Homework Problems IV

1. Chicanes can be used in ERLs to achieve two different goals: bunch compression and path length adjustment. Let ρ be the radius of the trajectory in the dipoles, L be the dipole length, a be the bending angle in the dipoles, λ be the spacing between the first and the second, and the third and the fourth dipoles, and d be the distance between the second and the third magnets. Such a chicane is an achromat due to its symmetry. Find expressions for **a**) the dispersion function in the middle of chicane and **b**) the momentum compaction ($R_{56}=\partial l/\partial\delta$, here l and δ are the path length difference between a trajectory and the reference particle trajectory and the relative momentum difference respectively). **c**) Design a chicane for path length adjustment corresponding to +/- 10º RF phase (frequency 1.3 GHz) at 5 GeV beam energy. What is the corresponding range of magnetic field and R_{56} for such a chicane?

2. Longitudinal phase space (δ, l) can often be represented as

$$\delta(l)=\delta_0+\left.\frac{\partial\delta}{\partial l}\right|_{l=0} l+\frac{1}{2!}\left.\frac{\partial^2\delta}{\partial l^2}\right|_{l=0} l^2$$

where δ_0 is uncorrelated energy spread, the first partial derivative represents the δ - l linear correlation coefficient, while the second derivative represents quadratic correlation or "curvature". Retaining only these two most significant terms, calculate energy spread and longitudinal emittance after 100 MeV linac for on-crest, 10º, and 20º RF phase, assuming that initially bunch had no δ - l correlation, 10 MeV beam energy, 20 keV uncorrelated energy spread, and the bunch length of 1º RF (all rms values). What is the

shortest bunch length achievable after compression for 10º and 20º RF phase? What is the corresponding R_{56} required in each case?

3. Design a beam dump for ERL (hint: follow the Cornell ERL prototype sketch). Use the parameters of the ERL prototype. The distance from quadrupole to collector is 8 m. Assume collecting angle is tilted 6°. What is quadrupole field strength and the beam spot size near the collector? What is the required water flow to cool the collector (use a water temperature difference of 20 °C)?

4. Consider the ERL prototype with injection kinetic energy of 5 MeV. At this energy the bunch moves with the speed $\beta =$ 0.9957. The SRF structure in the linac is optimized for $\beta = 1$. **a**) Calculate phase mismatch in 100 MeV linac for injected and energy recovered beams assuming a 5 m long linac. **b**) If linac consists of 5 cavities (1 m each) with independent phasing, what is the phase mismatch in the first / last cavites?