USPAS Course on
Recirculated and Energy Recovered Linear Accelerators

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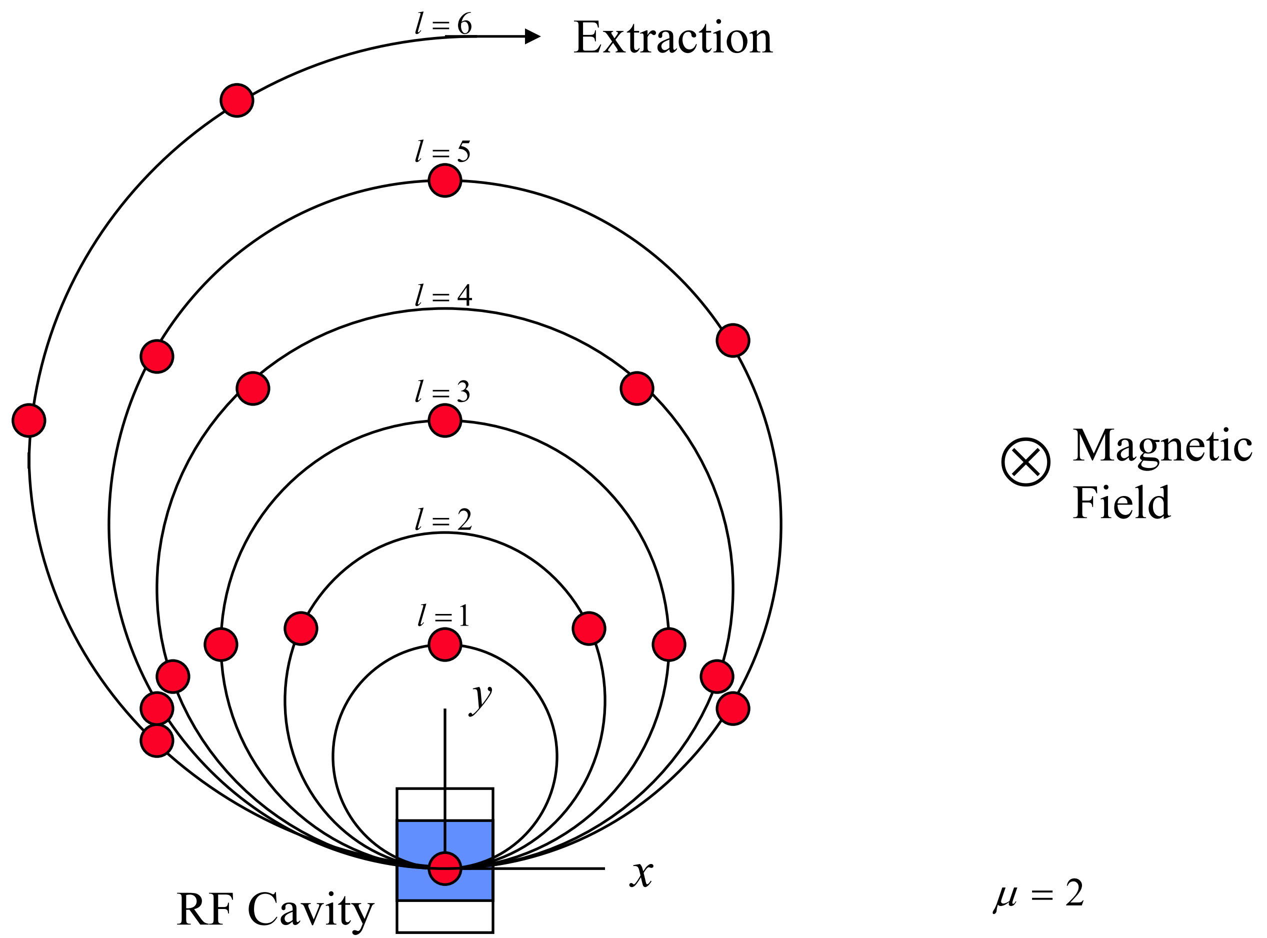
Lecture 3

Talk Outline

- "Classical" Microtrons
  - Basic Principles
  - Veksler and Phase Stability
  - Conventional Microtron
  - Performance
- Racetrack Microtrons
  - Basic Principles
  - Design Considerations
  - Examples
- Polytrons
  - Design Considerations
  - Argonne "Hexatron"

Classical Microtron: Veksler (1945)



$\mu = 2$

$\nu = 1$

Basic Principles

For the geometry given

$$\frac{d(\gamma m \vec{\mathrm{v}})}{dt} = -e\left[\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right]$$

$$\frac{d(\gamma m \mathrm{v}_x)}{dt} = e\mathrm{v}_y B_z$$

$$\frac{d(\gamma m \mathrm{v}_y)}{dt} = -e\mathrm{v}_x B_z$$

$$\frac{d^2\mathrm{v}_x}{dt^2} + \frac{\Omega_c^2}{\gamma^2}\mathrm{v}_x = 0 \qquad \frac{d^2\mathrm{v}_y}{dt^2} + \frac{\Omega_c^2}{\gamma^2}\mathrm{v}_y = 0$$

For each orbit, separately, and exactly

$$\mathrm{v}_x(t) = -v_{x0}\cos(\Omega_c t/\gamma) \qquad \mathrm{v}_y(t) = v_{x0}\sin(\Omega_c t/\gamma)$$

$$x(t) = -\frac{\gamma v_{x0}}{\Omega_c}\sin\left(\Omega_c t/\gamma\right) \qquad y(t) = \frac{\gamma v_{x0}}{\Omega_c} - \frac{\gamma v_{x0}}{\Omega_c}\cos\left(\Omega_c t/\gamma\right)$$

Non-relativistic cyclotron frequency: $\Omega_c = 2\pi f_c = eB_z / m$

Relativistic cyclotron frequency: Ω_c / γ

Bend radius of each orbit is: $\rho_l = \gamma_l \mathrm{v}_{x0,l} / \Omega_c \rightarrow \gamma_l c / \Omega_c$

In a conventional cyclotron, the particles move in a circular orbit that grows in size with energy, but where the relatively heavy particles stay in resonance with the RF, which drives the accelerating DEEs at the non-relativistic cyclotron frequency. By contrast, a microtron uses the "other side" of the cyclotron frequency formula. The cyclotron frequency decreases, proportional to energy, and the beam orbit radius increases in each orbit by precisely the amount which leads to arrival of the particles in the succeeding orbits precisely in phase.

Microtron Resonance Condition

Must have that the bunch pattern repeat in time. This condition is only possible if the time it takes to go around each orbit is precisely an integral number of RF periods

$$\gamma_1 = \mu \frac{f_c}{f_{RF}}$$

First Orbit

$$\Delta\gamma = \nu \frac{f_c}{f_{RF}}$$

Each Subsequent Orbit

For classical microtron assume can inject so that

$$\gamma_1 \approx 1 + \nu \frac{f_c}{f_{RF}}$$

$$\frac{f_c}{f_{RF}} \approx \frac{1}{\mu - \nu}$$

Parameter Choices

The energy gain in each pass must be identical for this resonance to be achieved, because once f_c/f_{RF} is chosen, $\Delta\gamma$ is fixed. Because the energy gain of non-relativistic ions from an RF cavity IS energy dependent, there is no way (presently!) to make a classical microtron for ions. For the same reason, in electron microtrons one would like the electrons close to relativistic after the first acceleration step. Concern about injection conditions which, as here in the microtron case, will be a recurring theme in examples!

$$f_c / f_{RF} = B_z / B_0 \qquad B_0 = \frac{2\pi mc}{\lambda e}$$

$$B_0 = 0.107\ \mathrm{T} = 1.07\ \mathrm{kG@10cm}$$

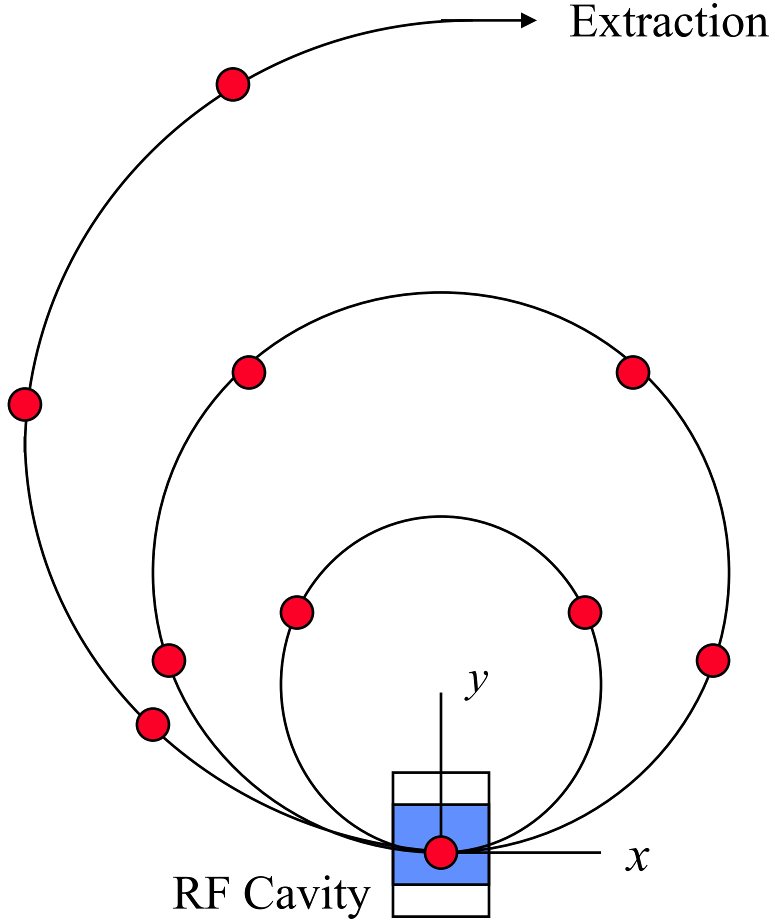
Notice that this field strength is NOT state-of-the-art, and that one normally chooses the magnetic field to be around this value. High frequency RF is expensive too!

Classical Microtron Possibilities

Assumption: Beam injected at low energy and energy gain is the same for each pass

$\frac{f_c}{f_{RF}}$	1	1/2	1/3	1/4	…
	$\mu, \nu, \gamma_1, \Delta\gamma$	$\mu, \nu, \gamma_1, \Delta\gamma$	$\mu, \nu, \gamma_1, \Delta\gamma$	$\mu, \nu, \gamma_1, \Delta\gamma$	…
	2, 1, 2, 1	3, 1, 3/2, 1	4, 1, 4/3, 1	5, 1, 5/4, 1	…
	3, 2, 3, 2	4, 2, 2, 2	5, 2, 5/3, 2	6, 2, 3/2, 2	…
	4, 3, 4, 3	5, 3, 5/2, 3	6, 3, 2, 3	7, 3, 7/4, 3	…
	5, 4, 5, 4	6, 4, 3, 4	7, 4, 7/3, 4	8, 4, 2, 4	…
	⋮	⋮	⋮	⋮	⋱

For same microtron magnet, no advantage to higher n; RF is more expensive because energy per pass needs to be higher



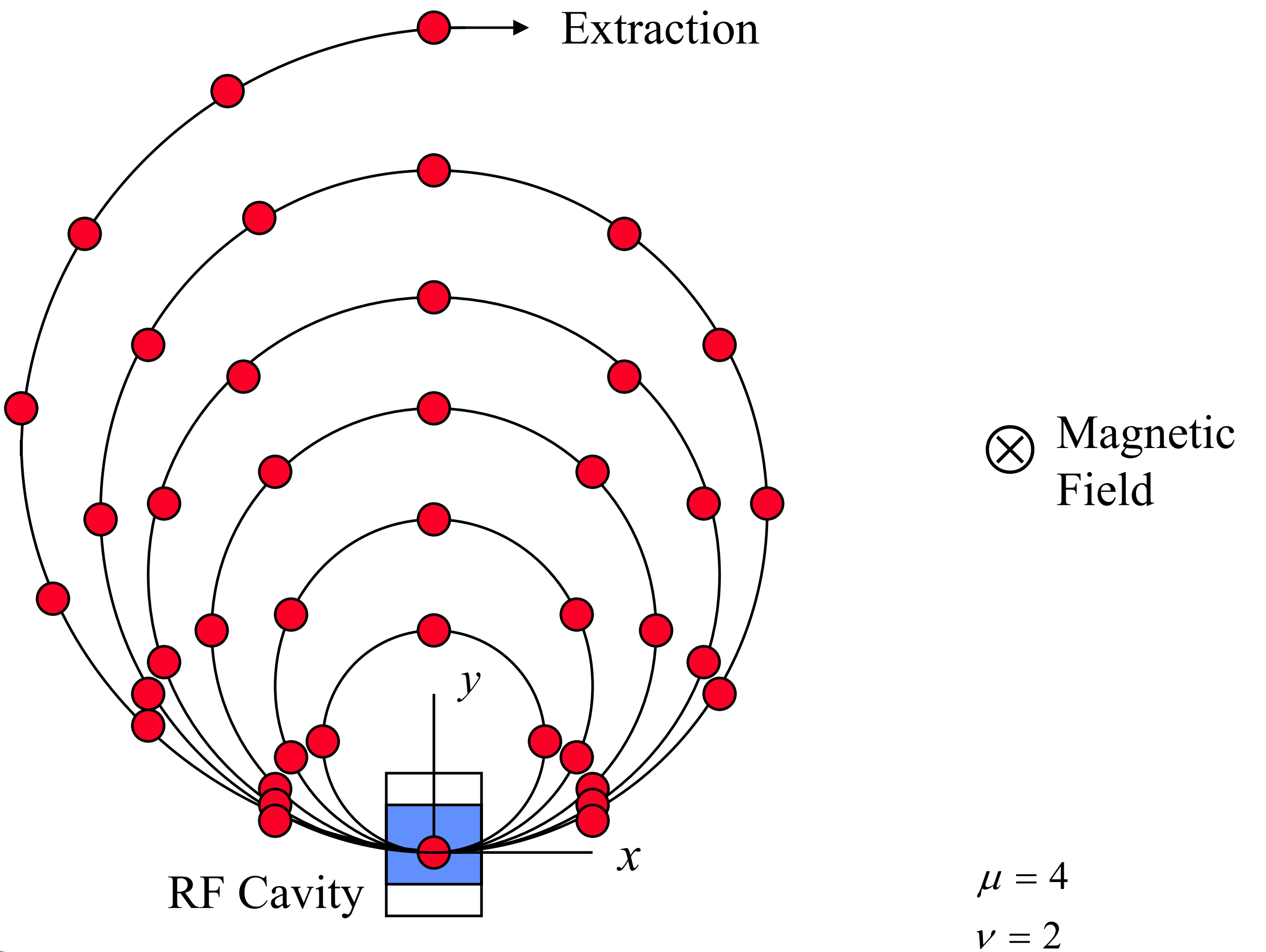
$\otimes$ Magnetic Field

$\mu = 3$

$\nu = 2$

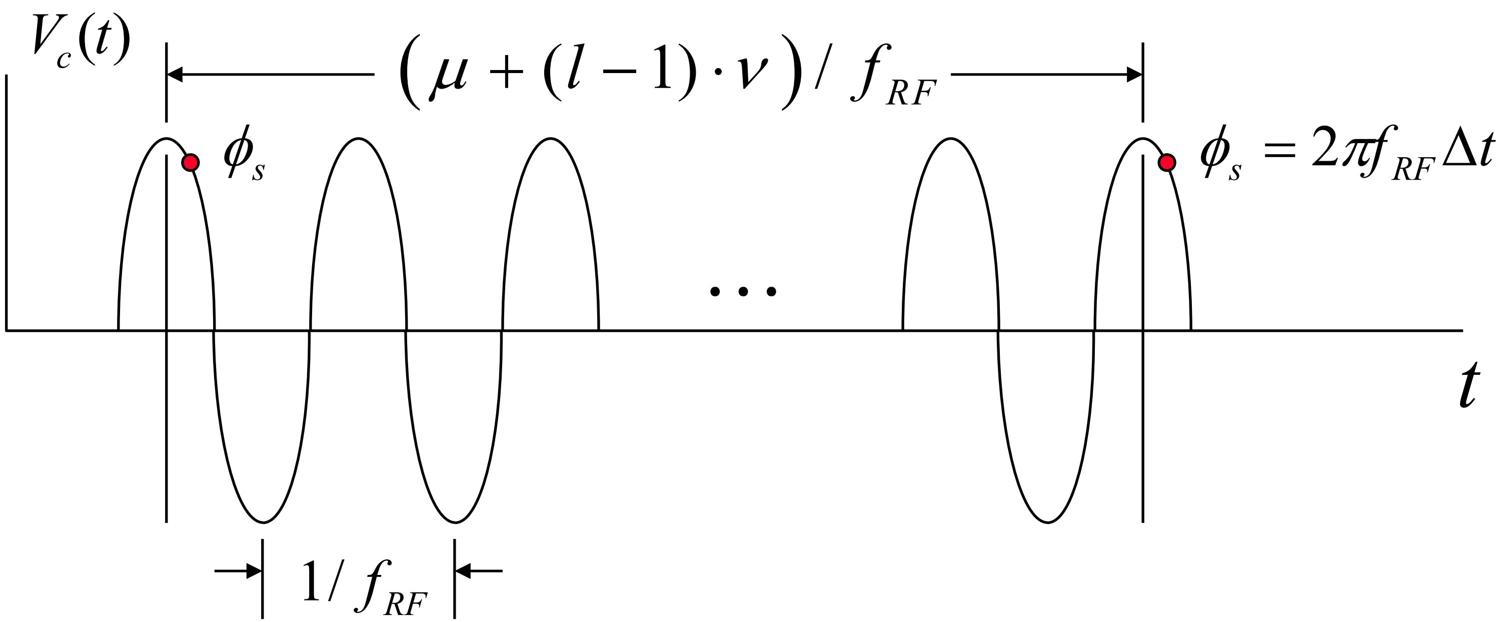
Going along diagonal changes frequency

To deal with lower frequencies, go up the diagonal



Phase Stability

Invented independently by Veksler (for microtrons!) and McMillan



Electrons arriving EARLY get more energy, have a longer path, and arrive later on the next pass. Extremely important discovery in accelerator physics. McMillan used same idea to design first electron synchrotron.

Phase Stability Condition

"Synchronous" electron has

$$\text{Phase} = \phi_s \qquad\qquad E_l = E_o + leV_c \cos\phi_s$$

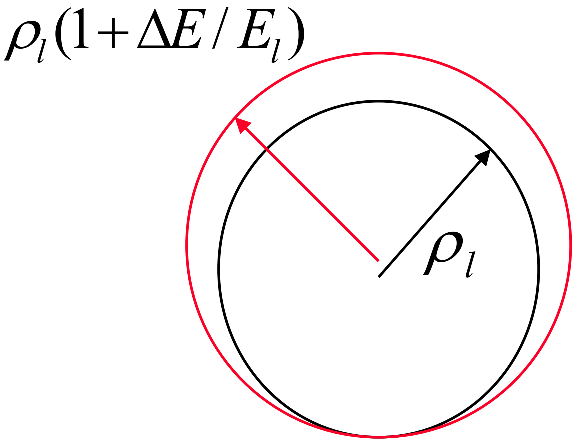
Difference equation for differences after passing through cavity pass $l + 1$:

$$\begin{pmatrix} \Delta\phi_{l+1} \\ \Delta E_{l+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -eV_c \sin\phi_s & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{2\pi M_{56}}{\lambda E_l} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

Because for an electron passing the cavity

$$\Delta E_{after} = \Delta E_{before} + eV_c\left(\cos(\phi_s + \Delta\phi) - \cos\phi_s\right)$$

Phase Stability Condition



$$\therefore M_{56} = 2\pi\rho_l$$

$$\begin{pmatrix} \Delta\phi_{l+1} \\ \Delta E_{l+1} \end{pmatrix} \approx \begin{pmatrix} 1 & \dfrac{4\pi^2\rho_l}{\lambda E_l} \\ -eV_c \sin\phi_s & 1-\dfrac{4\pi^2\rho_l eV_c}{\lambda E_l}\sin\phi_s \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

Phase Stability Condition

Have Phase Stability if

$$\left(\frac{\mathrm{Tr}\, M}{2} \right)^2 < 1$$

i.e.,

$$\boxed{0 < \nu\pi \tan\phi_s < 2}$$

Homework

Show that for any two-by-two unimodular real matrix M ($\det(M)=1$), the condition that the eigenvalues of M remain on the unit circle is equivalent to

$$\left(\frac{\mathrm{Tr}\, M}{2}\right)^2 < 1\,.$$

Show the stability condition follows from this condition on M, applied to the single pass longitudinal transfer matrix. Note ρ_l is proportional to E_l .

Compute the synchrotron phase advance per pass in the microtron as a function of ν and the synchronous phase ϕ_s

Problems with Classical Microtron

- Injection
- Would like to get magnetic field up to get to higher beam energy for same field volume, without increasing the RF frequency.

Solution: Higher "initial" energy

THE CONVENTIONAL MICROTRON

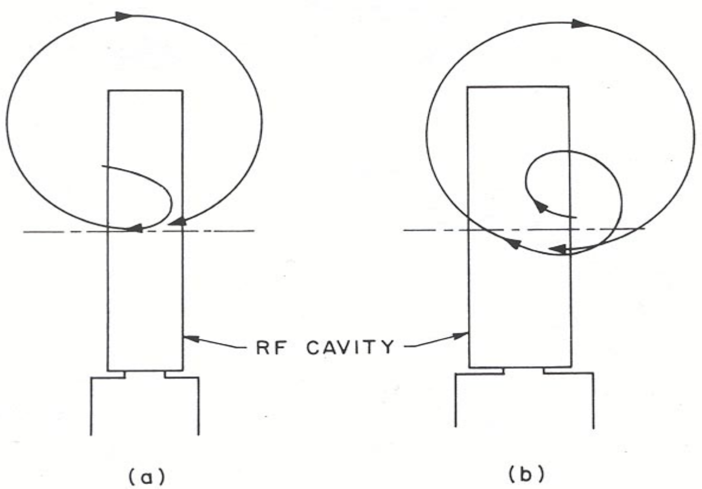


Figure 2.2 Conventional microtron injection schemes due to Melekhin, (a) first type, (b) second type. Electrons are produced by a thermionic emitter on the inner wall of the resonant cavity. Only the first complete orbits are shown (adapted from Kapitza and Melekhin, 1978; see also Kapitza *et al.*, 1961)

Conventional Microtron

Make

$$\gamma_1 = 1 + f\nu \frac{f_c}{f_{RF}} \qquad \text{with} \qquad f > 1$$

Now resonance conditions imply

$$\frac{f_c}{f_{RF}} = \frac{1}{\mu - f\nu},$$

$$\Delta\gamma = \frac{\nu}{\mu - f\nu}$$

And now it is possible to have (at the expense of higher RF power!)

$$f_c / f_{RF} > 1$$

Performance of Microtrons

The first and last entries are among the largest "classical" microtrons ever built.

Table B1 Notable examples of conventional microtrons $\quad \nu = 1$ for them all

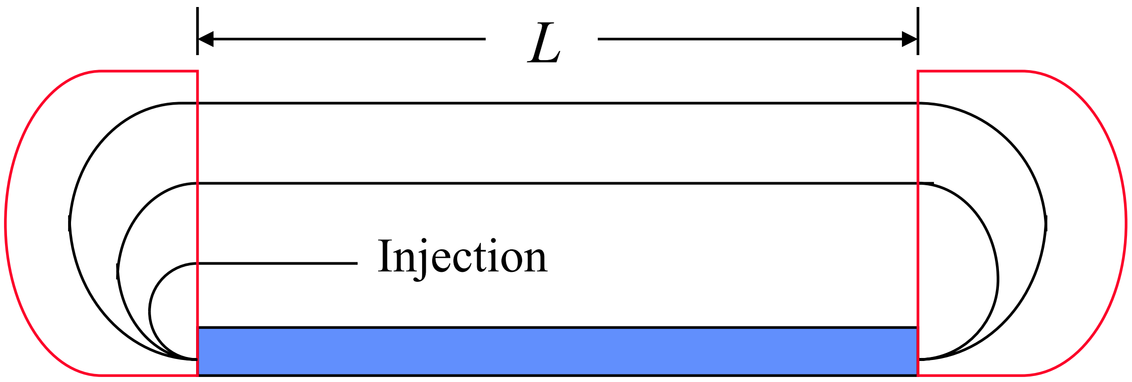
Institute Reference	Final energy (MeV)	Average beam current (μA)	No. of orbits	Magnet pole Diam. (m)	Application	
UC London Aitken *et al.* (1961)	29	0.01	56	2.0	NP (RD)	$f_c / f_{RF} = 1$
Lund Wernholm (1964)	6.4	50	10	0.50	Synchrotron injector	$f_c / f_{RF} = 1$
Moscow Kaptiza *et al.* (1965)	32	50	30	1.10	Particle dynamics radio-activation	$f_c / f_{RF} = 2$
Dubna Ananev *et al.* (1966)	30	60	30	1.10	Pulsed reactor injector	$f_c / f_{RF} = 2$
Wisconsin Rowe and Mills (1973)	44	15	34	1.37	SRI	$f_c / f_{RF} = 2.5$
Scanditronix Svensson *et al.* (1977) Egan-Krieger *et al.* (1983)	22.5	10	42	2.22	 MD SRI	$f_c / f_{RF} = 1$

All above microtrons operate with a RF wavelength of approximately 10 cm. A more complete survey of conventional microtrons is given by Kapitza and Melekhin (1978).

Racetrack Microtrons

- Basic idea: increase the flexibility of parameter choices while retaining the inherent longitudinal stability of the microtron geometry.
- Use the increased flexibility to make bending magnets better suited to containing higher energy beams than in conventional microtrons
- Solve "injection" problems by injection at relatively high energies

- Trick: split the two halves of the microtron



Revised Resonance Conditions

$$\gamma_1 \frac{f_{RF}}{f_c} + \frac{2L}{\lambda} = \mu >> 1$$

$$\Delta\gamma = \nu \frac{f_c}{f_{RF}}$$

To evaluate racetrack microtron longitudinal stability, use the same formulas as for classical microtron. For largest acceptance $\nu = 1$.

Huge advantage: because of the possibilities of long straights, long linacs operated in a longitudinally stable way are possible. In particular, there is now space for both CW normal conducting linacs and CW superconducting linacs.

Homework

Design a 30 MeV-200 MeV racetrack microtron. In particular, specify

(1) The bender fields
(2) The radius of largest orbit
(3) M56 of largest orbit
(4) Energy gain of linac section
(5) Linac length
(6) Range of stable synchronous phase

There are many “right” answers for the information given, and I insist on at least two passes! Assume that the accelerating structures have zero transverse extent.

Examples of Racetrack Microtrons

SOME RACE-TRACK MICROTRON PROJECTS

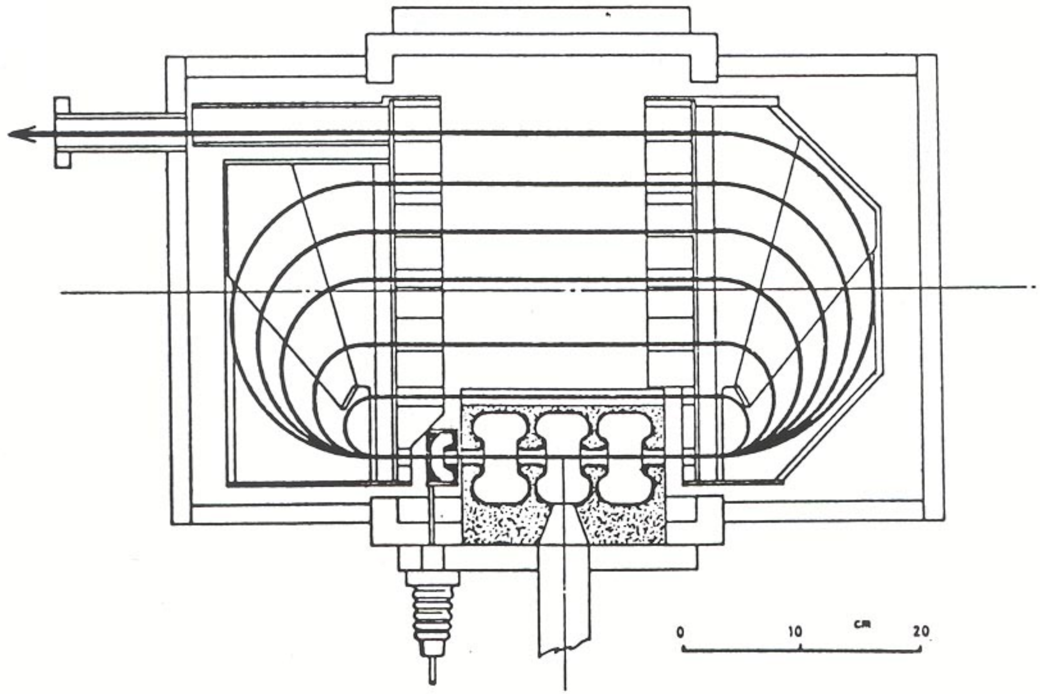


Figure 5.1 Configuration of the medical version of the University of Western Ontario race-track microtron (Froelich and Manca, 1975; © 1975 IEEE)

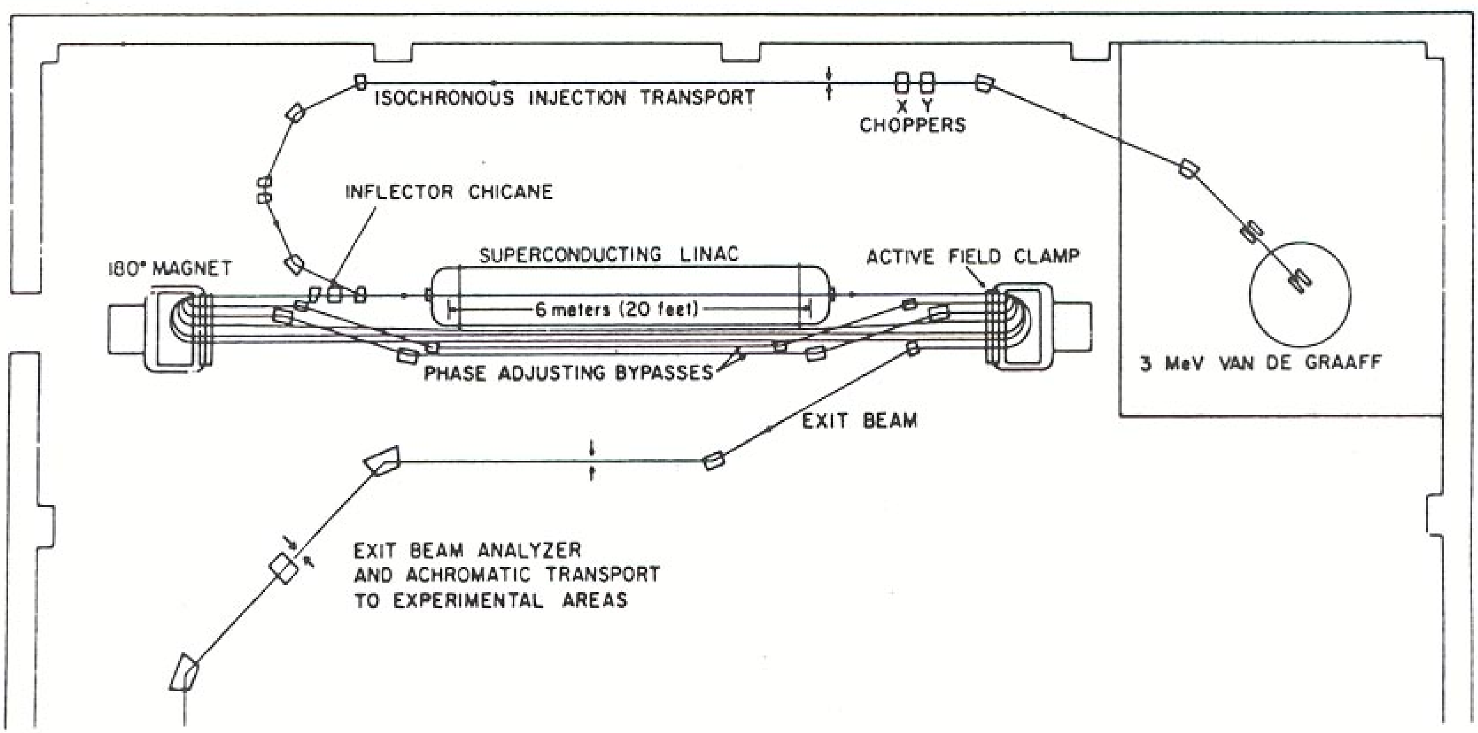


Figure 5.3 The second Illinois superconducting race-trace microtron, MUSL-2 (Axel *et al.*, 1977; © 1977 IEEE)

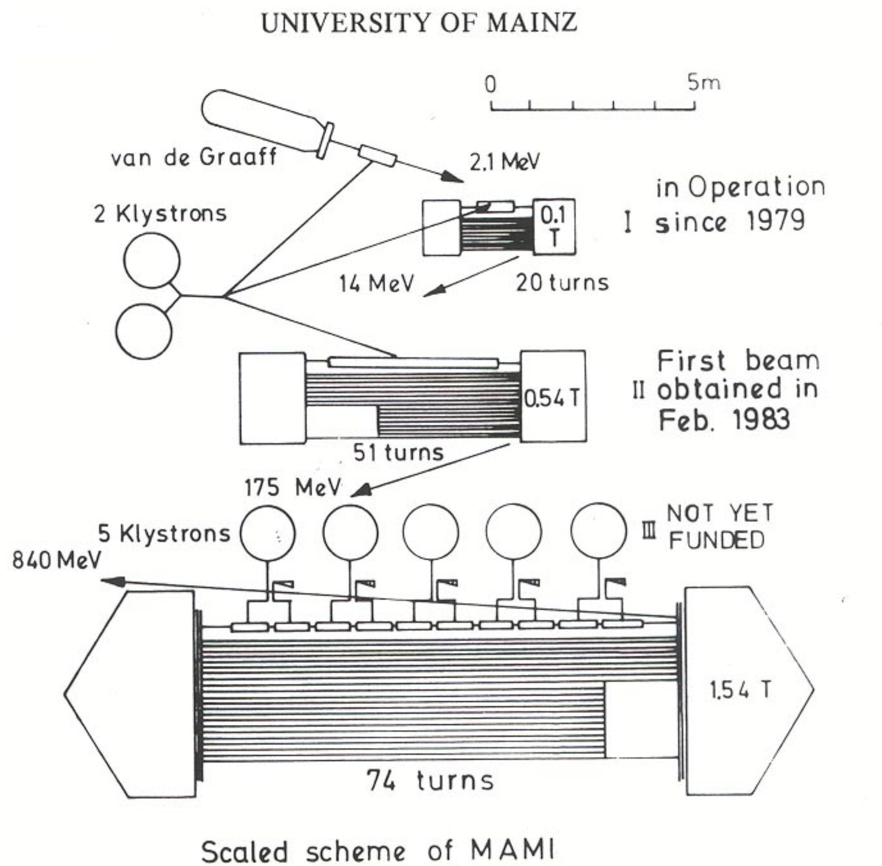


Figure 5.5 Schematic layout of the University of Mainz three-stage cascaded race-track microtron, MAMI (Herminghaus *et al.*, 1983; © 1983 IEEE)

Table 5.1 Parameters of MAMI, the Mainz cascaded race-trace microtron (adapted from Herminghaus *et al.*, 1983; © 1983 IEEE)

		Stage		
		I	II	III
General				
Input energy	(MeV)	2.1	14	175
Output energy	(MeV)	14	175	840
Number of passes		20	51	74
Total power consumption	(kW)	----- 280 -----		900
Magnet system				
Magnet separation	(m)	1.66	5.59	11.83
Magnetic field	(T)	0.10	0.54	1.54
Maximum orbit diameter	(m)	0.97	2.17	3.65
Magnet weight (each)	(ton)	1.3	43	305
Gap width	(cm)	6	7	12
RF system (2.449 GHz)				
Number of klystrons		----- 2 -----		5
Linac length	(m)	0.80	3.55	10.4
Total RF power	(kW)	9	64	197
Beam power	(kW)	1.2	16	67
Energy gain per pass	(MeV)	0.59	3.16	9.0
Beam performance (design, at 100 μA)				
Energy width	(keV)	± 9	± 18	± 60
Emittance: vert.	(mm mr)	0.17π	0.04π	0.01π
horiz.	(mm mr)	0.17π	0.09π	0.14π
Status		-- Operating --		Not yet funded

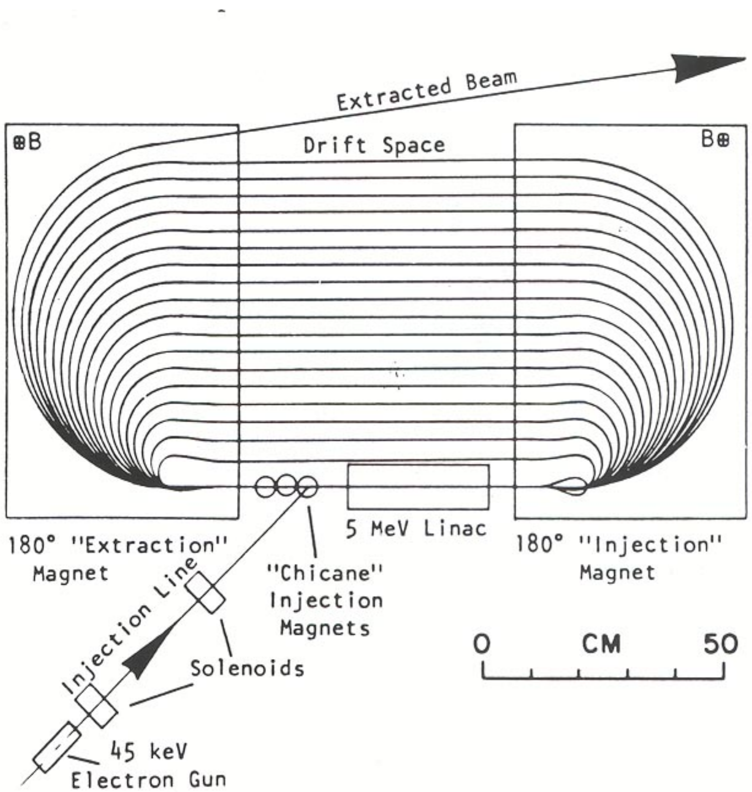


Figure 5.8 Schematic layout of the race-track microtron injector for the storage ring Aladdin at the Synchrotron Radiation Center of the University of Wisconsin (Green *et al.*, 1981; © 1981 IEEE)

Polytrons

For GeV scale energies or higher, the bend magnets for a racetrack microtron design become uneconomical. A way must be found to confine the active bending field to a relatively small bending area. A way to do this is illustrated in the idea of a polytron, which is a generalization of the racetrack microtron with the total bend between linacs of $360/p$, where p is an even integer.

To the best of my knowledge, no polytron has ever been built, although Argonne's hexatron was a serious competitor to the original NEAL proposal from SURA.

My guess is that superconducting machines like CEBAF will always be preferred to polytrons, although Herminghaus has given some reasons that one might expect to get smaller energy spread out of these devices.

Polytron Arrangements

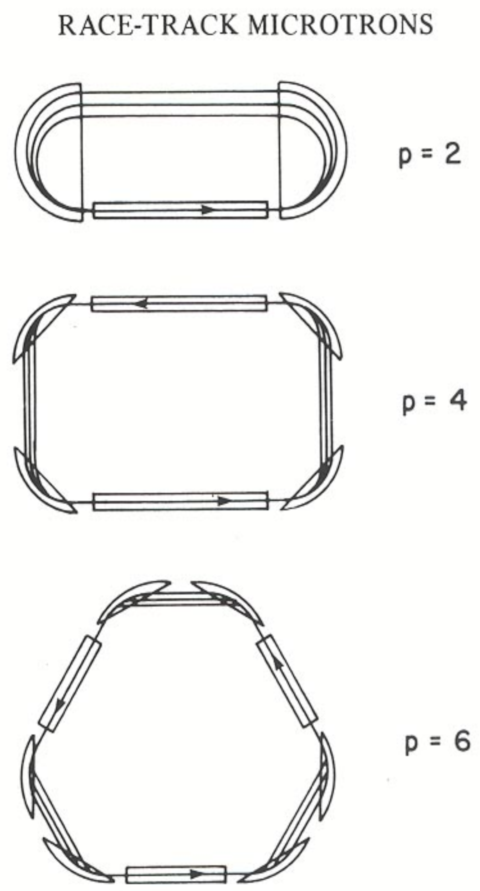
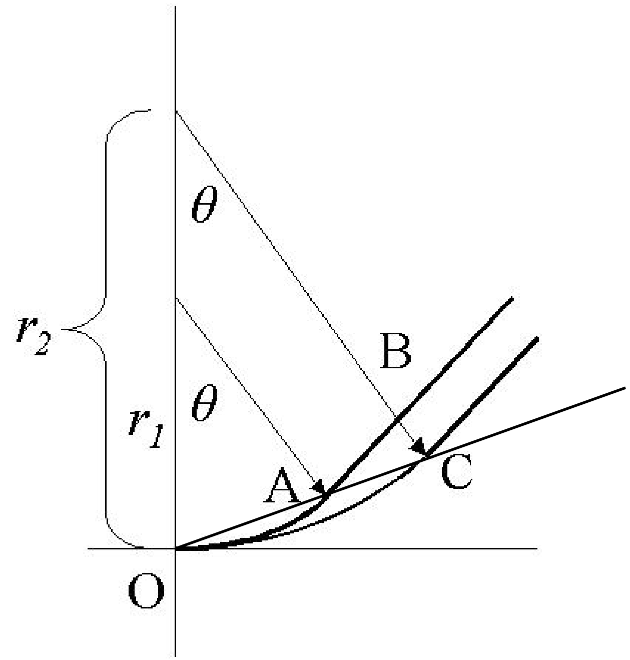


Figure 4.2 The standard race-track microtron, bicyclotron and hexatron as the first three members of the family of Polytrons

Bender Geometry



$$\Delta\gamma = \nu \frac{f_c}{f_{RF}} \frac{1}{1-(p/2\pi)\sin(2\pi/p)}$$

Polytron Properties

Polytrons have a greater phase stable area. Proof, examine the stability of

$$M = \begin{pmatrix} 1 & \dfrac{2\pi M_{56}}{\lambda E_l} \\ -eV_c \sin\phi_s & 1 - \dfrac{2\pi M_{56} eV_c \sin\phi_s}{\lambda E_l} \end{pmatrix}$$

But now the section bends only $720^\circ / p$

$$\therefore M_{56} = 4\pi\rho_l \left[1 - \left(p / 2\pi\right)\sin\left(2\pi / p\right)\right] / p$$

Stability Condition

$$0 < \nu\pi \tan\phi_s < p^2 / 2$$

Polytron Properties

Table 4.1 Properties of polytrons–multi-linac race-track microtrons

	Standard Race-Track Microtron	Bicyclotron	Hexatron	General
No. of straight sections	2	4	6	p
No. of magnets	2	4	6	p
Relative magnet pole area, A, for same B	1	0.363	0.173	$\frac{2\pi - p\sin(2\pi/p)}{2\pi}$
Relative total magnet weight	1	$\sim 1/5$	$\sim 1/14$	$A^{3/2}$
No. of linacs	1	2	3	$p/2$
Minimum incremental harmonic number, $\boldsymbol{\nu}$	1	2	3	$p/2$
Minimum $\frac{\Delta W}{m_0c^2}\frac{B_0}{B}$	1	5.51	17.34	$\frac{2\pi\boldsymbol{\nu}}{2\pi - p\sin(2\pi/p)}$
Maximum limit of p.s. region, ϕ_{max}	32.5°	51.9°	62.4°	$\arctan\cancel{(p^2/2\pi\boldsymbol{\nu})}$
$\frac{\text{Min orbit separation}}{\lambda}$	0.318	0.876	1.380	$\frac{\boldsymbol{\nu}[1-\cos(2\pi/p)]}{2\pi - p\sin(2\pi/p)}$

$\arctan\left(p/\pi n\right)$

NB, the numbers are right, just not the formula

Argonne "Hexatron"

MULTI-GeV ACCELERATORS

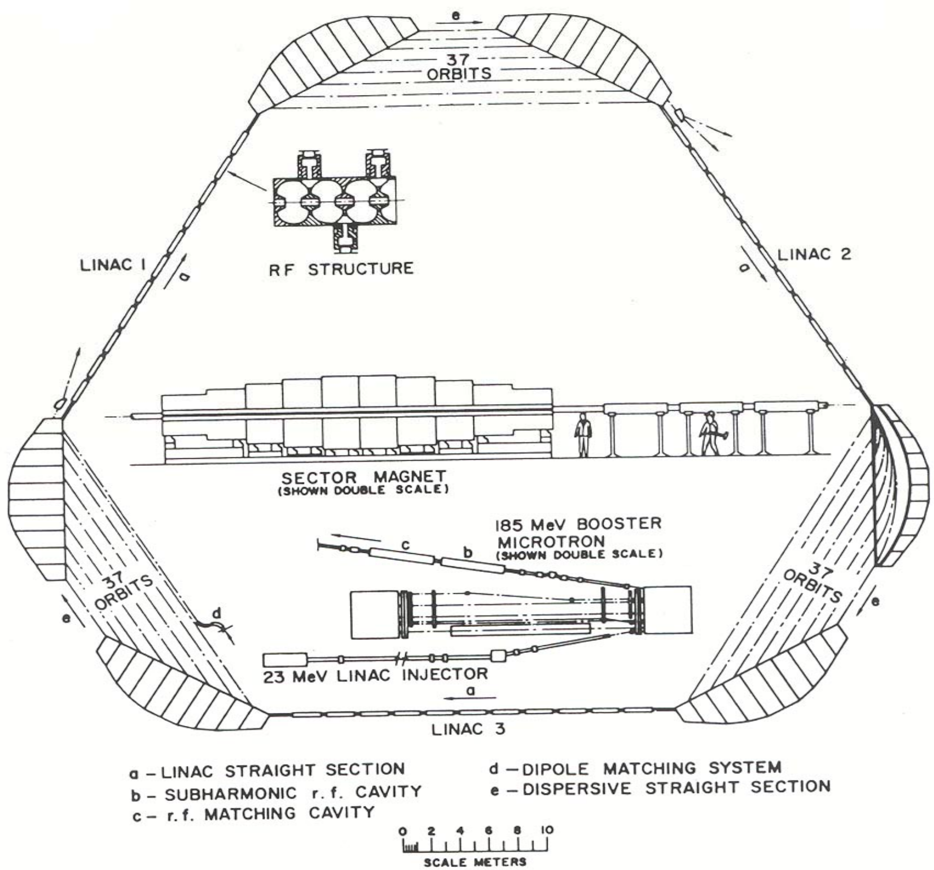
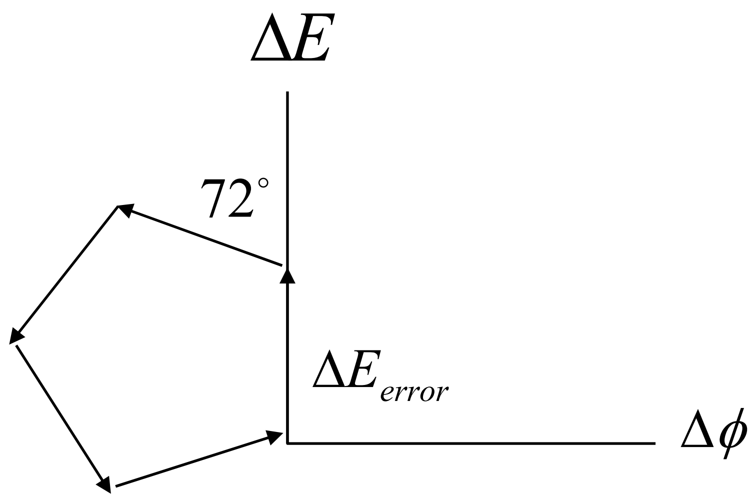


Figure 10.8 Layout of the Argonne 4 GeV hexatron design (Jackson *et al.*, 1982a)

Enhanced Longitudinal Stability (Herminghaus)

By proper choice of synchrotron frequency, it may be possible to cancel of RF phase and amplitude errors. For a 5-pass device and phase advance 1/5



Sum vanishes after fifth pass!!

One actually WANTS to run on the storage ring "linear resonance" for polytrons!

Summary

- Microtrons, racetrack microtrons, and polytrons have been introduced.
- These devices have been shown to be *Phase Stable.*
- Examples of these devices, including a superconducting racetrack microtron, have been presented.
- We're ready to take the next step, independent orbit recirculating accelerators.