USPAS Course
on
Recirculated and Energy Recovered Linear Accelerators

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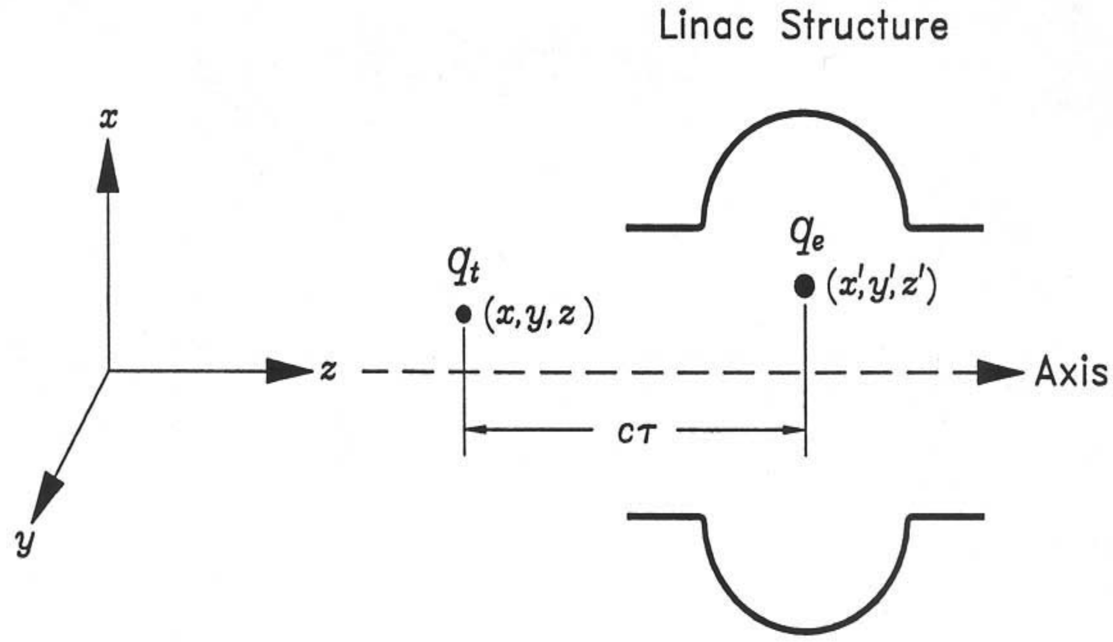
Lecture 11

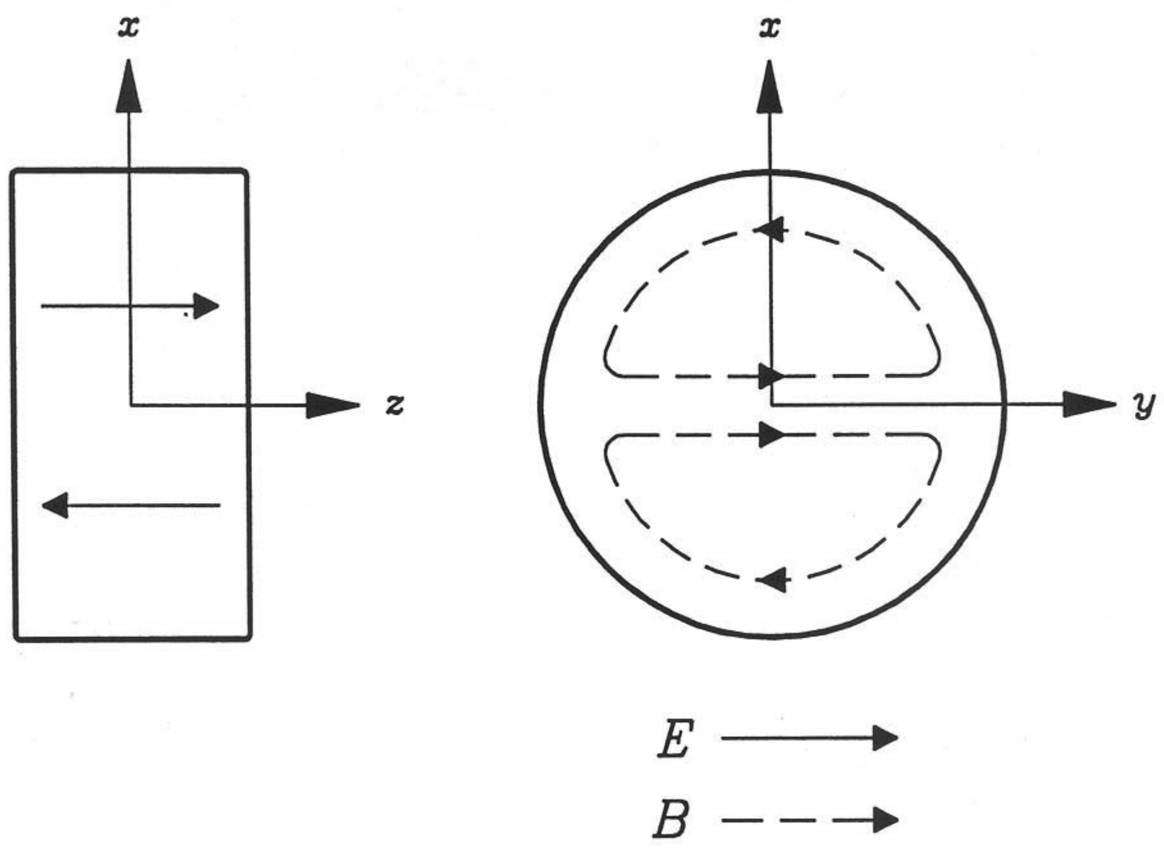
Multipass Beam Breakup

- Multibunch Stability
  - BBU/Wakes
  - Multipass BBU
  - Single Cavity Formula
  - Instability Threshold
  - General Analysis
  - Transfer Function Measurement
- Simulations
  - TDBBU
  - Single Bunch
- Recent Work (Pozdeyev and Tennant)
  - Theoretical Generalization of Single Pass BBU Threshold
  - Measurement Techniques and Results
  - Comparison of the Experimental Data to Simulations and the Analytical Formula

Wake Function Definition



EM Field of “Deflecting” Mode



BBU/Wakes

Recall the interaction between a particle and a trailing particle is characterized utilizing the longitudinal and transverse wake functions

$$W_l(\tau) = \frac{1}{q_e}\int E_z(z, z/c+\tau)\,dz$$

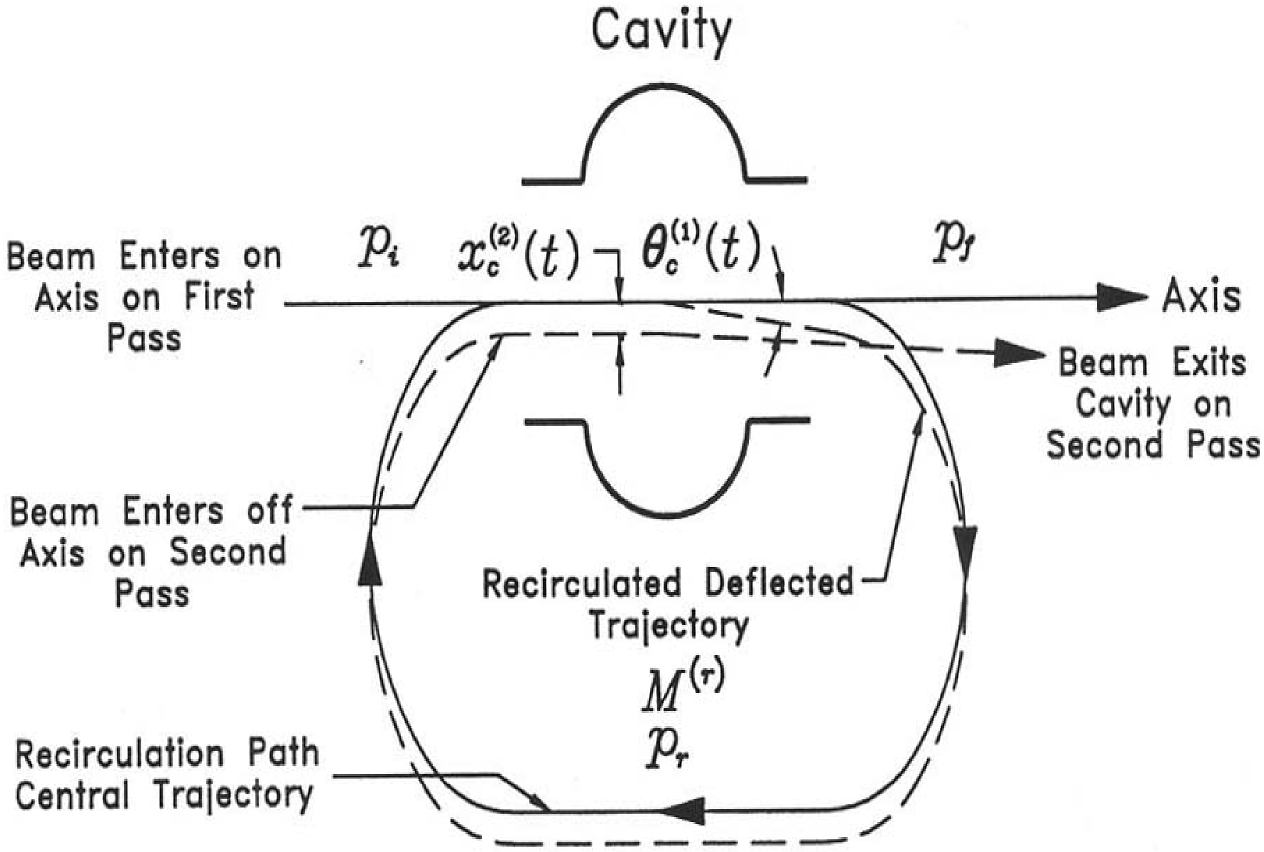
$$W_t(\tau) = \frac{1}{q_e r}\int \left[E_x(z, z/c+\tau) - cB_y(z, z/c+\tau)\right]dz$$

And that for frequencies below cutoff the wake may be treated as a sum of normal modes

$$W_l(\tau) = \sum_{\omega_\lambda} \left(\frac{R}{Q}\right)_\lambda \frac{\omega_\lambda}{2} e^{-\omega_\lambda \tau / 2Q_\lambda} \cos(\omega_\lambda \tau)$$

$$W_t(\tau) = \sum_{\omega_\lambda} \left(\frac{R}{Q}\right)_\lambda \frac{k_\lambda \omega_\lambda}{2} e^{-\omega_\lambda \tau / 2Q_\lambda} \sin(\omega_\lambda \tau)$$

Multipass BBU Instability



BBU/Wakes

The beam current in a CW recirculating linac is

$$I(t) = \sum_{m=-\infty}^{\infty} q_0 \delta(t - mt_0)$$

By the definition of wake, the deflecting voltage experienced by a particle at time t is

$$V(t) = \int_{-\infty}^{t} W_t(t-t')I(t')x(t')dt'$$

Multipass BBU

2-pass, single cavity model

Displacement of the beam on the second pass is

$$x(t) = M_{12} e V(t - t_r)/c$$

The following integral equation

$$V(t) = \frac{M_{12} e}{c} \int_{-\infty}^{t} W_t(t - t')I(t')V(t' - t_r)dt'$$

2-pass Single Cavity Case

Assume normal mode solution, clear the deltas for the current, and sum the geometric series

$$V\left(nt_0\right) = V_0 e^{-i\omega n t_0}$$

yields

$$1 = \kappa e^{i\omega t_r} \frac{e^{i\omega t_0} e^{-\omega_\lambda t_0 / 2Q_\lambda} \sin\left(\omega_\lambda t_0\right)}{1 + \left(e^{i\omega t_0} e^{-\omega_\lambda t_0 / 2Q_\lambda}\right)^2 - 2 e^{i\omega t_0} e^{-\omega_\lambda t_0 / 2Q_\lambda} \cos\left(\omega_\lambda t_0\right)}$$

where

$$\kappa = \left(R / Q\right)_\lambda k_\lambda^2 e T_{12} I_0 t_0 / 2$$

and T is the M matrix in momentum units

Threshold Current

If $\quad T_{12}\sin(\omega_\lambda t_r) < 0, \quad$ near threshold $\quad \kappa << 1$

Equation for the normal mode frequency is

$$e^{i\omega t_0} e^{-\omega_\lambda t_0 / 2Q_\lambda} \approx \left[1 \mp \frac{i\kappa}{2} e^{i\omega t_r}\right] e^{\pm i\omega_\lambda t_r}$$

Growth Rate

$$\mathrm{Im}(\omega) \approx -\frac{\kappa \sin(\omega_\lambda t_r)}{2t_0} - \frac{\omega_\lambda}{2Q_\lambda}$$

Threshold Current

If the average current exceeds the threshold current

$$I_{th} = \frac{1}{e} \frac{2\omega_\lambda}{(R/Q)_\lambda Q_\lambda k_\lambda^2 \left| T_{12} \sin(\omega_\lambda t_r) \right|}$$

have instability!

NB, For $T_{12}\sin(\omega_\lambda t_r) < 0,$ there is also a threshold current but it is not necessary that $\kappa << 1$. Perturbation analysis fails and the full dispersion relation must be solved.

Jefferson Lab

Generalization to Multiple Passes

Merminga result and Reference

Conclusions From Single Cavity Model

- Lower *R/Q* (geometry, cannot do to much about that!) and *Q* (HOM damping!) are much to be desired
- Because of the time delay factor, and that it will be in general different for different HOMs, it is generally not possible to choose the "right" time delay for all HOMs. What CAN be done is to choose the recirculation time properly to minimize the effect of one, presumably the most unstable, HOM.
- Optics with smaller *T*s helps. The problem is that one needs the *T*s small *Throughout the Linac!* It does no good to make it small one place and large elsewhere. Philosophy in the designs is the make betas small throughout the linac in hopes of minimizing the "average" *T*s.
- In reality, cumulative BBU amplification also contributes to the instability, necessitating more accurate simulation calculations of the effect.

General Multipass, Many Cavity Case

Let the bunch phase space vector be denoted by

$$V_i^I(k) = \begin{pmatrix} x_i^I(k) \\ p_{xi}^I(k) \\ y_i^I(k) \\ p_{yi}^I(k) \end{pmatrix}$$

Coordinates AFTER cavity i on pass I of bunch k

General Case, Cont.

$$V_i^{\,I}(k)=T_{i,i-1}^{\,II}V_{i-1}^{\,I}(k)+\begin{pmatrix} 0 \\ \kappa\displaystyle\sum_{l=1}^{k+t(I)-1} s_{k+t_i(I)-l}\,\bar{x}_i(l) \\ 0 \\ \kappa\displaystyle\sum_{l=1}^{k+t(I)-1} s_{k+t_i(I)-l}\,\bar{y}_i(l) \end{pmatrix}$$

$$V_1^{\,I}(k)=T_{1,N}^{\,I,I-1}V_N^{\,I-1}(k)+\begin{pmatrix} 0 \\ \kappa\displaystyle\sum_{l=1}^{k+t(I)-1} s_{k+t_i(I)-l}\,\bar{x}_1(l) \\ 0 \\ \kappa\displaystyle\sum_{l=1}^{k+t(I)-1} s_{k+t_i(I)-l}\,\bar{y}_1(l) \end{pmatrix}$$

General Case, Cont.

where

$$s_k = e^{-\omega\tau/2Q} \sin\left(m\omega\tau\right)$$

$$\bar{x}_i = \sum_{J=1}^{N_P} x_i^J \left(l - t_i\left(J\right)\right)$$

and the time delays of succeeding passes is given by

$$t_i\left(1\right) = 0$$

$$t_i\left(2\right) = \text{number of RF periods until 2nd crossing}$$

$$t_i\left(3\right) = \text{number of RF periods until 3rd crossing}$$

etc.

General Case, Cont.

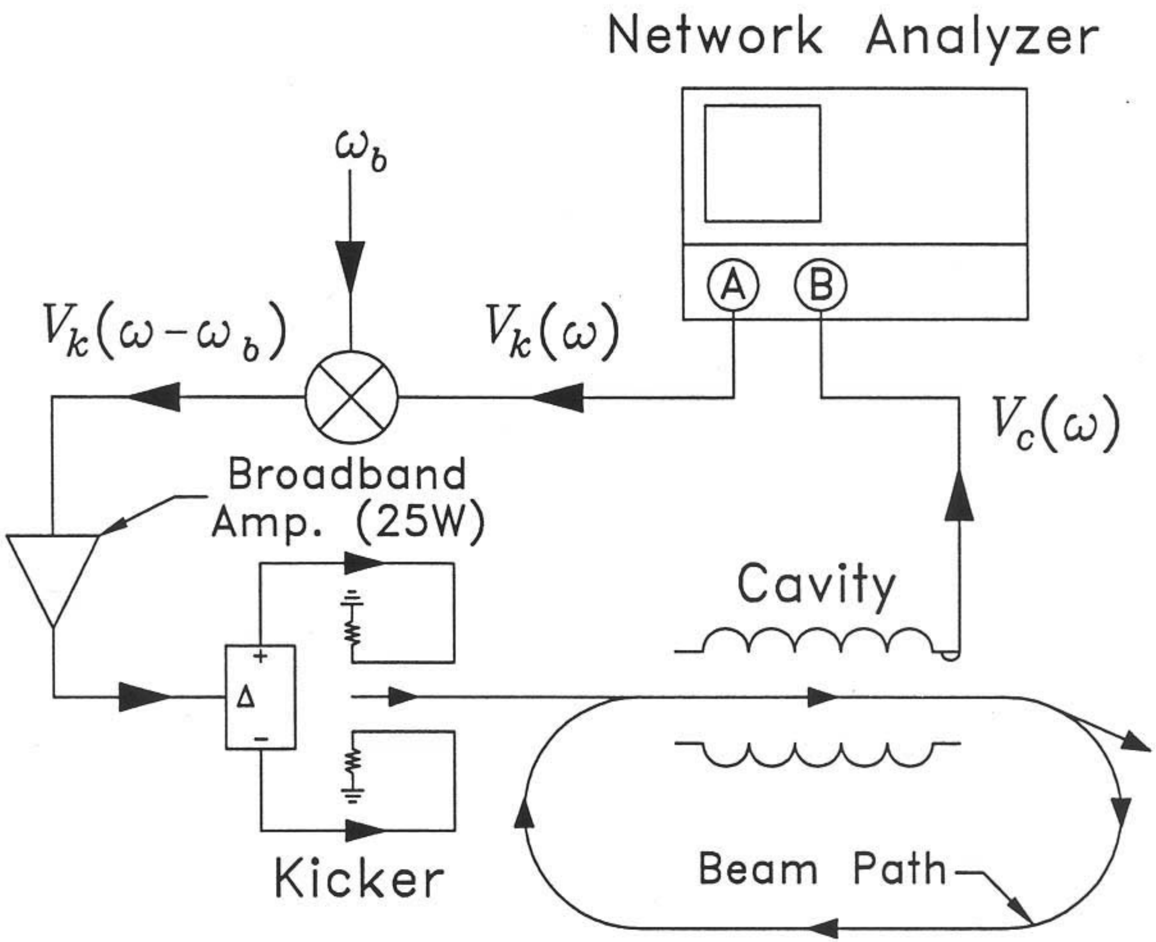
$$V_i^I(k) = V_{0i}^I e^{2\pi i \nu k}$$

$$D_{xi} = \sum_{J=1}^{N_p} \hat{x}_i^J e^{-2\pi i \nu t_i(J)}$$

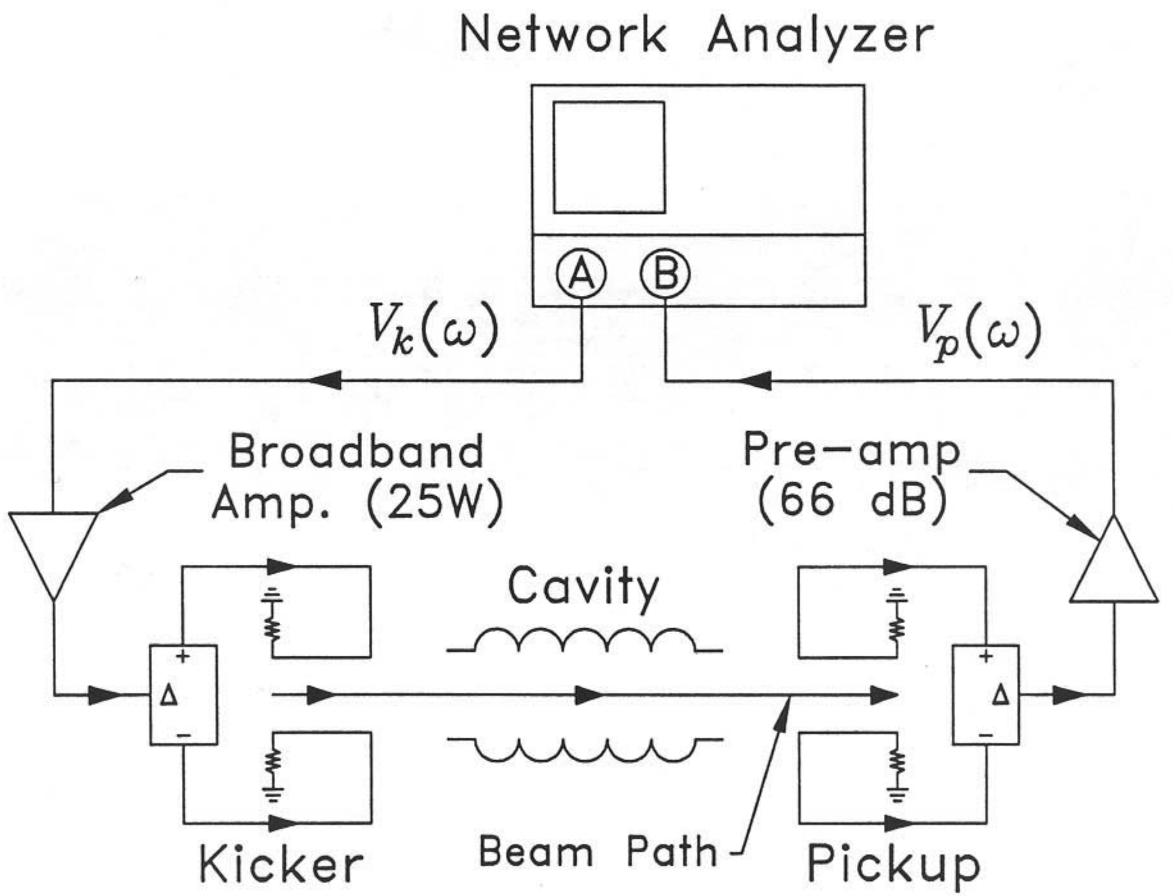
$$D_{xi} = \sum_{I=2}^{N_P} \sum_{J<I} \sum_{l=1}^{N} \left(T_{il}^{IJ}\right)_{12} e^{2\pi i o(t_l(J)-t_i(I))} h_l(\nu) D_{xl} + \sum_{I=1}^{N_P} \sum_{l=1}^{i-1} \left(T_{il}^{II}\right)_{12} e^{2\pi i o(t_l(I)-t_i(I))} h_l(\nu) D_{xl}$$

$$h_i(\nu) = \frac{(R/Q)_i k_i^2 e I_0 \tau}{2} \frac{e^{-2\pi i \nu} e^{-\omega_i \tau / 2Q_i} \sin(\omega_i \tau)}{1 + \left(e^{-2\pi i \nu} e^{-\omega_i \tau / 2Q_i}\right)^2 - 2 e^{-2\pi i \nu} e^{-\omega_i \tau / 2Q_i} \cos(\omega_i \tau)}$$

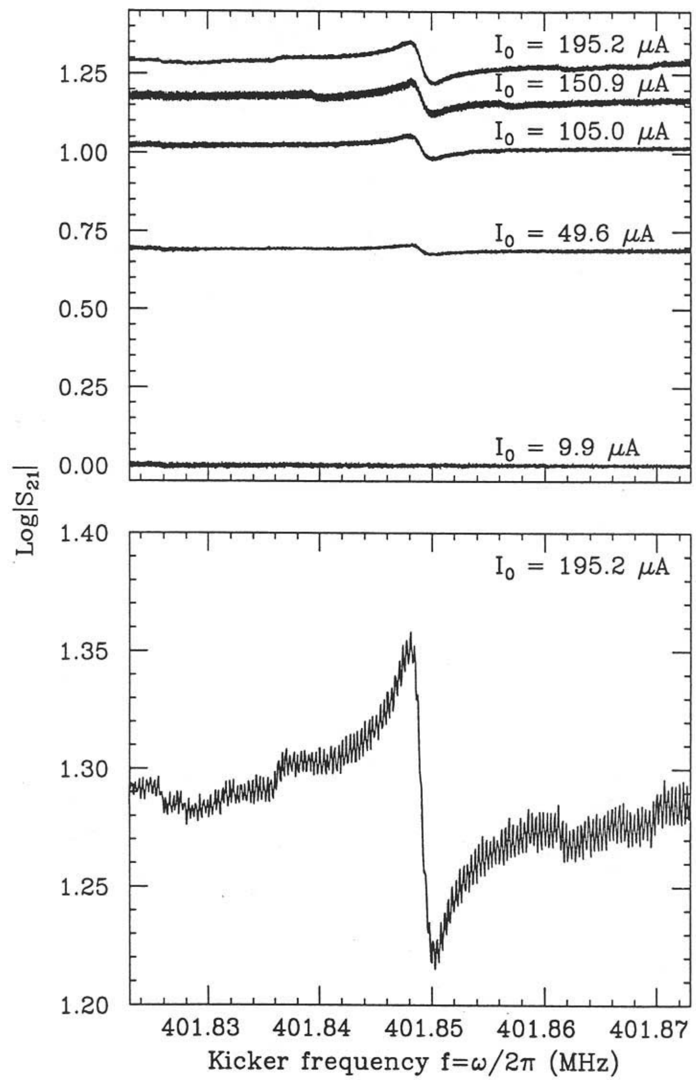
HOM Coupling Measurement



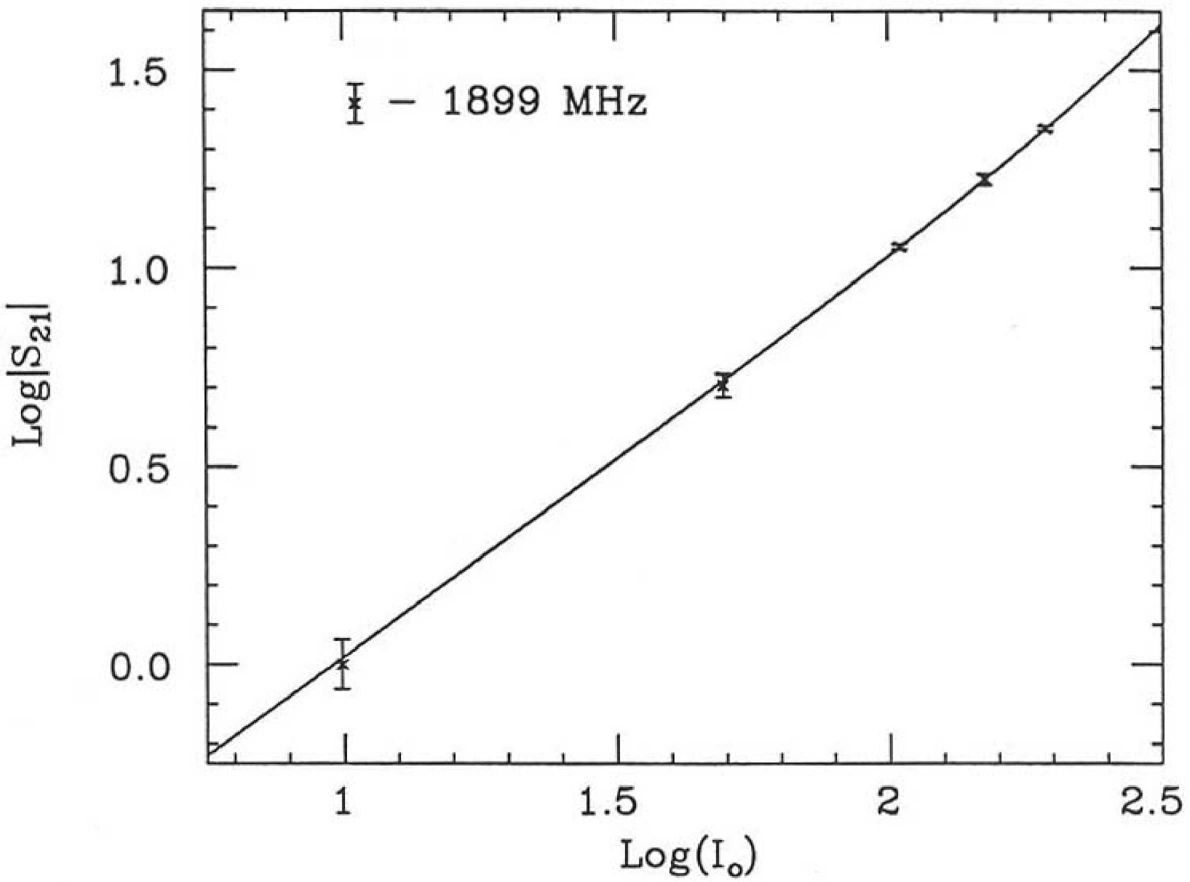
Beam Transfer Function



Measurement Results



Current Dependence of Measurement Results



TDBBU Simulation

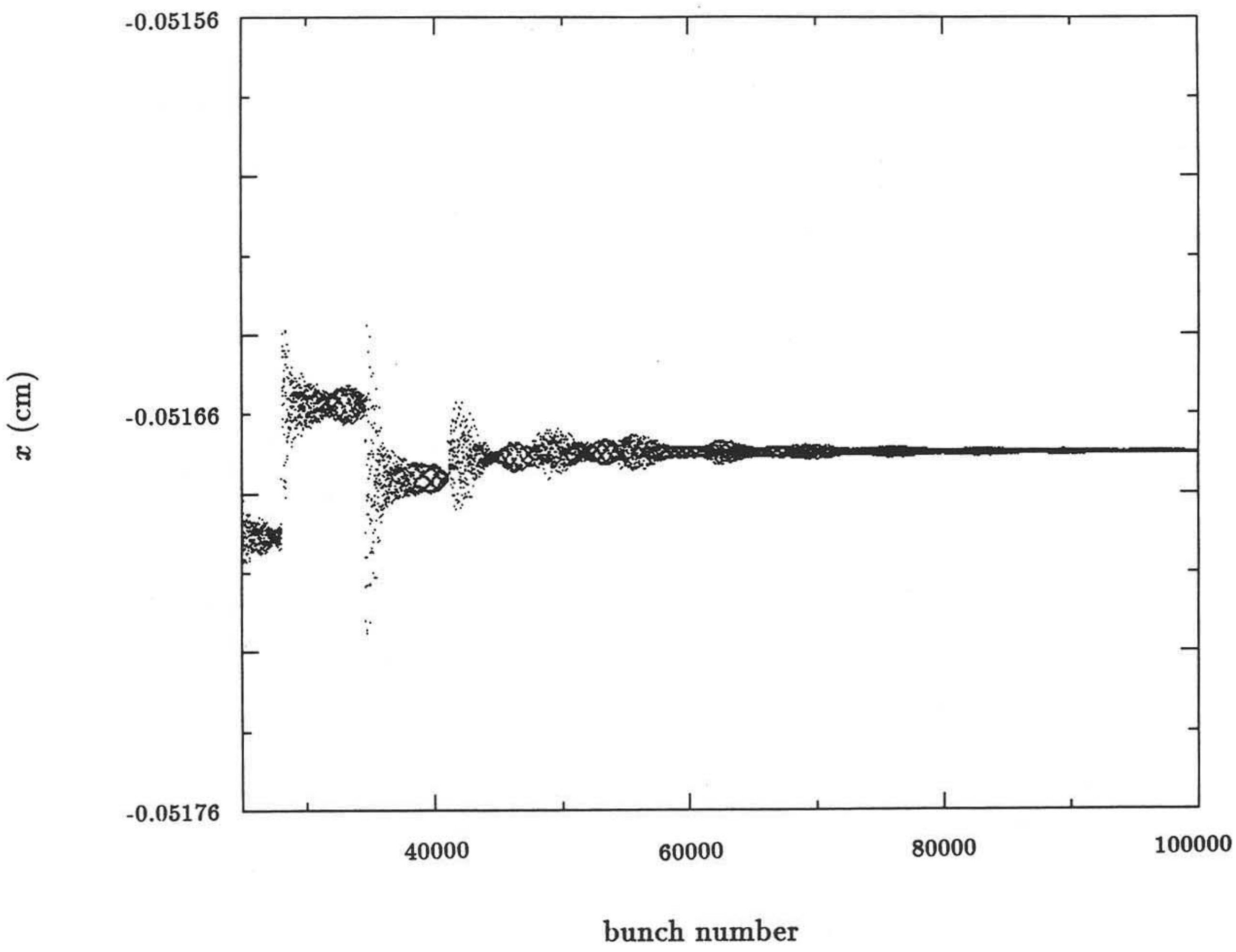
- Short bunch simulation of Multibunch BBU assuming multiple cavity deflecting modes
- Multipass accelerators may be simulated
- Accelerators with several linac segments may be simulated
- Accelerators with accelerating passes and decelerating passes may be simulated
- Simulations include effects of differing path lengths from differing linac segments
- The current in successive bunches may be varied in a programmed manner

One iteration of code corresponds to one fundamental RF period

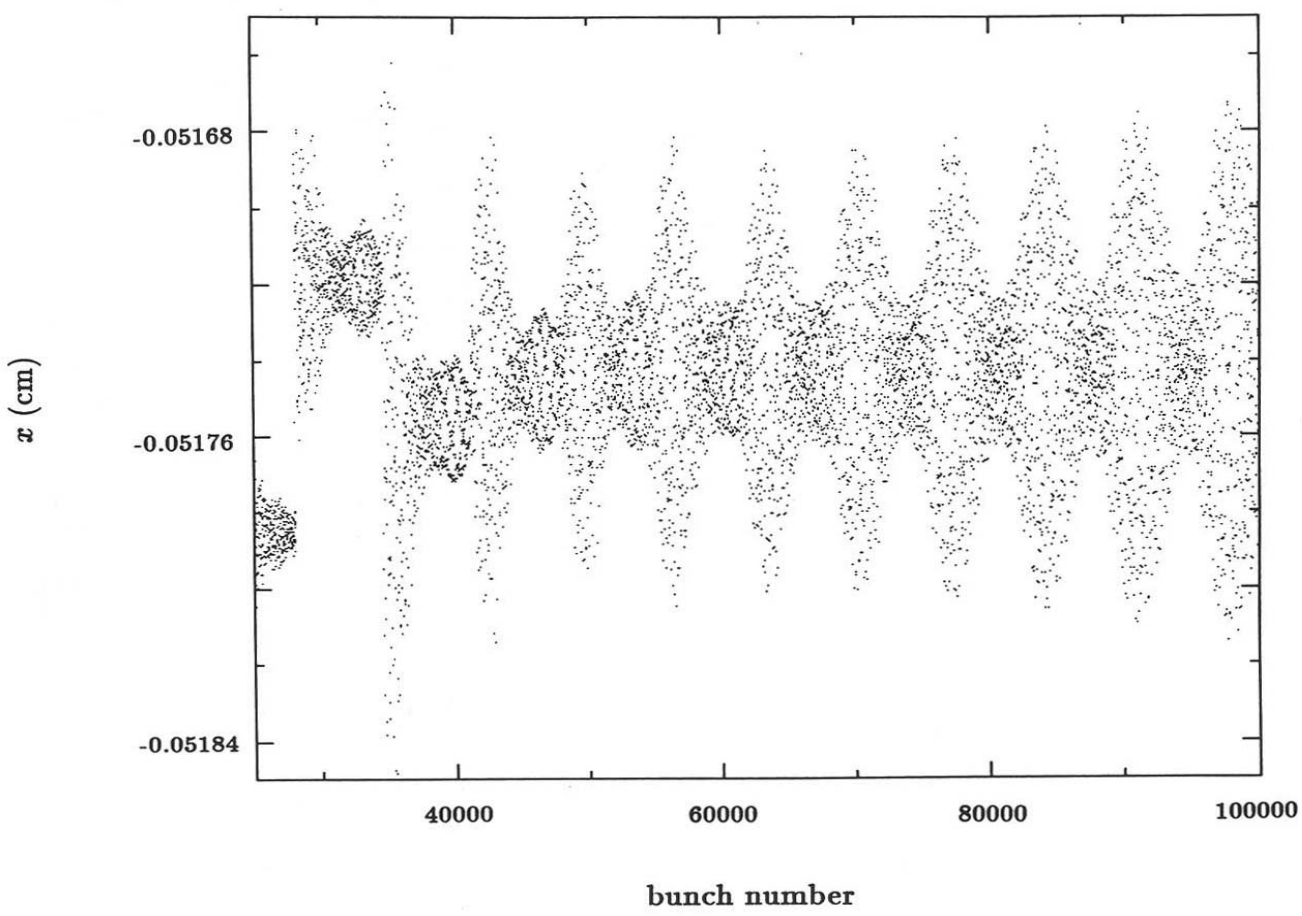
TDBBU Simulation Parameters

f (MHz)	polarization	$R/Q\,(\Omega)$	Q
1890	x, y	25.0	32000
1969	x, y	54.2	4000
2086	x, y	14.7	10000
2110	x, y	28.7	13000

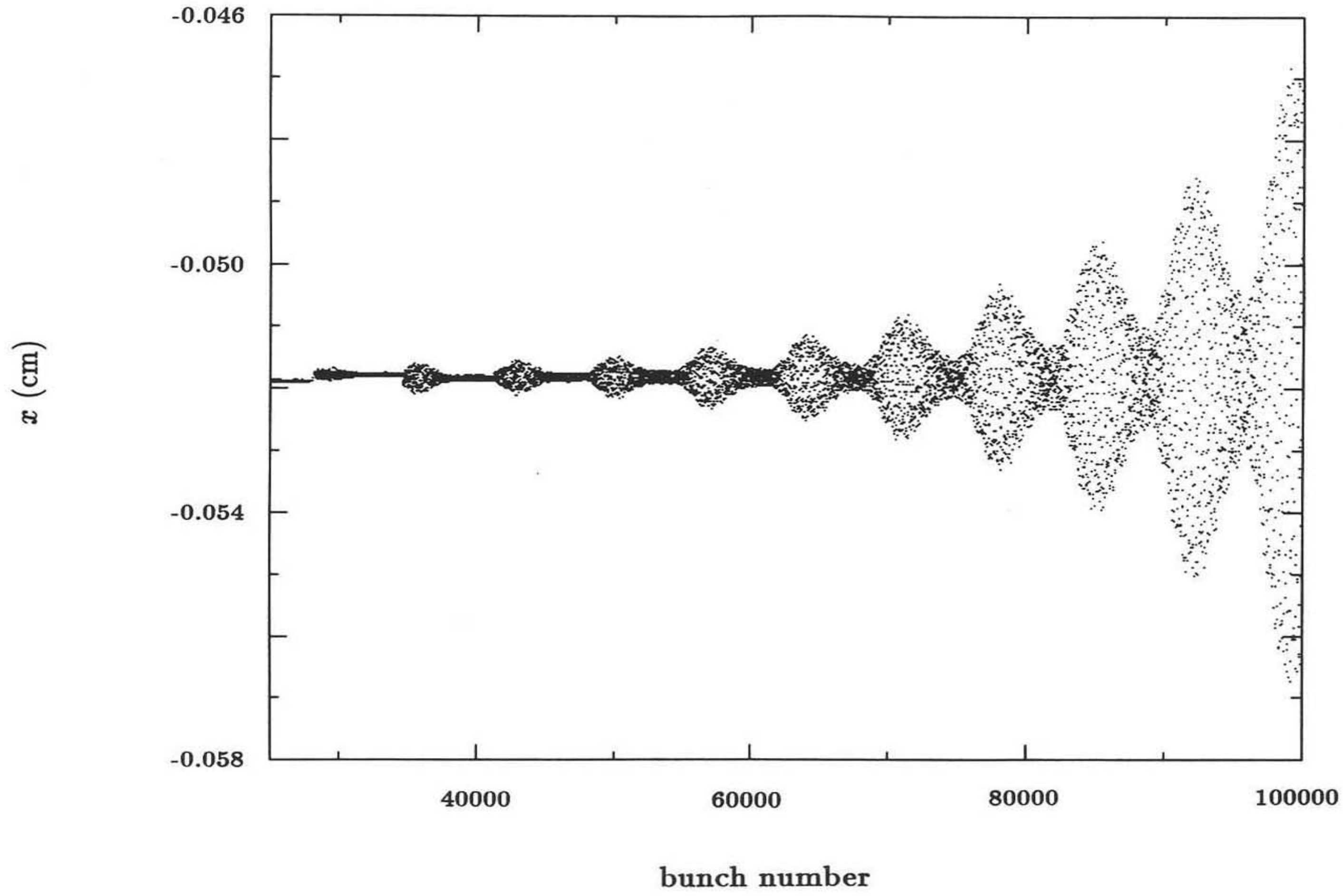
10 mA, Below Threshold



20 mA, Just Beyond Theshold



30 mA, Above Threshold



Randomization of HOM Frequencies

Threshold current depends on detailed HOM frequency choices. For flat 1 MHz HOM frequency distribution get following threshold current table.

Seed	2 GeV	4 GeV
1	11 mA	21 mA
2	13 mA	22 mA
3	12 mA	22 mA
4	14 mA	19 mA
5	12 mA	24 mA

G. A. Krafft and J. J. Bisognano, PAC 1989, 1356

Effect of First Pass Rotation

Rand and Smith proposed starting current may be increased by rotation.

Seed	2 GeV	4 GeV
1	22 mA	34 mA
2	23 mA	36 mA
3	24 mA	38 mA
4	21 mA	33 mA
5	21 mA	38 mA

G. A. Krafft and J. J. Bisognano, PAC 1989, 1356

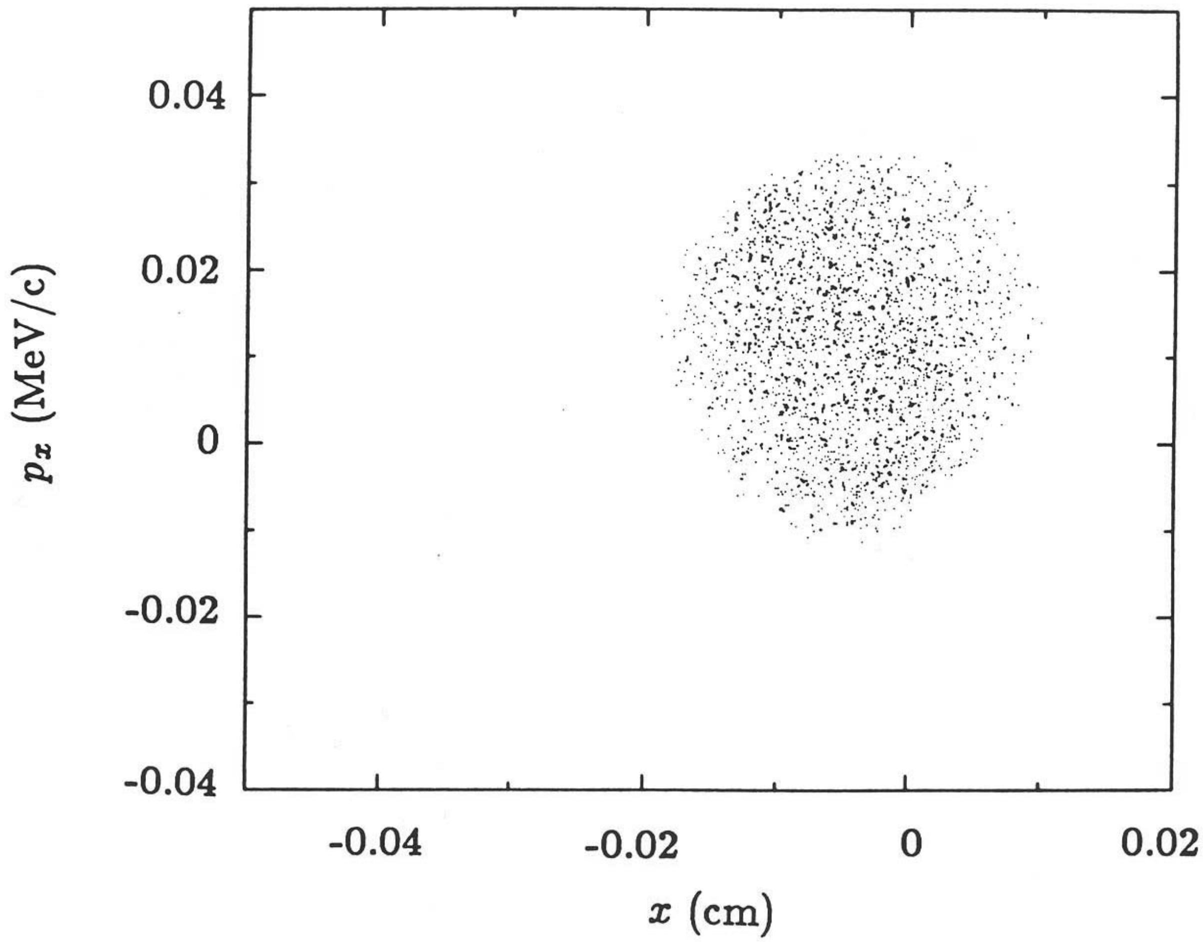
Single Bunch Simulations

Can do "single bunch" calculation using same simulation algorithm by assuming that fundamental period of the simulation is a small fraction of the bunch length (this approximation is OK because we have very little longitudinal motion in a CEBAF-type machine!)

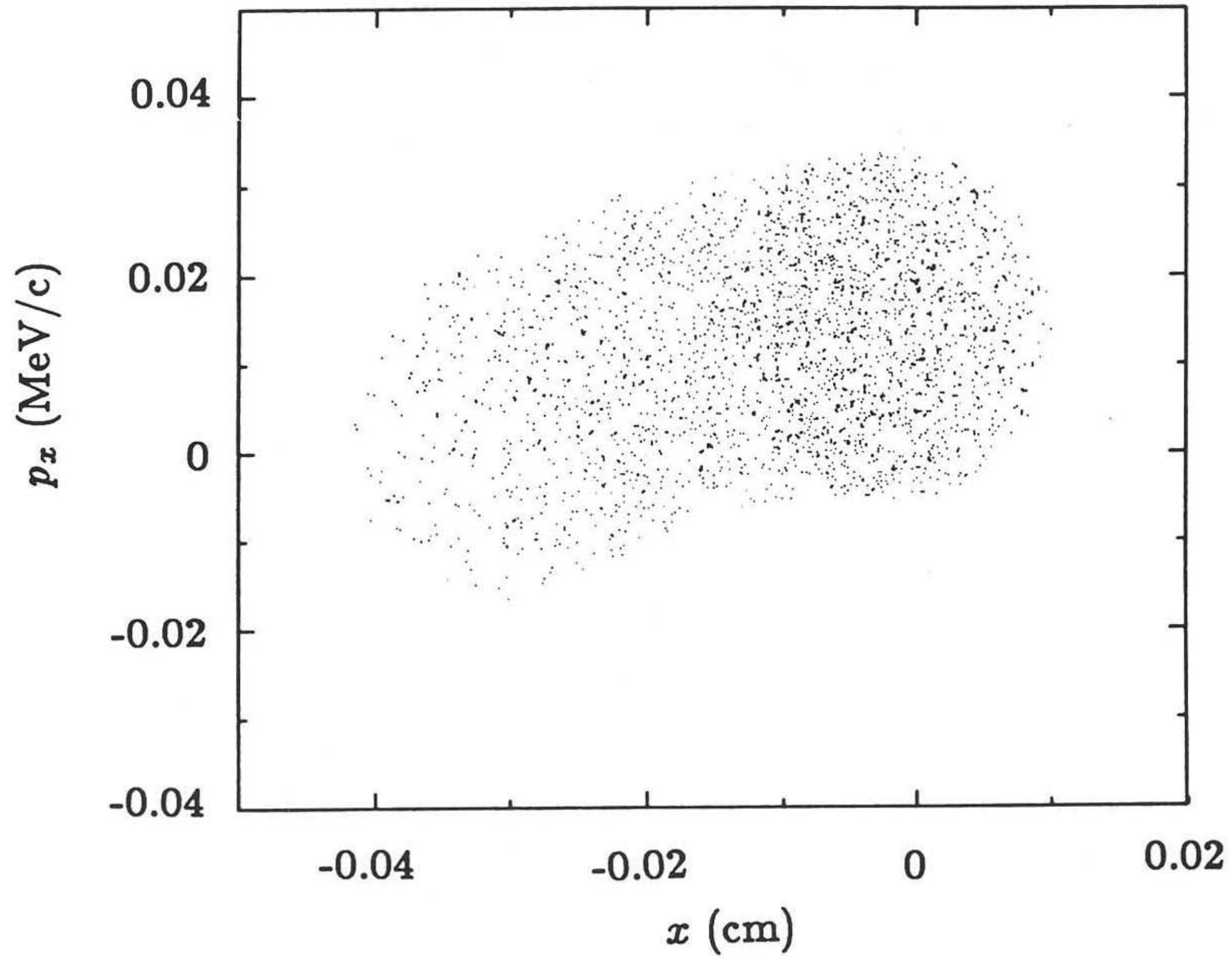
Simulation Parameters for CEBAF

Emittance	1	mm mrad
Transverse Wake Slope	30	V/pc cm^2 per cavity
Longitudinal Wake	41	V/pC per cavity
Bunch Length	2.2	psec

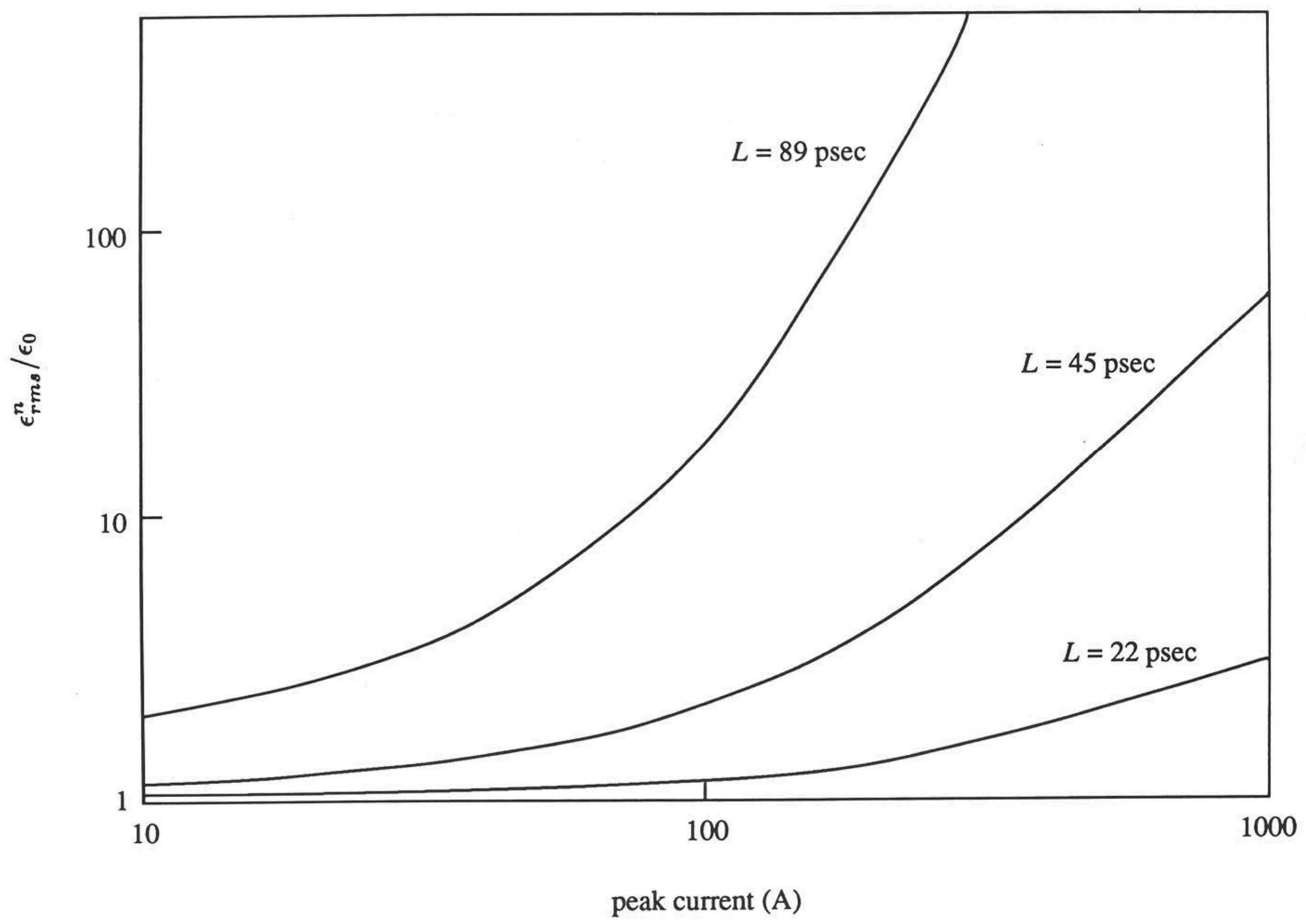
Phase Space at end I=0



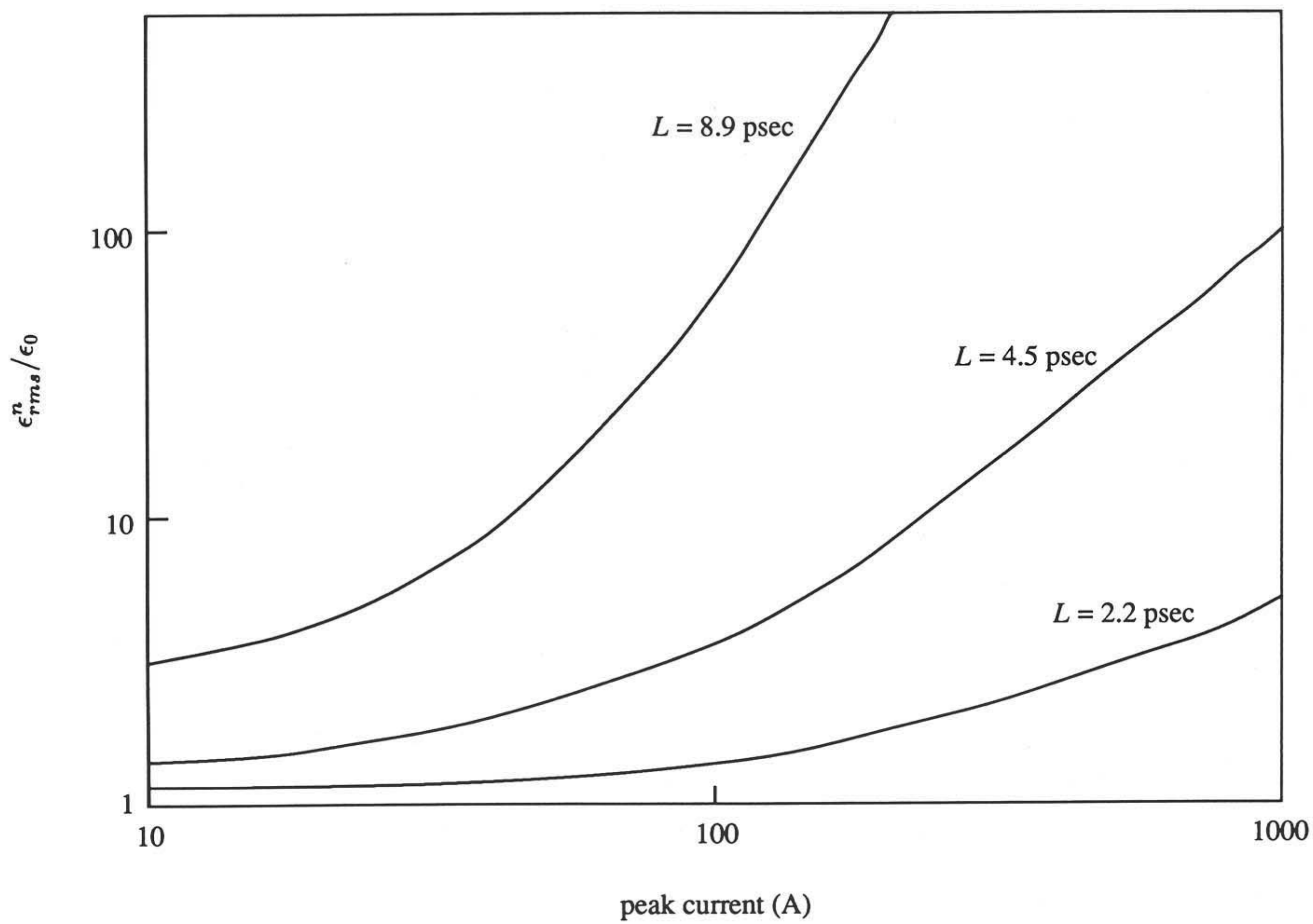
Phase Space At End



Emittance vs. Peak Current



Emittance vs. Current Including BNS Damping



Studies of the regenerative BBU at the JLab FEL Upgrade

Eduard Pozdeyev, Chris Tennant

HOM Energy Equation

At the equilibrium, the stored HOM energy does not change (dU/dt=0)

$$\frac{dU}{dt} = -\frac{V_a^2}{a^2}\left(I_b \frac{m_{12}}{V_b}\frac{c}{\omega}\frac{\sin(\omega T_r)}{2} + \frac{1}{(\omega/c)^2\left(\frac{R}{Q}\right)Q_L} \right)$$

The formula yields two regions:

$m_{12}\sin(\omega T_r)<0$ – unstable

$m_{12}\sin(\omega T_r)>0$ – "pseudo" -stable

$$I_{th} = -\frac{2V_b}{(\omega/c)\left(\frac{R}{Q}\right)Q_L\, m_{12}\sin(\omega T_r)}$$

(Thorough analysis by J. Bisognano, G. Krafft, S. Laubach,1987 Hoffstaetter, Bazarov, 2004)

Two dimensional case (single mode)

Single mode, two-pass recirculator,
arbitrary m(4x4), arbitrary mode polarization α

$$x \to \vec{d} \cdot \vec{n} = x\cos(\alpha) + y\sin(\alpha)$$

$$\boxed{I_{th} = -\frac{2V_b}{(\omega / c)\left(\frac{R}{Q}\right) Q_L\, m^* \sin(\omega T_r)}}$$

$$\boxed{m^* = m_{12}\cos^2(\alpha) + (m_{14} + m_{32})\sin(\alpha)\cos(\alpha) + m_{34}\sin^2(\alpha)}$$

(Pozdeyev, 2004)

Two dimensional case (degenerate modes)

Two degenerate dipole modes polarized in x and y.

$$M\ (4\times 4) = \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$$

for $M_{14}\,M_{32} > 0$

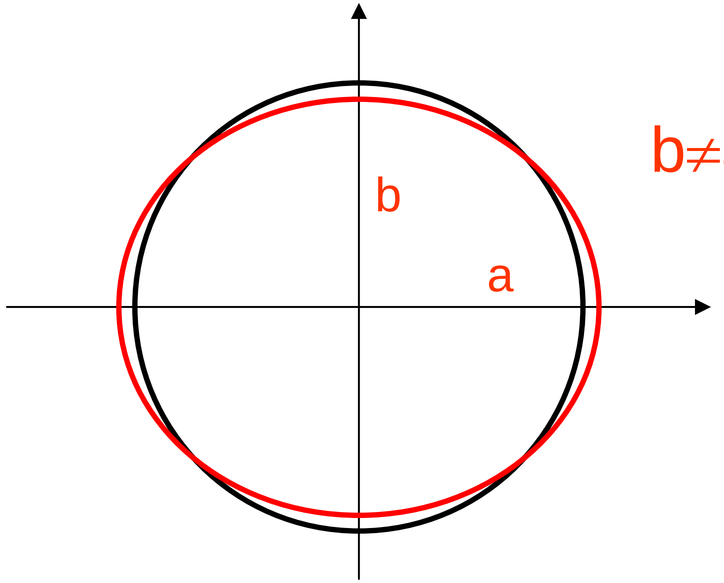
$$I_{th} = \frac{2E\,\omega\exp\left(-\dfrac{\omega\, t_\tau}{2Q}\right)}{ec\left(\dfrac{Z''T^2}{Q}\right)Q\sqrt{M_{14}\,M_{32}}\left|\sin\omega\, t_\tau\right|}$$

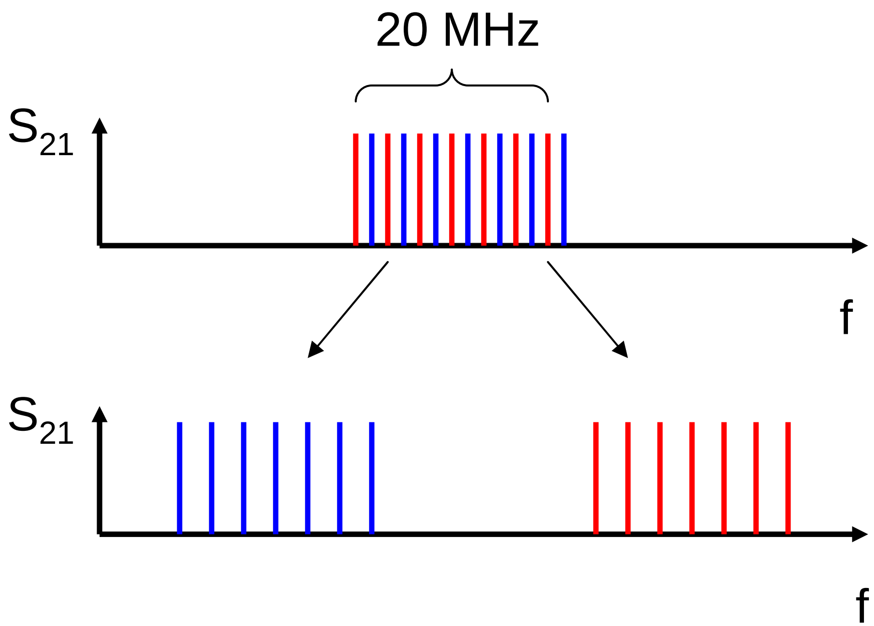
$M_{14}\,M_{32} < 0$

$$I_{th} = \frac{2E\,\omega\exp\left(-\dfrac{\omega\, t_\tau}{2Q}\right)}{ec\left(\dfrac{Z''T^2}{Q}\right)Q\sqrt{-M_{14}\,M_{32}}\left|\cos\omega\, t_\tau\right|}$$

(B. Yunn, 2005)

Splitting degenerate modes for effective BBU suppression by 90°-rotation/reflection





Frequency separation can be estimated using formula for a square cavity

$$\frac{\delta f}{f} = \frac{6}{5}\frac{\delta d}{d}$$

where $\pm\delta d$ is the variation of the cavity transverse size

Voltage evolution above and below I_{th}

$$\frac{V_a^2}{a^2} = \omega\left(\frac{\omega}{c}\right)^2\left(\frac{R}{Q}\right)U$$

$$\boxed{\frac{dU}{U} = -dt\,\frac{\omega}{Q_L}\frac{I_{th}-I}{I_{th}}}$$

$$\boxed{U = U_0 \exp\left(-t\,\frac{\omega}{Q_L}\frac{I_{th}-I_b}{I_{th}}\right)} \qquad \boxed{V = V_0 \exp\left(-t\,\frac{\omega}{2Q_L}\frac{I_{th}-I_b}{I_{th}}\right)}$$

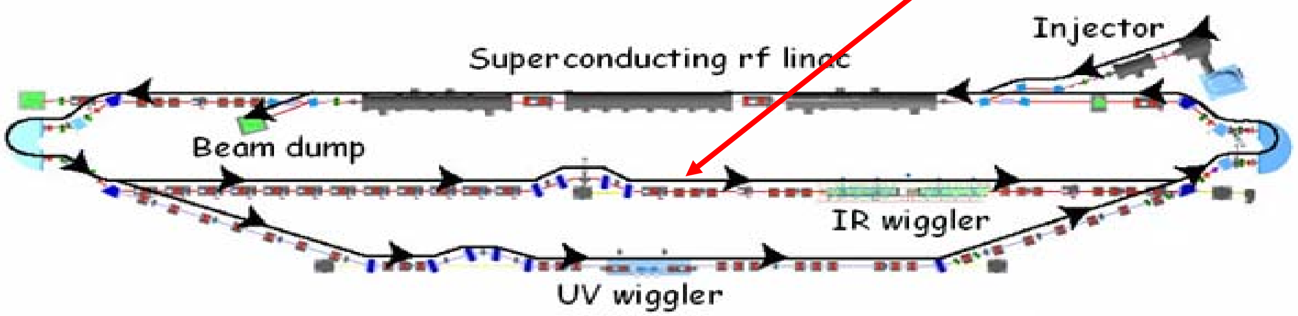
The system HOM+beam can be described by the effective quality factor:

$$\boxed{Q_{eff} = Q_L\,\frac{I_{th}}{I_{th}-I}} \quad \Leftrightarrow \quad \boxed{\tau_{eff} = \tau_0\,\frac{I_{th}}{I_{th}-I}}$$

JLab FEL Upgrade

Energy(MeV)	80-200
Charge per bunch (pC)	135
Bunch rep.rate (MHz)	4-75
Average current (mA)	10
Laser power (kW)	10

Cavities of Zone 3 have higher accel. gradient than Zone 2,4. The Q of dipole HOMs is also higher. HOMs of Zone 3 impose BBU limit.



IR wiggler

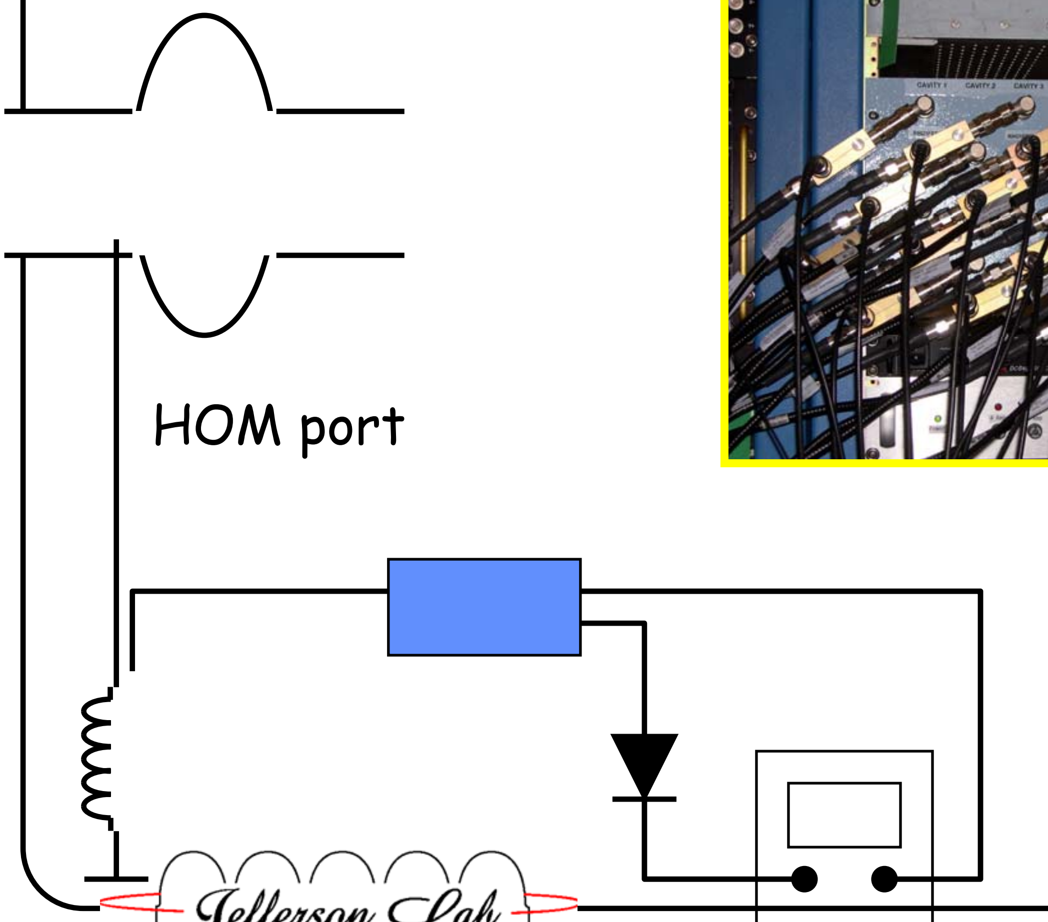
Questions we tried to answer

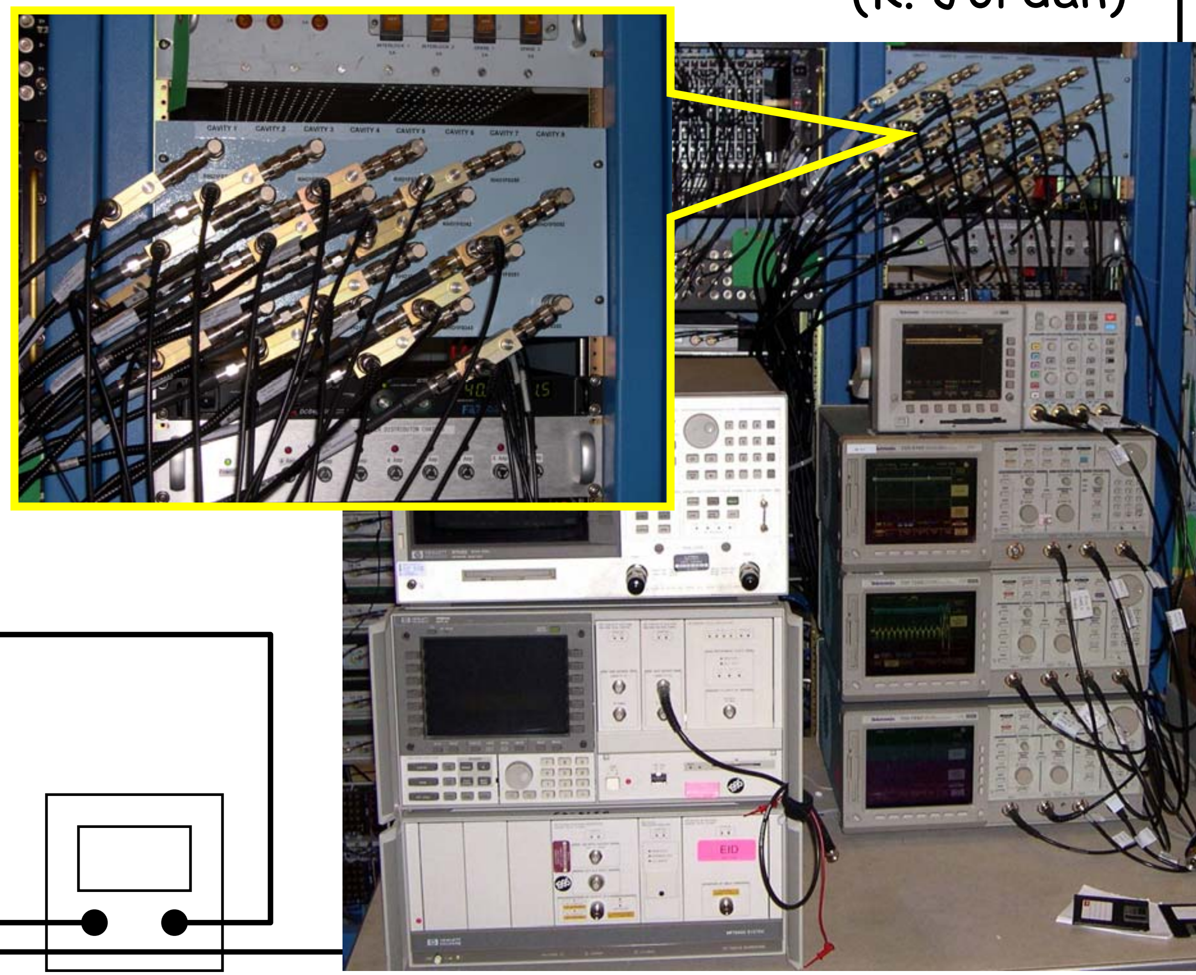
- How well do the model and simulations describe the BBU and the beam behavior
- Can we experimentally measure (predict) the BBU threshold doing measurements below the threshold
- Can we suppress BBU (C. Tennant, next talk)

Direct observation of the BBU threshold

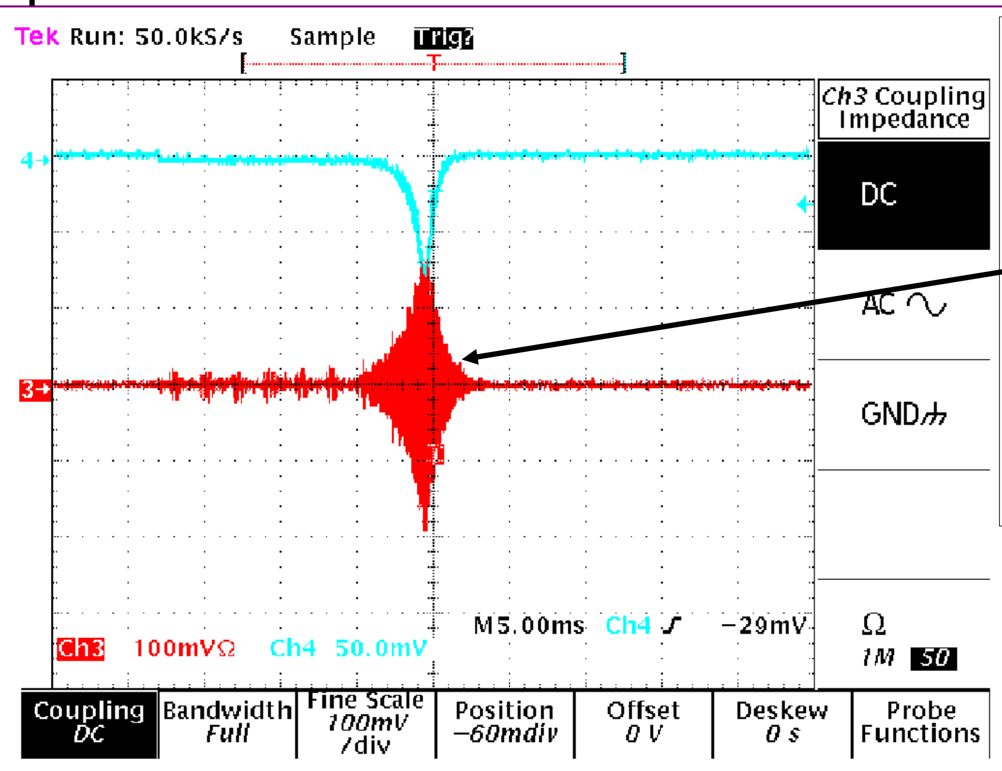
Schottky diodes where used to measure HOM power from the HOM ports.

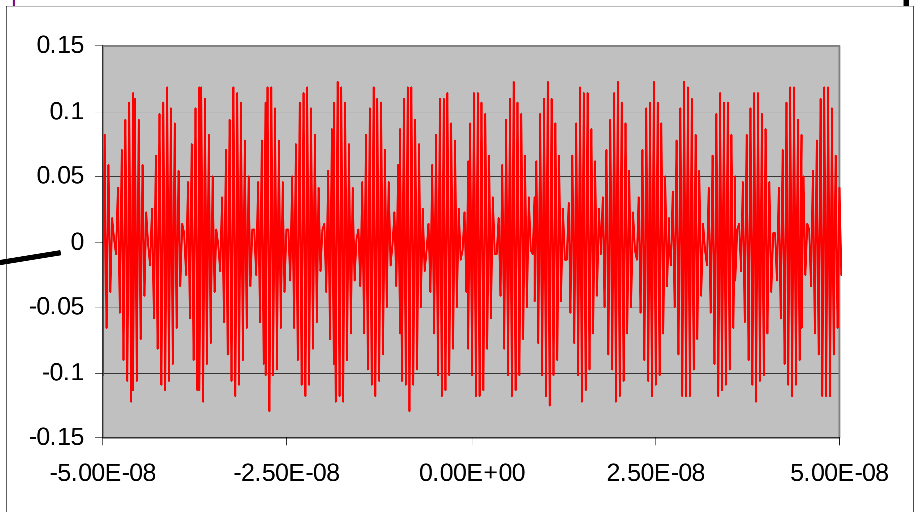
(K. Jordan)





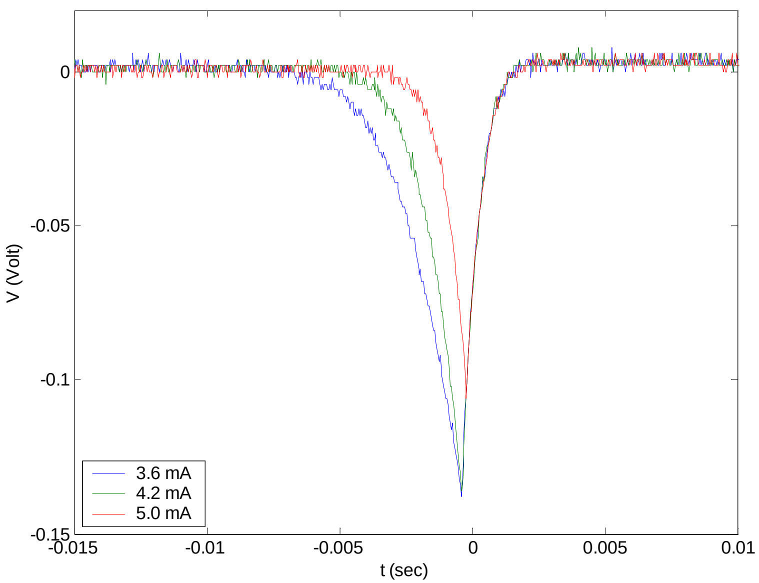
Direct observation of the BBU threshold



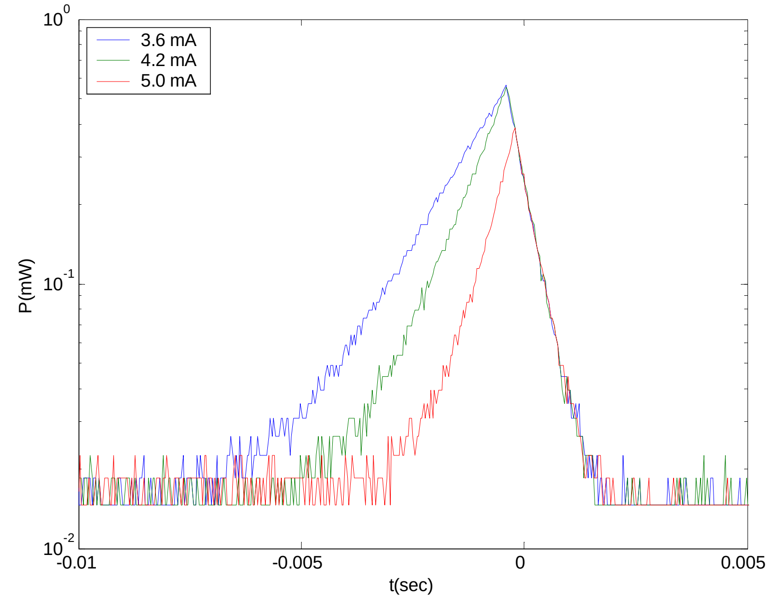


Cav 7, F_{hom}=2106 MHz, Ith=2.7 mA

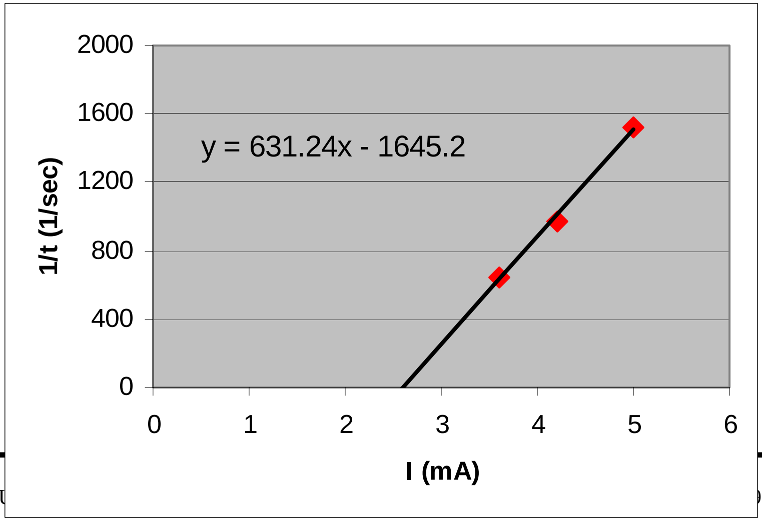
HOM voltage growth rate measurements



invert
+
adjust
+
log

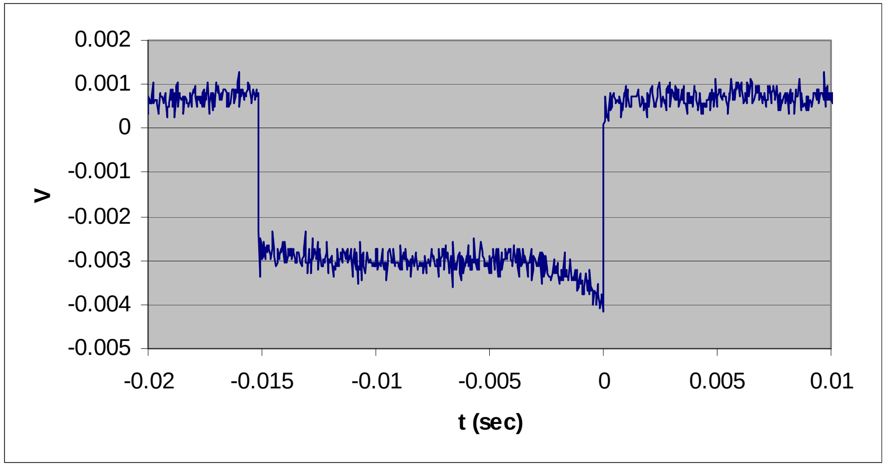


$$\tau_{eff} = \tau_0 \frac{I_{th}}{I_{th} - I}$$



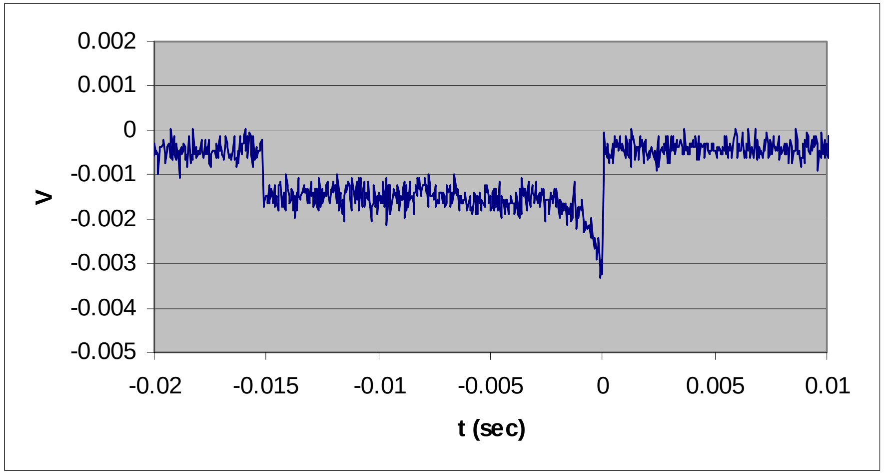
Cav 7, F_{hom}=2106 MHz,
Ith=2.61 mA

What about other HOMs?



I=5mA

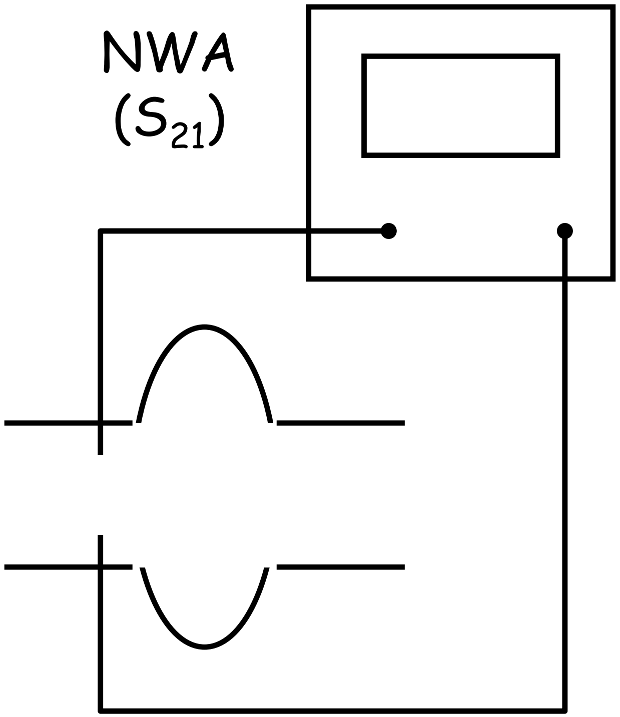
Cav. 3, F=1786.206
BTF measurements: the HOM is very far from the threshold (BTF-predicted I_{th}=34 mA)



Cav. 8, F=1881.481
BTF measurements inconclusive. Cross-talk prevented us from taking accurate BTF data.

We are not sure what causes this voltage rise

Beam Transfer Function (BTF) measurements



Measuring Q(I) for several beam current values and using the formula

$$Q_{eff} = Q_L \frac{I_{th}}{I_{th} - I}$$

one can predict the BBU threshold below the threshold.

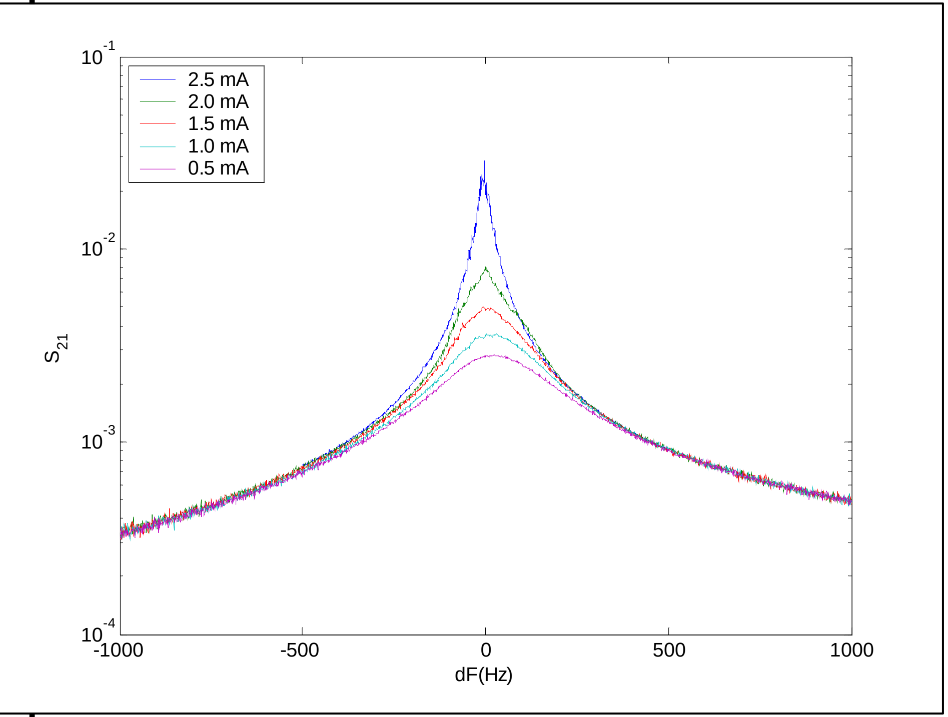
Port-to-port BTF:

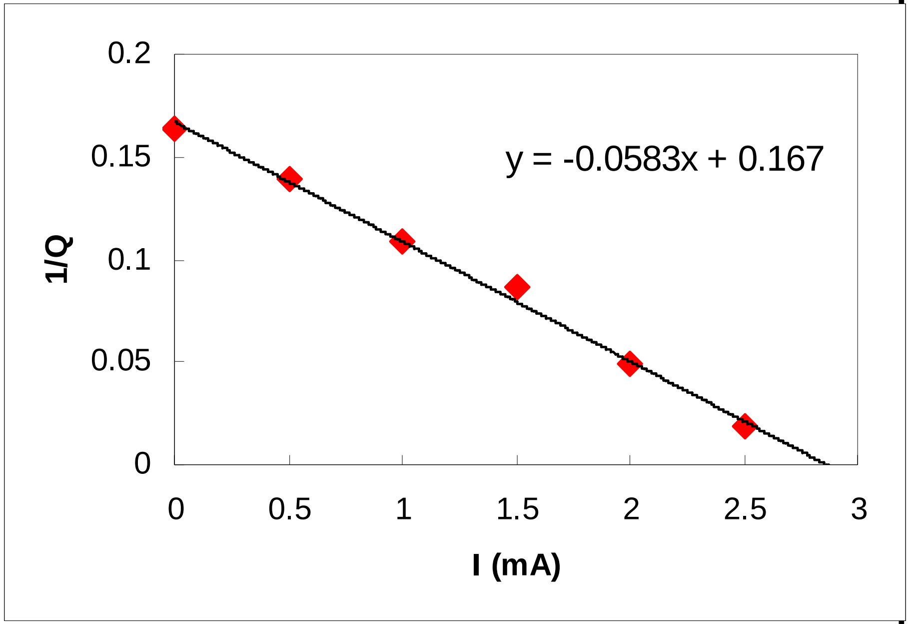
+'s: 1) stronger signal
2) no need for RF amplifier
3) no need for kicker

-'s: cross-talk can complicate Q-measurements

Beam Transfer Function (BTF) measurements

Cav 7, F_{hom}=2106 MHz

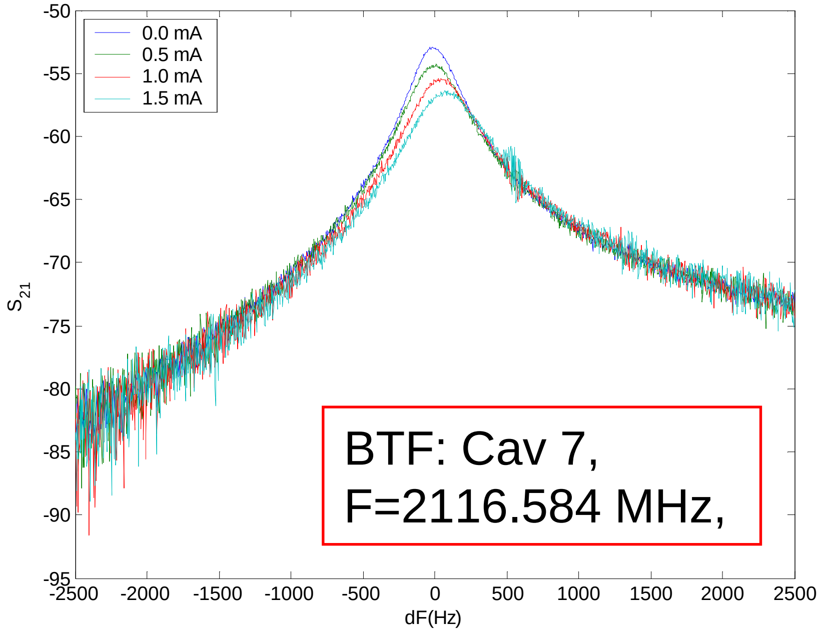


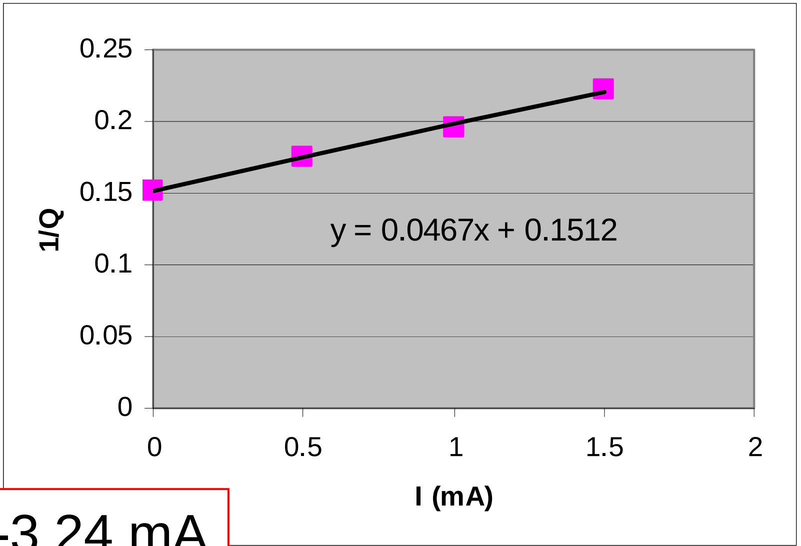


Projected threshold current is 2.86 mA

The “pseudo”-stable region ($m_{12}\sin(\omega T_r)>0$)

$$Q_{eff} = Q_L \frac{I_{th}}{I_{th} - I} = Q_L \frac{-|I_{th}|}{-|I_{th}| - I} = Q_L \frac{|I_{th}|}{|I_{th}| + I}$$





I_{th}=-3.24 mA

For $m_{12}\sin(\omega T_r)>0$, BBU still can happen at very high currents (~10A).
(J. Bisognano, G. Krafft, S. Laubach (1987), Hoffstaetter, Bazarov (2004))

Comparison to simulations and the threshold formula

May 2004: TDBBU, MATBBU, ERLBBU simulations:
Simulated threshold 2.7 mA, Measured threshold 2.5 mA

Dave Douglas' optics file (Nov.2004) with "All Save" quadrupole values

			Formula	Measured
Cavity	f_{hom}(mA)	Orientation	I_{th} (mA)	I_{th} (mA)
7	2106	Y	2.5	2.7
7	2116.58	Y	-3.1	-3.1
4	2114.15	X	-27	-9.5
3	1786.2	X	156	34

(C. Tennant)

Conclusions and Plans

- The dipole HOM in Zone 3 Cav. 7 with F=2106 had the lowest BBU threshold in the machine (2.7 mA).
- Behavior of the HOM+beam system can be described by the effective quality factor, given by:

$$\boxed{Q_{eff} = Q_L \frac{I_{th}}{I_{th} - I}}$$

(This formula can fail for extremely high currents or/and larger accelerators)

- Measuring the Q as a function of current (BTF) below the threshold and measuring the rise time above the threshold, we were able to accurately predict the threshold.

Conclusions and Plans

- Programs TDBBU, MATBBU, and ERLBBU accurately predicted the threshold in the JLab FEL Upgrade. More work is needed for accurate comparison of the experimental data to simulations.

- Measurement of HOM polarization and betatron coupling is required for accurate comparison of the experimental data with simulations and theory. Interesting modes are Cav.7 f=2106, Cav.7 f=2116.584

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CONCLUSIONS

- Described Multipass Multibunch BBU Instability
- Described Analytical Techniques and Simulation Techniques for studying this instability
- Discussed Some Recent Measurements at the Jefferson Lab FEL on this Instability
- Reviewed where we are in understanding this instability