Homework Problems I
PHYS 854: Accelerator Physics

.

1. Suppose a particle orbit as a function of time $\vec{x}(t)$ is given in an inertial frame *K* and the integral

$$\tau = \int_{t_1}^{t_2} \frac{dt}{\gamma(t)} = \int_{t_1}^{t_2} \sqrt{1-\beta_x^2(t)-\beta_y^2(t)-\beta_z^2(t)}\,dt$$

is evaluated. Using the invariance of the space-time interval, show that if the same calculation is done in a frame *K′* moving uniformly with respect to *K*, then $\tau' = \tau$. In other words, the proper time τ along a particle orbit is a Lorentz invariant quantity.

2. From the relativistic Lorentz Force Equation derive

$$\vec{\mathrm{v}} \cdot \frac{d(\gamma m \vec{\mathrm{v}})}{dt} = q\vec{\mathrm{v}} \cdot \vec{E}.$$

From the usual expression

$$\gamma = \frac{1}{\sqrt{1-\vec{\mathrm{v}} \cdot \vec{\mathrm{v}} / c^2}},$$

show

$$\frac{d(\gamma m c^2)}{dt} = q\vec{E} \cdot \vec{\mathrm{v}}.$$

Therefore, even at relativistic energies, magnetic fields cannot change the particle energy when radiation reaction is neglected. Verify that the relativistic force law, written as $\frac{dp^\alpha}{d\tau} = qF^\alpha{}_\nu u^\nu$ (ν summation implied), where

$$F^\alpha{}_\nu = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix},$$

yields the relativistic Lorentz force equation when evaluated on the space components $\alpha = 1,2,3$.

3. Repeat, using the relativistic equations of motion, the derivation in class of the cyclotron frequency. Show the relativistic cyclotron angular frequency is

$$\Omega_c = \frac{qB}{\gamma m}.$$

Show the radius ρ of the cyclotron motion is

$$\rho = \frac{\beta c}{qB/\gamma m},$$

and conclude

$$B\rho = \frac{p}{q}.$$

4. The so-called East Arcs in the CEBAF electron accelerator at Jefferson Lab consist of five separate beam-lines that operate at different electron beam energies. The rectangular dipoles in each beam-line are identical and all together bend the beam through 180 degrees. More and longer magnets are needed to bend the higher beam energy electrons. When the total electron energy is 5500 MeV, the electron energy in each of the arcs is given in the table. With the information given, calculate the other entries in this table:

Arc	Electron Energy (MeV)	Number of Dipoles	Dipole Length (m)	Bend Angle (rad)	Magnetic Field (T)
1	605	16	1		
2	1693	32	1		
3	2781	32	2		
4	3868	32	3		
5	4956	32	3		

5. Consider a FODO cell, what is the phase advance would be needed to construct a $-I$ transformation matrix out of this cell.