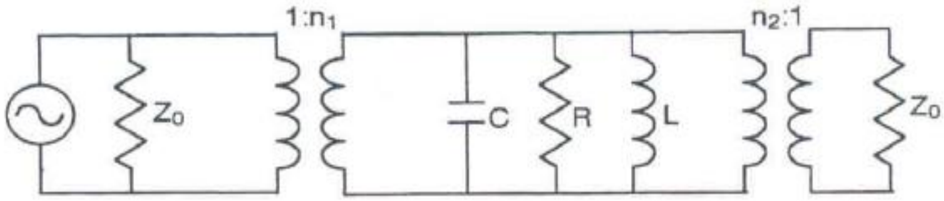
Homework Problems V
Physics 854 Accelerator Physics
Due November 16, 2017

1. Assume a 2-port cavity with coupling coefficients β_1 (input) and β_2 (output).



   a. Calculate the dissipated, transmitted, and reflected power for a given incident power.
   b. What happens if we interchange input and output?
   c. What happens when $\beta_1 = \beta_2$?

2. A single cell 1.5 GHz superconducting cavity has an accelerating mode with $R/Q = 70\ \Omega$, $Q_0 = 2.5\times10^{10}$, $Q_{\mathrm{ext}} = 5\times10^8$, and is operating at $V_c = 5$ MV.

   a. Calculate the stored energy in the cavity.
   b. Calculate the power dissipated in the fundamental operating mode.
   c. Calculate the loaded Q (Q_L) and the loaded bandwidth (FWHM).
   d. Calculate the generator power with no beam loading and for a beam current of 0.1 mA with no detuning.
   e. If microphonics noise affects the cavity resonance frequency and peak detuning by 10 Hz, calculate the generator power for the same beam current of 0.1 mA.

3. Normalize, and compute the *rms* emittance of the following distributions:

Gaussian
$$f\left(x, x'\right) = A\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{x'^2}{2\sigma_{x'}^2}\right)$$

Waterbag
$$f\left(x, x'\right) = A\Theta\left(1 - \frac{x^2}{\Delta x^2} - \frac{x'^2}{\Delta x'^2}\right)$$

K-V, or microcanonical
$$f\left(x, x'\right) = A\delta\left(1 - \frac{x^2}{\Delta x^2} - \frac{x'^2}{\Delta x'^2}\right)$$

Klimontovich $$f\left(x,x'\right)=A\sum_{i=1}^{N}\delta\left(x-x_i\right)\delta\left(x'-x'_i\right)$$

Treat σ_x, $\sigma_{x'}$, Δx, $\Delta x'$, x_i, x'_i as parameters. Θ Unit step, δ Dirac's delta

For distributions (1)-(3), what does the projected distribution, e.g., $p(x)=\int f(x,x')dx'$ look like?

4. Normalize the Gaussian-elliptical phase space distribution
$$\rho\left(x,x'\right)=A\exp\left(-\left(\gamma x^2+2\alpha xx'+\beta x'^2\right)/2\varepsilon\right)$$
assuming $\beta\gamma-\alpha^2=1$. Show the statistical average definitions of $\alpha,\beta,$ and γ evaluate to exactly the correct values for this distribution, and $\varepsilon_{rms}=\varepsilon$.