

Graduate Accelerator Physics

G. A. Krafft
Jefferson Lab
Old Dominion University
Lecture 1

Course Outline



- **Course Content**
- Introduction to Accelerators and Short Historical Overview
 - Basic Units and Definitions
 - Lorentz Force
 - Linear and Circular Accelerators
- Particle Motion in EM Fields
 - Linear Beam Dynamics
 - Periodic Systems
 - Magnetic Multipoles
 - Nonlinear Perturbations
 - Coupled Motion

- Synchrotron Radiation
 - Radiation Power and Distribution
 - Insertion Devices
 - X-ray Sources
 - Free Electron Lasers
- Technical Components
 - Particle Acceleration Cavities and RF Systems
 - Spin and Spin Manipulation
- Collective Effects
 - Particle Distributions
 - Vlasov Equation
 - Self-consistent Fields

Landau Damping

Beam-Beam Effects

- Relaxation Phenomena

Radiation Damping

Toushek effect/IBS

Beam Cooling

Energy Units



- When a particle is accelerated, i.e., its energy is changed by an electromagnetic field, it must have fallen through an Electric Field (we show later by very general arguments that Magnetic Fields cannot change particle energy). For electrostatic accelerating fields the energy change is

$$\Delta E = q\Delta\Phi = q(\Phi_a - \Phi_b)$$

q charge, Φ , the electrostatic potentials before and after the motion through the electric field. Therefore, particle energy can be conveniently expressed in units of the “equivalent” electrostatic potential change needed to accelerate the particle to the given energy. Definition: 1 eV, or 1 electron volt, is the energy acquired by 1 electron falling through a one volt potential difference.

Energy Units



$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ MeV} = 10^6 \text{ eV} = 1.6 \times 10^{-13} \text{ J}$$

To convert rest mass to eV use [Einstein relation](#)

$$E_0 = mc^2$$

where m is the rest mass. For electrons

$$\begin{aligned} E_{electron,0} &= 9.1 \times 10^{-31} \text{ kg} \left(3 \times 10^8 \text{ m/sec} \right)^2 = 81.9 \times 10^{-15} \text{ J} \\ &= 0.512 \text{ MeV} \end{aligned}$$

Recent “best fit” value 0.51099906 MeV

Some Needed Relativity



Accelerator Physics is “Applied Special Relativity”

Following Maxwell Equations, which exhibit this symmetry, assume all Laws of Physics must be of form to guarantee the invariance of the space-time interval

$$(ct')^2 - x'^2 - y'^2 - z'^2 = (ct)^2 - x^2 - y^2 - z^2$$

Coordinate transformations that leave interval unchanged are the usual rotations and Lorentz Transformations, e.g. the z boost

$$ct' = \gamma(ct - \beta z)$$

$$x' = x$$

$$y' = y$$

$$z' = \gamma(z - \beta ct)$$

Relativistic Factors



Following Einstein define the relativistic factors

$$\vec{\beta} = \frac{\vec{v}}{c} \quad \beta = \frac{|\vec{v}|}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

Easy way to accomplish task of defining a *Relativistic Mechanics*: write all laws of physics in terms of 4-vectors and 4-tensors, i.e., quantities that transform under Lorentz transformations in the same way as the coordinate differentials.

Four-vectors



Four-vector transformation under z boost Lorentz Transformation

$$v^{0\prime} = \gamma(v^0 - \beta v^3)$$

$$v^{1\prime} = v^1$$

$$v^{2\prime} = v^2$$

$$v^{3\prime} = \gamma(v^3 - \beta v^0)$$

Important example: Four-velocity. Note that interval

$$d\tau \equiv \sqrt{1 - \beta^2} dt$$

Lorentz invariant. So the following is a 4-vector

$$cu^\alpha \equiv \left(\frac{dct}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right) = c\gamma(1, \beta_x, \beta_y, \beta_z)$$

4-Momentum



Single particle mechanics must be defined in terms of Four-momentum

$$p^\alpha \equiv mcu^\alpha = mc\gamma(1, \beta_x, \beta_y, \beta_z) = (E/c, p_x, p_y, p_z)$$

Norms, which must be Lorentz invariant, are ($g_{\mu\nu} = g^{\mu\nu} = (1, -1, -1, -1)$)

$$\sqrt{u_\alpha u^\alpha} \equiv 1, \sqrt{p_\alpha p^\alpha} \equiv mc$$

What happens to Newton's Law $\vec{F} = m\vec{a} = d\vec{p} / dt$?

$$\frac{dp^\alpha}{d\tau} \equiv F^\alpha$$

But need a Four-force on the RHS!!!

Electromagnetic Field



Described by the Four-vector potential

$$A^\mu = (\phi, cA_x, cA_y, cA_z)$$

Field Tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$
$$= \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{pmatrix}$$

Electromagnetic (Lorentz Force)



Non-relativistic

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Relativistic Generalization (ν summation implied)

$$F^\alpha = qF^\alpha{}_\nu u^\nu$$

Electromagnetic Field with lower second index

$$F^\alpha{}_\nu \equiv F^{\alpha\beta} g_{\beta\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix}$$

Relativistic Mechanics in E-M Field



Energy Exchange Equation (Note: no magnetic field!)

$$\frac{d\gamma}{dt} = \frac{q\vec{E} \cdot \vec{v}}{mc^2}$$

Relativistic Lorentz Force Equation (you verify in HW!)

$$\frac{d(\gamma m \vec{v})}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

Methods of Acceleration



- Acceleration by Static Electric Fields (DC) Acceleration
 - Cockcroft-Walton
 - van de Graaf Accelerators
 - Limited by voltage breakdowns to potentials of under a million volts in 1930, and presently to potentials of tens of millions of volts (in modern van de Graaf accelerators). Not enough to do nuclear physics at the time.
- Radio Frequency (RF) Acceleration
 - Main means to accelerate in most present day accelerators because one can get to 10-100 MV in a meter these days. Reason: alternating fields don't cause breakdown (if you are careful!) until much higher field levels than DC.
 - Ideas started with Ising and Wideröe

Cockcroft-Walton



Proton Source at Fermilab, Beam Energy 750 keV

van de Graaf Accelerator



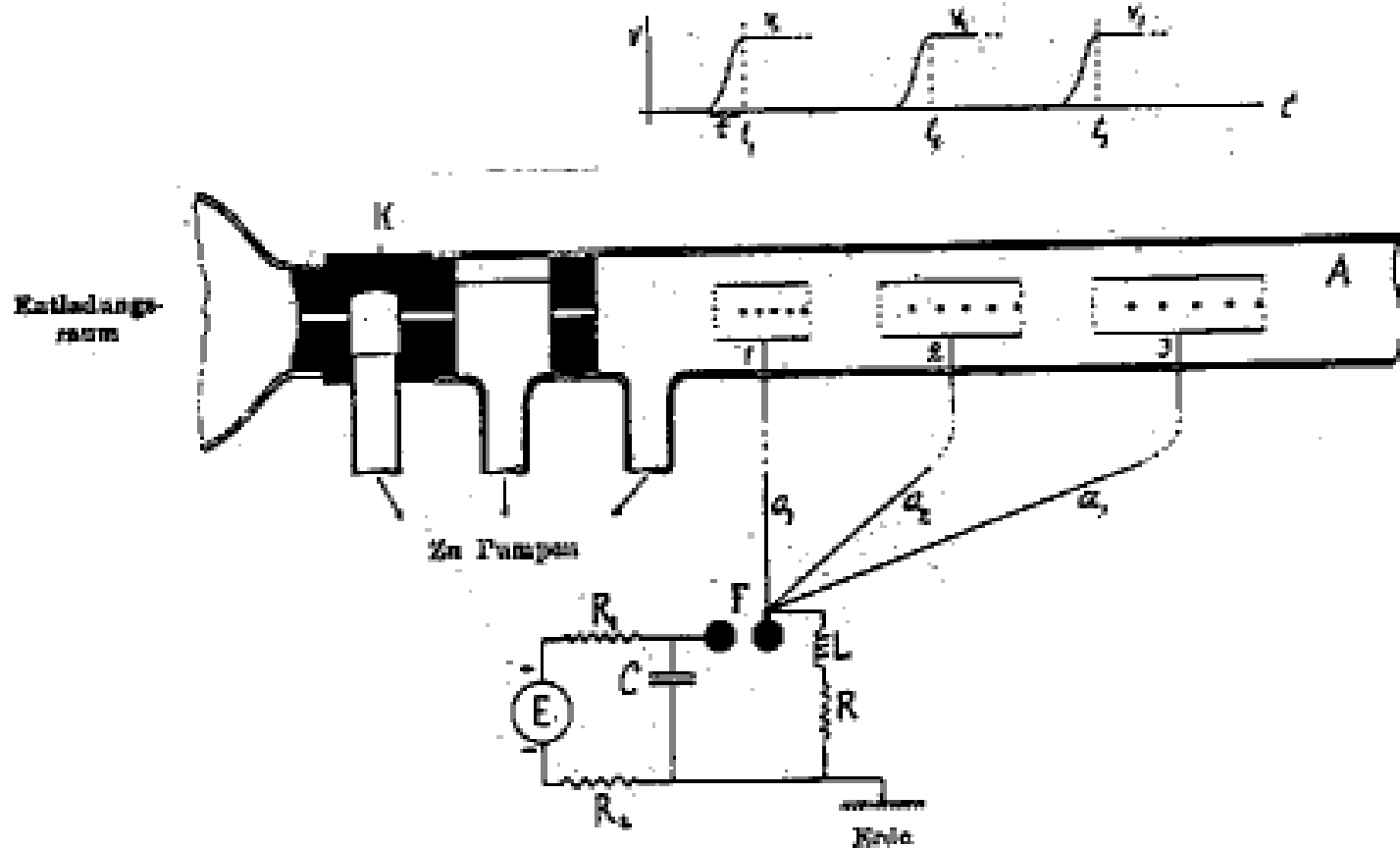
Generator



Brookhaven
Tandem
van de Graaf
~ 15 MV

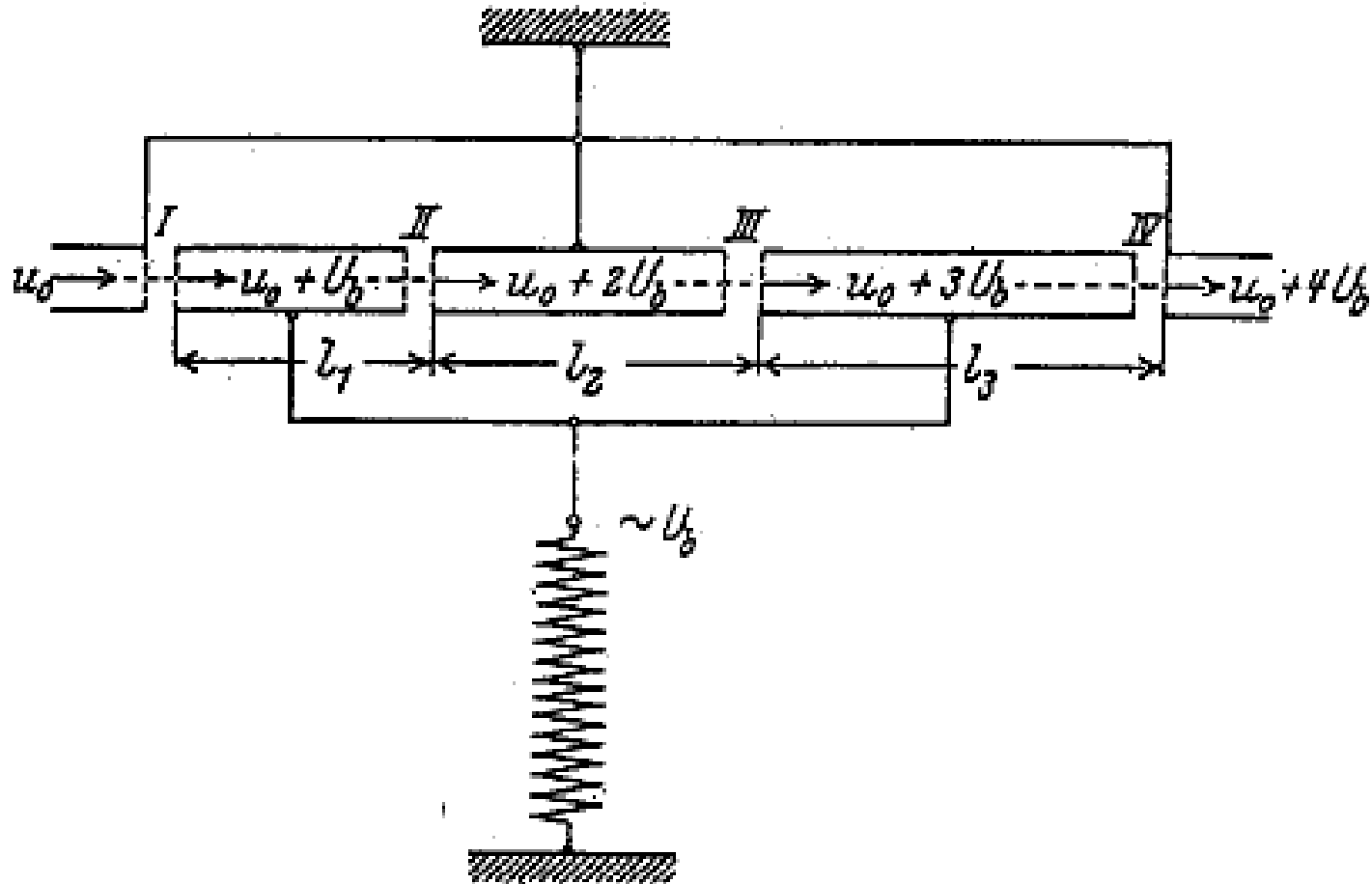
Tandem trick multiplies
the output energy

Ising's Linac Idea



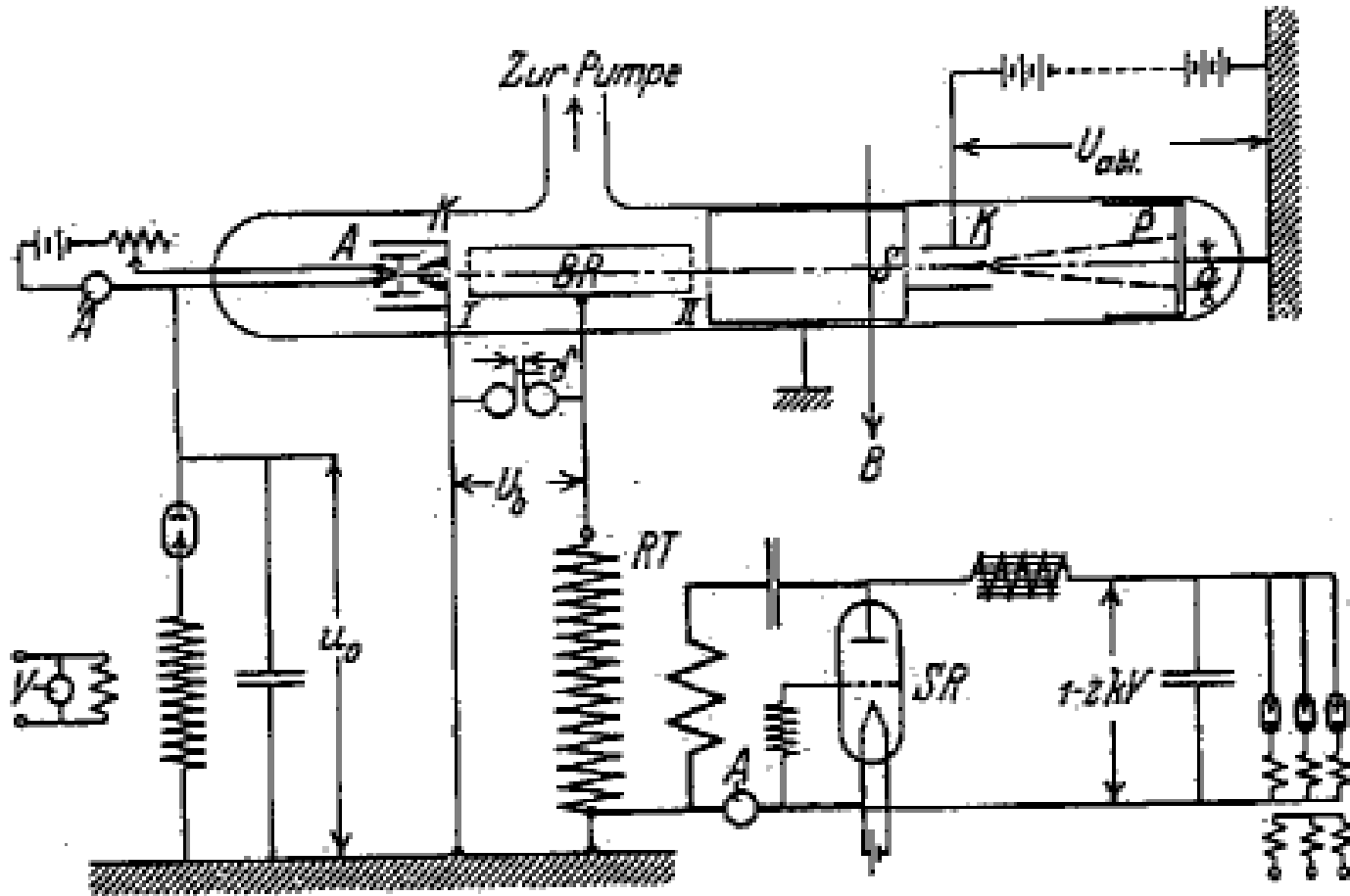
Prinzip einer Methode zur Herstellung von Kanalstrahlen hoher Voltzahl' (in German), Arkiv för matematik o. fysik, 18, Nr. 30, 1-4 (1924).

Drift Tube Linac Proposal



Idea Shown in Wideröe Thesis

Wideröe Thesis Experiment



Über ein neues Prinzip zur Herstellung hoher Spannungen, *Archiv für Elektrotechnik* **21**, 387 (1928)

(On a new principle for the production of higher voltages)

Sloan-Lawrence Heavy Ion Linac

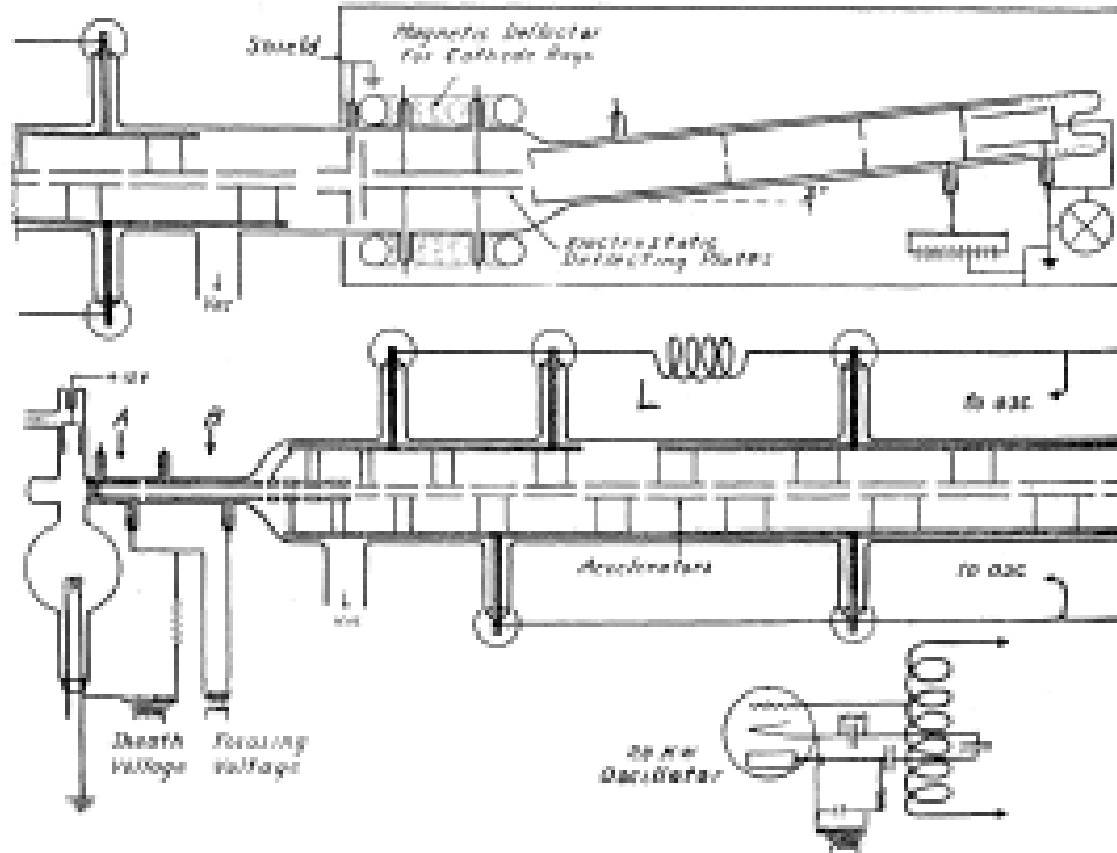


Fig. 1. Diagram of apparatus.

The Production of Heavy High Speed Ions without the Use of High Voltages
David H. Sloan and Ernest O. Lawrence Phys. Rev. **38**, 2021 (1931)

Alvarez Drift Tube Linac

- The first large proton drift tube linac built by Luis Alvarez and Panofsky after WW II
- (1945-1955) Alvarez Proton Linac

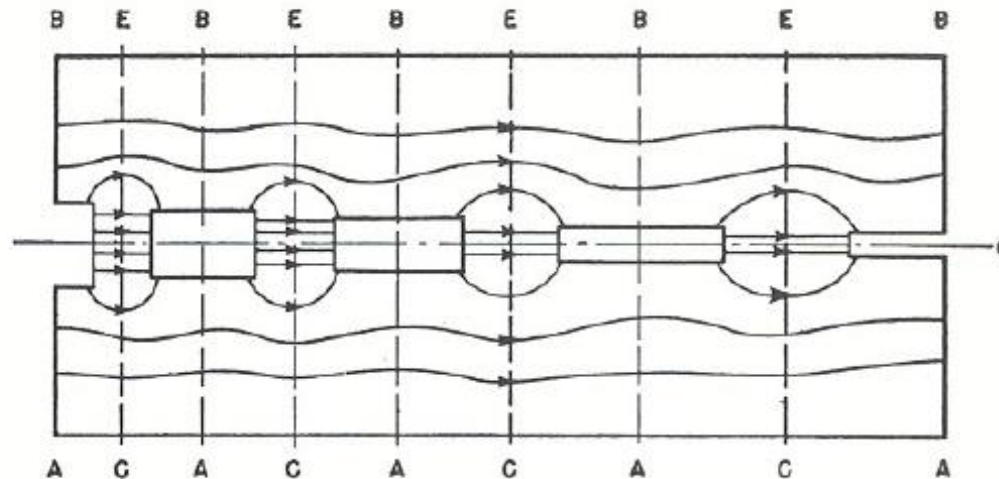


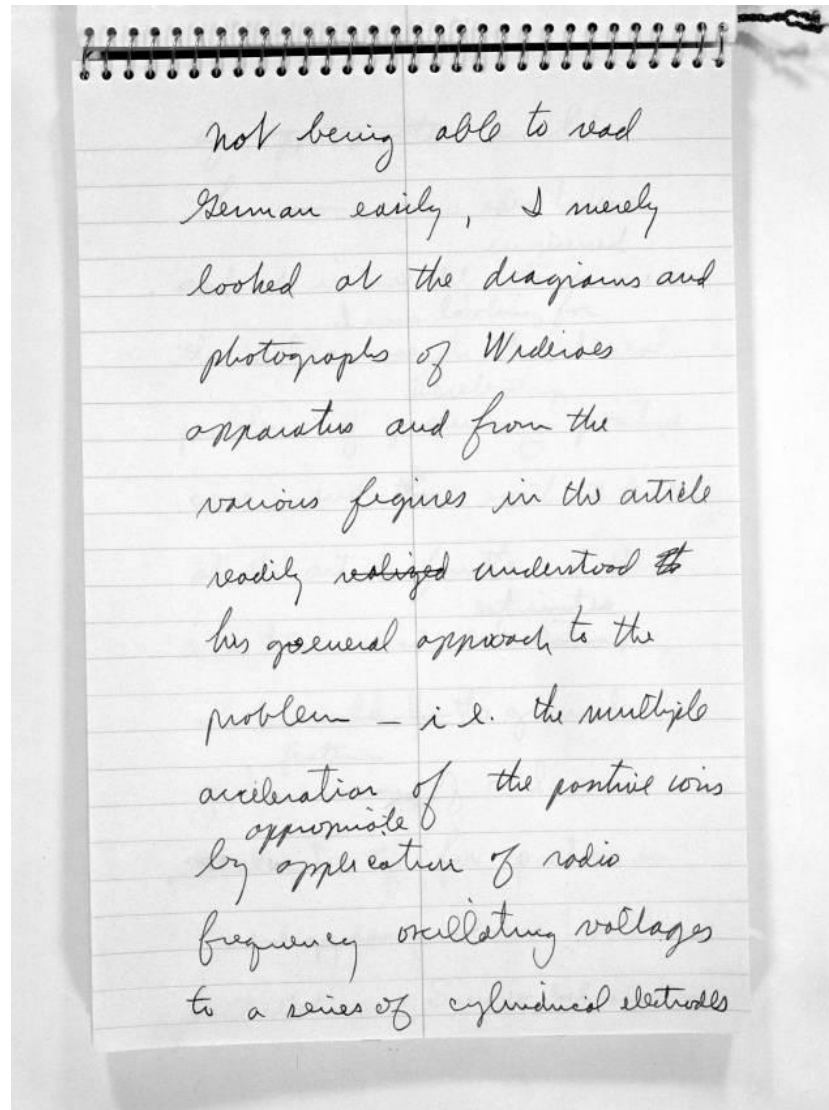
FIG. 2. Linear accelerator produced by introducing drift tubes into cavity excited as in Fig. 1. Division into unit cells.

Alvarez, Bradner, Frank, Gordon, Gow, Marshal, F. Oppenheimer, Panofsky, Richman, and Woodyard, *Rev. Sci. Instrum.*, **26**, 111-133, (1955)

Earnest Orlando Lawrence



Germ of Idea*



not being able to read
German easily, I merely
looked at the diagrams and
photographs of Weiskopff's
apparatus and from the
various figures in the article
readily realized and understood ~~the~~
his general approach to the
problem - i.e. the multiple
acceleration of the positive ions
by ^{appropriate} application of radio
frequency oscillating voltages
to a series of cylindrical electrodes

*Stated in
E. O. Lawrence
Nobel Lecture

Lawrence's Question

- Can you re-use “the same” accelerating gap many times?

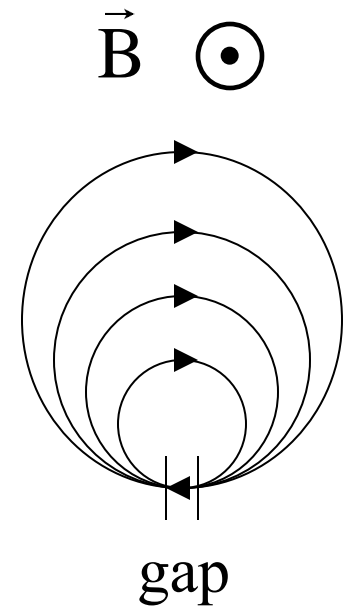
$$\vec{F} = m\vec{a} = q\vec{v} \times \vec{B}$$

$$\frac{d^2 x}{dt^2} = \frac{qB}{m} v_y \rightarrow \frac{d^2 v_x}{dt^2} + \Omega_c^2 v_x = 0$$

$$\frac{d^2 y}{dt^2} = -\frac{qB}{m} v_x \rightarrow \frac{d^2 v_y}{dt^2} + \Omega_c^2 v_y = 0$$

$$\frac{d}{dt} (v_x^2 + v_y^2) = \frac{qB}{m} (v_x v_y - v_y v_x) = 0$$

$$v_0 = \sqrt{v_x^2(t) + v_y^2(t)} \text{ is a constant of the motion}$$



Cyclotron Frequency



$$v_x(t) = v_0 \cos(\Omega_c t + \delta); v_y(t) = -v_0 \sin(\Omega_c t + \delta)$$

$$x(t) = x_0 + \frac{v_0}{\Omega_c} \sin(\Omega_c t + \delta); y(t) = y_0 + \frac{v_0}{\Omega_c} \cos(\Omega_c t + \delta)$$

The radius of the oscillation $r = v_0/\Omega_c$ is proportional to the velocity after the gap. Therefore, the particle **takes the same amount of time to come around to the gap, independent of the actual particle energy!!!!** (only in the non-relativistic approximation). Establish a resonance (equality!) between RF frequency and particle transverse oscillation frequency, also known as the Cyclotron Frequency

$$f_{rf} = f_c = \Omega_c / 2\pi = \frac{qB}{2\pi m}$$

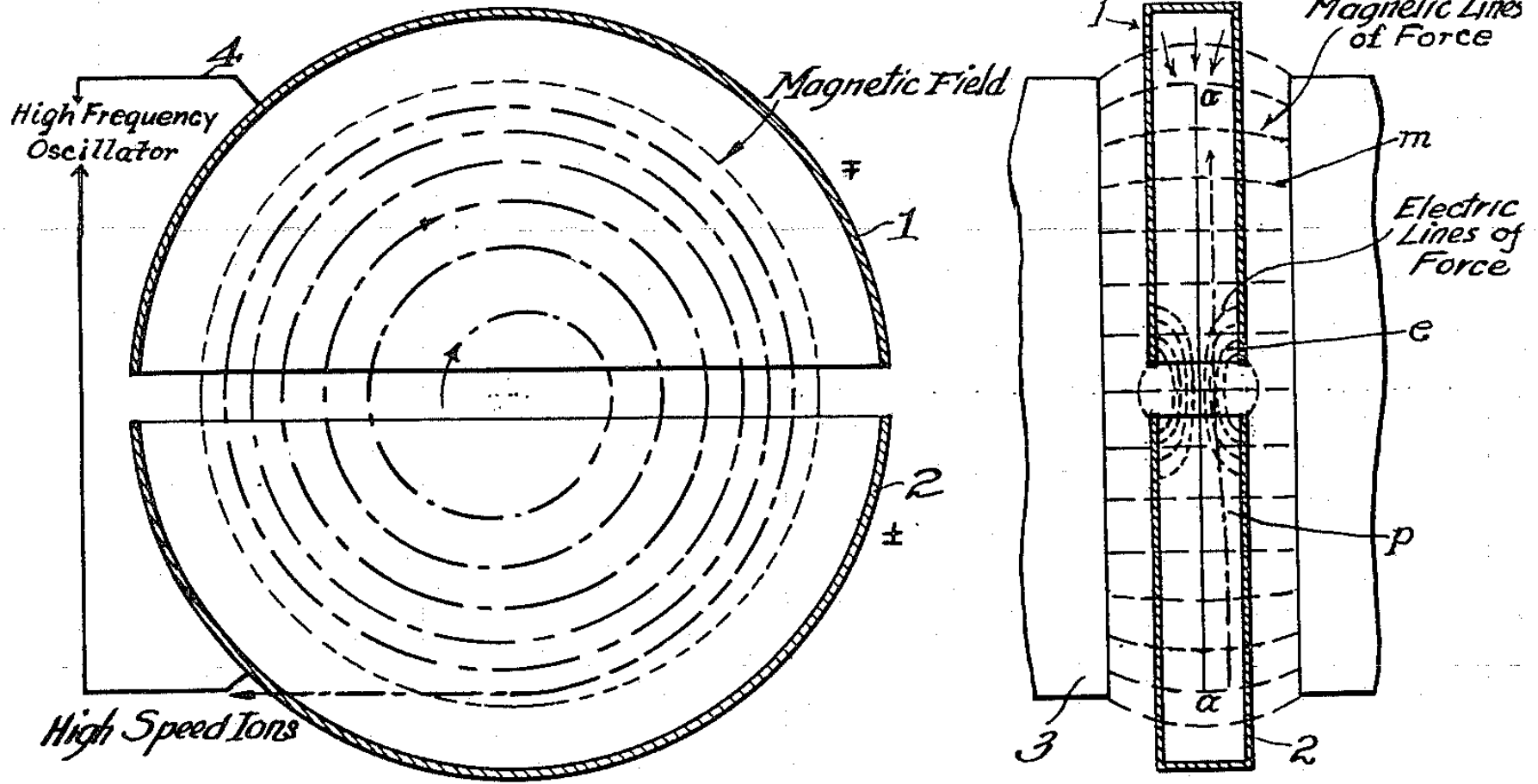
What Correspond to Drift Tubes?



- Dee's!



U. S. Patent Diagram



Magnet for 27 Inch Cyclotron (LHS)

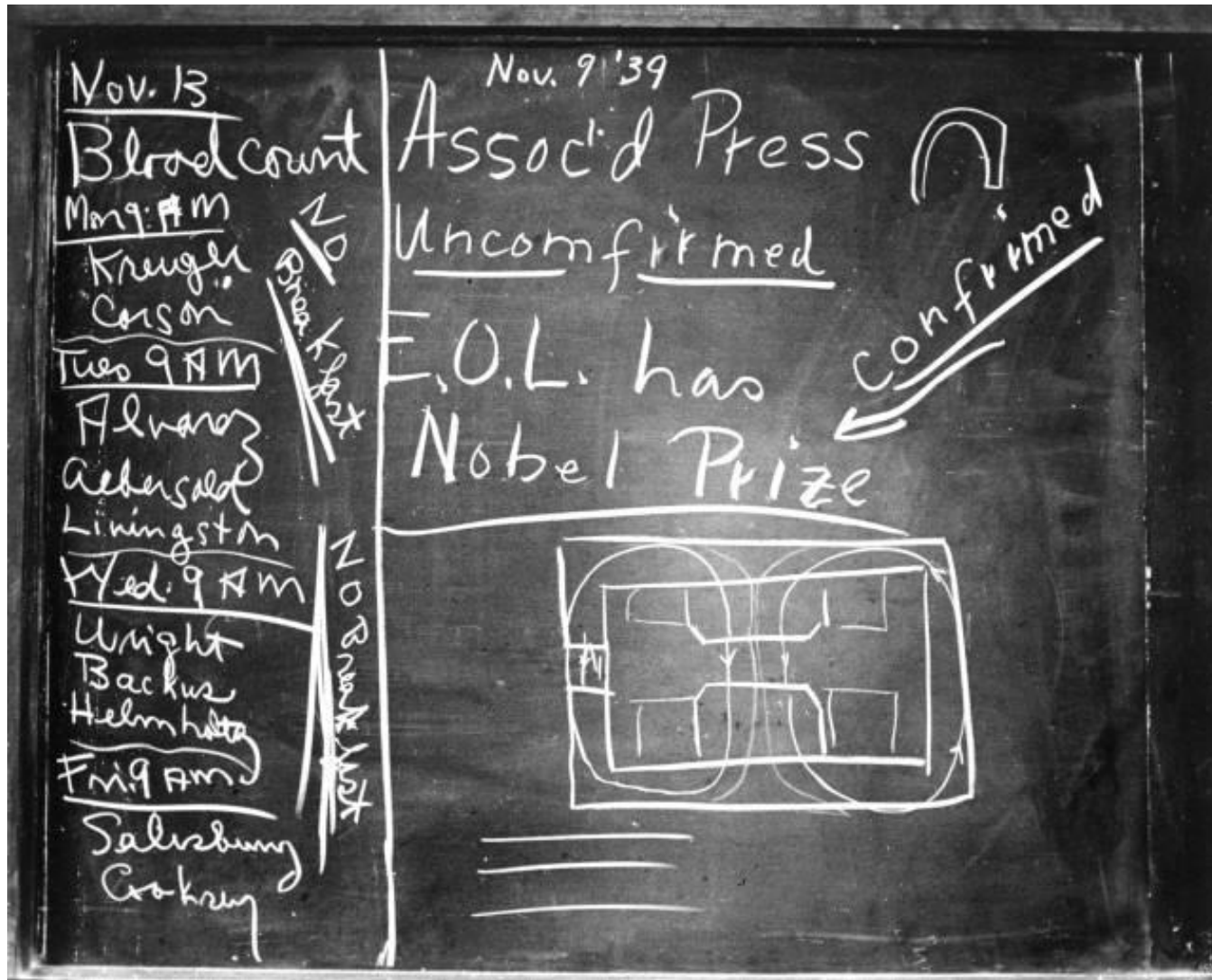


Lawrence and “His Boys”

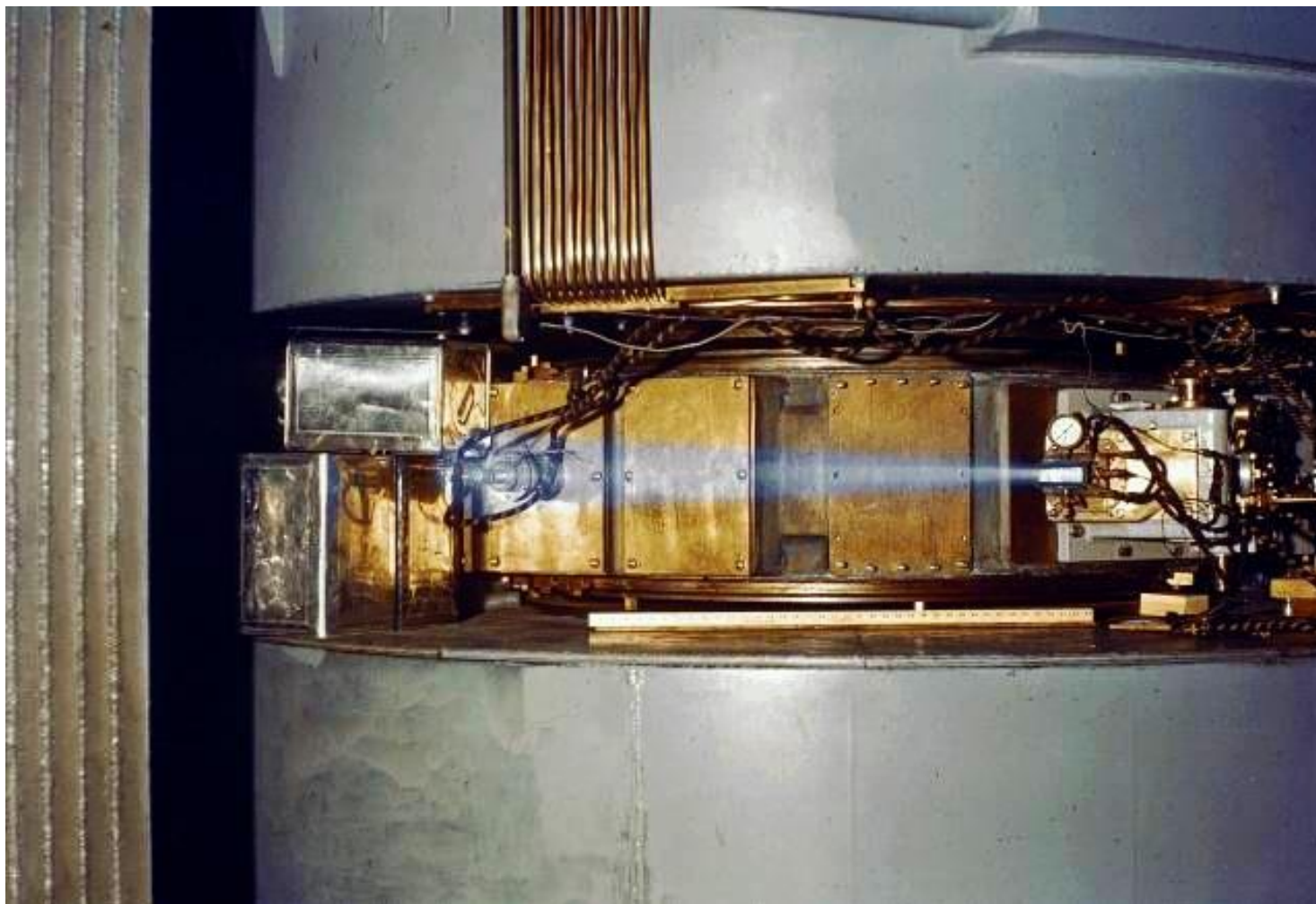


1-4: Jack Livingood, Frank Exner, M.S.Livingston, David Sloan, E.O.Lawrence, Milton White;
Wesley Coates, L.Jackson Laslett and Commander T. Lucci - 1933

And Then!

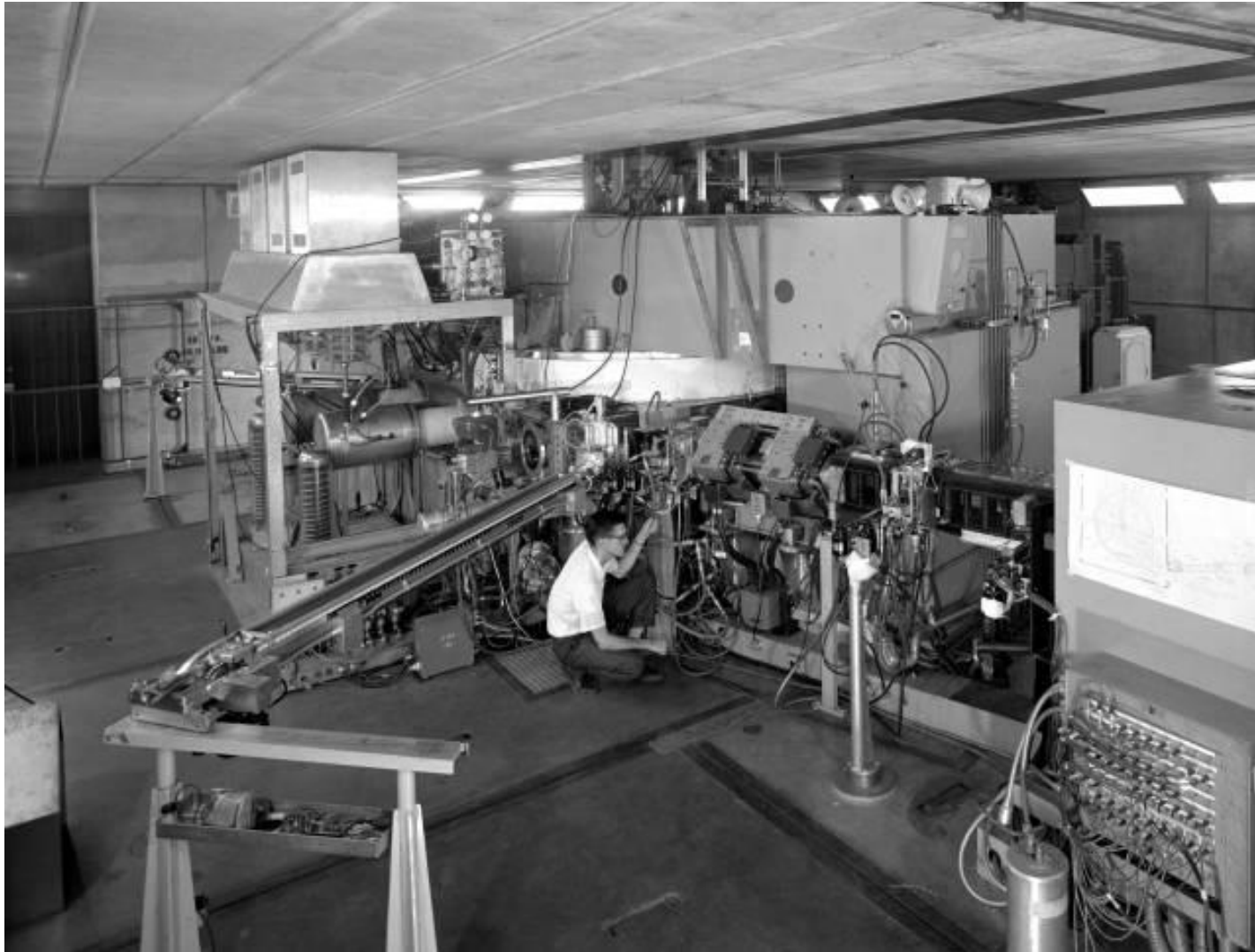


Beam Extracted from a Cyclotron



Radiation Laboratory 60 Inch Cyclotron, circa 1939

88 Inch Cyclotron at Berkeley Lab



Relativistic Corrections



When include relativistic effects (you'll see in the HW!) the “effective” mass to compute the oscillation frequency is the relativistic mass γm

$$f_c = \Omega_c / 2\pi = \frac{qB}{2\pi\gamma m}$$

where γ is Einstein's relativistic γ , most usefully expressed as

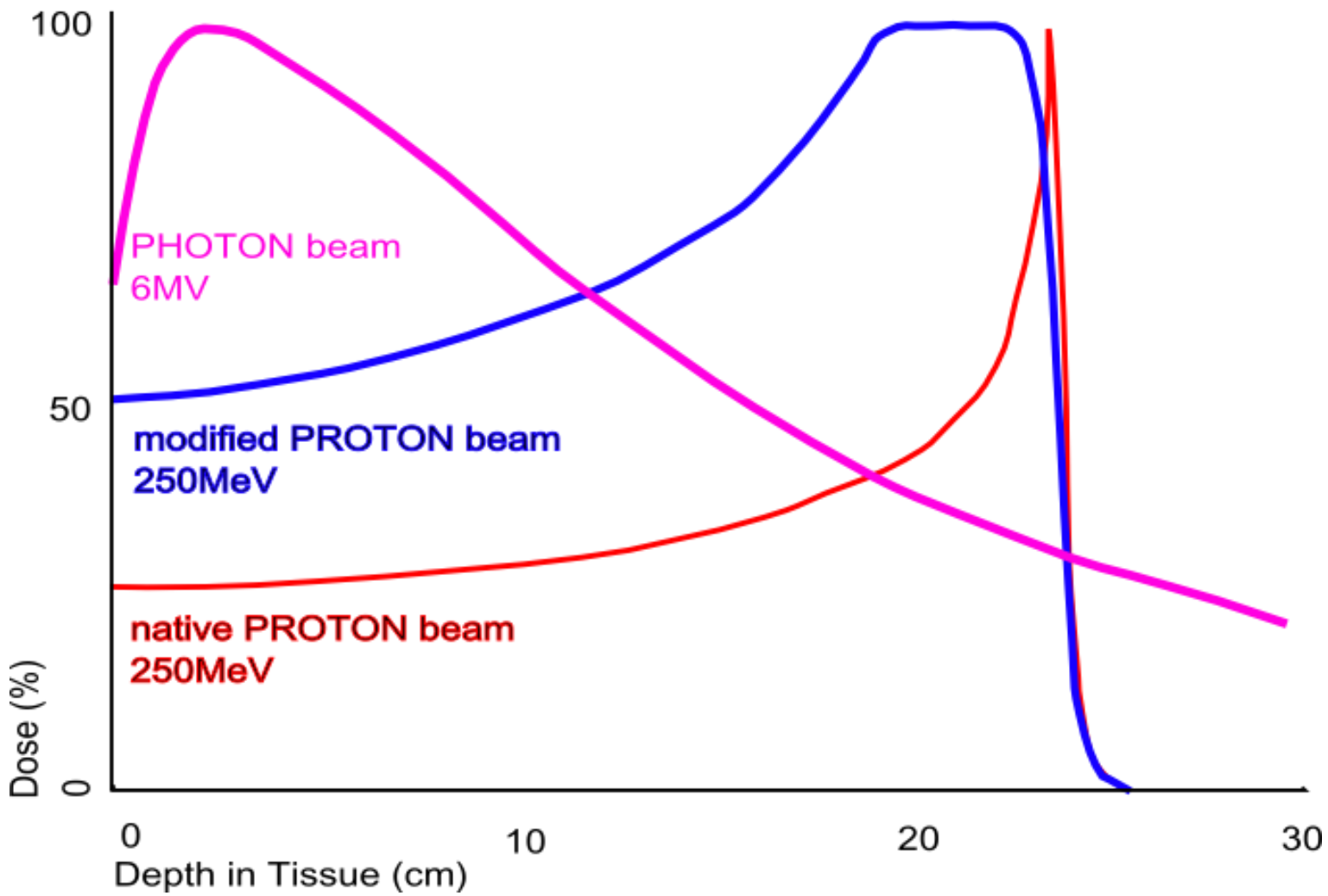
$$\gamma = \frac{E_{tot}}{E_0} = \frac{E_0 + E_{kin}}{E_0} = \frac{mc^2 + E_{kin}}{mc^2}$$

m particle rest mass, E_{kin} particle kinetic energy

Cyclotrons for Radiation Therapy



Bragg Peak



Lorentz Group: Linear Lie Group

- The group of Lorentz Transformations is a Linear (Matrix) Lie group. It can therefore be analyzed by standard methods. First find the generators

$$\Lambda = e^G \approx I + G + \dots$$

$$\Lambda^t g \Lambda = g \rightarrow G^t g + g G = 0 \rightarrow (g G)^t = -g G$$

- General anti-symmetrical matrix yields

$$gG = \begin{pmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & -\zeta_z & \zeta_y \\ \omega_y & \zeta_z & 0 & -\zeta_x \\ \omega_z & -\zeta_y & \zeta_x & 0 \end{pmatrix} \rightarrow G = \begin{pmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ -\omega_x & 0 & \zeta_z & -\zeta_y \\ -\omega_y & -\zeta_z & 0 & \zeta_x \\ -\omega_z & \zeta_y & -\zeta_x & 0 \end{pmatrix}$$

Generators



Six Fundamental Generators

Rotations

$$S_{1,2,3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Boosts

$$K_{1,2,3} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Commutation Relations



$(S_{1,2,3})^2$ and $(K_{1,2,3})^2$ are straightforward to compute
 $(\boldsymbol{\varepsilon} \cdot \boldsymbol{S})^3 = -\boldsymbol{\varepsilon} \cdot \boldsymbol{S}$ and $(\boldsymbol{\varepsilon} \cdot \boldsymbol{K})^3 = \boldsymbol{\varepsilon} \cdot \boldsymbol{K}$ for any unit three-vector $\boldsymbol{\varepsilon}$

Commutation Relations are

$$\left[S_i, S_j \right] = \varepsilon_{ijk} S_k \quad \left[S_i, K_j \right] = \varepsilon_{ijk} K_k \quad \left[K_i, K_j \right] = -\varepsilon_{ijk} S_k$$

Arbitrary Lorentz transformation connected to the identity is

$$G = -\boldsymbol{\omega} \cdot \boldsymbol{S} - \boldsymbol{\zeta} \cdot \boldsymbol{K} \quad \Lambda = \exp(G)$$

$$mc \frac{du^\alpha}{d\tau} = qF^\alpha{}_\nu u^\nu \rightarrow u^\alpha(\tau) = \exp(qF\tau / mc) u(\tau = 0)$$

- Electric Fields give boosts
- Magnetic Fields give rotations