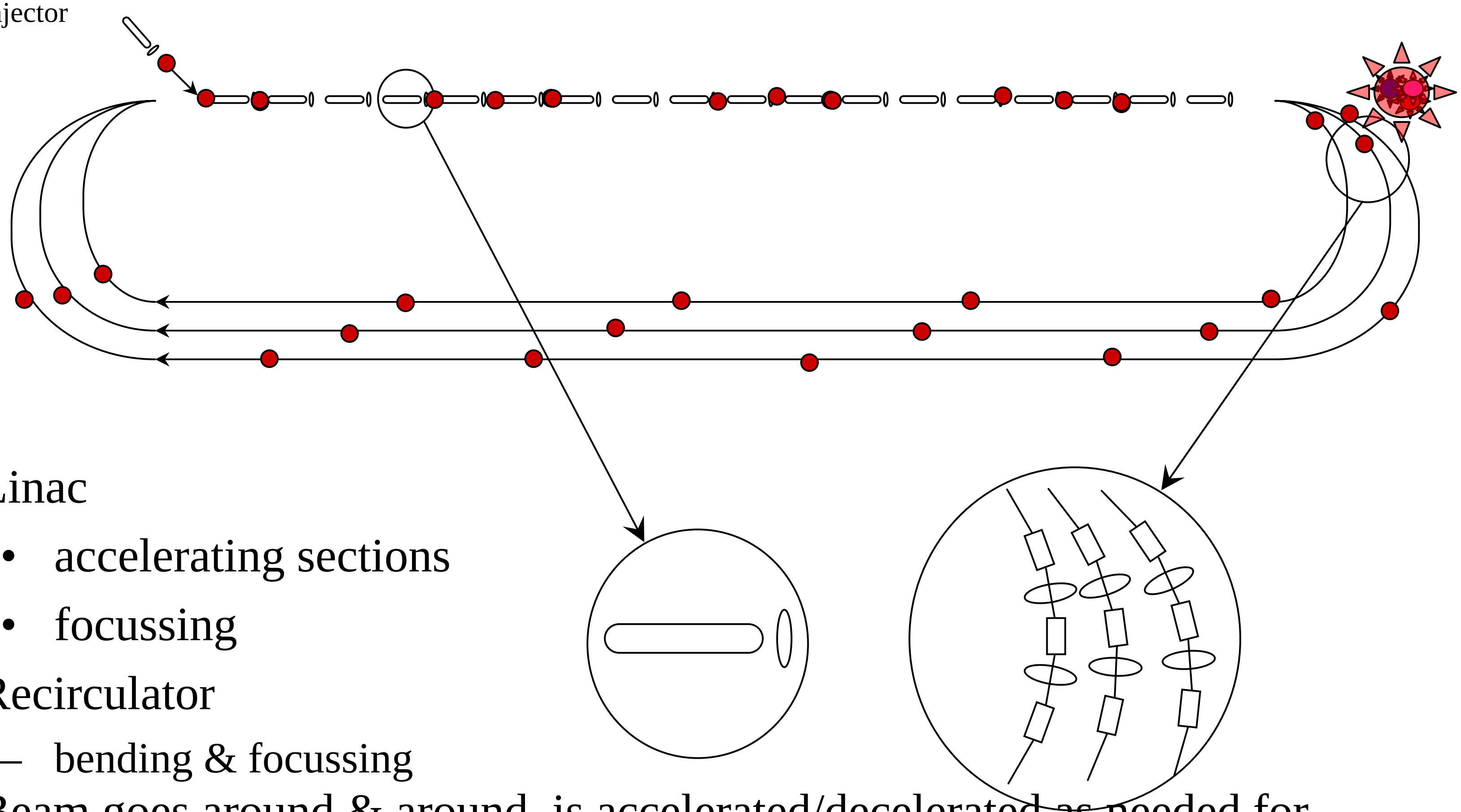




D. Douglas

The Naïve Recirculator

- Injector



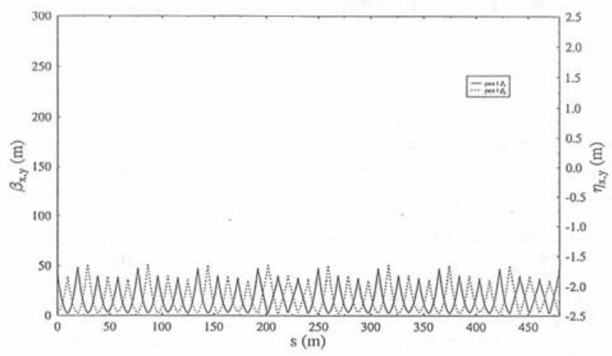
- Linac
  - accelerating sections
  - focussing
- Recirculator
  - bending & focussing
- Beam goes around & around, is accelerated/decelerated as needed for the application at hand

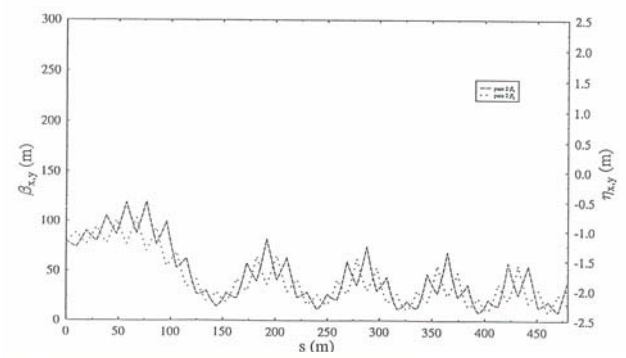
Beam envelope/spot size control is the *transverse optical issue in recirculating linacs*

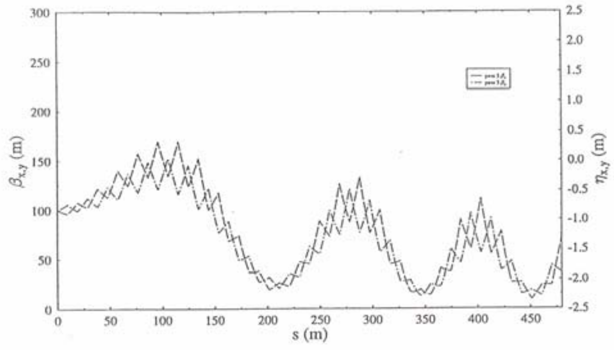
- Recirculation leads to mismatch between beam energy and excitation of focusing elements
  - set focusing for first pass $\Rightarrow$ higher passes get “no” focusing/blow up (linac looks like a drift, β_{max} ~ linac length)
  - set focusing for higher passes $\Rightarrow$ first pass *over-focused*/blows up
  - Large envelopes lead to scraping, error sensitivity, lower instability thresholds
- Imposes limits on
  - injection energy (higher is better but costs more),
  - linac length (shorter is better but gives less acceleration), and/or
  - achievable control over β_{max}

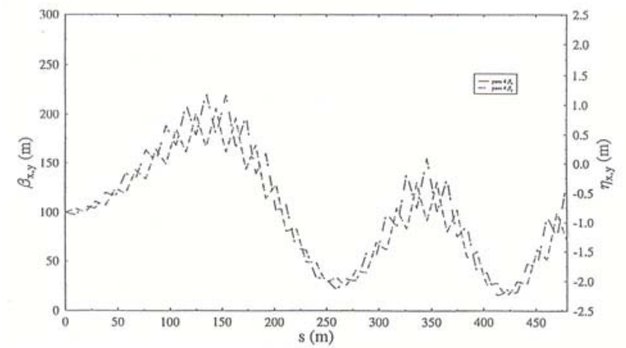
CEBAF Envelopes

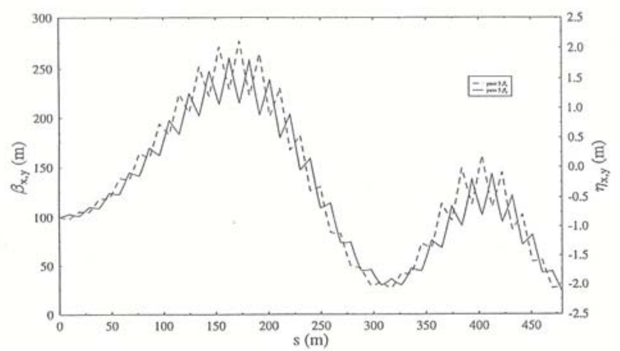
– FODO quad lattice with 120° phase advance on 1st pass









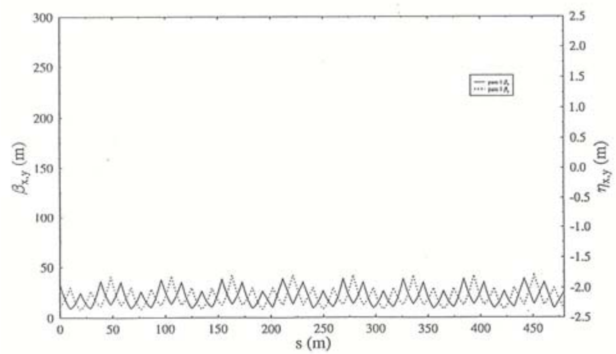


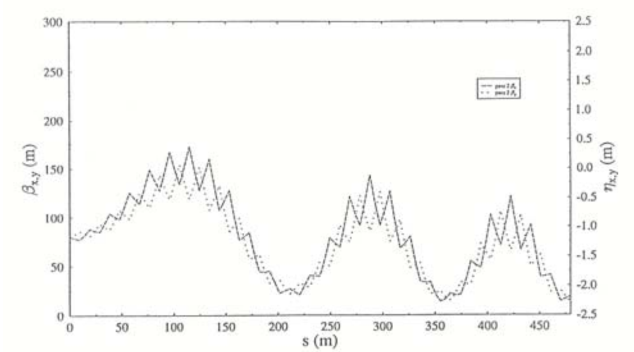
Panaceas

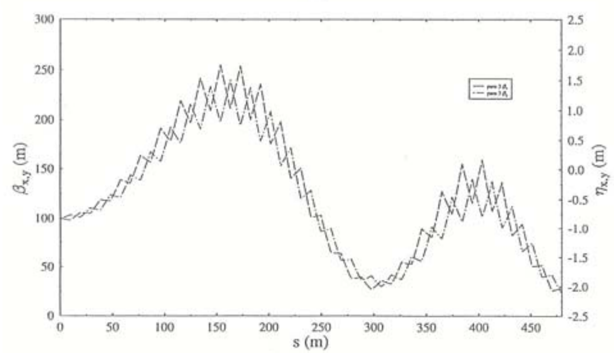
- Focus 1st pass as much as possible (whilst maintaining adequate betatron stability)
- Use a “split linac” – 2 halves rather than 1 whole
  - Shorter linac $\Rightarrow$ lower peak envelopes (“shorter drift length”)
- *Linac interruptus*
- High injection energy
- “Graded gradient” focusing in energy-recovering linacs
- Use high gradient RF
- Use an “inventive” linac topology
  - “Counter-rotated” linacs
  - “Bisected” linac topology
  - “Asymmetric” linac topology

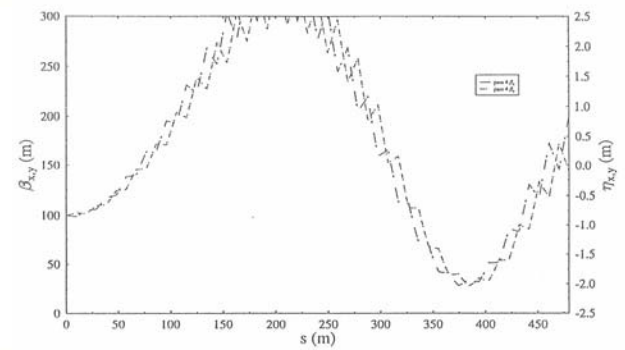
CEBAF Envelopes, reduced focusing

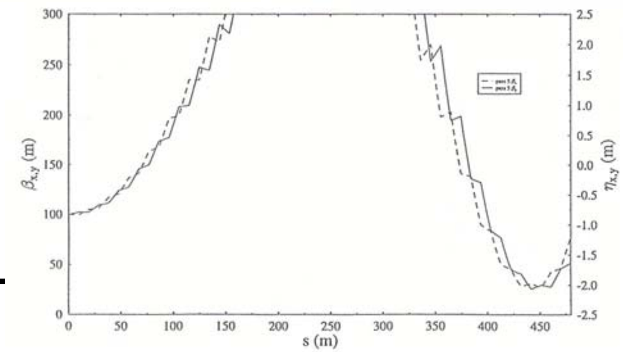
– FODO quad lattice with 60^o phase advance on 1st pass











Injection Energy

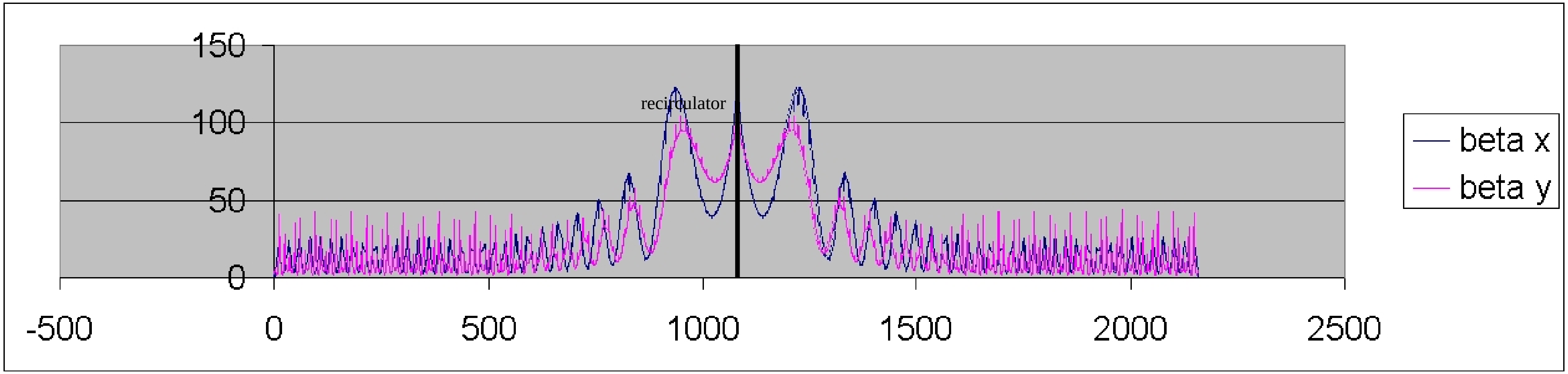
- Injection energy "must" be high enough to avoid significant levels of pass-to-pass RF phase slip
  - CEBAF E_{inj} = 45 MeV, $\delta\phi_{RF} \sim$ 1-2^o on 1st pass, little thereafter
  - IR Demo FEL E_{inj} = 10 MeV, $\delta\phi_{RF} \sim 10^o$ from pass to pass
- Injection energy "should" be high enough to allow adequate pass-to-pass focusing in a single transport system
  - "adequate" is system dependent
    - CEBAF (45 MeV $\Rightarrow$ 4 GeV): $\beta_{max} \sim$ 200 m – adequate to run 200 μA
    - IR Demo (10 MeV $\Rightarrow$ 45 MeV): $\beta_{max} \sim$ 25 m – adequate to run 5 mA
  - Higher is better (front end focusing elements stronger) but more expensive
    - SUPERCEBAF (1 GeV $\Rightarrow$ 16 GeV), using same type of linac focusing as in CEBAF: $\beta_{max} \sim$ 130 m
- Naïve figure of merit: E_{final}/E_{inj}, with smaller being better

“Graded-gradient” Focusing

- There are 2 common focusing patterns:
  - constant gradient (all quads have same pole tip field; sometimes used in microtrons)
  - constant focal length (quad excitation tracks energy; often used in linacs)
- Neither works well for energy recovering linacs
  - Beam envelopes blow up, limiting linac length & tolerable E_{final}:E_{in} ratio
- “Graded-gradient” focusing $\Rightarrow$ match focal length of quads to beam of lowest energy
  - Excitation of focusing elements increases with energy to linac midpoint, then declines to linac end
  - Allows “exact” match for half of linac, produces “adiabatically induced” mismatch in second half

"Graded-gradient" Focusing, cont.

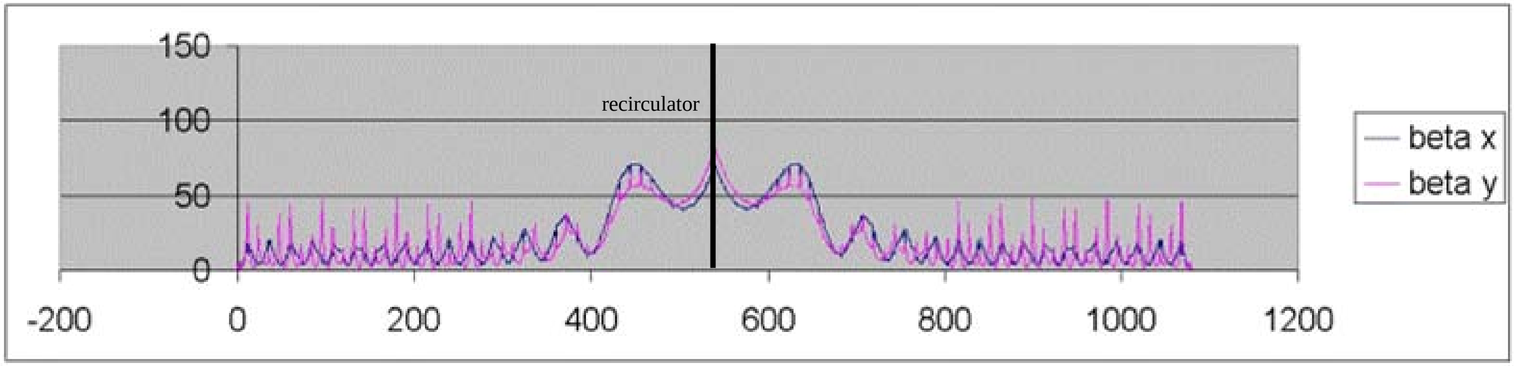
1 km, 10 MeV→10 GeV linac (111 MV/module), triplet focusing:



222 MV

High Accelerating Gradient

- Higher accelerating gradient very helpful in limiting beam envelope mismatch
  - Shortens linac
  - Increases excitation of front end (after 1st accelerating section) focusing elements, reducing mismatch on higher passes
  - ½ km 10 MeV→10 GeV linac (~222 MV/module), using triplets:



111 MV

High Accelerating Gradient, cont.

- "Focal Failure Factor"
  - Ratio of energies after 1st/before final accelerating section
  - Figure of merit for multipass mismatch – more descriptive than ratio of injected to final energies
  - For the two example machines:

Average Gradient	E after 1st	E before last	"FFF"
111 MeV/module	121 MeV	9889 MeV	~82
222 MeV/module	232 MeV	9778 MeV	~42

(compare to E_{out}/E_{in} = 1000…)

Why it Matters ("Halo")

Performance of recirculated linacs may ultimately be limited by loss of "halo" – particles far from the beam core

- There is "stuff" in the beam not necessarily well described by core emittance, rms spot sizes, gaussian tails, *etc.*
- This "stuff" represents a small fraction ($<10^{-4}$? 10^{-5} ?) of the total current, but it can get scraped away locally, causing heating, activation, and damaging components
- Heuristically:
  - Higher current leads to more such loss
  - Smaller beam pipe results in greater loss
  - Bigger beam envelopes encourage increased loss

$$I_{loss} \propto I \times \frac{\beta}{a}$$

Phenomenology

- In CEBAF, BLM/BCMs induced trips ⇒ losses of ~1 μA out of 100 μA in 1 cm aperture where β ~100 m
  ⇒ proportionality const. ~ (1 μA/100 μA)x (0.01 m/100 m) ~ 10^{-6}
- In the IR Demo FEL, BLMs induce trips ⇒ losses of ~ 1 μA out of 5000 μA in 2.5 cm aperture where β ~5 m
  ⇒ proportionality const. ~ (1 μA/5000 μA)x (0.025 m/5 m) ~ 10^{-6}
- One might then guess

$$I_{loss} = 10^{-6} I \times \frac{\beta}{a}$$

which, in a 100 mA machine tolerating 5 μA loss in a 2.5 cm bore, implies you must have β ~ 1.25 m (*ouch!)*

Moral: There will be great virtue in clean beam and small beam envelope function values!

- Recirculation arc design
  - Functional modularity
  - Beam separation (extraction)/recombination (reinjection) geometry
    - Single step
    - Staircase
    - Overshoot
  - Beam quality preservation
    - Incoherent synchrotron radiation control
      - Energy spread ~g5/r2
      - Emittance excitation ~ <H>g7/r2, H ~ b2, h2
    - CSR control & compensation (e.g ½ betatron wavelength correction in IR Demo; don’t squeeze entire phase at one time; keep betas, etas small)
    - Space charge control (don’t squeeze entire phase space at one time)
  - Matching
    - Transverse – linac to recirculator, vice versa
    - Longitudinal phase space management
      - Orthogonal knobs useful: e.g. IR Demo – path length, M_{56}, T_{566} all decoupled & more or less separate from transverse