USPAS Course on
Recirculated and Energy Recovered Linear Accelerators

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Lecture 7

Lecture Outline

- Some General Rules Regarding Recirculated Linac Design
- Longitudinal Single Particle Dynamics
  - Longitudinal Gymnastics
  - Correcting RF Curvature with RF
  - Correcting RF Curvature with Sextupoles
  - Correction RF Curvature with Linearizers
  - Longitudinal Tune Choices
  - Energy Spread Estimates
- Transverse Single Particle Dynamics
  - Basic Considerations
  - Betatron Motion Damping and Antidamping
  - RF Focussing
  - Energy Ratio Limits
  - Beam Loss

Some General Rules on Recirculated Linac Design

- Design the nonlinear development of the longitudinal phase space first, adjust the transverse phase space "appropriately" based on the longitudinal design. This is actually a pretty important rule and saves much work if possible because the longitudinal control elements (i.e., RF cavities and places where M_{56} is introduced) can have substantial effects on the transverse dynamics (e.g. through RF focussing or generating dispersion!) BUT the elements controlling the transverse design tend to have somewhat less effect on the longitudinal dynamics, at least insofar as non-linear effects are concerned

- Work to achieve linear (elliptical!) longitudinal phase space densities at specific locations in the design. This may be accomplished by adjusting specific non-linear distortions in the phase space with offsetting distortions introduced as correction elements

- Become familiar with and quantify specific non-linear distortions and the library of "tools" available to correct them

Longitudinal Phase Space

- Begin with the action principle [Jackson, Section 12.1]

$$\text{Action } = \int_{t_1}^{t_2} L\left[q_i(t), \dot{q}_i(t), t\right] dt$$

- Action integral must be Lorentz invariant, as is the proper time, $d\tau = dt/\gamma$

$$\therefore \quad \gamma L$$

must be a Lorentz invariant. So, as we've already used previously, the Lagrangian for particle in an EM field must be (MKS units)

$$L = -mc^2\sqrt{1-\beta^2} - e\Phi + eA\cdot v$$

You've already seen the E-L Equations yield the correct relativistic equations of motion

The canonical momenta congugate to the position coordinates are

$$P_i = \frac{\partial L}{\partial v_i} = \gamma m v_i + eA_i$$

The Hamiltonian $H = P \bullet v - L$ is

$$H(q_i, P_i, t) = \sqrt{(P - eA)^2 + m^2 c^4} + e\Phi$$

and the standard relativistic EM force law may be derived from Hamilton's canonical equations

$$\dot{q}_i = \frac{\partial H}{\partial P_i} \qquad \dot{P}_i = -\frac{\partial H}{\partial q_i}$$

The coordinate pairs (q_i, P_i), e.g. (z, P_z), form 2-D spaces called phase space.

Usually in accelerators, the longitudinal velocity is much bigger than the transverse velocity, and one can treat the longitudinal dynamics separately from the transverse dynamics.

$$H_{eff}(z, P_z) = \sqrt{\left(P_z - eA_z(z)\right)^2 + m^2c^4} + e\phi(z)$$

$$\approx P_z - eA_z(z) + e\phi(z)$$ Extreme Relativistic

$$\approx mc^2 + \frac{\left(P_z - eA_z(z)\right)^2}{2m} + e\phi(z)$$ Nonrelativistic

Which is clearly close to the total energy in this case!

Canonical Variables for Longitudinal Motion

In extreme relativistic limit

$$\left(z, E\right)$$

or

$$\left(\phi, \Delta E\right)$$

Form a canonically conjugate pair.

Liouville's Theorem

For a 3-dimensional Hamiltonian system, the sum of the projected phase space area is preserved

Pf:

$$\frac{d}{dt} g_t^* \left(\sum_i dP_i \Lambda dq_i \right) = \sum_i \left(\frac{d}{dt} g_t^* dP_i \Lambda dq_i + dP_i \Lambda \frac{d}{dt} g_t^* dq_i \right)$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} \left(- \frac{\partial^2 H}{\partial q_j \partial q_i} dq_j \Lambda dq_i - \frac{\partial^2 H}{\partial P_j \partial q_i} dP_j \Lambda dq_i \right)$$

$$\sum_{i=1}^{3} \sum_{j=1}^{3} \left(\frac{\partial^2 H}{\partial q_j \partial P_i} dP_i \Lambda dq_j + \frac{\partial^2 H}{\partial P_j \partial P_i} dP_i \Lambda dP_j \right) = 0$$

Here

$$g_t : P \to P$$

is the phase flow function.

Liouville's Theorem

Have also used, e.g.

$$\frac{d}{dt} g_t^* dP_i = -\sum_{j=1}^{3} \frac{\partial^2 H}{\partial q_j \partial q_i} dq_j - \sum_{j=1}^{3} \frac{\partial^2 H}{\partial P_j \partial q_i} dP_j$$

This equation follows from the Hamilton equations of motion

Corollary 1 (Liouville's Theorem): The local full 6-d phase volume is preserved

$$\frac{d}{dt} g_t^* (dq_1 \Lambda dq_2 \Lambda dq_3 \Lambda dP_1 \Lambda dP_2 \Lambda dP_3) = 0$$

Corollary 2 (Longitudinal Liouville's Theorem): In the case that the longitudinal motion uncoupled from the transverse motion, the longitudinal phase space density is preserved

Longitudinal Emittance

Definition: Utilizing the single particle distribution function to define averages as before, we define the longitudinal emittance to be the following phase space average:

$$\varepsilon_z = \sqrt{\left\langle \left(z - \langle z \rangle\right)^2 \right\rangle \left\langle \left(P_z - \langle P_z \rangle\right)^2 \right\rangle - \left\langle \left(z - \langle z \rangle\right)\left(P_z - \langle P_z \rangle\right)\right\rangle^2} \; / \; mc$$

Units in this definition are m. For perfectly linear restoring forces, one can show that this quantity is preserved with acceleration. However, this quantity can go both up and down depending on manipulations done to “straighten out” curvatures in phase space.

In linacs and recirculating linacs especially, this quantity provides a great “metric” for evaluating and comparing different accelerator designs. Smaller longitudinal emittance implies that one is able to compress to smaller bunches.

Cautions

This definition is good enough for our purposes, but there is no real "standard" definition in the field. For example, sometimes emittances are computed with subsets of the total number of particles. When "chirping" is discussed, sometimes the intrinsic phase and energy spread is being talked about, etc.

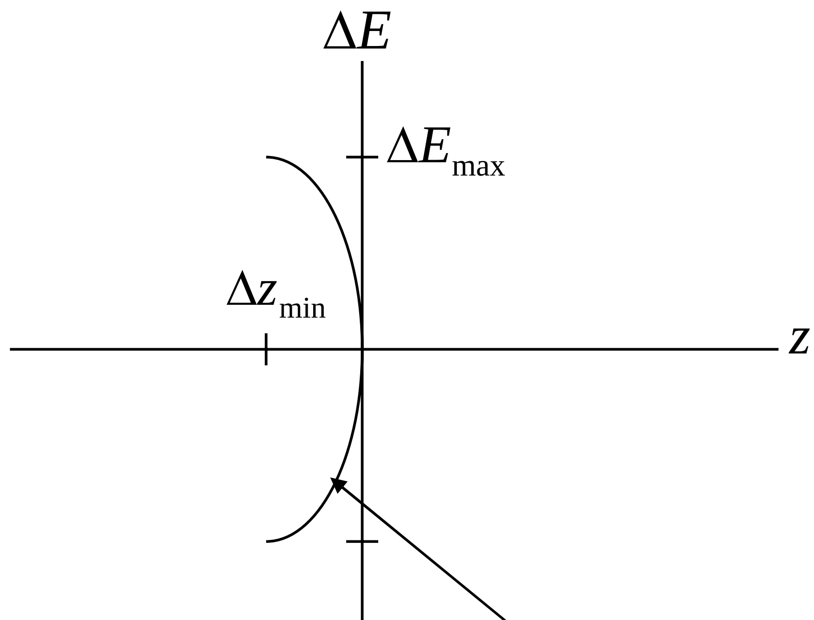
Ideally, one would like to obtain a final accelerated emittance at the same level as comes out of the gun. Cannot do that exactly, but quantifying various sources of emittance growth in the linacs can provide a path to optimize the end use emittance.

I, personally, like the units keV degrees for my emittance reporting because bunch durations are easily measured in RF degrees and energy spreads are typically 10s of keVs. To convert

$$\varepsilon_z[\text{keV degrees}] = \varepsilon_z[\text{m}]\frac{511\,\text{keV}\cdot 360\text{ degrees}}{\lambda_{RF}[\text{m}]}$$

Homework

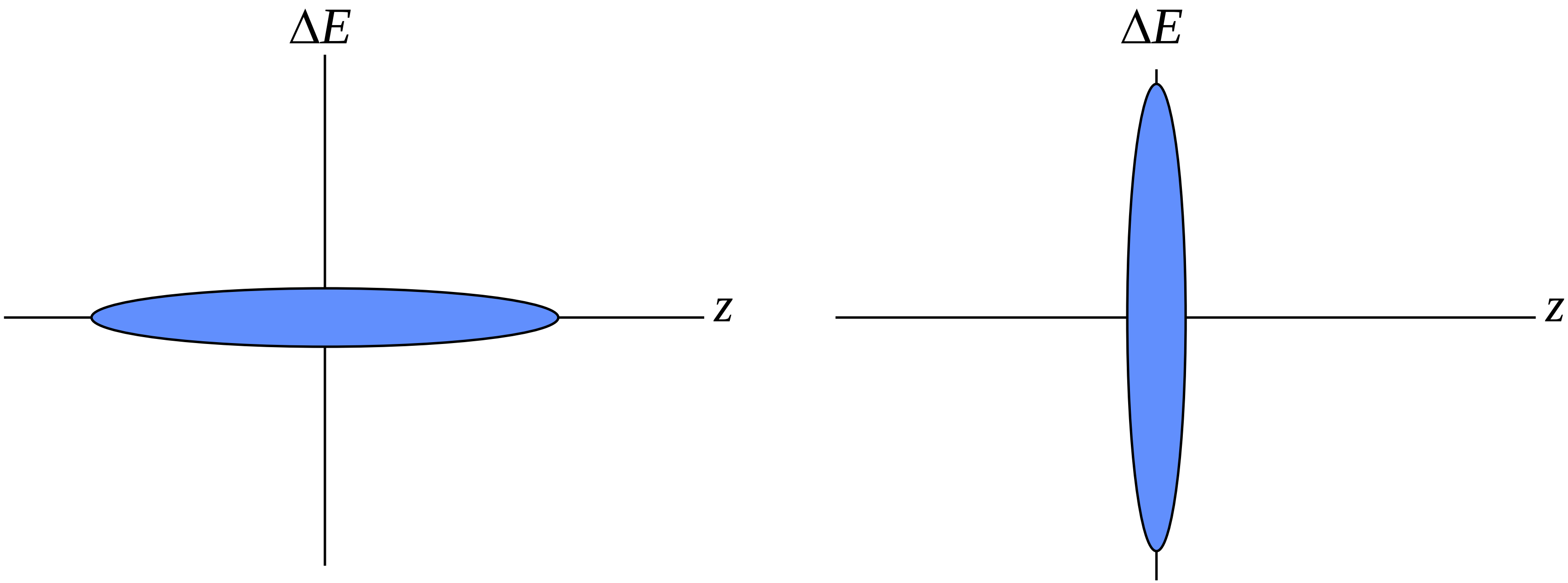
Suppose for a moment that one could create a distribution with no intrinsic spread but which had a parabolic distortion in the phase space. Compute the longitudinal emittance as a function of the parabolic distortion. Does your result approach the proper limit as Δz_{min} goes to zero?



$$f(z,\Delta E) = A\delta\left(z + z_{min}(\Delta E/\Delta E_{max})^2\right)\left[\Theta(\Delta E + \Delta E_{max}) - \Theta(\Delta E - \Delta E_{max})\right]$$

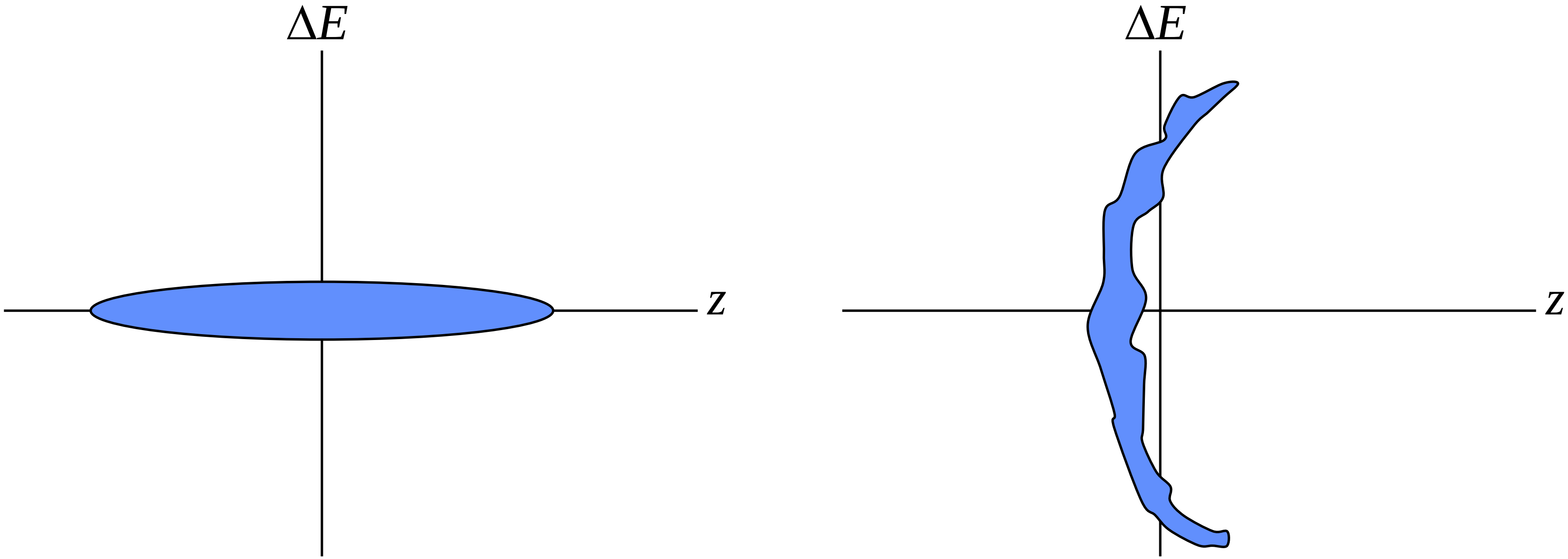
Bunching

Fundamental problem: Take the first phase space distribution to the second one



What Tends to Happen!

Usually, it doesn't work out the first time because of distortions



Bunching Elements

RF Bunchers

$\phi_s \approx 90^\circ$ at the bunching zero crossing

Relatively low gradient
Usually done at low energy
Not too much curvature distortion introduced

Other Linac Cavities

$\phi_s \approx 1-10^\circ$ offset in the bunching direction

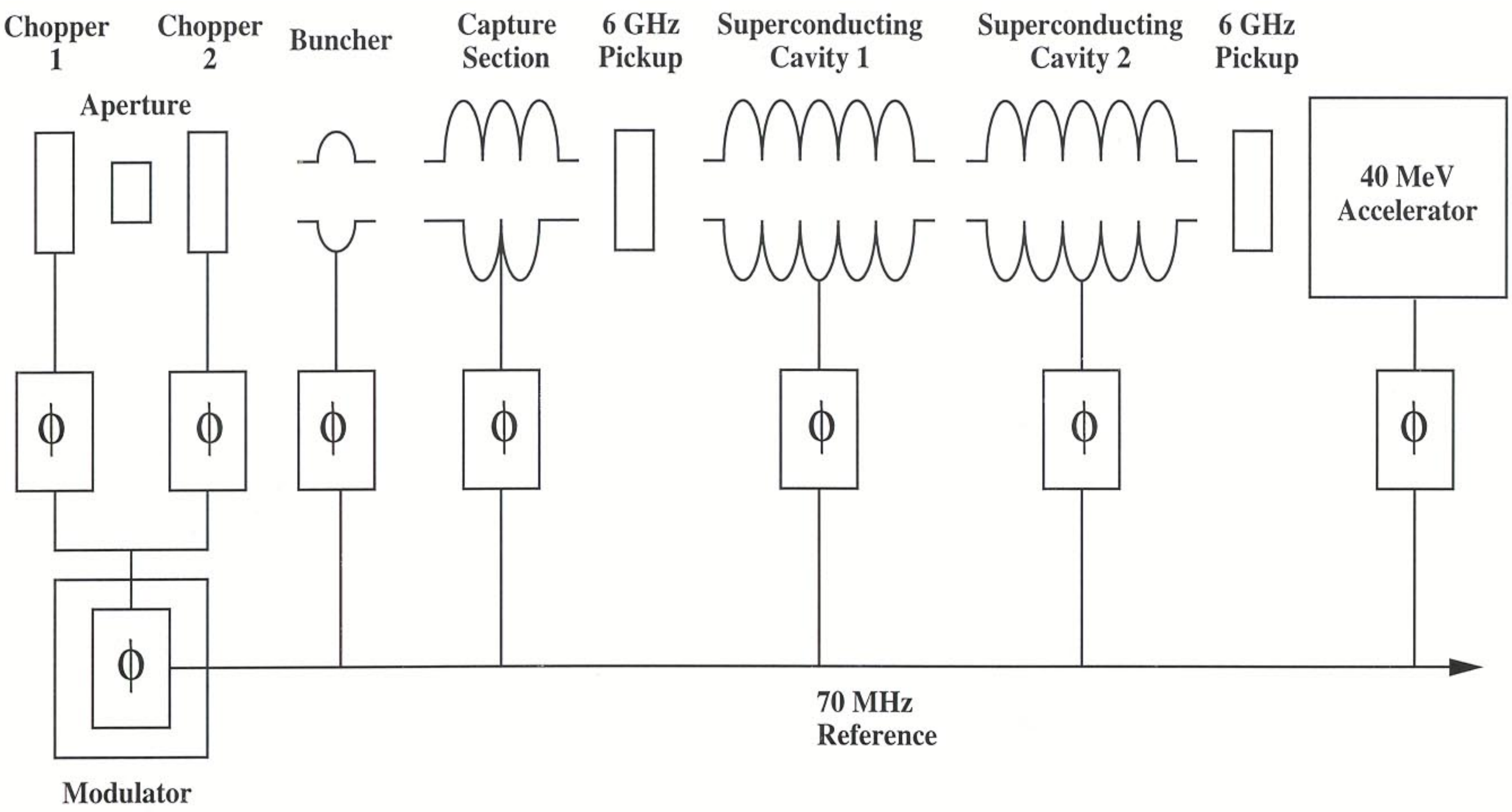
Relatively high gradient
Can be done at high energy
Lots of curvature distortion introduced because at peak of cos

Bend regions can generate M_{56} as discussed previously

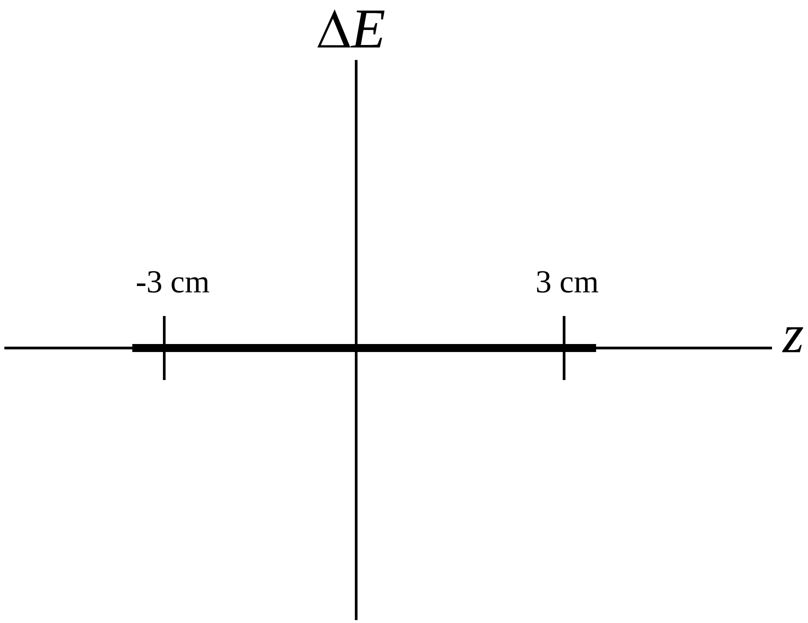
Some design rules

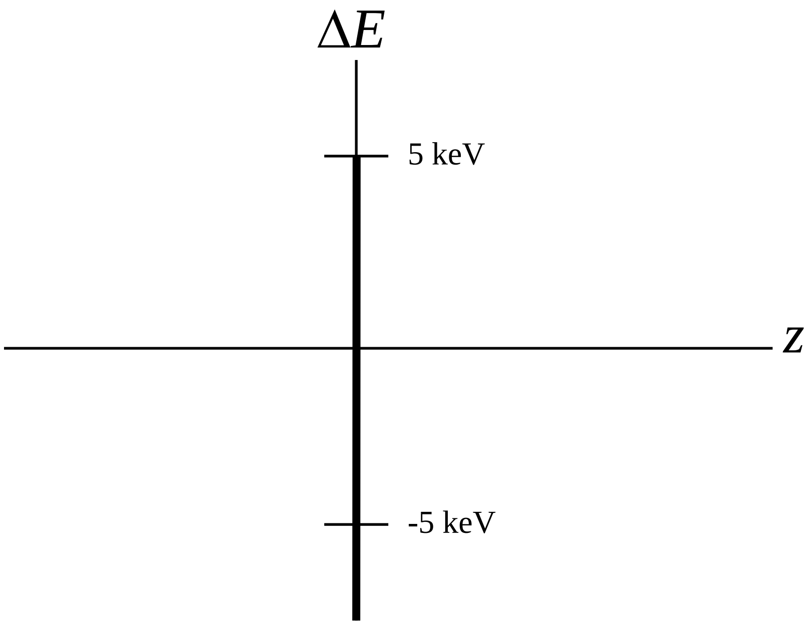
- Avoid overbunching, especially at non-relativistic energies
- When possible, reduce any phase space non-linearities with available correction elements.
- When possible, choose setups that are easy to set up and rapidly diagnose
  - Favorite positions are zero crossings and crests of RF elements
- Try to reduce sensitivities to RF drifts by making beta functions small at locations where RF cavities do substantial acceleration. This saves on a lot of operational setup and troubleshooting grief.
- Design bunching program first, and then the transverse optics because generally the transverse optics depend quite a bit on the RF cavity settings and phases, whereas the longitudinal dynamics is by contrast insensitive to the details of the transverse settings

Schematic of CEBAF Injector

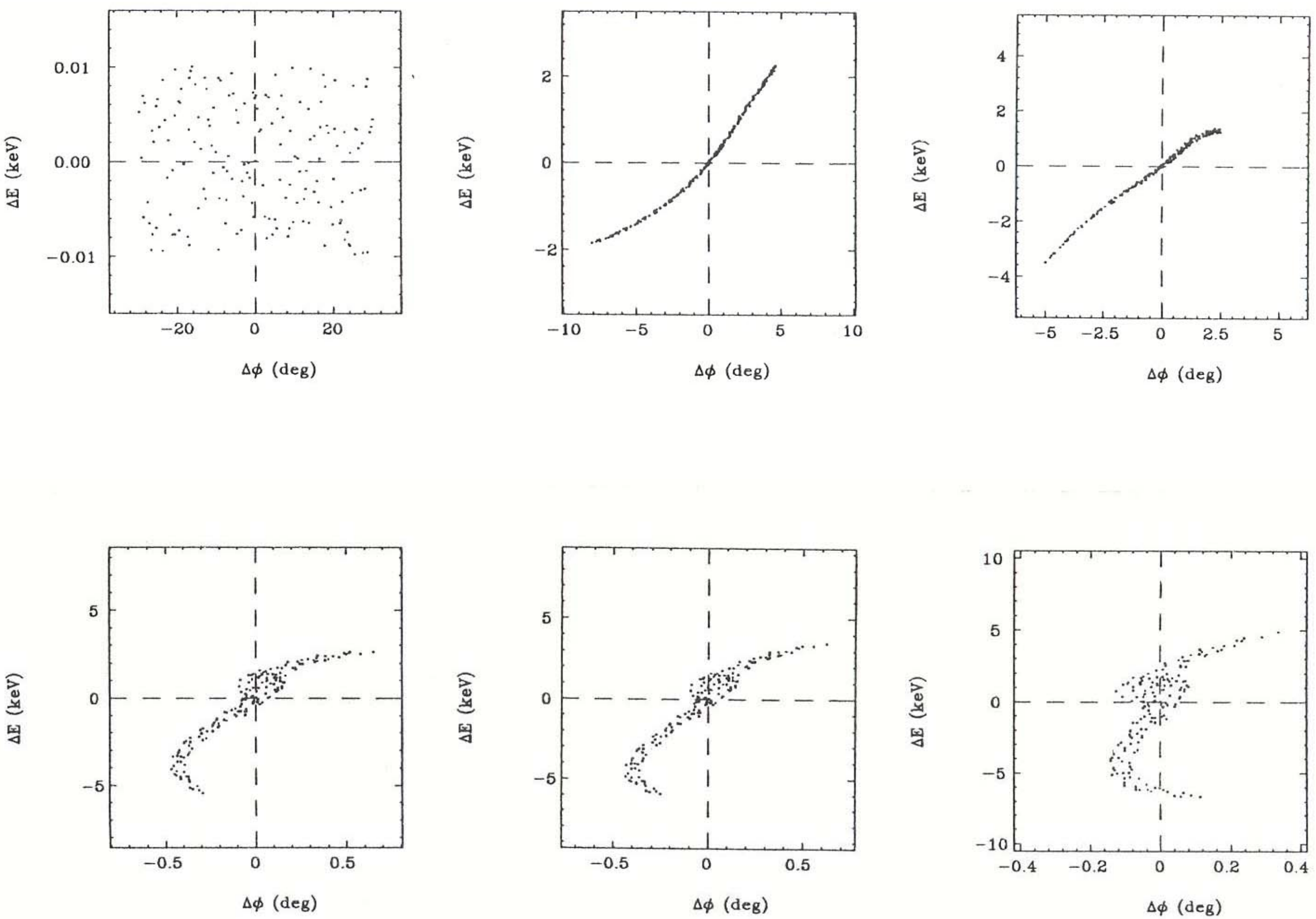


Phase Space from CEBAF Bunching





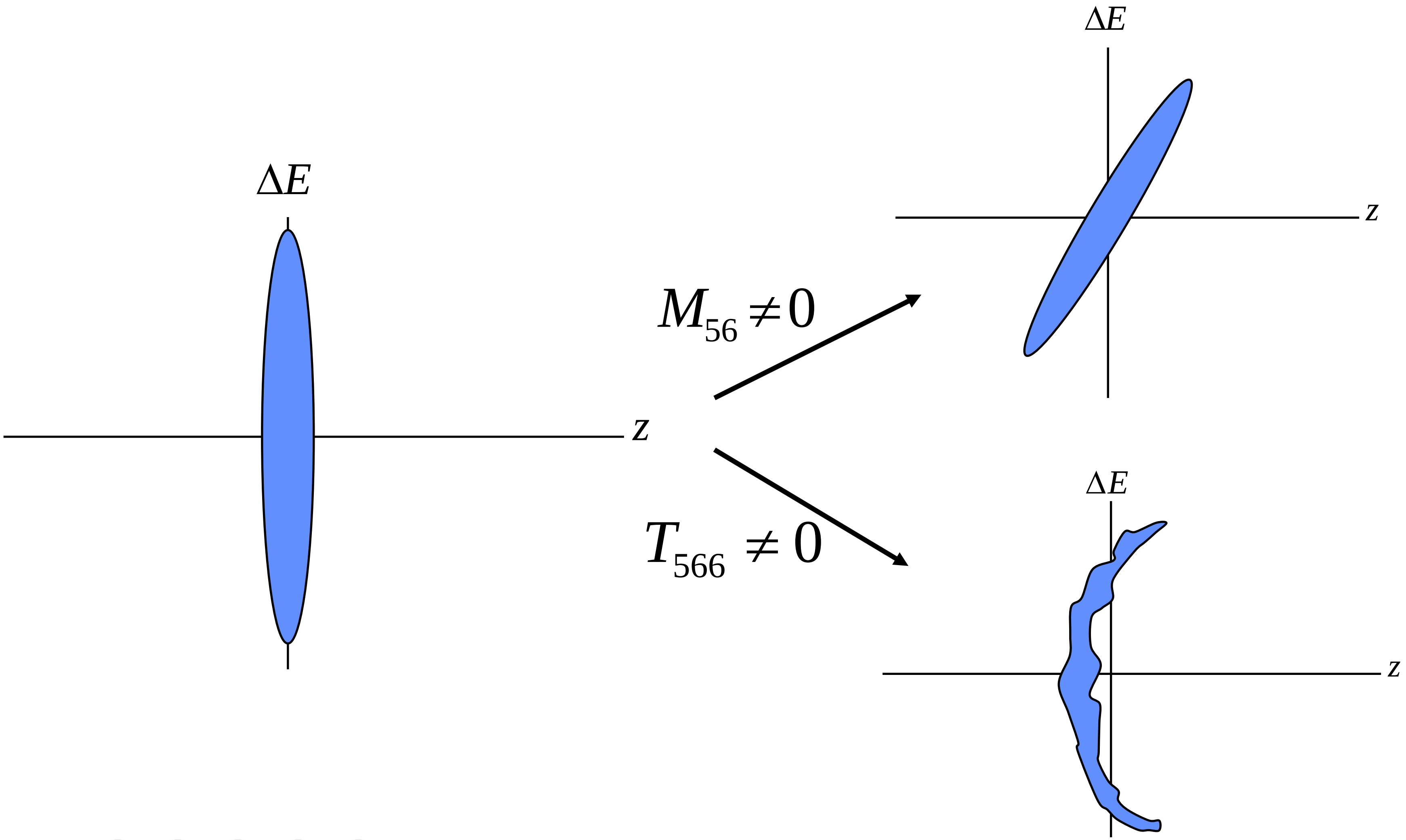
Calculated Longitudinal Phase Space



Injector Phasing Procedure

Quantity		Setup Measurement
Buncher Phase	-19 deg from zero	Spectrometer to find 0
Buncher Gradient	40 kV/m	Phase Transfer
Capture Section Phase	+16.5 deg from crest	Max. 500 keV energy
Capture Section Gradient	1.3 MV/m	Energy @ 500 keV
Unit Phases	-7.5 deg crested	Max. 5 MeV energy
40 MeV Accelerator Phases	crested	Max. 45 MeV energy

Effects of Nonlinearities

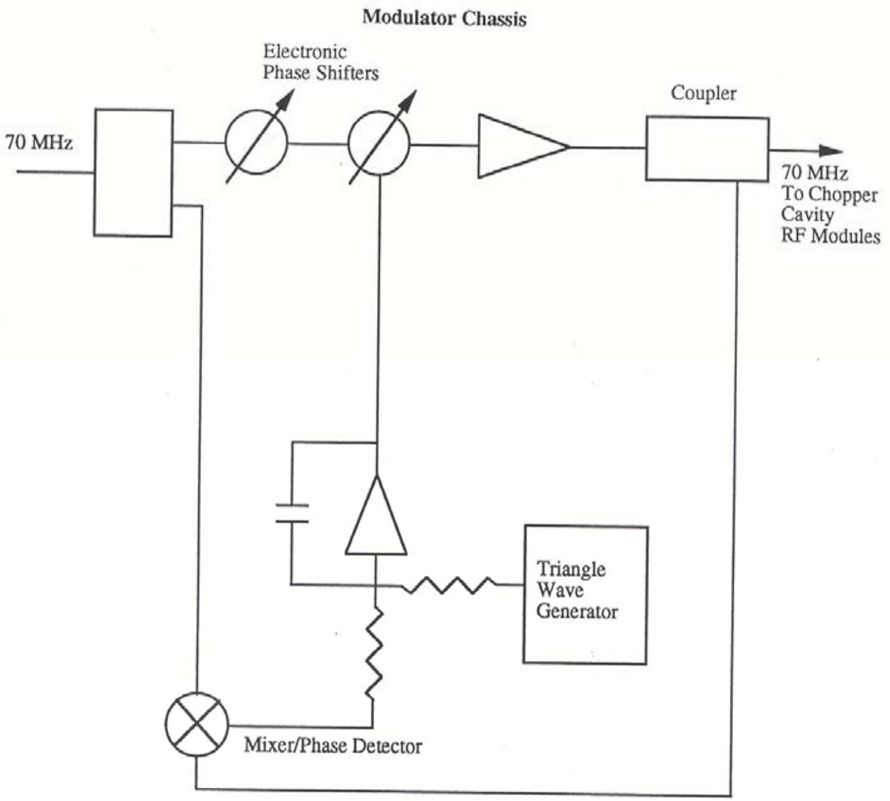


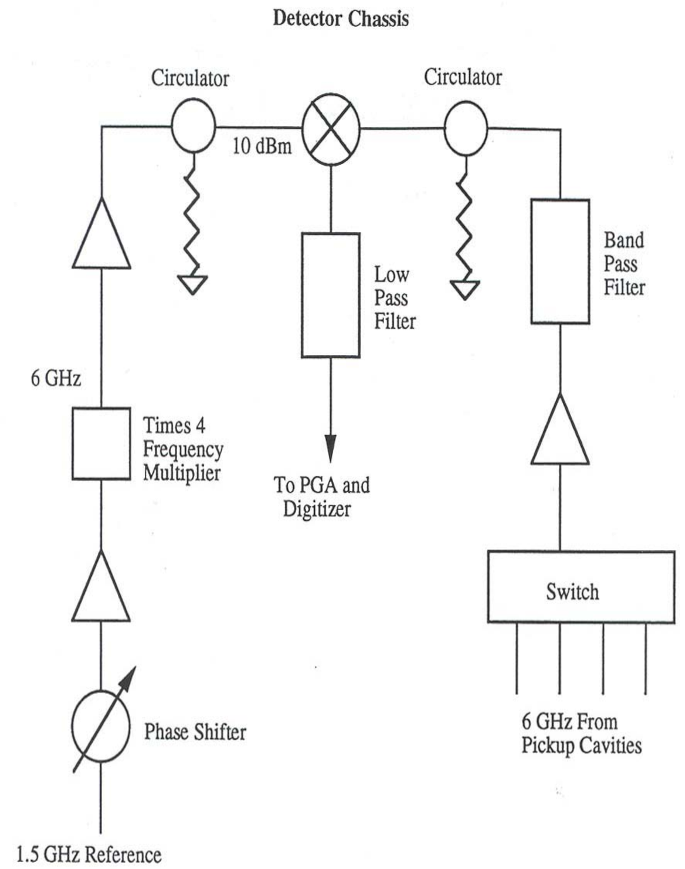
Homework

Utilizing the non-relativistic velocity-energy relation, compute the T_{566} for a drift space of length L as a function of γ

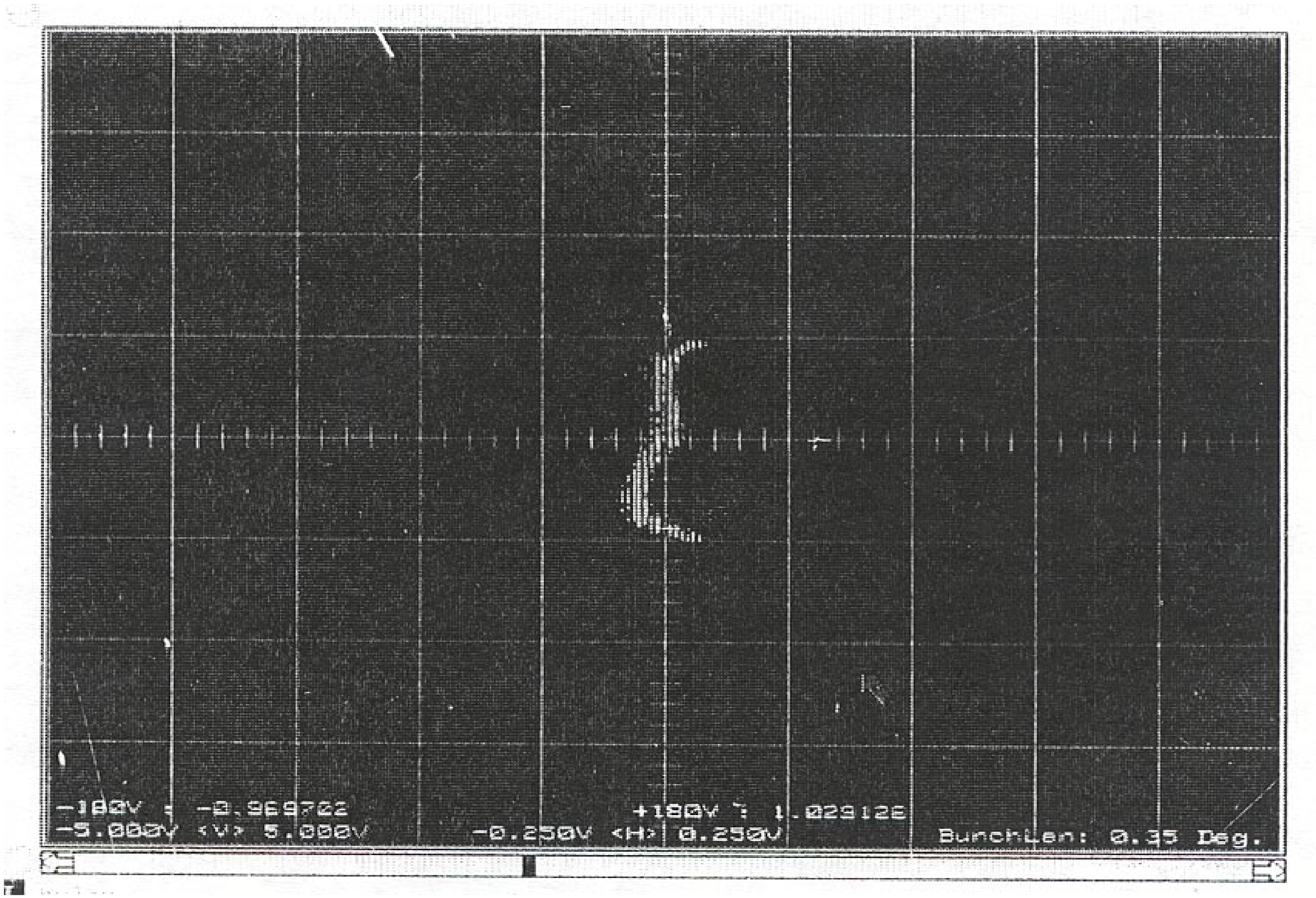
Phase Transfer Technique

Simultaneously, digitize phase modulation and arrival time determined by a phase detector

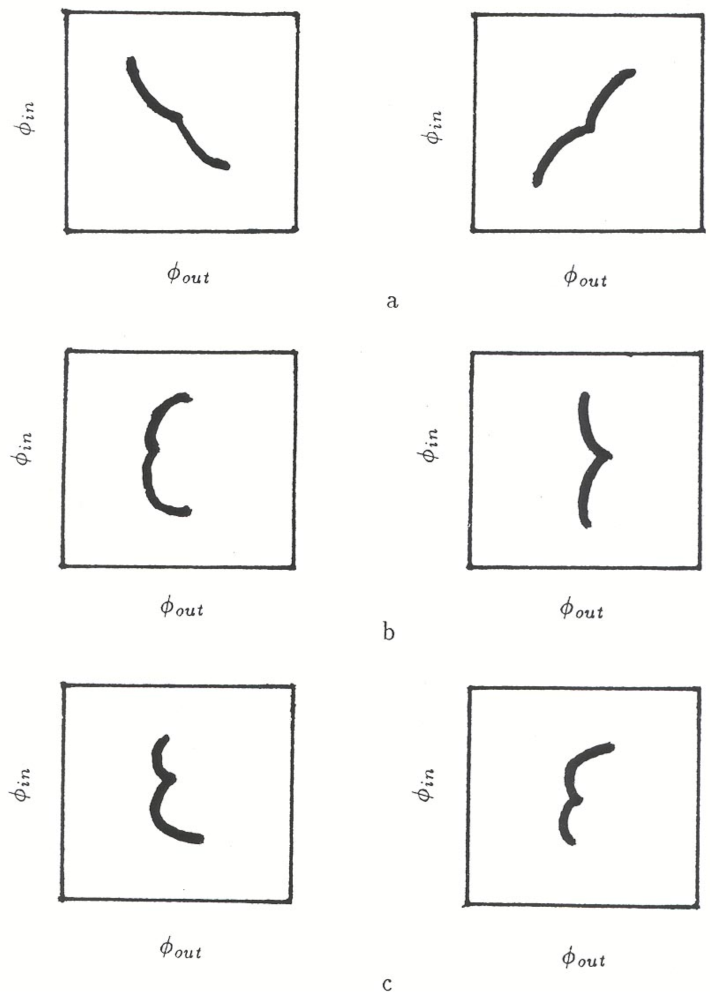




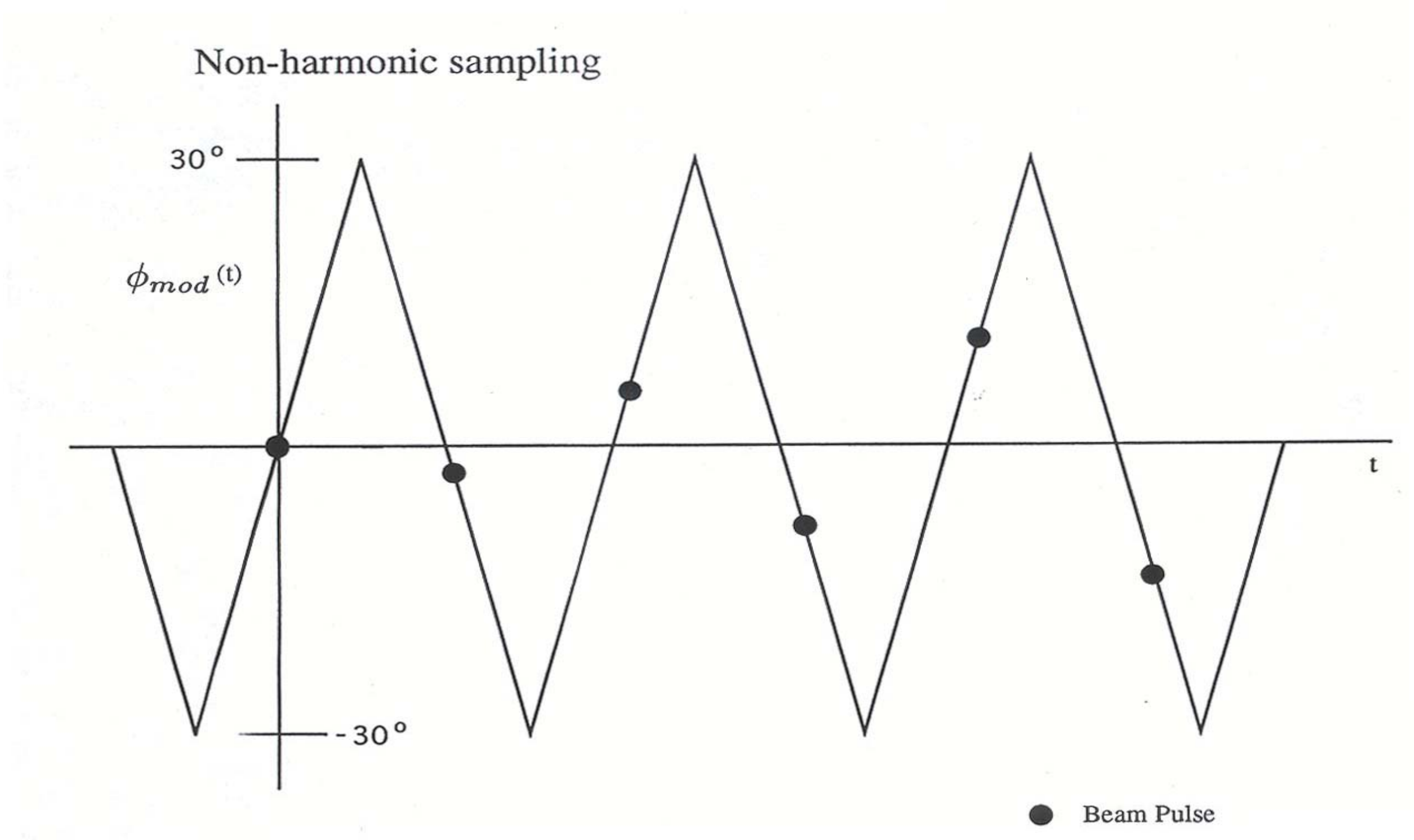
Some Early Results



Phase Space Correction Scheme

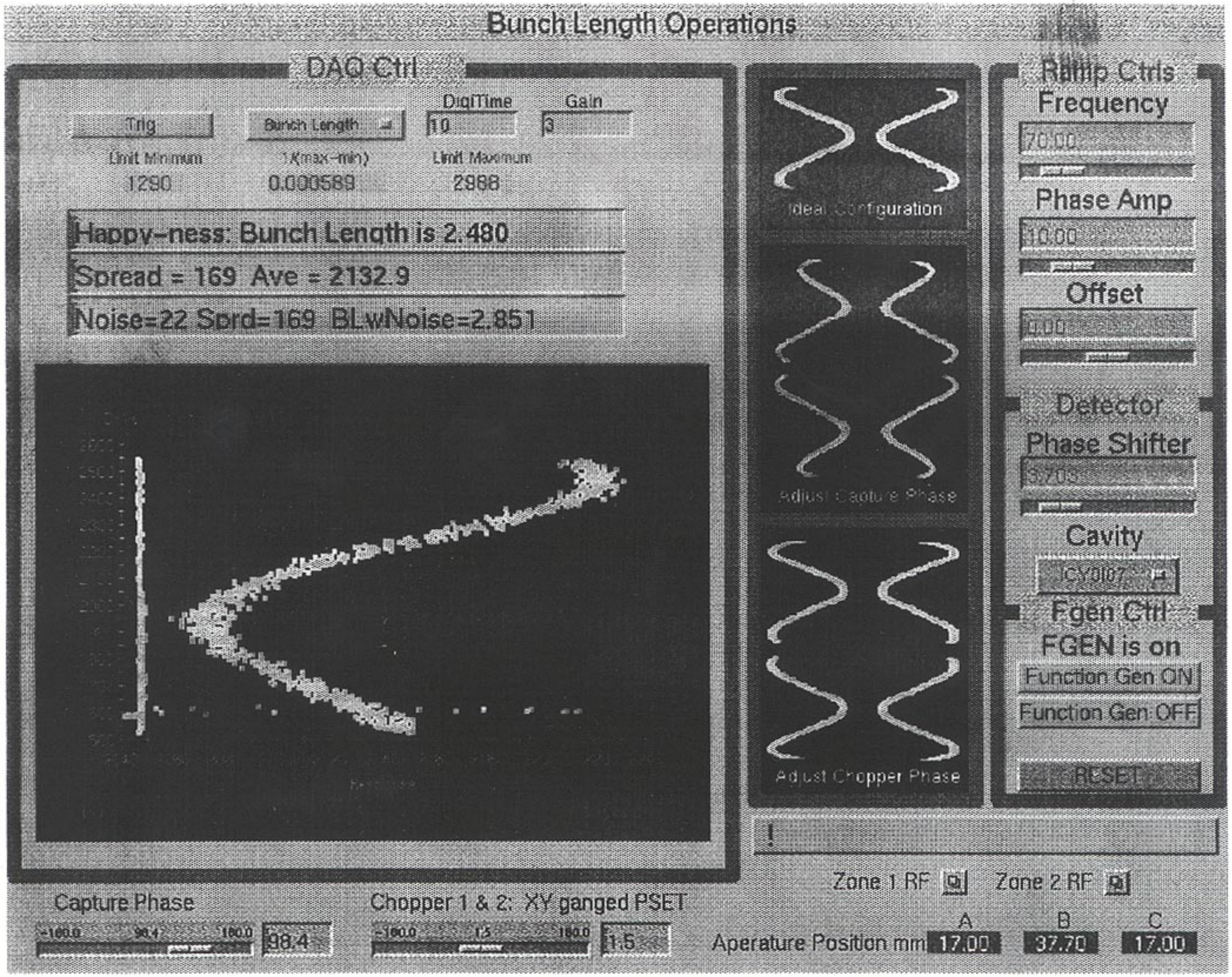


Triangle Wave Modulation



Simultaneous detection and digitization of the arrival phase from the mixer output

Phase Transfer Function, more recently



Tschebyshev Analysis of Nonlinear Transfer Maps

- Concentrate on problem: how can one easily aquire and intelligently analyze and organize, information about the optics of (i.e., the transport maps of) the accelerator including the nonlinearities?
- Basic philosophy: perturb the beam around the operating point, varying one or more variables in a systematic way.
- Triangle wave modulation as in the phase transfer device is very good for producing pictures to compare to the phase space plots, but the Fourier transform of a triangle wave has harmonics of the fundamental frequency mixed in. For example, a tilt in the phase space distribution would produce signals at odd harmonics of the fundamental in the mixer output.
- Question: is there a better way? Yes
- Question: Is there a function set and modulation pattern, such that the function set is cleanly distinguished by Fourier analysis of the modulation pattern applied to the function set? Yes, Tschebyshev polynomials and sinusoidal modulation do the trick!

Definition of Tschebyshev Polynomials

Defining relation

$$T_n(\cos\theta) = \cos(n\theta)$$

$n=1$ $$T_1(x) = x$$

$n=2$ $$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

$$T_2(x) = 2x^2 - 1$$

$n=3$ $$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

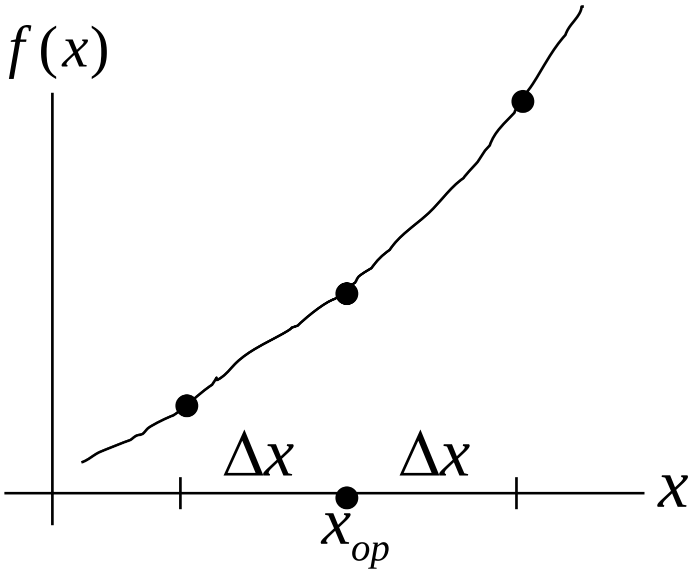
$$T_3(x) = 4x^3 - 3x$$

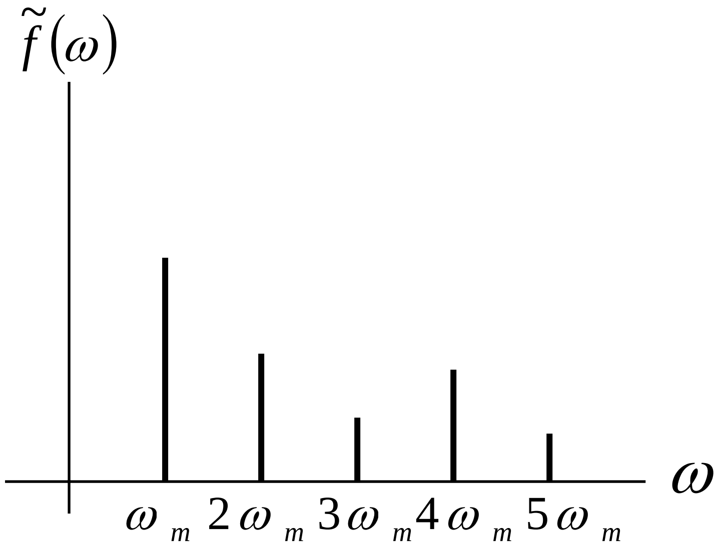
$n=4$ $$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta - 1$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

$$\vdots$$

General Measurement





$$x = x_{op} + \Delta x \cos(\omega_m t)$$

Orthogonality and Tschebyshev Expansions

A general continuous function, $f(x)$, defined on the domain [-1,1] may be expanded in a uniformly convergent series

$$f(x) = \sum_{n=0}^{\infty} a_n T_n(x)$$

The expansion coefficients may be obtained by the overlap integral

$$a_n = \int_{-1}^{1} \frac{f(x) T_n(x)}{\left(1 - x^2\right)^{1/2}} dx$$

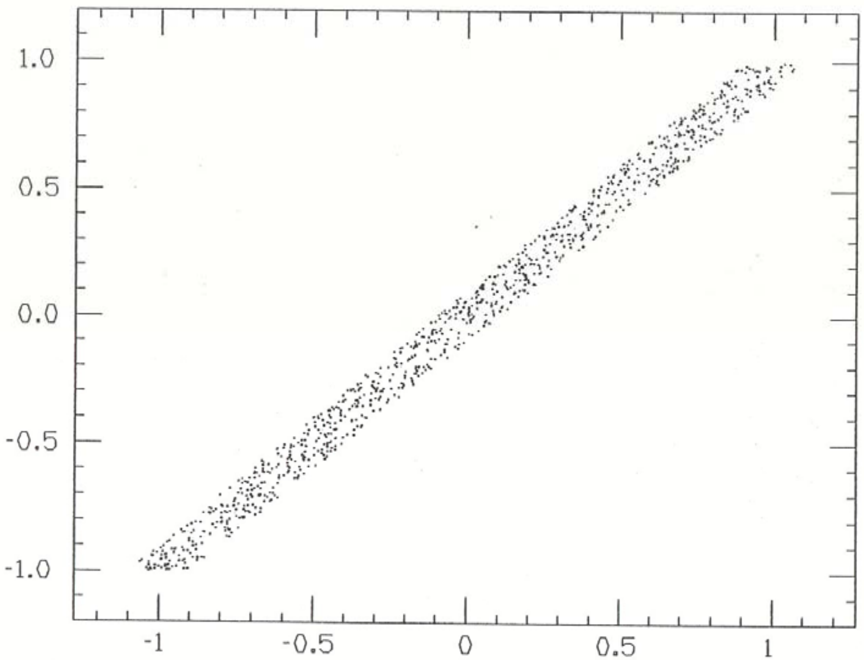
Alternatively, and this is the main idea of the analysis, the expansion coefficients may be obtained by performing a very simple measurement, namely, modulate the input with a sinusoidal oscillation throughout [-1,1] and Fourier tranform the resulting data.

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\cos\omega t)\cos(n\omega t)\,d(\omega t)$$

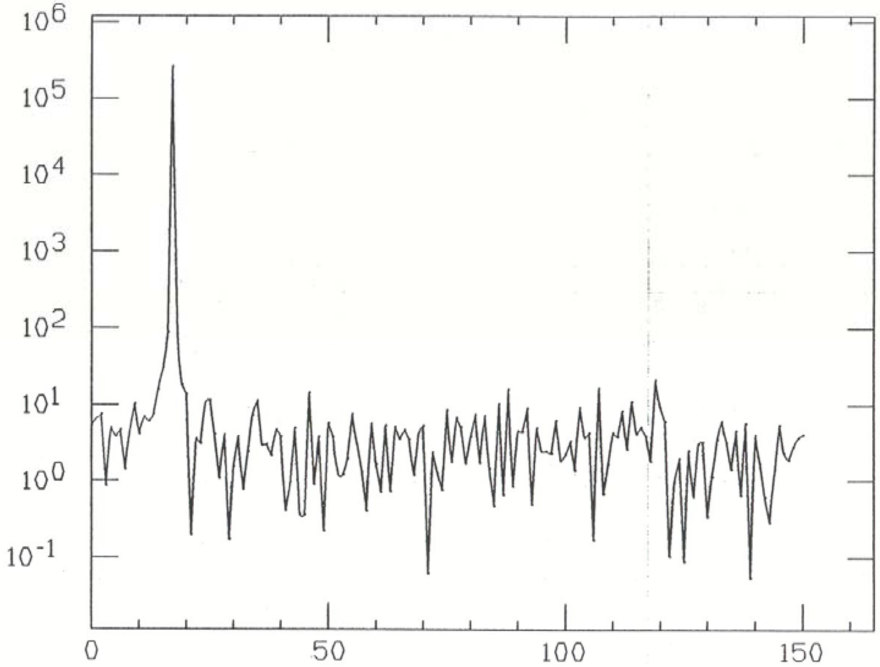
The amplitudes of the expansion coefficients appear directly as the size of the peaks in the Fourier analysis of the output data, because

$$T_n(\cos\theta) = \cos(n\theta)$$

$n = 1$

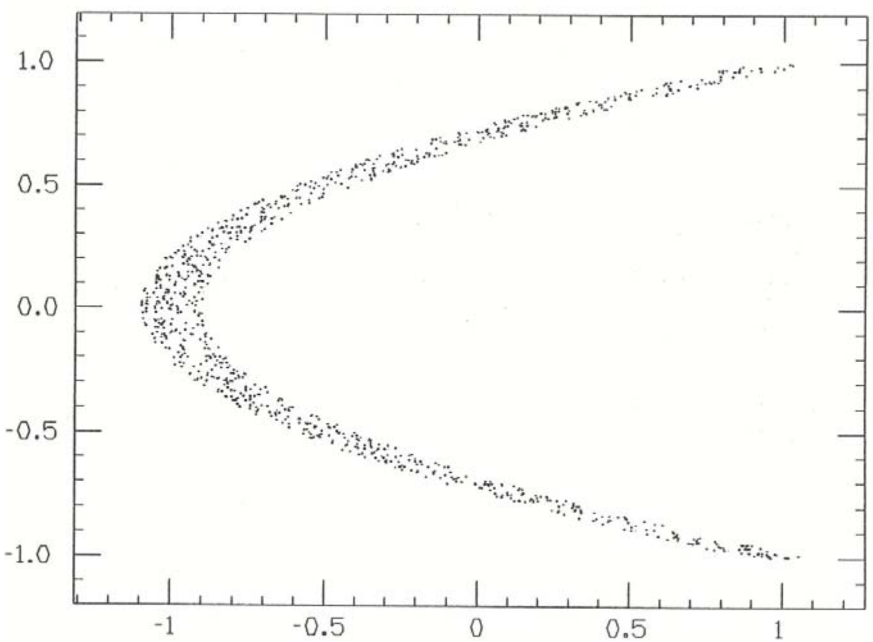


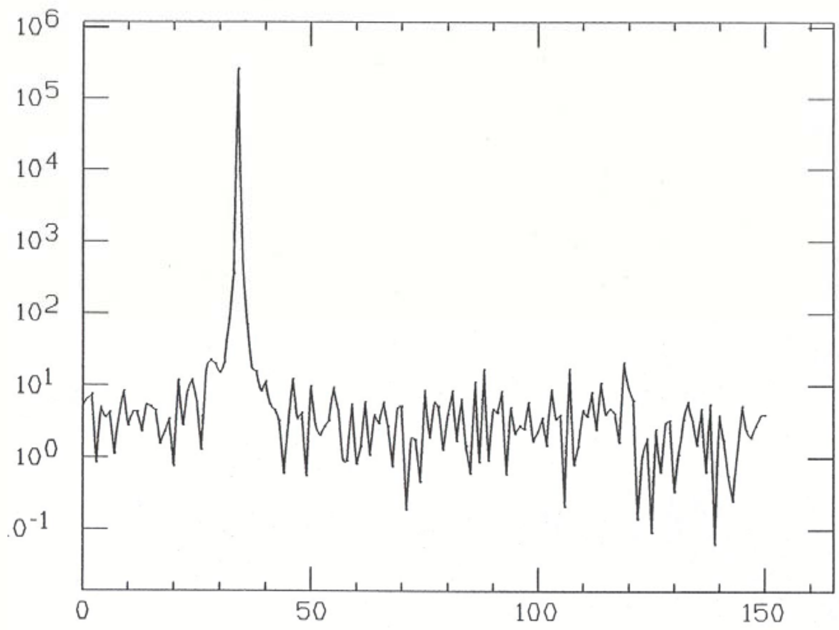
1.0
0.5
0.0
-0.5
-1.0
-1
-0.5
0
0.5
1



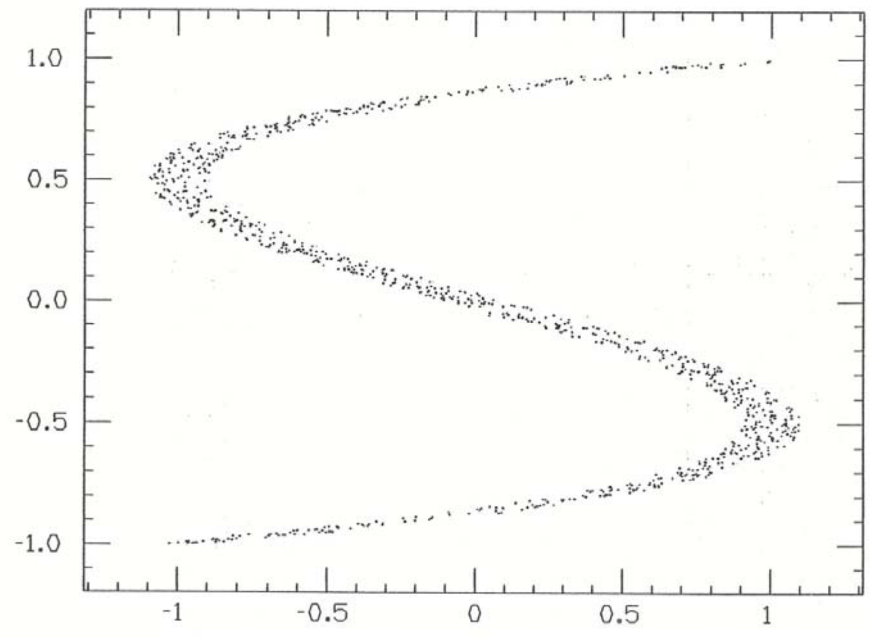
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
0
50
100
150

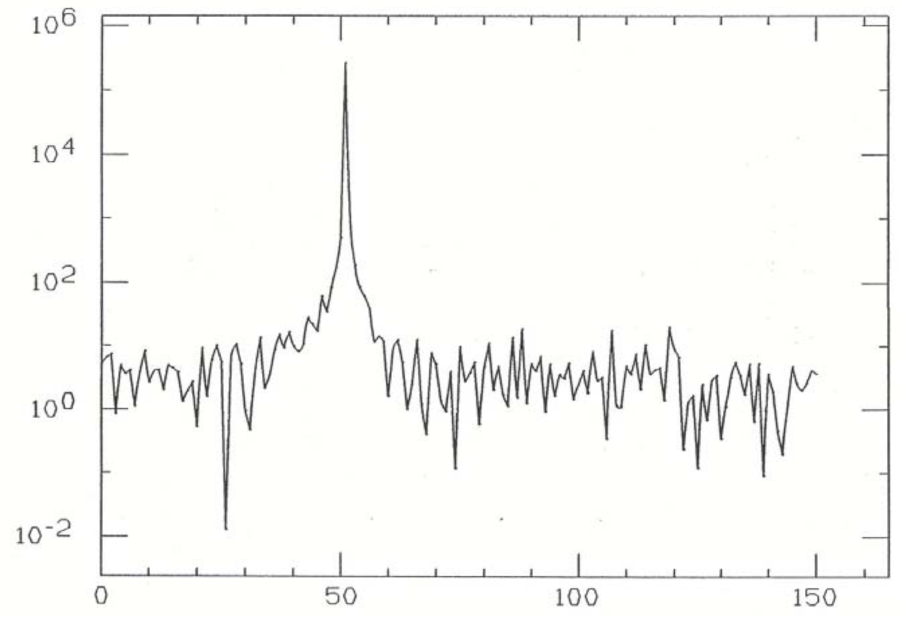
$n = 2$


1.0
0.5
0.0
-0.5
-1.0
-1
-0.5
0
0.5
1

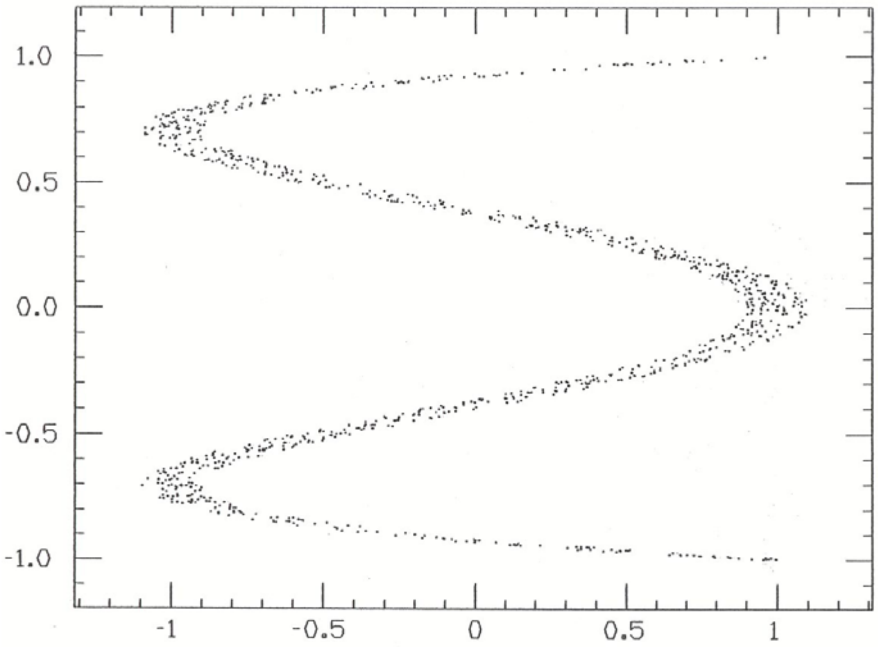

10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
0
50
100
150

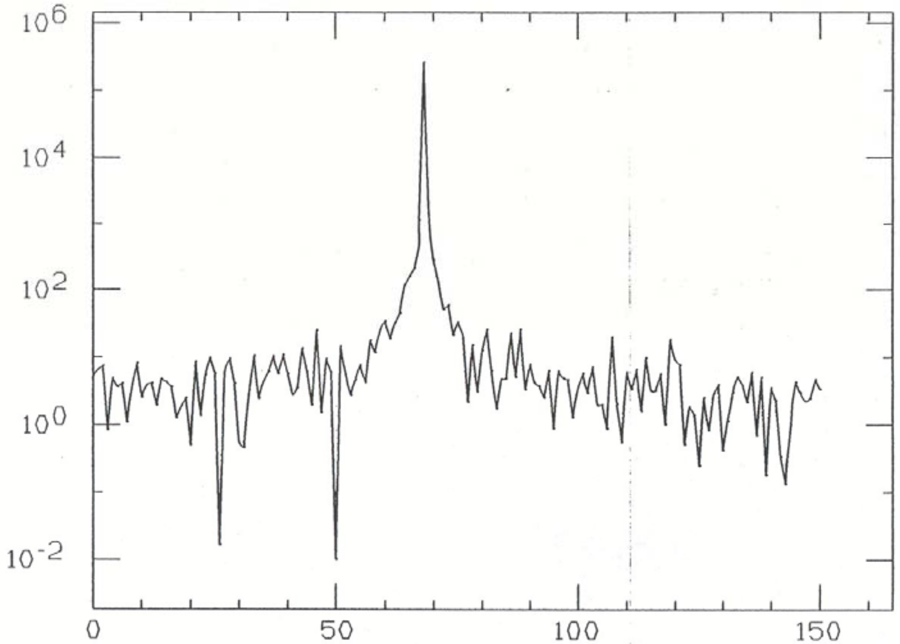
$n = 3$



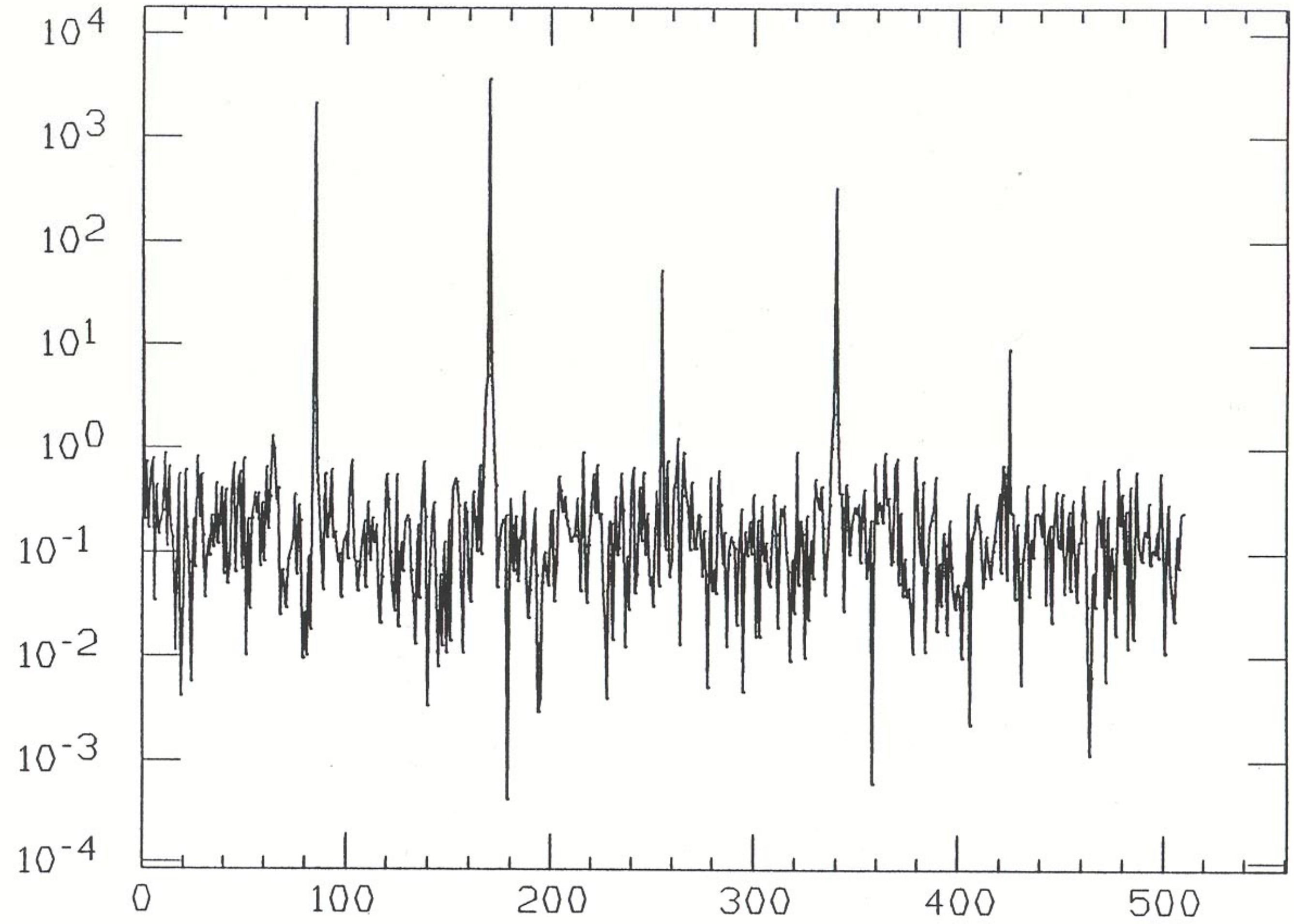


$n = 4$





Spectrum from sinusoidal phase modulation

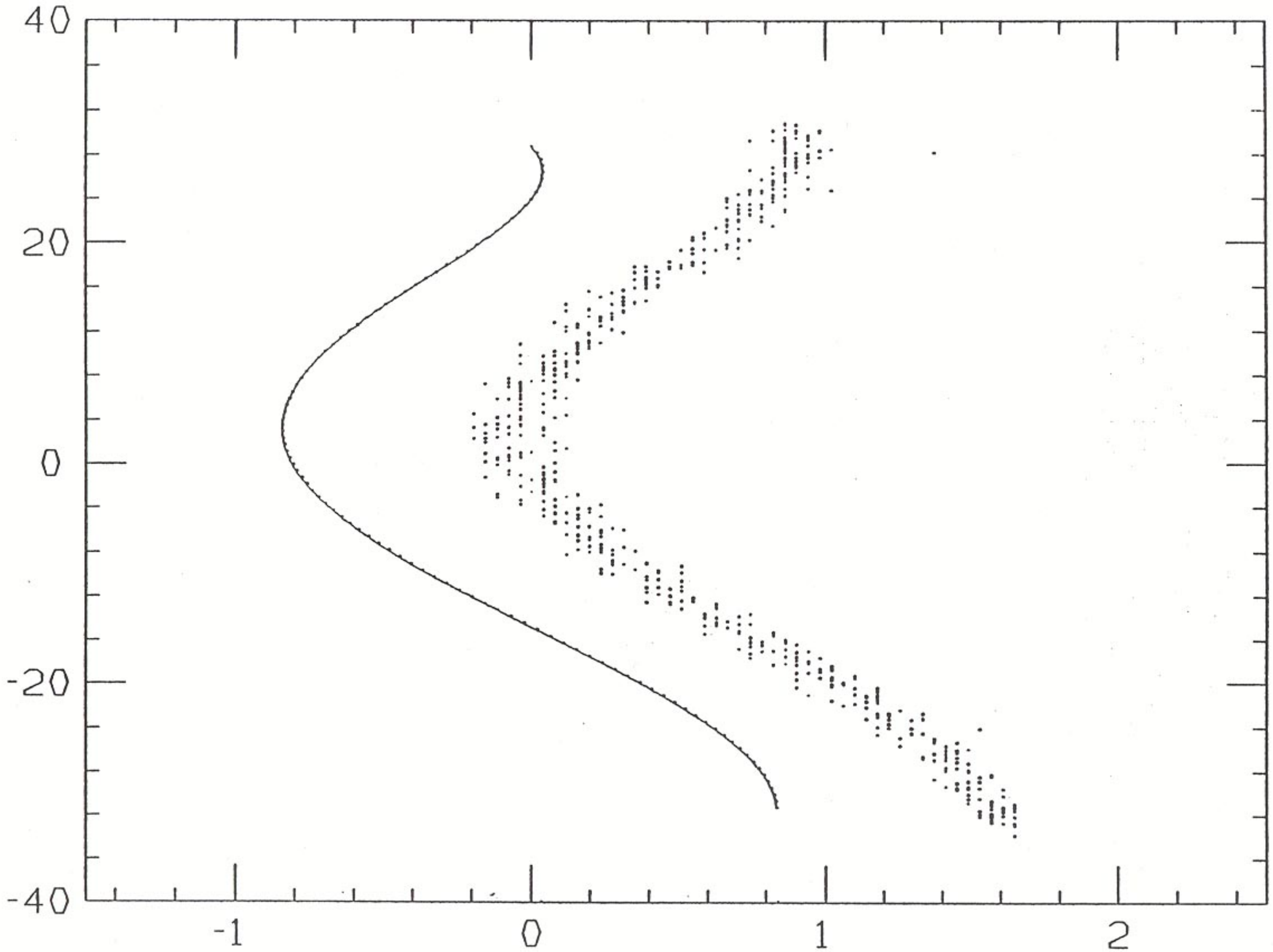


Analysis Results

n	$a_n(^\circ)$
1	-0.458
2	0.599
3	0.072
4	-0.182
5	-0.031

Expansion Coefficients for phase-phase correlation

Comparison Analysis Results and Original Data



Summary

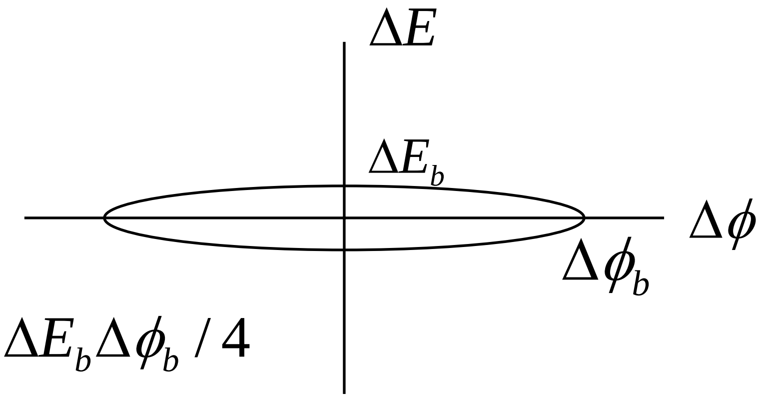
The notion of longitudinal emittance and longitudinal phase space have been introduced

We have given an indication how longitudinal phase space may be manipulated in order to achieve design goals

We have indicated some of the reasoning pertinent to the design of the CEBAF injector at Jefferson Lab

We have introduced a means of quantifying nonlinear distrotions in phase space and a means of quatifying the nonlinear transfer characteristics of nonlinear maps

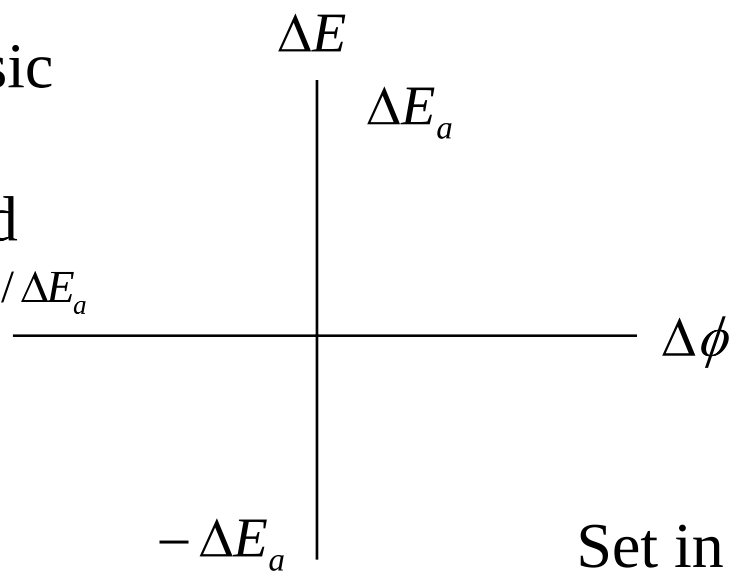
Energy Spread, Intrinsic



$$\varepsilon_L \approx \Delta E_b \Delta\phi_b / 4$$

Intrinsic
Phase
Spread

$$\sigma_\phi \approx 2\varepsilon_L / \Delta E_a$$



Set in injector/initial bunching

Longitudinal Optimization

Energy spread from perfectly phased linac

$$\frac{\sigma_E}{E} = \sqrt{\sigma_{E,inj}^2 / E^2 + \sigma_\phi^4 / 2}$$

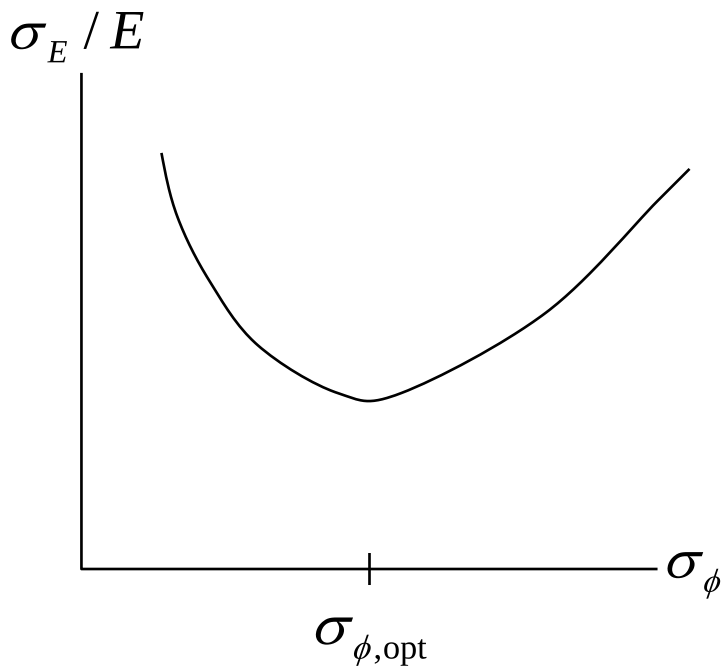
Using definition of longitudinal emittance, derive an optimum

$$\sigma_{\phi,opt} = \left(\frac{\varepsilon_L}{E} \right)^{1/3}$$

And minimum energy spread out the end

$$\frac{\sigma_E}{E} = \sqrt{\frac{3}{2}} \left(\frac{\varepsilon_L}{E} \right)^{2/3}$$

Longitudinal Optimization

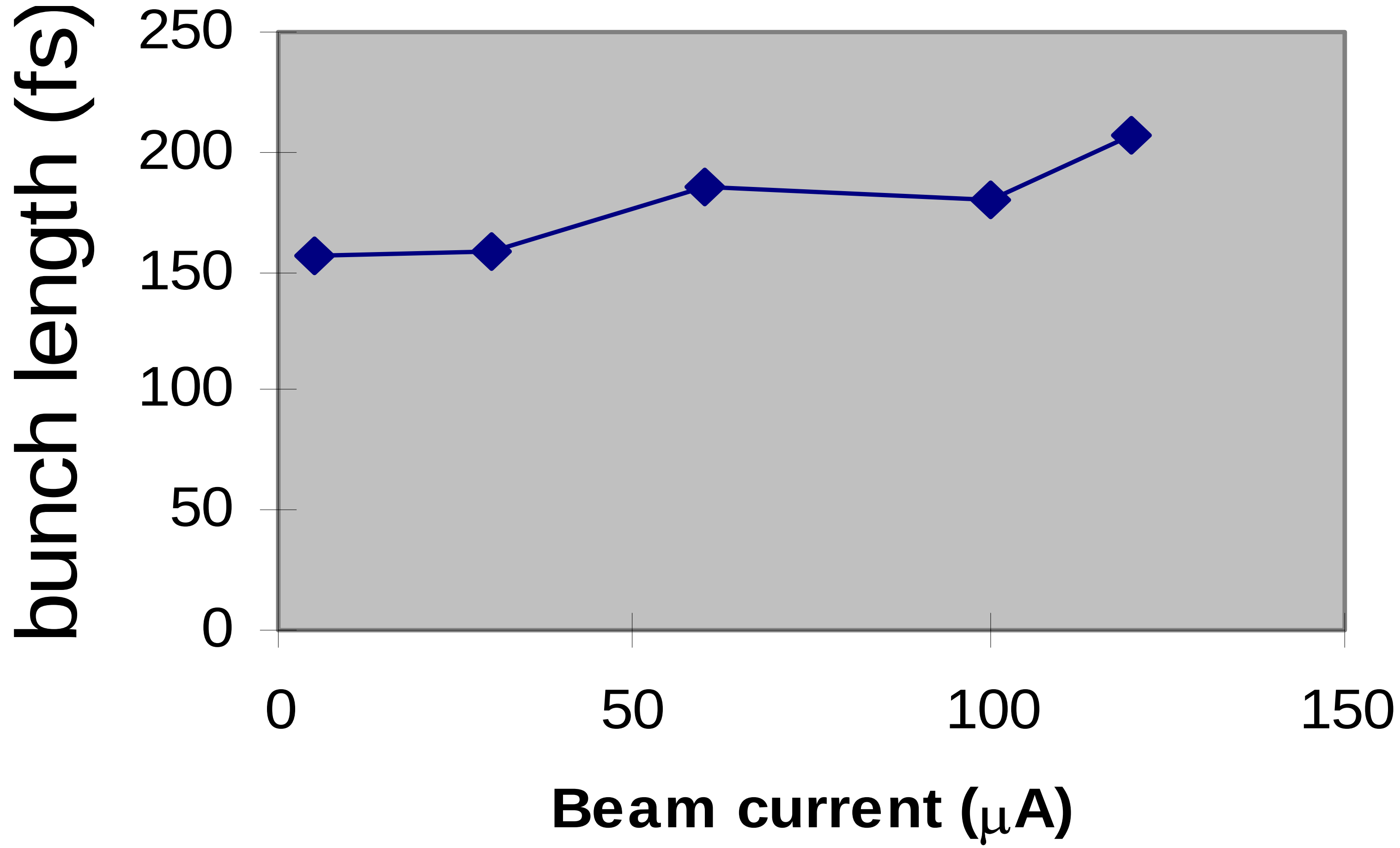


Measurements of longitudinal emittance

Injector	45 MeV	<6.7 keV deg
Nlinac exit	500 MeV	<12.5 keV deg

Optimal spread using larger number 2E-5 @ 1 GeV (5E-6 @ 4 GeV)

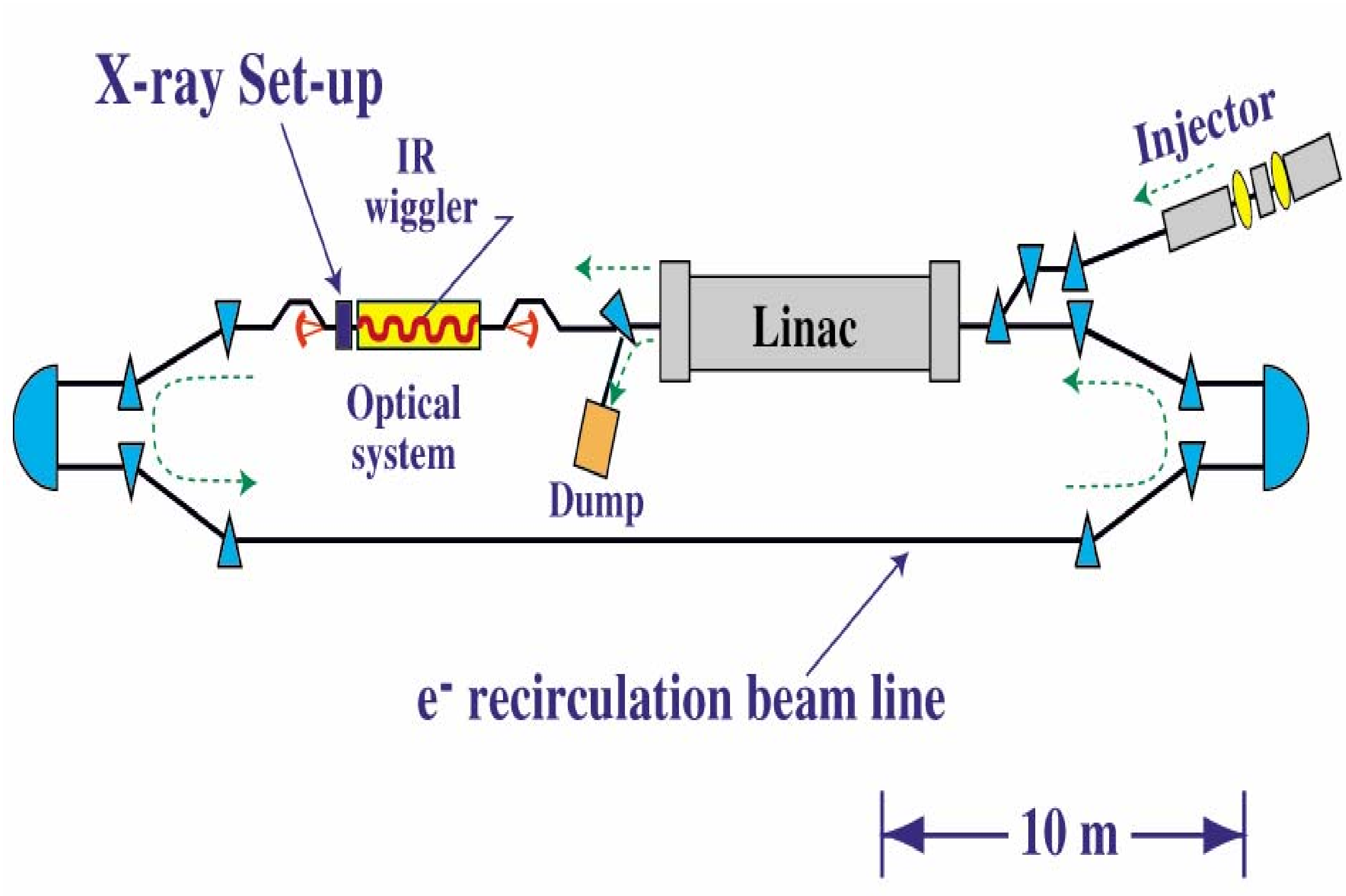
Short Bunch Configuration



High Charge Accelerator Design

- Must include effects of space charge (interparticle interactions!) in the design.
- These effects are calculated by self-consistently solving Poisson's equations, or more generally the full Maxwell equations, with the particle motion, which gives the charge densities and currents for the source terms of the self fields.
- Usually, some kind of "leap-frogging" is done in the calculations
- At Jefferson Lab we've been happy with PARMELA simulations, in contrast to people that exist in the nC regime.
- One advantage of the regime we operate in, around 100 pC, seems to have much less predictive trouble than higher charge-per-bunch. My inclination, (see our ERL talks in the future!), is to avoid going to higher bunch charges as much as possible because there seems to be very few clean results once heavy space-charge gets involved (perhaps this is just a statement about how complicated space-charge is, no one wants to take the time to do it right!)

Jefferson Lab FEL

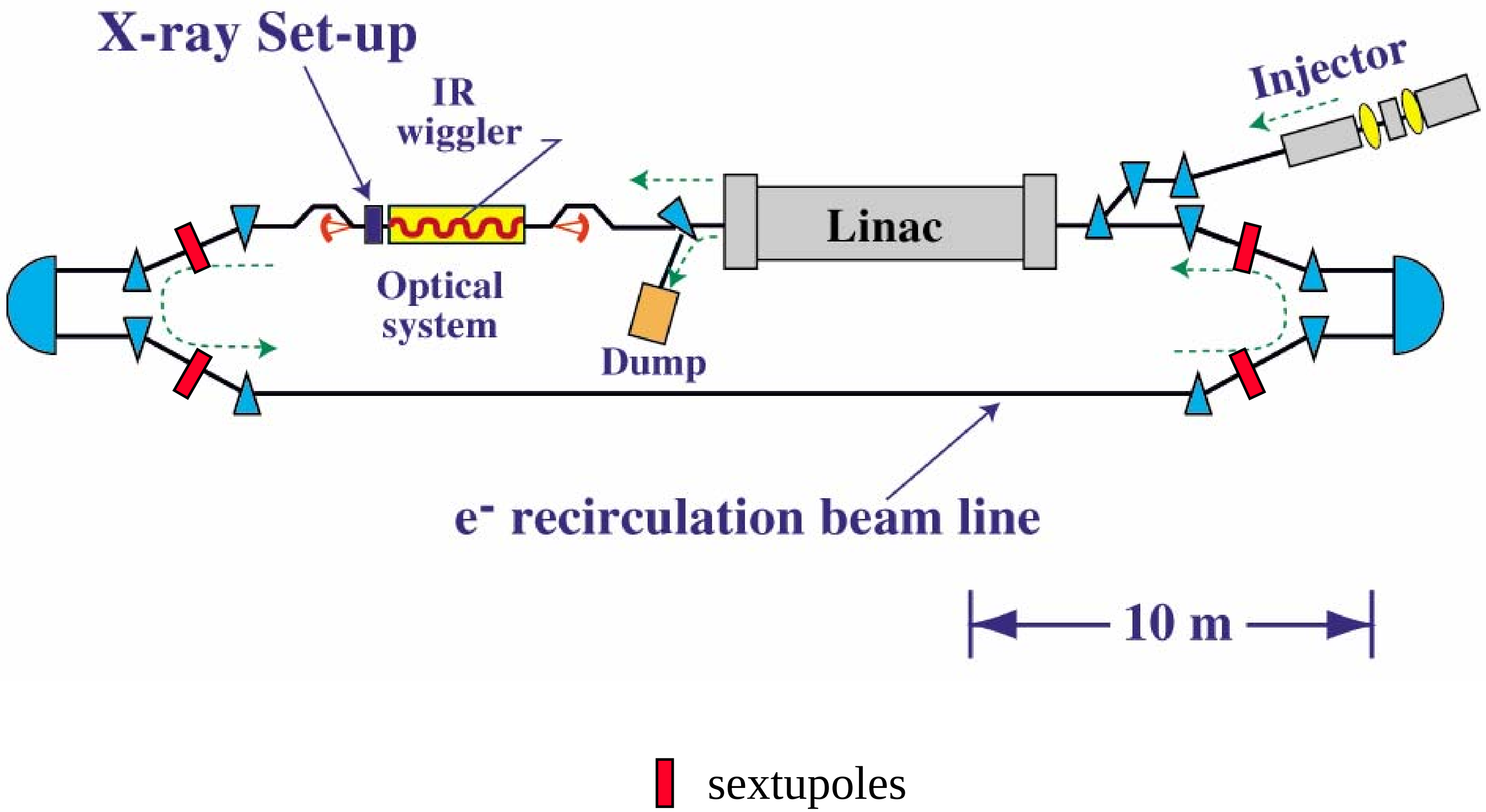


FEL Parameters

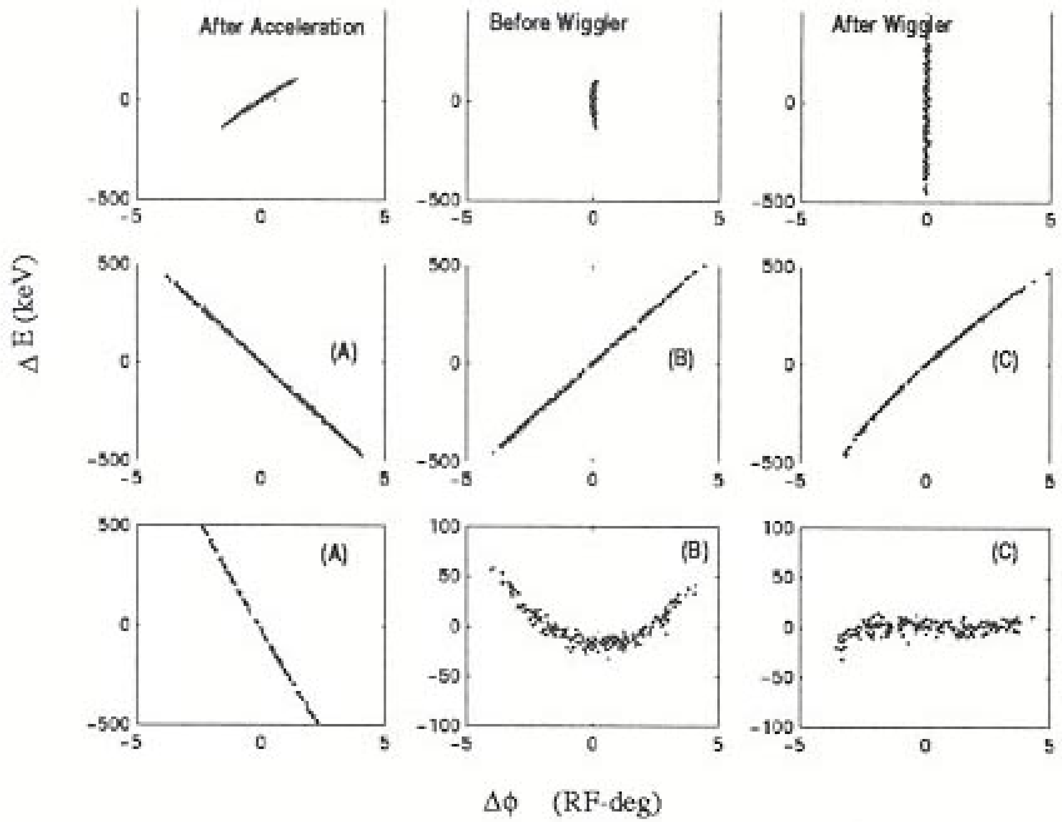
Parameter	Designed	Measured
Kinetic Energy	48 MeV	48.0 MeV
Average current	5 mA	4.8 mA
Bunch charge	60 pC	Up to 60 pC
Bunch length (rms)	<1 ps	0.4±0.1 ps
Peak current	22 A	Up to 60 A
Trans. Emittance (rms)	<8.7 mm-mr	7.5±1.5 mm-mr
Long. Emittance (rms)	33 keV-deg	26±7 keV-deg
Pulse repetition frequency (PRF)	18.7 MHz, x2	18.7 MHz, x0.25, x0.5, x2, and x4

Correction of Nonlinearities by Sextupoles

Basic Idea: Use sextupoles to get T_{566} in a bending arc to compensate any curvature induced terms.

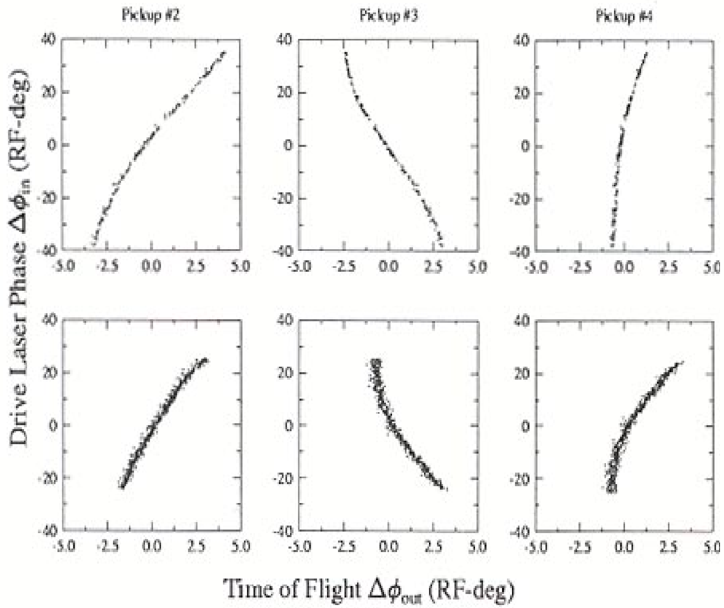


Longitudinal Phase Space Manipulations



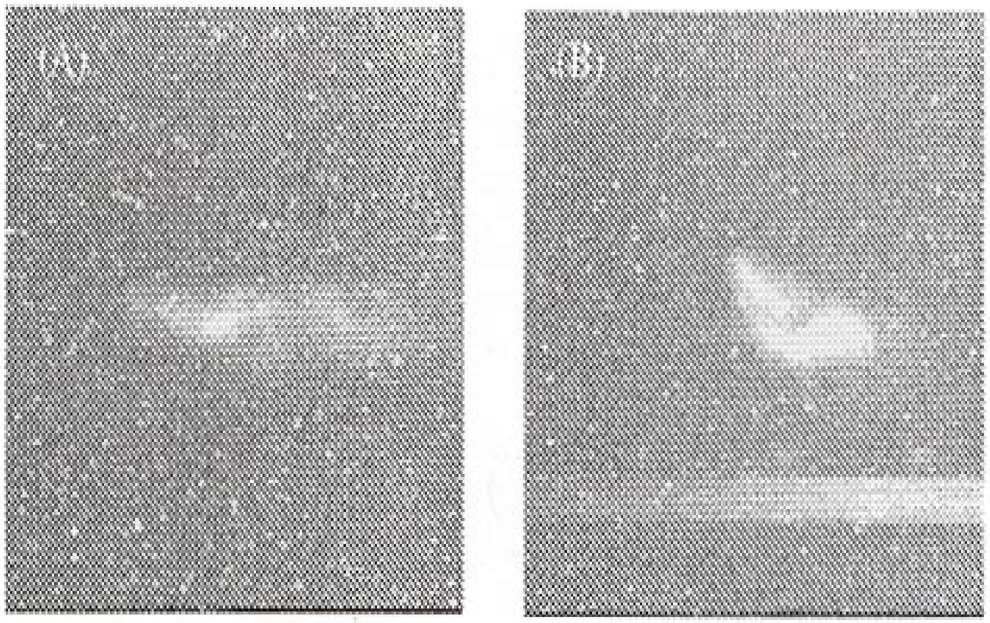
Simulation calculations of longitudinal dynamics of JLAB FEL

Transfer Function Measurements



Experiment		
# 2	0.1172	0.0008
# 3	-0.0801	0.0016
# 4	0.0911	0.0006
Simulation		
# 2	0.1070	0.0007
# 3	-0.0834	0.0003
# 4	0.0256	0.0004

Correction of Nonlinearities by Sextupoles



Basic Idea is to use sextupoles to get T_{566} in the bending arc to compensate any curvature induced terms.

Correction of Nonlinearities by "Linearizers"

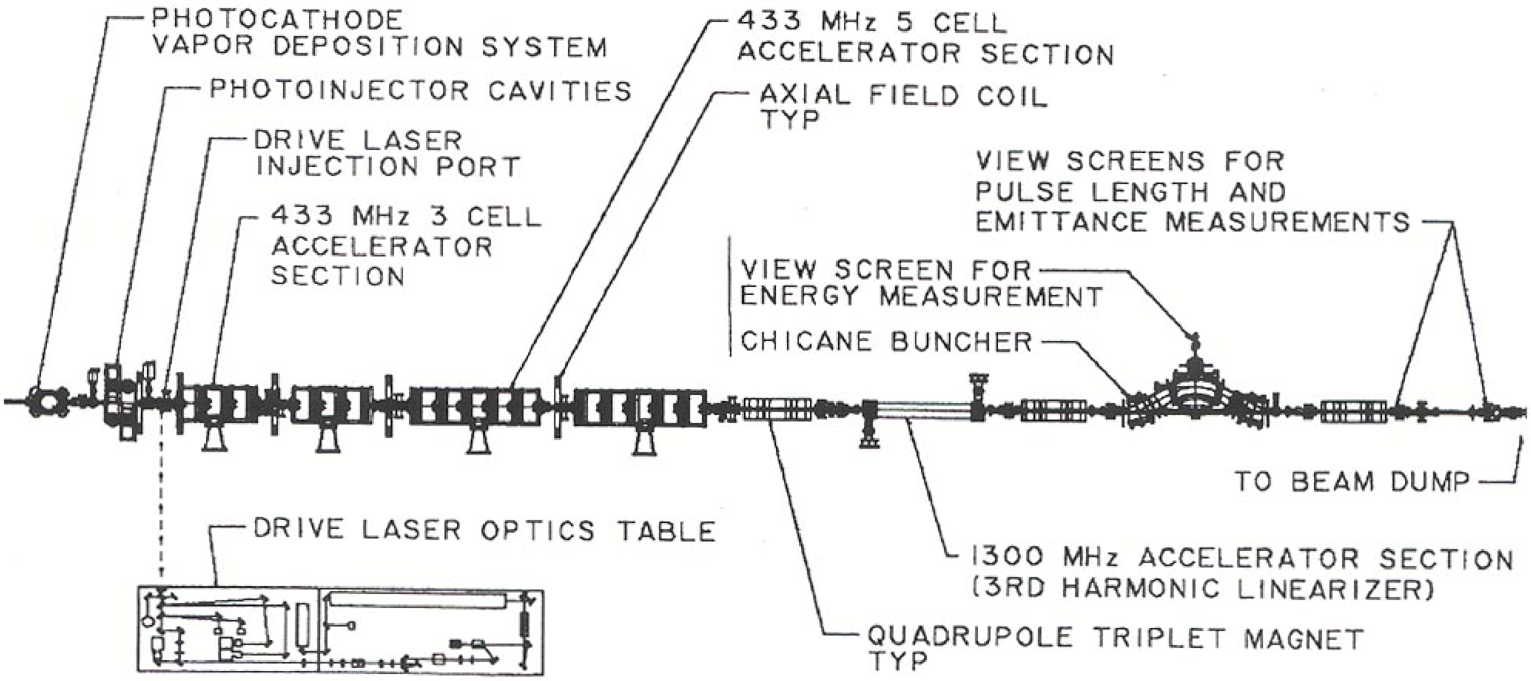
$$V_c = V_0 \cos\theta \approx V_0\left(1 - \frac{\theta^2}{2} + \cdots\right)$$

$$V_{lin} = \frac{V_0}{9}\cos 3\theta \approx \frac{V_0}{9}\left(1 - \frac{9\theta^2}{2} + \cdots\right)$$

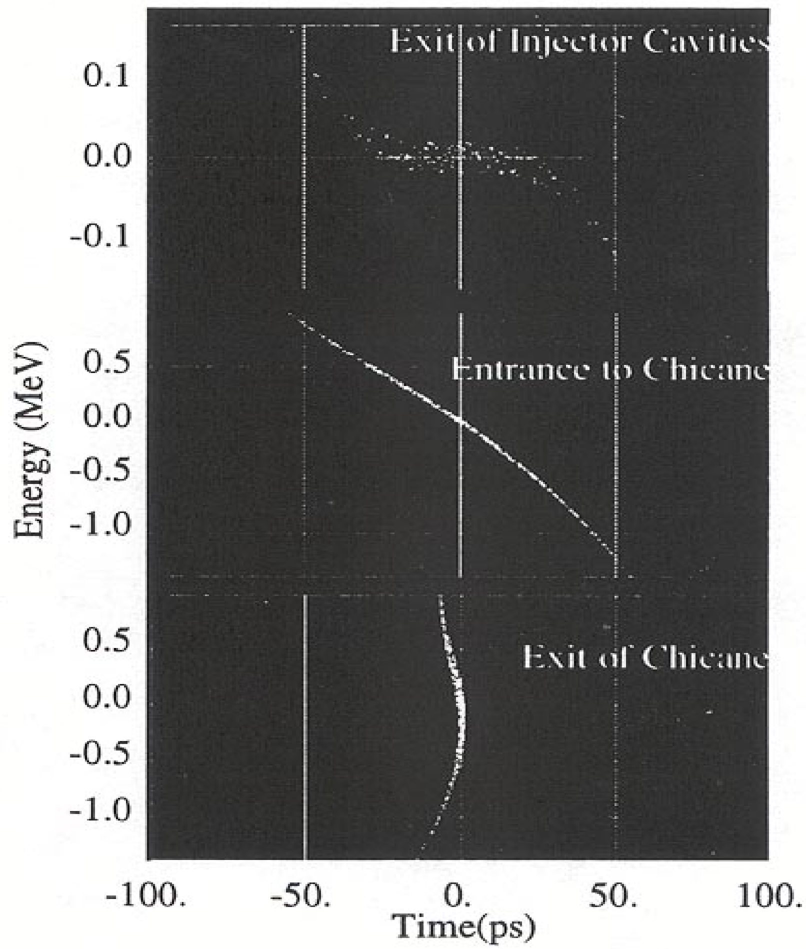
$$V_c - V_{lin} = \frac{8V_0}{9} + o\left(\theta^4\right),$$ independent of phase!

T. Smith, Proc. 1986 Int. Linac Conf., p. 421 (1986)
Dowell, D., et. al., Proc. 1995 PAC, p. 992 (1995)

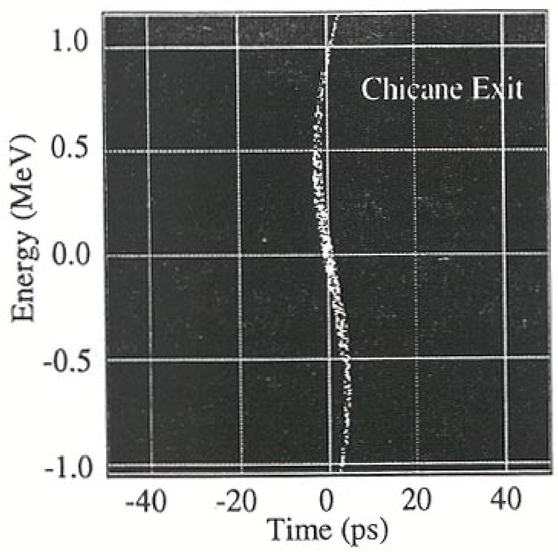
Boeing High Average Power FEL

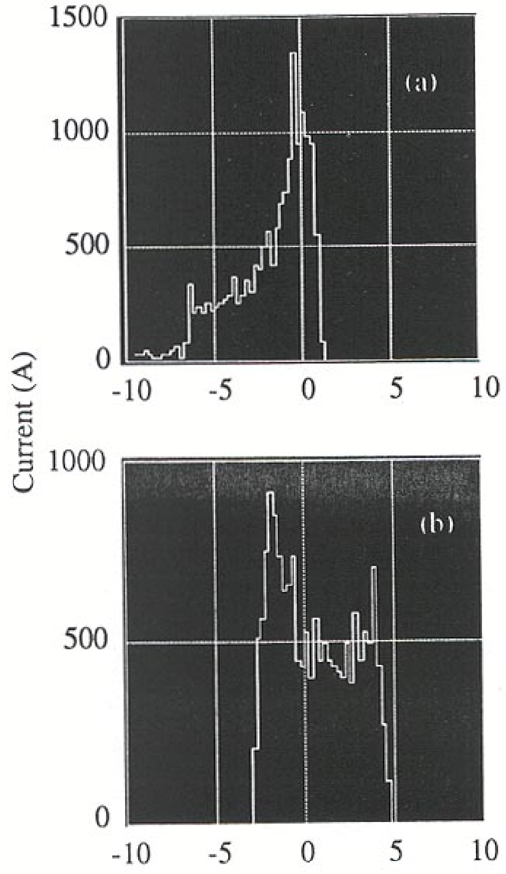


Phase Space Evolution Without Linearizer



Correction of Nonlinearities by "Linearizers"





Summary

We have shown how proper manipulation of the longitudinal phase space can lead to accelerators with superior beam characteristics.

We have shown how phase space tends to be degraded by generation of "curvatures" in longitudinal phase space, and a means to quantify such effects.

In this lecture and the preceding one, we've discussed some of the ways that people have combated this effect through (1) proper RF phase choices, (2) adding sextupoles in recirculation optics, and (3) RF linearization cavities.