Homework Problems VI
Physics 854
Accelerator Physics

1. Assume a 1 A 100 GeV proton beam is in an accelerator with transition energy 50 GeV. What is the instability growth time for a space charge negative mass instability. Assume the beam size is 1 cm, the pipe radius is 3 cm, and the revolution frequency is 100 kHz. How much energy spread is needed to stabilize the instability?

2. The Argonne Lab Advanced Photon Source (APS) has an electron storage ring that operates at 7 GeV. Using the FEL/undulator resonance condition, and the fact that APS "Undulator-A" designs can operate with K between 0.1 and 2, depending on the pole-gap size, and the undulator period of 2.3 cm, to estimate the range of wavelengths that can be produced from such undulators installed as insertion devices in the APS ring. Suppose one wanted to produce 2 THz (= 2.0×10^{12} sec^{-1}) electromagnetic radiation from an undulator with the same period and $K=1$. How much electron beam energy is needed?

3. From $d\tau = dt/\gamma$, and the expression for the Lorentz-invariant power
$$P = -\frac{q^2}{6\pi\varepsilon_0 c}\frac{du^\mu}{d\tau}\frac{du_\mu}{d\tau}$$
show the Lienard expression applies
$$P(t) = \frac{q^2}{6\pi\varepsilon_0 c}\gamma^6\left(\dot{\vec{\beta}}^2 - \left[\vec{\beta}\times\dot{\vec{\beta}}\right]^2\right).$$

4. Two exact integrals involving the modified Bessel function $K_{5/3}$ are
$$\int_0^\infty \xi'^2 K_{5/3}(\xi')\,d\xi' = \frac{16\pi}{9\sqrt{3}}$$
$$\int_0^\infty \xi' K_{5/3}(\xi')\,d\xi' = \frac{5\pi}{3}.$$
From these two formulas show
$$P = \int_0^\infty \frac{dP}{d\omega}d\omega = \frac{\sqrt{3}}{8\pi^2\varepsilon_0}\frac{e^2}{\rho}\omega_c\gamma\int_0^\infty \xi\int_\xi^\infty K_{5/3}(x)\,dx\,d\xi = \frac{e^2 c}{6\pi\varepsilon_0\rho^2}\gamma^4$$
and
$$\langle\hbar\omega\rangle = \frac{8}{15\sqrt{3}}\hbar\omega_c.$$
Hint: Apply Fubini's Theorem.