Homework Problems III
PHYS 854: Accelerator Physics

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1. Problem 10.3: Consider a ring composed of an even number of $2n_c$ of FODO cells. To provide two component free spaces, we cut the ring at a symmetry line through the middle of two quadrupoles on opposite sides of the ring and insert a drift space of length $2l$ which is assumed to be much shorter than the value of the betatron function at this symmetry point $l \ll \beta_0$. Derive the transformation matrix for this ring and compare with that of the unperturbed ring. What is the tune change of the accelerator? The betatron functions will be modified. Derive the new value of the horizontal betatron function at the symmetry point in units of the unperturbed betatron function. Is there a difference to whether the free section is inserted in the middle of a focusing or defocusing quadrupole? How does the D-function change?

2. Problem 10.4: Sometimes two FODO channels of different parameters must be matched. Show that a lattice section can be designed with the phase advance of $\Delta\psi_x = \Delta\psi_y = \pi/2$, which will provide the desired matching of the betatron functions from the symmetry point of one FODO cell the the symmetry point of the other cells. Such a matching section is also called a quarter wavelength transformer and is applicable to any matching of symmetry points. Does this transformer also work for curved FODO channels, where the dispersion is finite?

3. Problem 10.6: For one example determine the real quadrupole length required to produce the quoted betatron phase advances per FODO cell in Table 10.1. Compare with thin lens quadrupole strengths.

4. Problem 10.7: Calculate the values of the betatron functions in the center of the quadrupoles for #1 and #2 FODO cells in Table 10.1 and compare with the actual thick lens betatron functions in Figs. 10.6 and 10.7 (The values given are roughly 0.8 m and 4.5 m in Fig. 6 (#1 case) and about 3 m and 21.5 m in Fig. 7 (#2 case)). Discuss the difference.

Table 10.1

Example	#1	#2	#3	#4
Energy, E (GeV)	10	50	4	20,000
Half cell length, L (m)	6.0	2.6	3.6	114.25
Quadrupole length, l_q (m)	0.705	1.243	0.15	3.64
Bending magnet length, l_b (m)	3.55	2.486	2.5	99.24
Phase advance per cell, ψ	101.4	108.0	135.0	90.0
Quadrupole strength, k (m^{-2})	…	…	…	…
Lattice type (FODO)	sf	cf	sf	sf