Accelerator Physics
Weak Focusing

G. A. Krafft
Jefferson Lab
Old Dominion University

Lecture 3

Evaluate the constant

$$\delta\ddot{r} + (1-n)\Omega_c^2 \delta r = \Omega_c const + R\Omega_c^2 (\delta\gamma / \gamma_0)$$

For a time independent solution $\delta r = \Delta R$ (orbit at larger radius)

$$(1-n)\Omega_c^2 \Delta R = \Omega_c const + R\Omega_c^2 (\delta\gamma / \gamma_0)$$

$$const = \Omega_c R \frac{\Delta p}{p} - \Omega_c R \frac{\delta\gamma}{\gamma_0} = \Omega_c R \frac{\Delta p}{p} (1-\beta_0^2)$$

General Betatron Oscillation equations

$$\delta\ddot{r} + (1-n)\Omega_c^2 \delta r = \Omega_c^2 R \frac{\Delta p}{p}$$

$$\delta\ddot{z} + n\Omega_c^2 \delta z = 0$$

No Longitudinal Focusing

$$R\delta\dot{\theta} + \Omega_c \delta r = \Omega_c R \frac{\Delta p}{p}\left(1 - \beta_0^2\right)$$

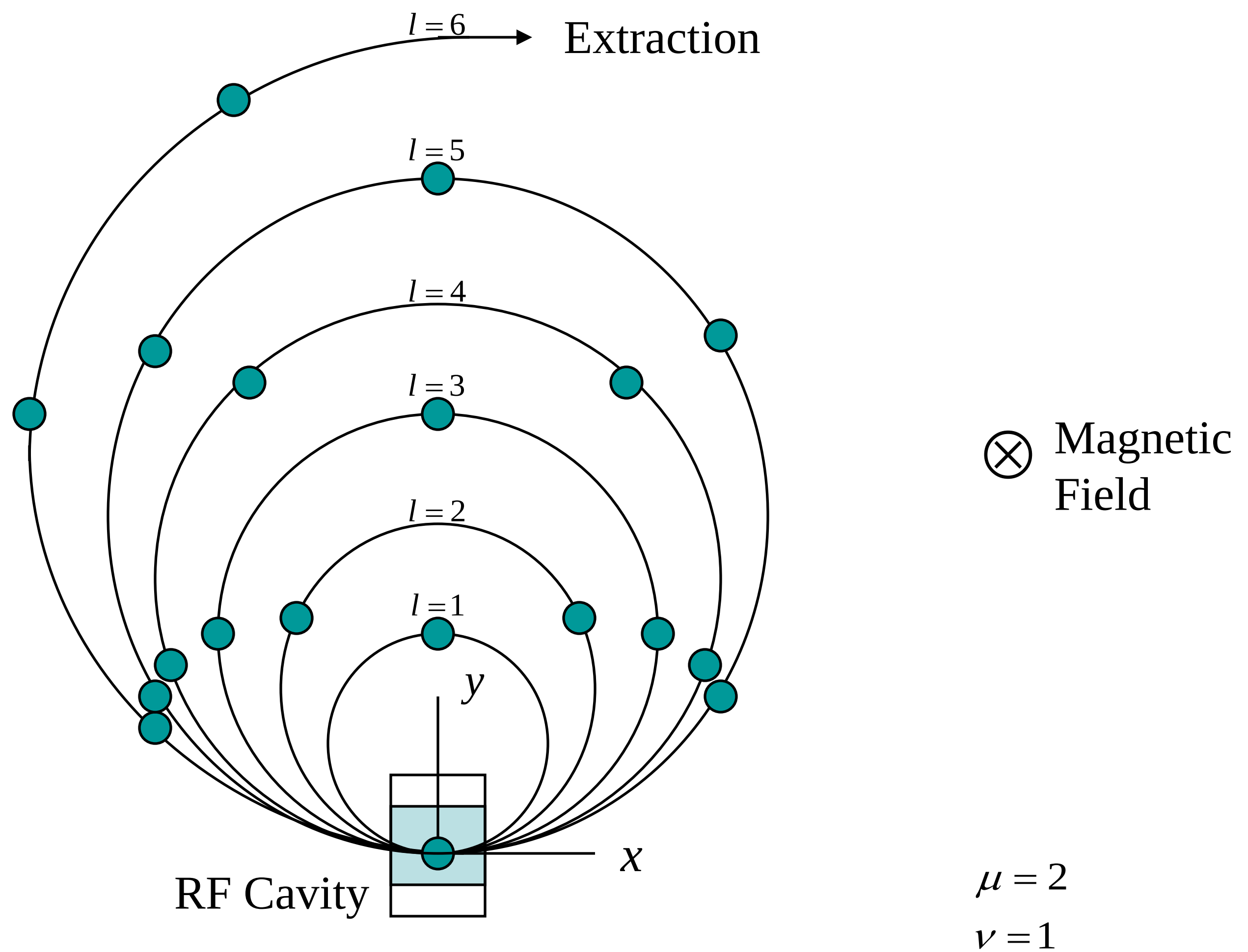
$$\theta = \theta_0 + \Omega_c t + \int \left[\Omega_c \frac{\Delta p}{p} \frac{1}{\gamma_0^2} - \Omega_c \frac{\Delta R}{R} \right] dt$$

$$= \theta_0 + \Omega_c t + \int \Omega_c \frac{\Delta p}{p} \left[\frac{1}{\gamma_0^2} - \frac{1}{1-n} \right] dt$$

Speed Change

Path increase from displaced orbit

Classical Microtron: Veksler (1945)



Basic Principles

For the geometry given

$$\frac{d(\gamma m\vec{\mathrm{v}})}{dt} = -e\left[\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right]$$

$$\frac{d(\gamma m\mathrm{v}_x)}{dt} = e\mathrm{v}_y B_z$$

$$\frac{d(\gamma m\mathrm{v}_y)}{dt} = -e\mathrm{v}_x B_z$$

$$\frac{d^2\mathrm{v}_x}{dt^2} + \Omega_c^2\mathrm{v}_x = 0 \qquad \frac{d^2\mathrm{v}_y}{dt^2} + \Omega_c^2\mathrm{v}_y = 0$$

For each orbit, separately, and exactly

$$\mathrm{v}_x(t) = -v_{x0}\cos(\Omega_c t) \qquad \mathrm{v}_y(t) = v_{x0}\sin(\Omega_c t)$$

$$x(t) = -\frac{v_{x0}}{\Omega_c}\sin\left(\Omega_c t\right) \qquad y(t) = \frac{v_{x0}}{\Omega_c} - \frac{v_{x0}}{\Omega_c}\cos\left(\Omega_c t\right)$$

Non-relativistic cyclotron frequency: $2\pi f_c = eB_z / m$

Relativistic cyclotron frequency: $\Omega_c = eB / \gamma m$

Bend radius of each orbit is: $\rho_l = \mathrm{v}_{x0,l} / \Omega_c \to c / \Omega_c$

In a conventional cyclotron, the particles move in a circular orbit that grows in size with energy, but where the relatively heavy particles stay in resonance with the RF, which drives the accelerating DEEs at the non-relativistic cyclotron frequency. By contrast, a microtron uses the “other side” of the cyclotron frequency formula. The cyclotron frequency decreases, proportional to energy, and the beam orbit radius increases in each orbit by precisely the amount which leads to arrival of the particles in the succeeding orbits precisely in phase.

Microtron Resonance Condition

Must have that the bunch pattern repeat in time. This condition is only possible if the time it takes to go around each orbit is precisely an integral number of RF periods

$$\gamma_1 = \mu \frac{f_c}{f_{RF}}$$

First Orbit

$$\Delta\gamma = \nu \frac{f_c}{f_{RF}}$$

Each Subsequent Orbit

For classical microtron assume can inject so that

$$\gamma_1 \approx 1 + \nu \frac{f_c}{f_{RF}}$$

$$\frac{f_c}{f_{RF}} \approx \frac{1}{\mu - \nu}$$

Parameter Choices

The energy gain in each pass must be identical for this resonance to be achieved, because once f_c/f_{RF} is chosen, $\Delta\gamma$ is fixed. Because the energy gain of non-relativistic ions from an RF cavity IS energy dependent, there is no way (presently!) to make a classical microtron for ions. For the same reason, in electron microtrons one would like the electrons close to relativistic after the first acceleration step. Concern about injection conditions which, as here in the microtron case, will be a recurring theme in examples!

$$f_c / f_{RF} = B_z / B_0 \qquad B_0 = \frac{2\pi mc}{\lambda e}$$

$$B_0 = 0.107\mathrm{T} = 1.07\mathrm{kG} @ 10\mathrm{cm}$$

Notice that this field strength is NOT state-of-the-art, and that one normally chooses the magnetic field to be around this value. High frequency RF is expensive too!

Classical Microtron Possibilities

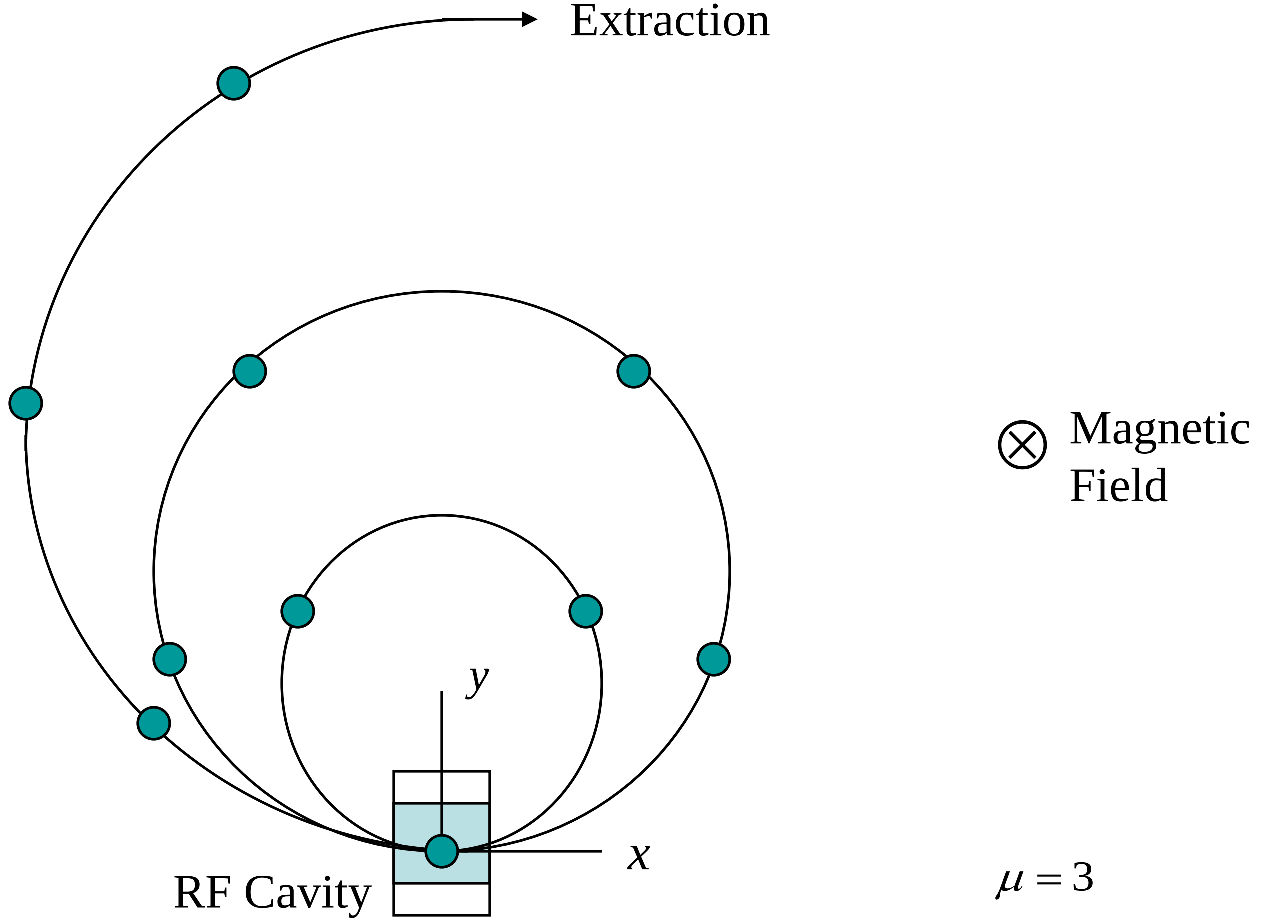
Assumption: Beam injected at low energy and energy gain is the same for each pass

$\frac{f_c}{f_{RF}}$	1	1/2	1/3	1/4	
	$\mu,\nu,\gamma_1,\Delta\gamma$	$\mu,\nu,\gamma_1,\Delta\gamma$	$\mu,\nu,\gamma_1,\Delta\gamma$	$\mu,\nu,\gamma_1,\Delta\gamma$	$\cdots$
	2, 1, 2, 1	3, 1, 3/2, 1/2	4, 1, 4/3, 1/3	5, 1, 5/4, 1/4	$\cdots$
	3, 2, 3, 2	4, 2, 2, 1	5, 2, 5/3, 2/3	6, 2, 3/2, 1/2	$\cdots$
	4, 3, 4, 3	5, 3, 5/2, 3/2	6, 3, 2, 1	7, 3, 7/4, 3/4	$\cdots$
	5, 4, 5, 4	6, 4, 3, 2	7, 4, 7/3, 4/3	8, 4, 2, 1	$\cdots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

$\Delta\gamma$ lower

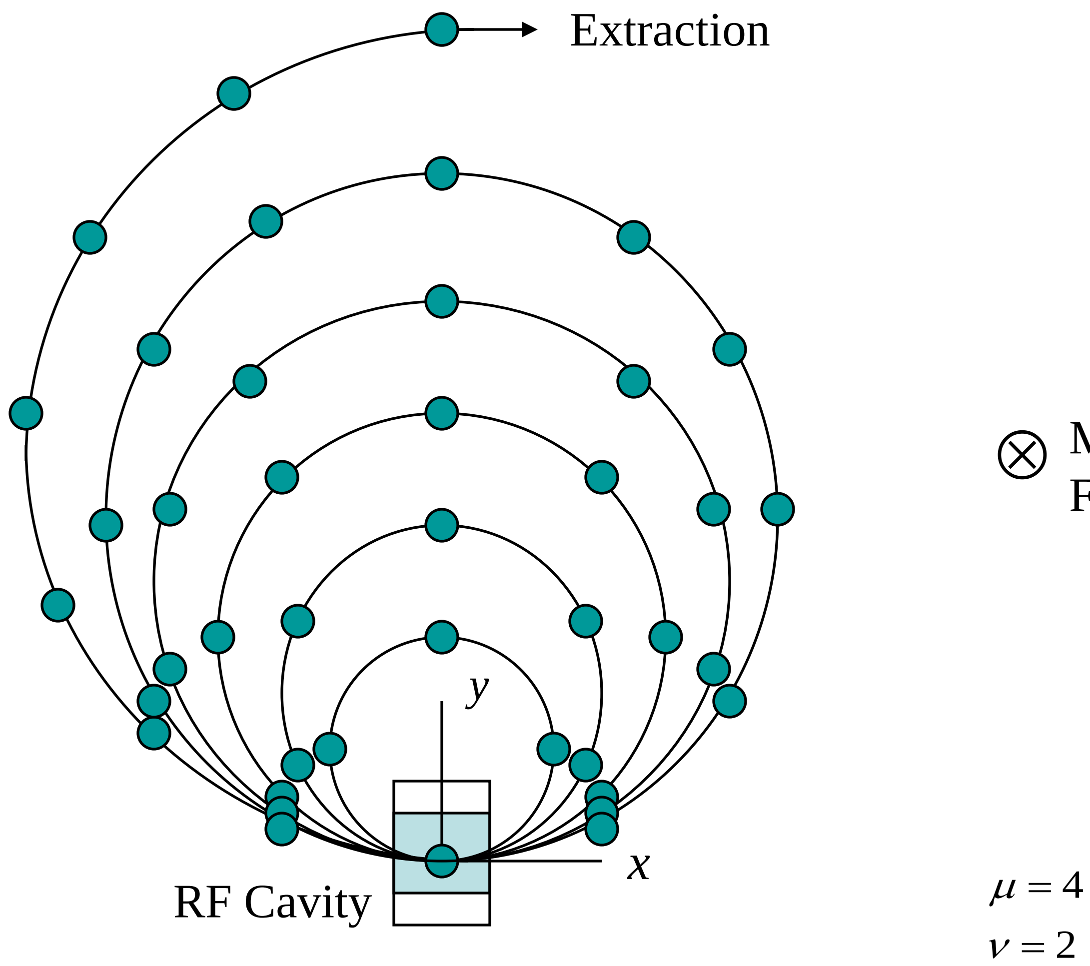
B bigger

For same microtron magnet, no advantage to higher n; RF is more expensive because energy per pass needs to be higher



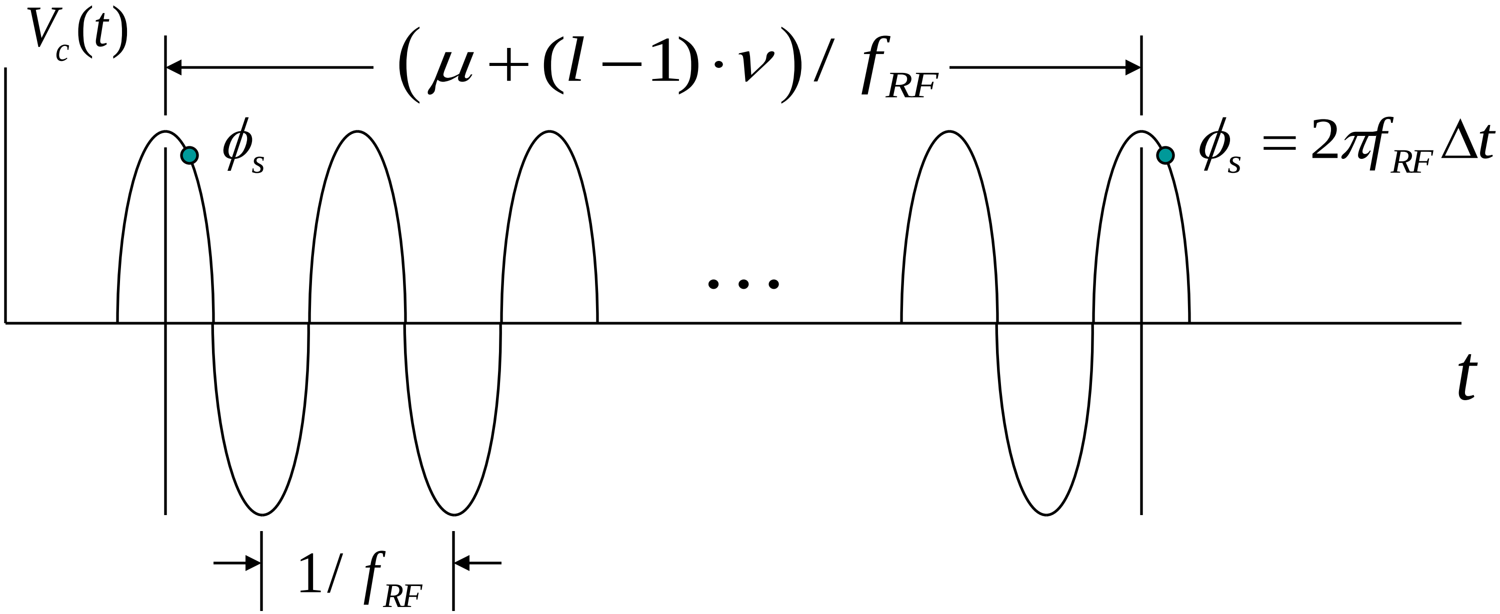
Going along diagonal changes frequency

To deal with lower frequencies, go up the diagonal



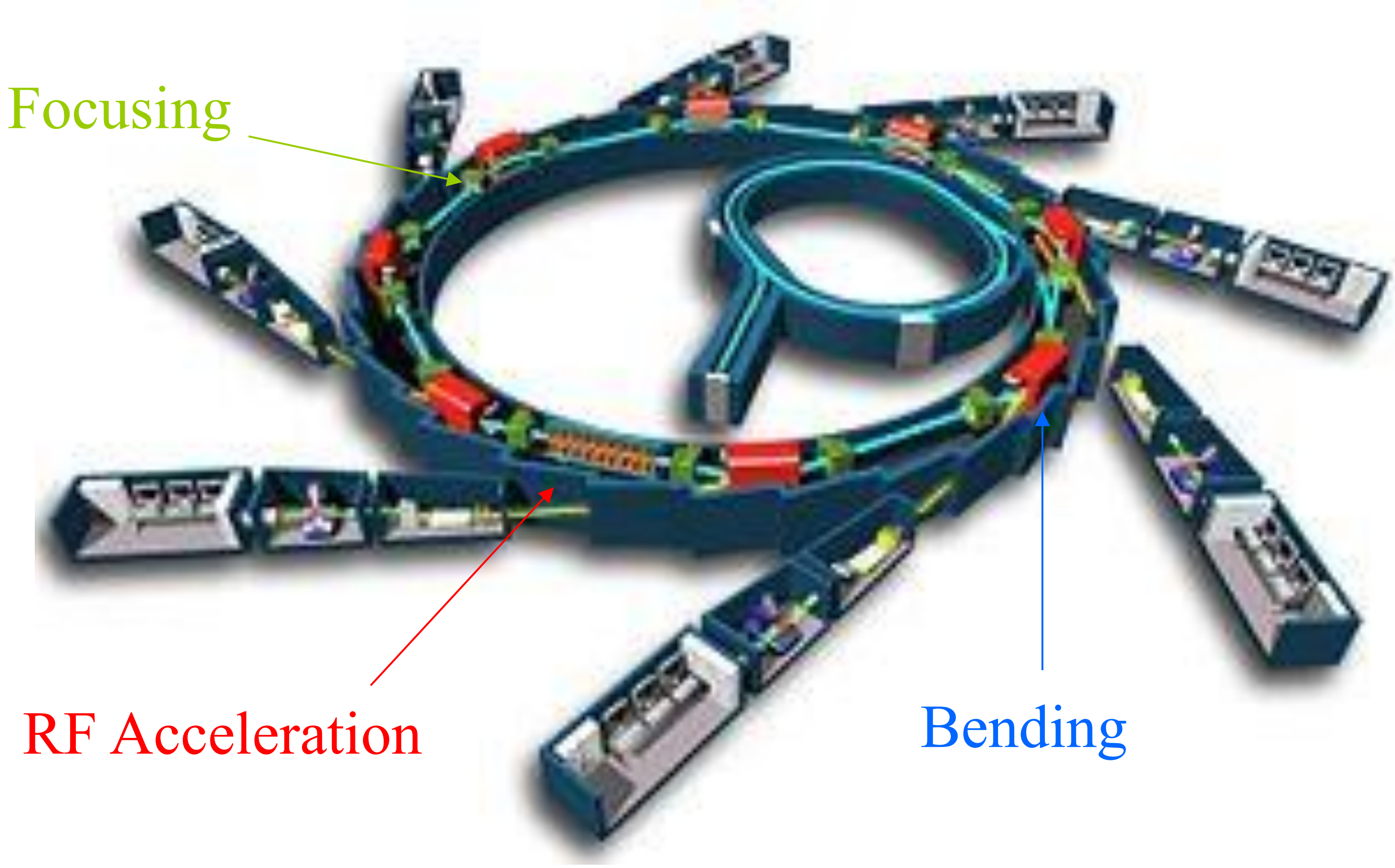
Phase Stability

Invented independently by Veksler (for microtrons!) and McMillan



Electrons arriving EARLY get more energy, have a longer path, and arrive later on the next pass. Extremely important discovery in accelerator physics. McMillan used same idea to design first electron synchrotron.

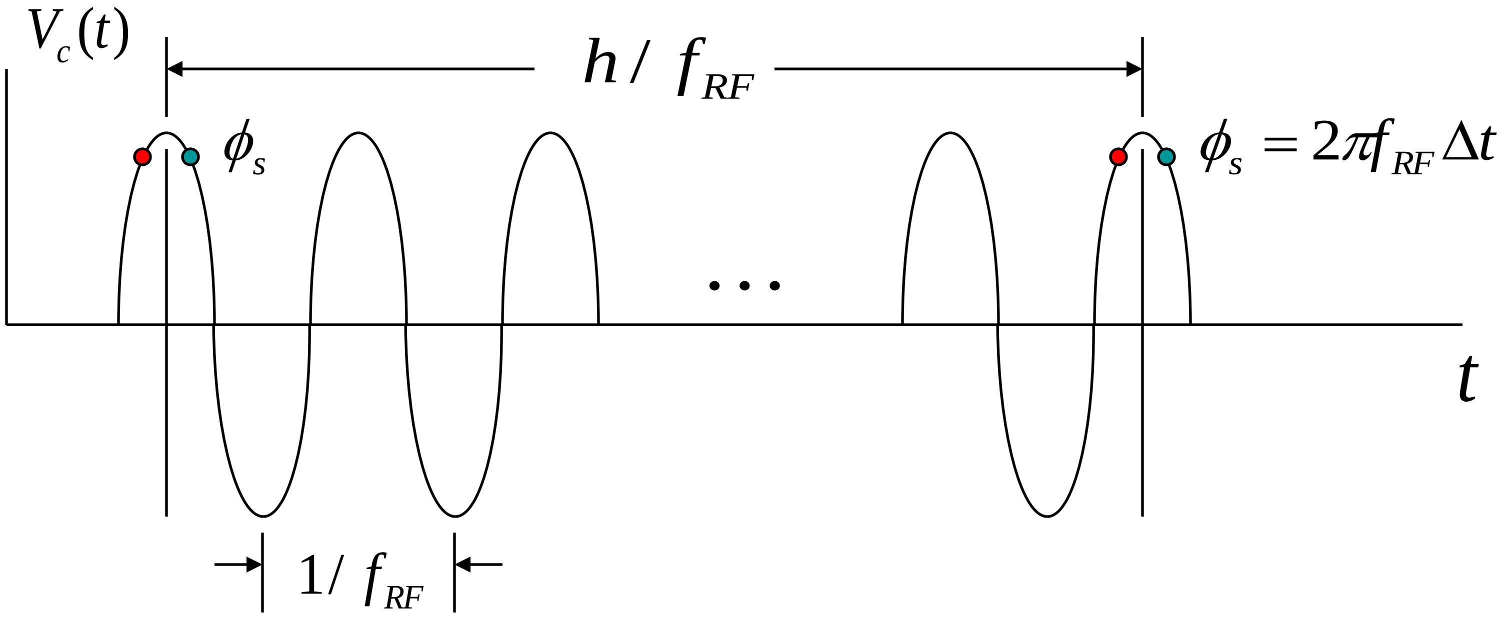
Generic Modern Synchrotron



Spokes are user stations for this X-ray ring source

Synchrotron Phase Stability

Edwin McMillan discovered phase stability independently of Veksler and used the idea to design first large electron synchrotron.



$$h = Lf_{RF} / \beta c$$

Harmonic number: # of RF oscillations in a revolution

Transition Energy

Beam energy where speed increment effect balances path length change effect on accelerator revolution frequency. Revolution frequency independent of beam energy to linear order. We will calculate in a few weeks

- Below Transition Energy: Particles arriving EARLY get less acceleration and speed increment, and arrive later, with repect to the center of the bunch, on the next pass. Applies to heavy particle synchrotrons during first part of acceleration when the beam is non-relativistic and accelerations still produce velocity changes.

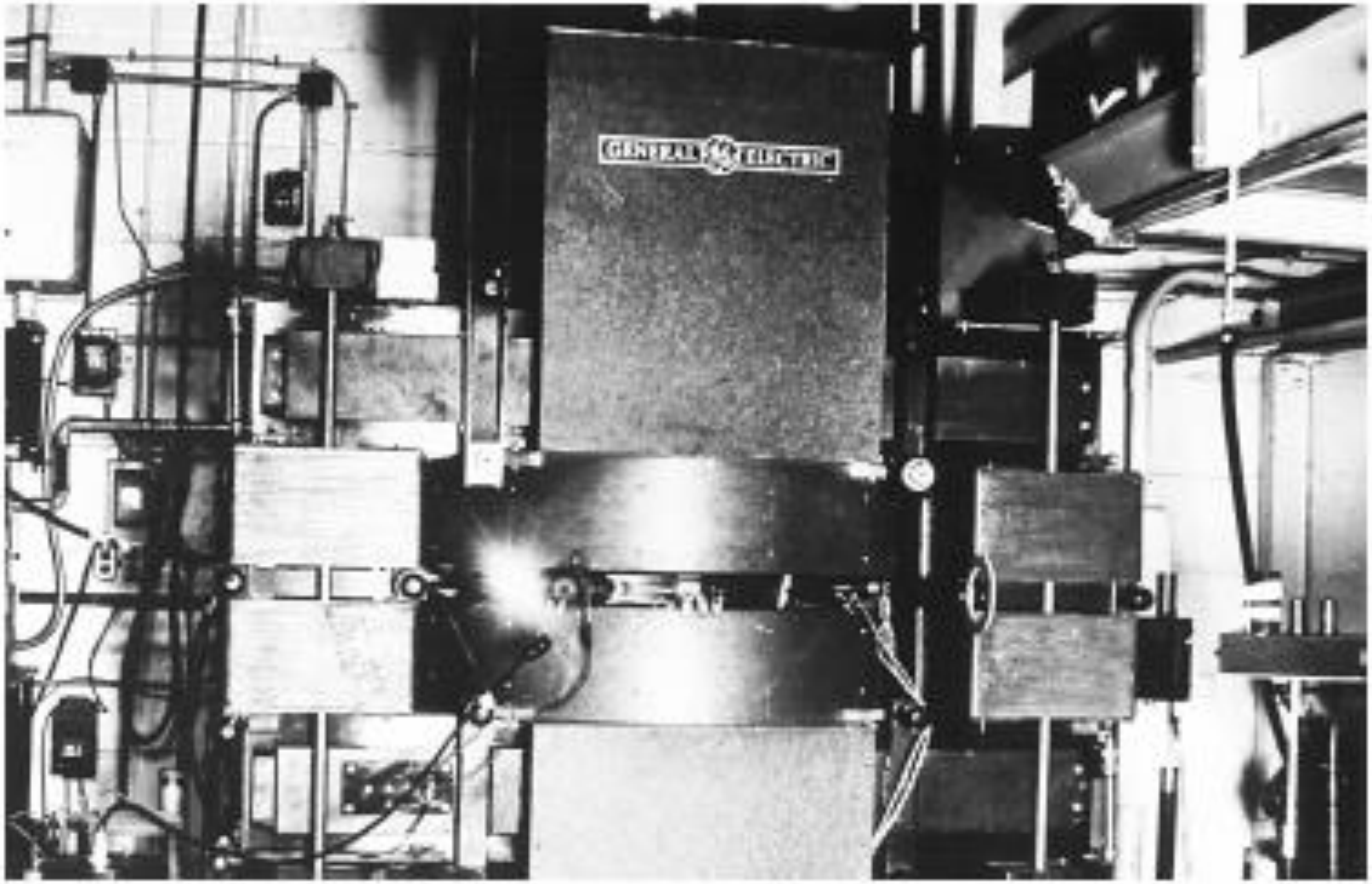
- Above Transition Energy: Particles arriving EARLY get more energy, have a longer path, and arrive later on the next pass. Applies for electron synchrotrons and heavy particle synchrotrons when approach relativistic velocities. As seen before, Microtrons operate here.

Ed McMillan

Vacuum chamber for electron synchrotron being packed for shipment to Smithsonian

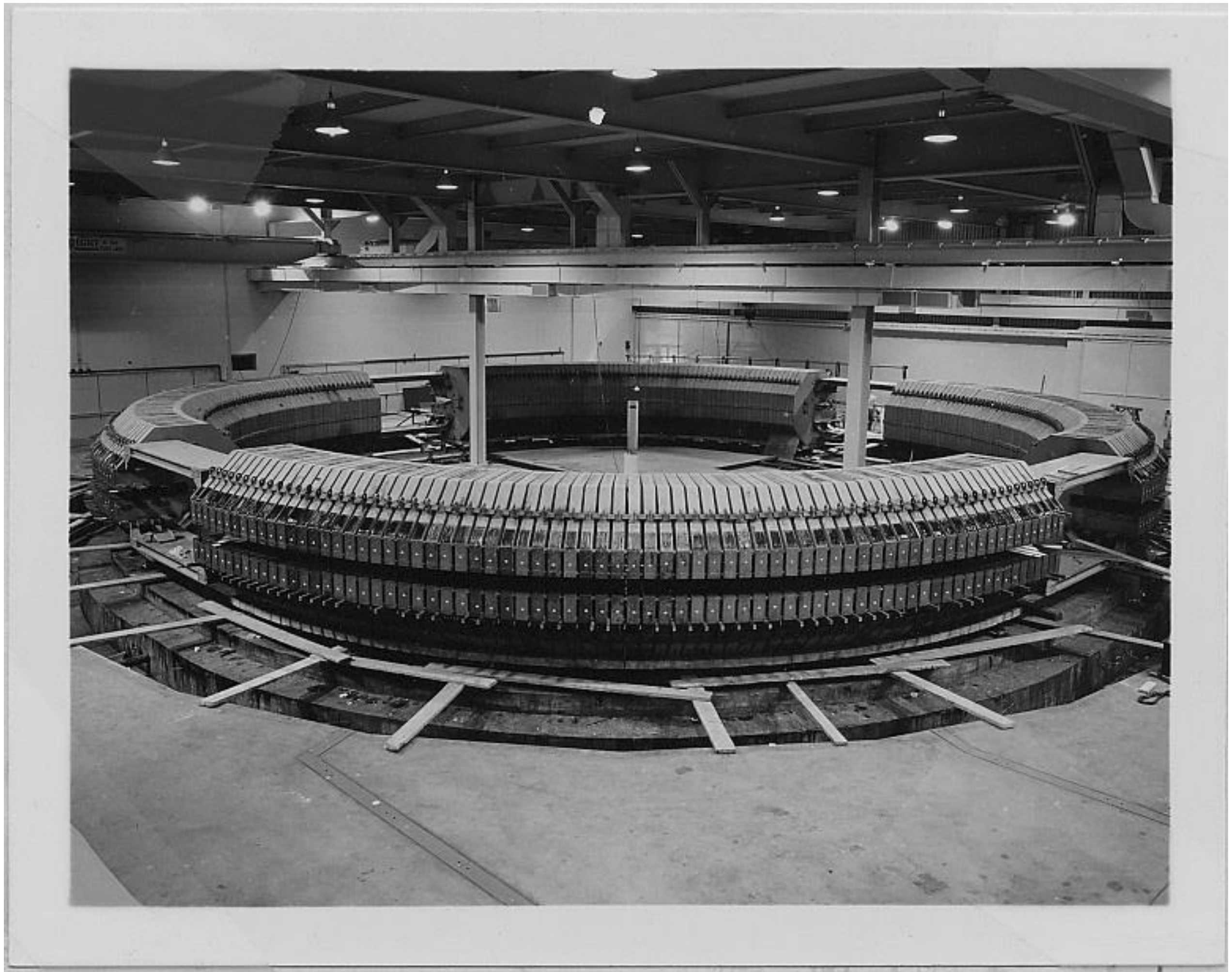
Full Electron Synchrotron

GE Electron Synchrotron



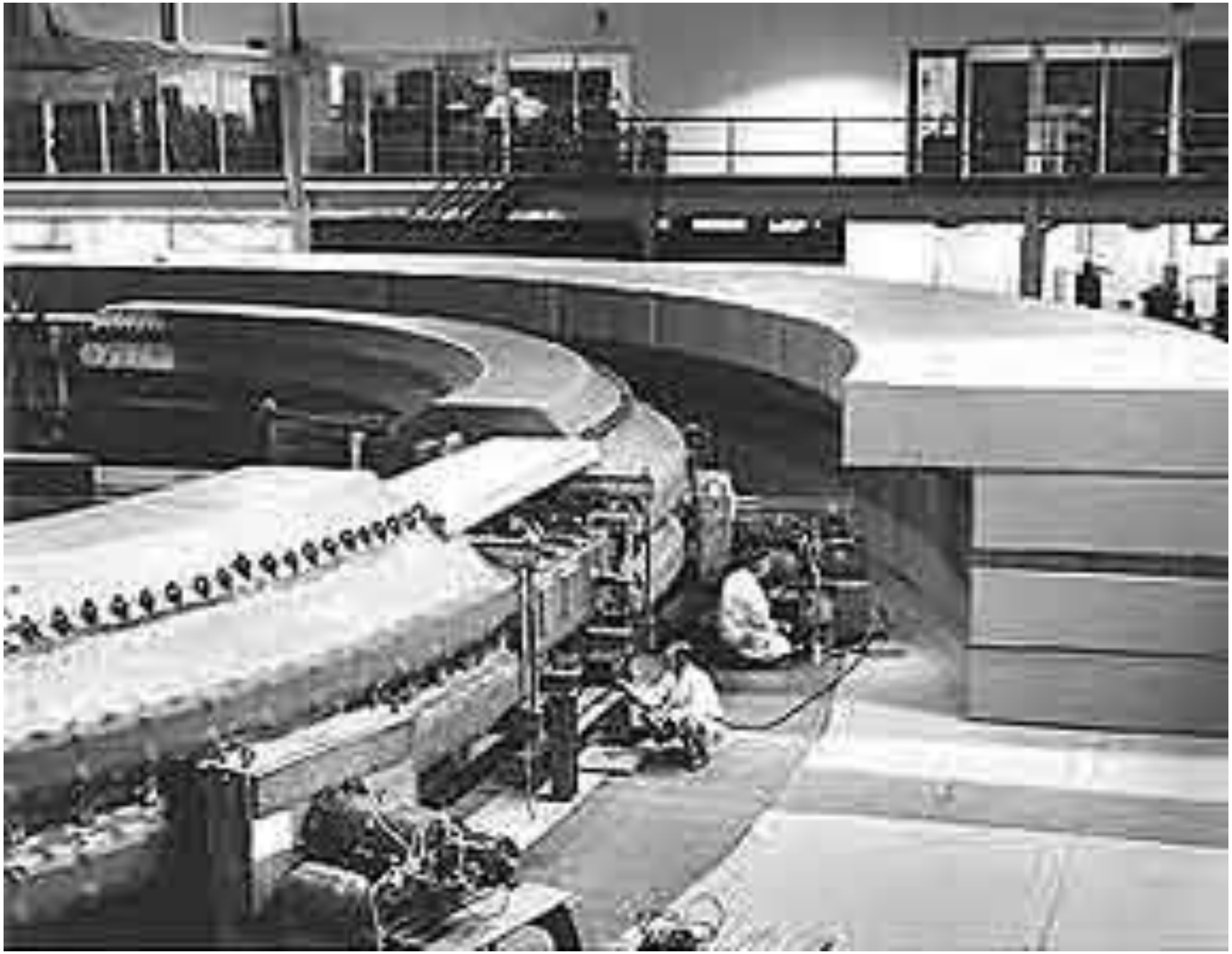
Elder, F. R.; Gurewitsch, A. M.; Langmuir, R. V.; Pollock, H. C., "Radiation from Electrons in a Synchrotron" (1947) *Physical Review*, vol. 71, Issue 11, pp. 829-830

Cosmotron (First GeV Accelerator)

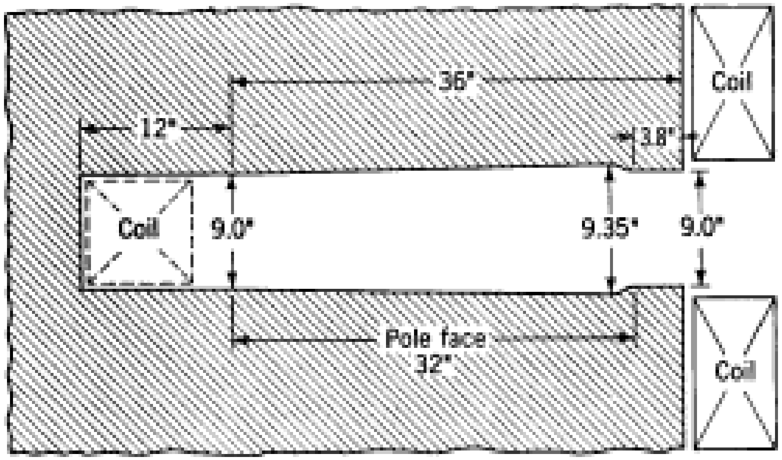


6/15/50 Neg. No. 6-151-0
View of Cosmotron Magnet Blocks after Leveling and Spacing

BNL Cosmotron and Shielding



Cosmotron Magnet



The Cosmotron magnet

Cosmotron People

E.Courant -Lattice Designer

Stan Livingston - Boss

Snyder -theorist

Christofilos - inventor

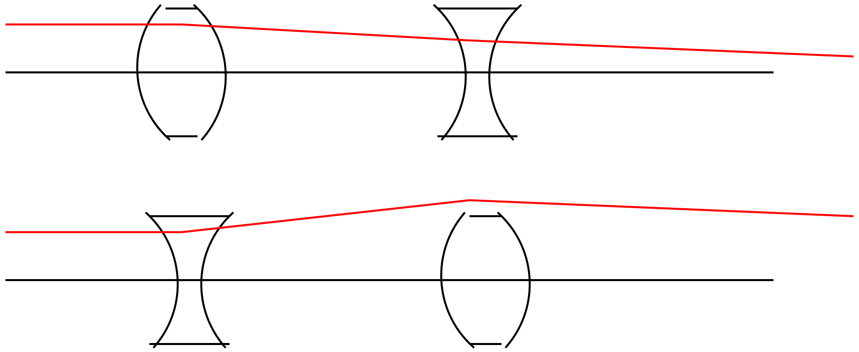
Bevatron

Designed to discover the antiproton; Largest Weak Focusing Synchrotron

Strong Focusing

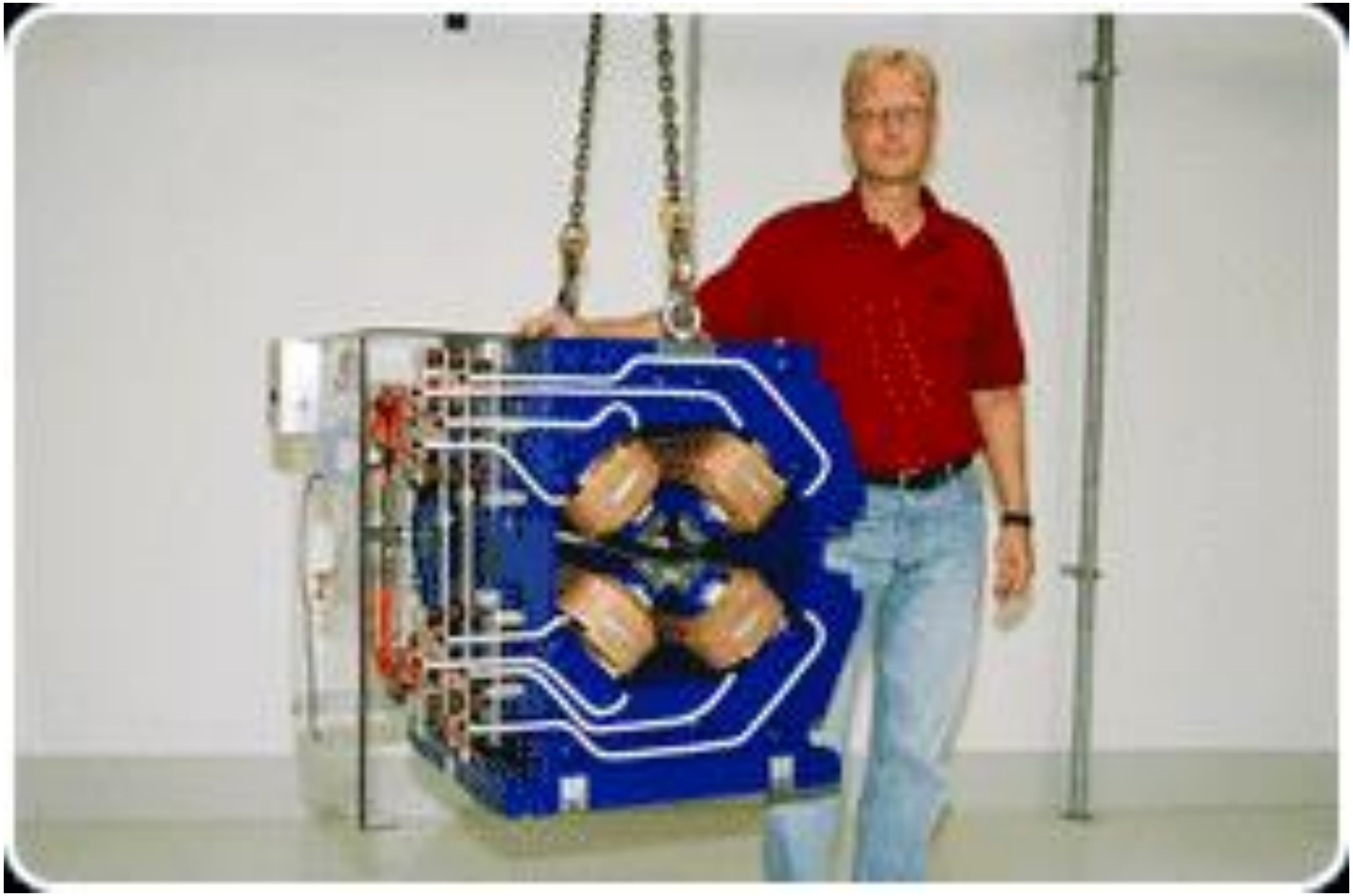
- Betatron oscillation work has showed us that, apart from bend plane focusing, a shaped field that focuses in one transverse direction, defocuses in the other
- Question: is it possible to develop a system that focuses in both directions simultaneously?

- Strong focusing: alternate the signs of focusing and defocusing: get net focusing!!



Order doesn't matter

Linear Magnetic Lenses: Quadrupoles



Source: Danfysik Web site

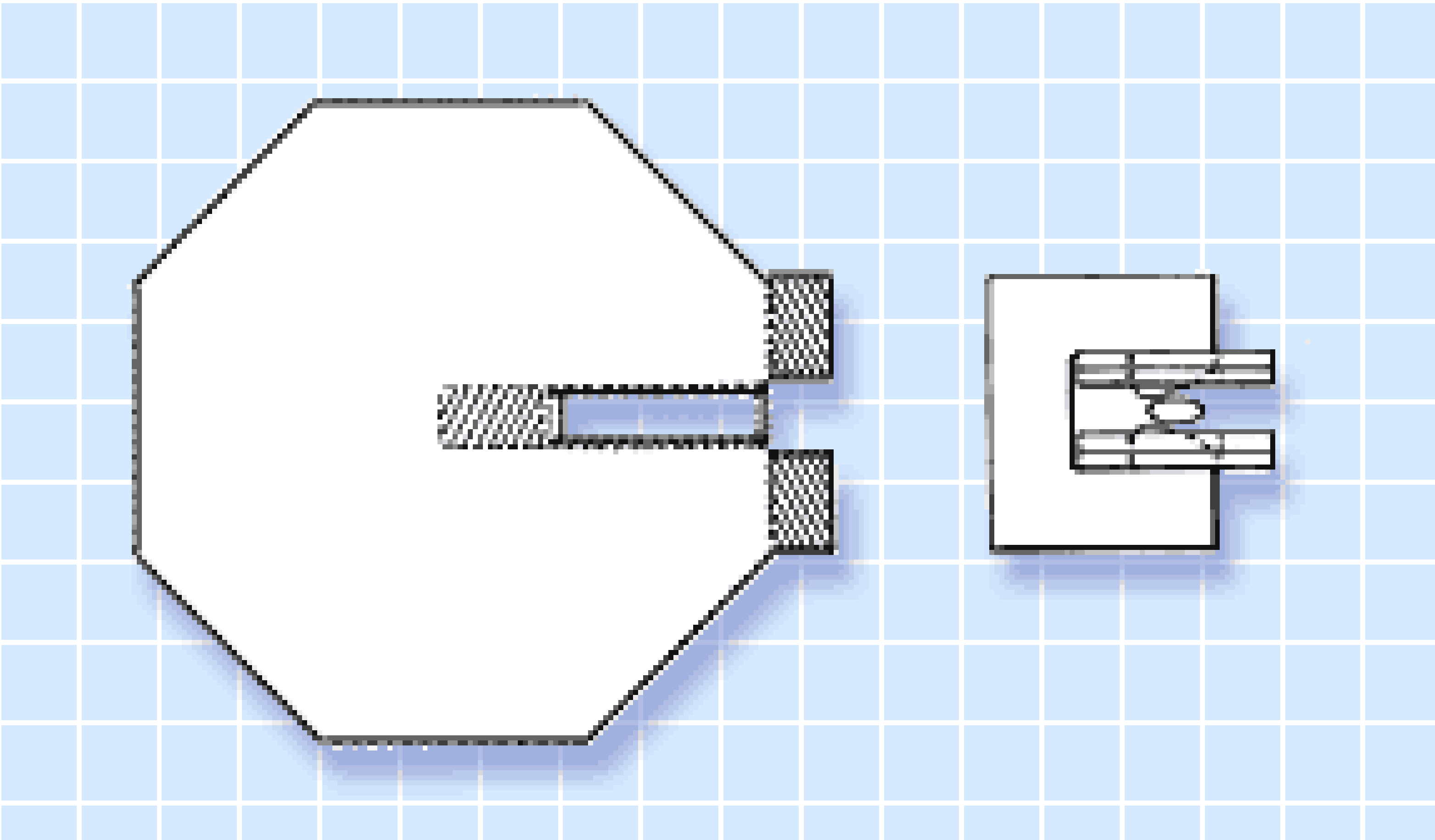
Comment on Strong Focusing

One main advantage of strong focusing. In weak focusing machines, $n < 1$ for stability. Therefore, the fall-off distance, or field gradient cannot be too high. There is no such limit for strong focusing.

$$n \gg 1$$

is now allowed, leading to large field gradients and relatively short focal length magnetic lenses. This tighter focusing is what allows smaller beam sizes. Focusing gradients now limited only by magnet construction issues (pole magnetic field limits).

Weak vs. Strong Benders

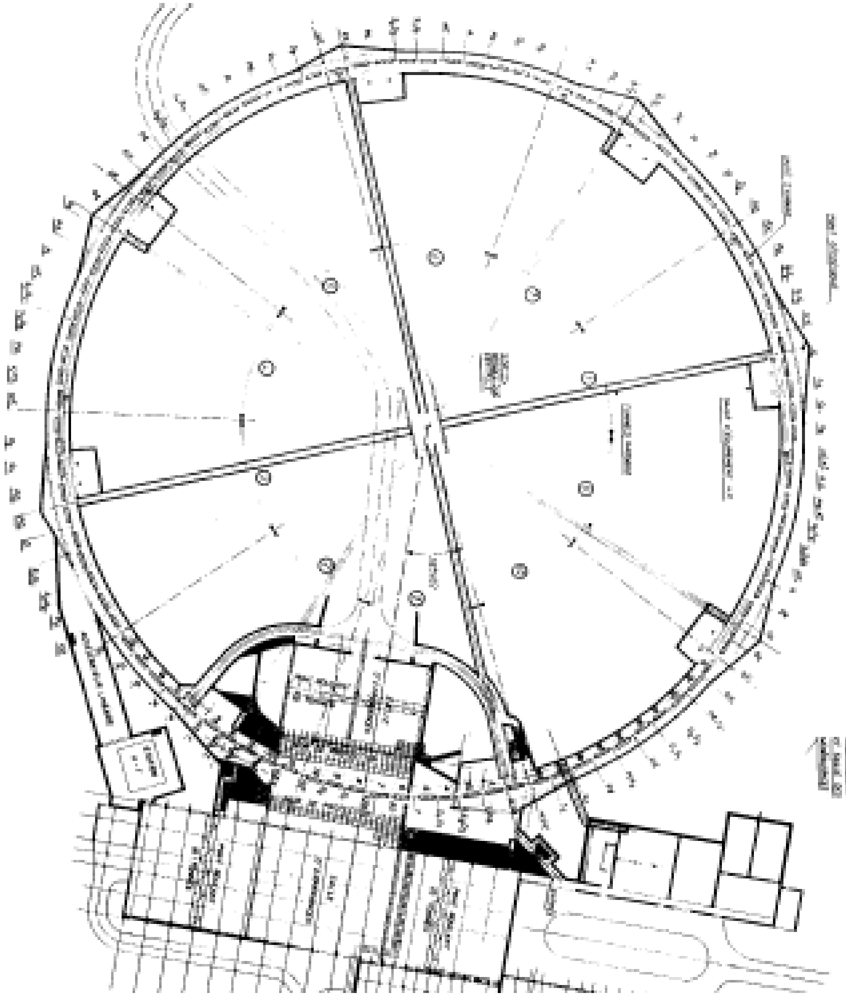


First Strong-Focusing Synchrotron

Cornell 1 GeV Electron Synchrotron (LEPP-AP Home Page)

Alternating Gradient Synchrotron (AGS)

CERN PS



25 GeV Proton Synchrotron

CERN SPS

Eventually 400 GeV protons and antiprotons

FNAL

First TeV-scale accelerator; Large Superconducting Benders

LEP Tunnel (Now LHC!)

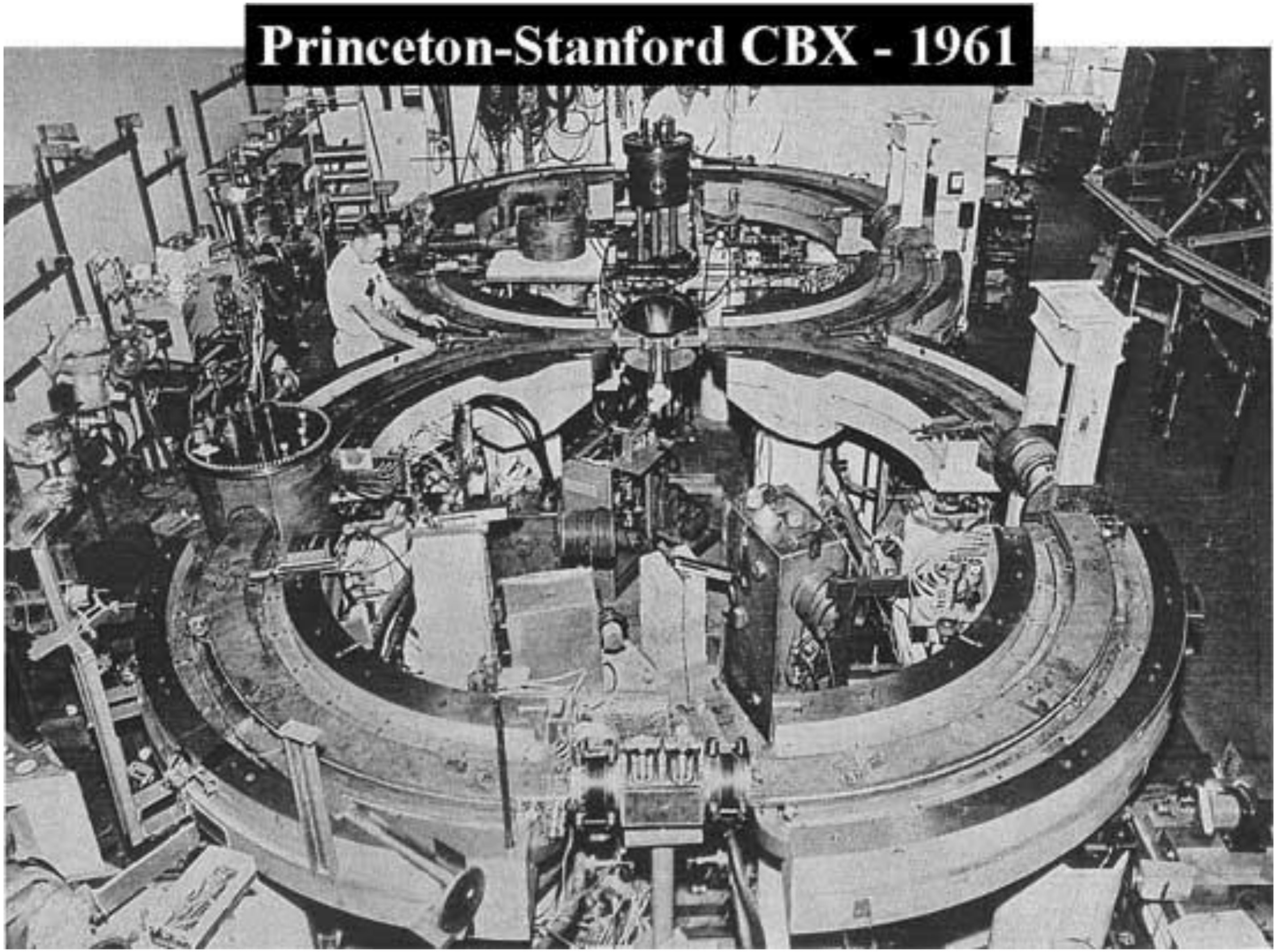
Empty

LHC

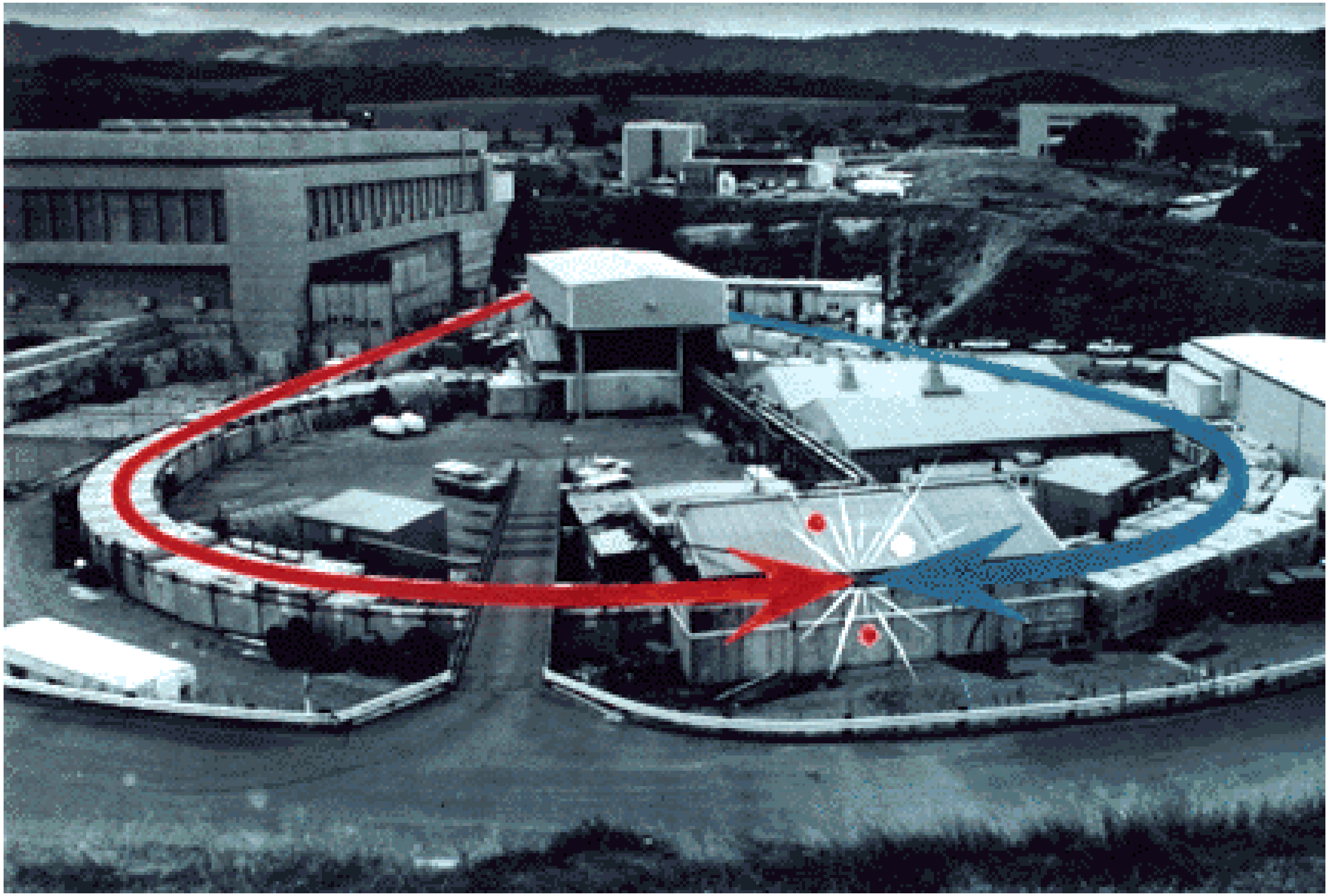
Storage Rings

- Some modern accelerators are designed not to "accelerate" much at all, but to "store" beams for long periods of time that can be usefully used by experimental users.
  – Colliders for High Energy Physics. Accelerated beam-accelerated beam collisions are much more energetic than accelerated beam-target collisions. To get to the highest beam energy for a given acceleration system design a collider
  – Electron storage rings for X-ray production: circulating electrons emit synchrotron radiation for a wide variety of experimental purposes.

Princeton-Stanford Collider



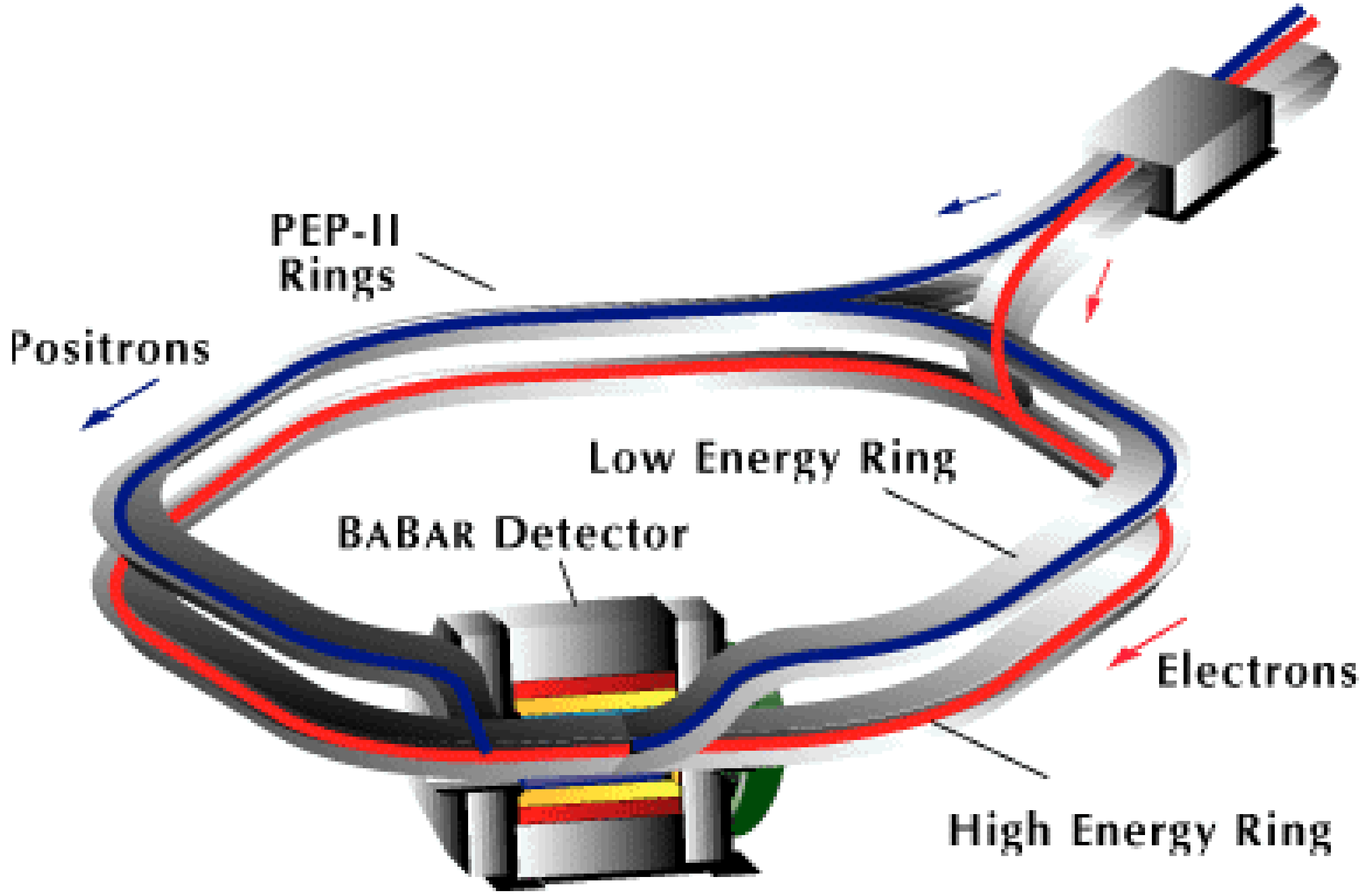
SPEAR



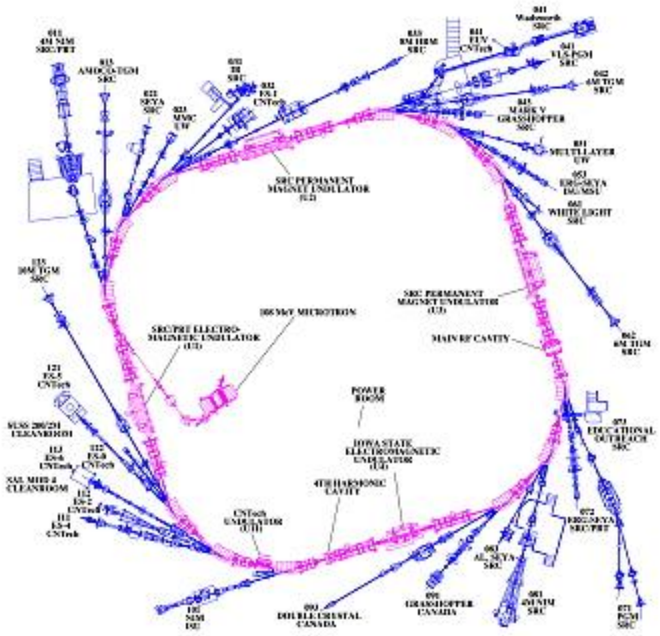
Eventually became leading synchrotron radiation machine

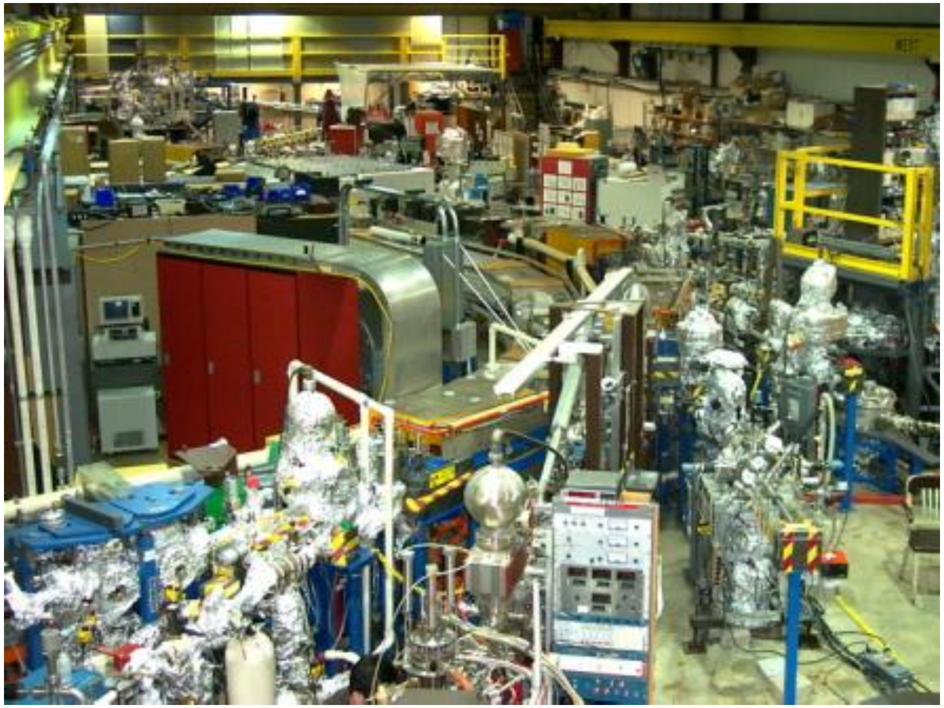
Cornell 10 GeV ES and CESR

SLAC's PEP II B-factory



ALADDIN at Univ. of Wisconsin

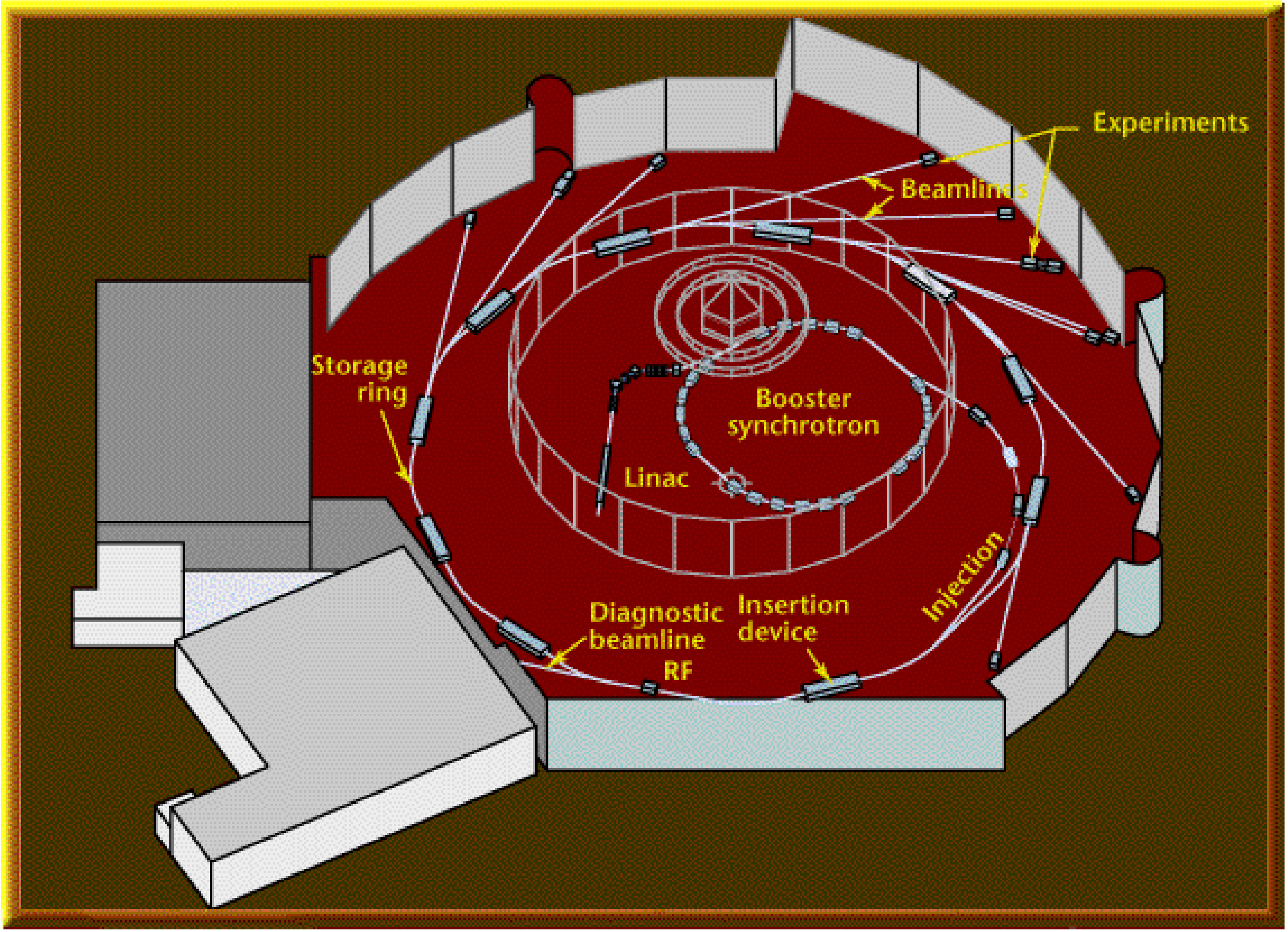




VUV Ring at NSLS

VUV ring "uncovered"

Berkeley's ALS



Argonne APS

ESRF