Mid Term Examination
Physics 854
Thursday, October 19, 2017

1. Consider a relativistic electron circulating in a betatron at radius *R* with field index $n = 0.6$.
   a. If the electron makes 10 revolutions, how many radial and vertical oscillations does it make?
   b. What is the dispersion function at all locations in the betatron?
   c. Suppose a particle has a relative momentum error of 10^{-3}. What is the additional path length per turn?
   d. Is this result consistent with the formula

$$M_{56} = \int \frac{D(s)}{\rho(s)} ds ?$$

2. Suppose a periodic focusing channel has focusing as in Figure 1, where $\psi = \sqrt{K}L = \pi/4$.
   a. What is the (thick lens!) phase advance per full cell?
   b. What are $\beta(s=0)$, $\beta(s=L/2)$, $\beta(s=L)$, and $\beta(s=3L/2)$, expressed in units of L (e.g. 2.34 L)?

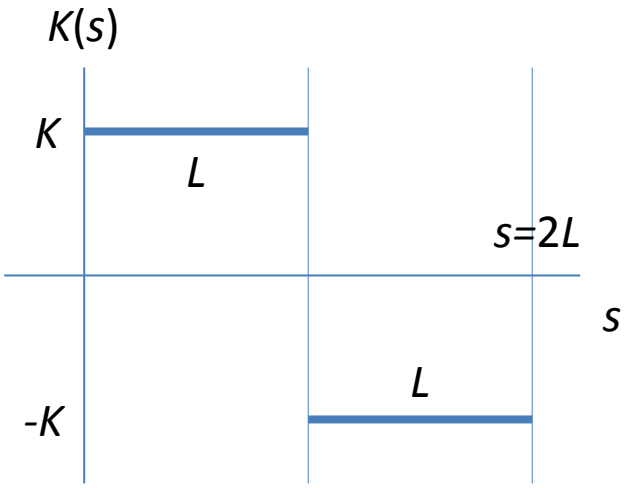

Figure 1: Periodic Focusing Function for Problem 2

3. Consider the following matrix

$$M = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}.$$

a. Is it unimodular?
b. What is the phase advance of this matrix?
c. What are the $\alpha, \beta,$ and γ (Twiss parameters) for this matrix?
d. What are M^4, M^{100}, M^{103}, M^{104}, and M^{105}, i. e., what are the individual matrix elements of these powers of *M*?

4. Recall our expressions from the homework for the transfer matrix of a FODO system. The one-period transfer matrix starting with the middle of the focusing magnet was

$$M_f = \begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1-L^2/(2f^2) & 2L+L^2/f \\ -L/(2f^2)+L^2/(4f^3) & 1-L^2/(2f^2) \end{pmatrix}.$$

and M_d is obtained from M_f by replacing f with $-f$

a. How should one choose *f*, in terms of *L*, so that the phase advance through one period of the FODO system is 90 degrees (corresponding to 1/4 of a transverse oscillation per period)?
b. For the matched phase space (x, x') ellipse in the 90 degree phase advance system, what is the beta-function in the middle of the focusing lens?
c. For the matched phase space ellipse in the 90 degree phase advance system, what is the beta-function in the middle of the defocusing lens?
d. What are the alpha-functions for the matched ellipses at these same two locations?
e. Suppose *L* is 3 m, and the area of a matched phase space ellipse is $\pi\varepsilon = 2\pi \times 10^{-6}$ m radian, what are the maximum extents (x_{max}) of the matched ellipses in the focusing and defocusing lenses of the 90 degrees phase advance system?