Accelerator Physics
Negative Mass Instability

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Lecture 15
Negative Mass Instability

- Simplified argument: assume longitudinal clump on otherwise uniform beam
- Particles pushed away from clump centroid
- If above transition, come back LATER if ahead of clump center and EARLIER if behind it
- The clump is therefore enhanced!
- INSTABILITY; particles act as if they have negative mass (they accelerate backward compared to force!)
Longitudinal Impedance

$W_{||}$ longitudinal wake function

$\xi$ distance between exciting charge $q$ and test charge

$W_{||}(\xi) \equiv \frac{1}{q_{\text{ring}}} \int E_z(z, t_{q \text{ arrival}} + \xi / \beta c) \, dz \quad \text{units V/C}$

trailing particle (singly charged) picks up voltage per turn of

$\Delta V(\bar{z}) = -e \int_{\bar{z}} \lambda(z) W_{||}(z - \bar{z}) \, dz$

total energy loss

$\Delta U = -\int_{-\infty}^{\infty} e \lambda(\bar{z}) \, d\bar{z} \int_{\bar{z}}^{\infty} e \lambda(z) W_{||}(z - \bar{z}) \, dz$
Frequency Domain

\[ I(\bar{z}, t) = \beta c \lambda(\bar{z}, t) \]

note the coordinate \( \bar{z} \) moves with beam

\[ \Delta V(\bar{z}, t) = -\frac{1}{\beta c} \int_{\bar{z}}^{\infty} I \left( z, t + \frac{z - \bar{z}}{\beta c} \right) W_\parallel (z - \bar{z}) \, dz \]

Fourier Transform

\[ \Delta V(\omega) = -I(\omega) \frac{1}{\beta c} \int_{\bar{z}}^{\infty} e^{-i\omega \xi/\beta c} W_\parallel (\xi) \, d\xi \equiv -Z_\parallel(\omega) I(\omega) \]

\[ Z_\parallel(\omega) = \frac{1}{\beta c} \int_{-\infty}^{\infty} e^{-i\omega \xi/\beta c} W_\parallel (\xi) \, d\xi \]

\[ W_\parallel (\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega z/\beta c} Z_\parallel (\omega) \, d\omega \]

Loss factor

\[ k = \frac{\Delta U}{q^2} = \frac{2}{q^2} \int_{0}^{\infty} \Re \left[ Z(\omega) \right] |I|^2(\omega) \, d\omega \]
NMI Simple Analysis

\( \omega \) revolution frequency of particle

\[
\frac{d \omega}{dt} = \frac{\partial \omega}{\partial t} + \frac{\partial \omega}{\partial \theta} \frac{\partial \theta}{\partial t}
\]

\[
\frac{d \omega}{dt} = \frac{d \omega}{dE} \frac{dE}{dt} = \eta_c \omega_0 \frac{dE}{dt}
\]

\[
\frac{dE}{dt} = qV_{zn} \frac{\omega_0}{2\pi} = -qZ||I_n e^{i(n\theta - \omega t)} \frac{\omega_0}{2\pi}
\]

\[
\omega = \omega_0 + \omega_n e^{i(n\theta - \omega t)} \quad \Omega \text{ oscillation frequency of disturbance}
\]

\[
\omega_n (\Omega - n\omega_0) = -i \frac{q\eta_c \omega_0^2}{2\pi \beta^2} \frac{Z||I_n}{E_0}
\]
Linearized Continuity Equation

\[ I = v_z \rho \pi r_b^2 = v_z \lambda \]

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z} (v_z \rho) = 0 \]

\[ \frac{\partial \lambda}{\partial t} + \frac{1}{R} \frac{\partial}{\partial \theta} (v_z \lambda) = \]

\[ \frac{\partial \delta \lambda}{\partial t} + \omega_0 \frac{\partial \delta \lambda}{\partial \theta} + \lambda_0 \frac{\partial \delta \omega}{\partial \theta} = 0 \]

\[ (\Omega - n\omega_0) I_n = \omega_n nI_0 \]
\[ \Delta \Omega^2 = (\Omega - n\omega_0)^2 = -i \frac{nq\eta_c \omega_0^2 I_0}{2\pi\beta^2 E_0} Z_{||} \]

\( \text{Re} \, Z_{||} \neq 0 \rightarrow 1 \text{ mode has positive imaginary part} \)
\( \rightarrow \text{instability} \)

Resistive impedance has positive real part
"Resistive wall instability"

If \( \text{Re} \, Z_{||} = 0 \) (e.g. space charge impedance at long wavelengths)

stability/instability depends on sign of RHS

\( \text{Im} \, Z_{||} < 0 \) (inductive, stable if \( \eta_c < 0 \), unstable if \( \eta_c > 0 \))

\( \text{Im} \, Z_{||} > 0 \) (capacitive, space charge is this way,
stable if \( \eta_c > 0 \), unstable if \( \eta_c < 0 \))

Later case is negative mass instability