Accelerator Physics
Negative Mass Instability

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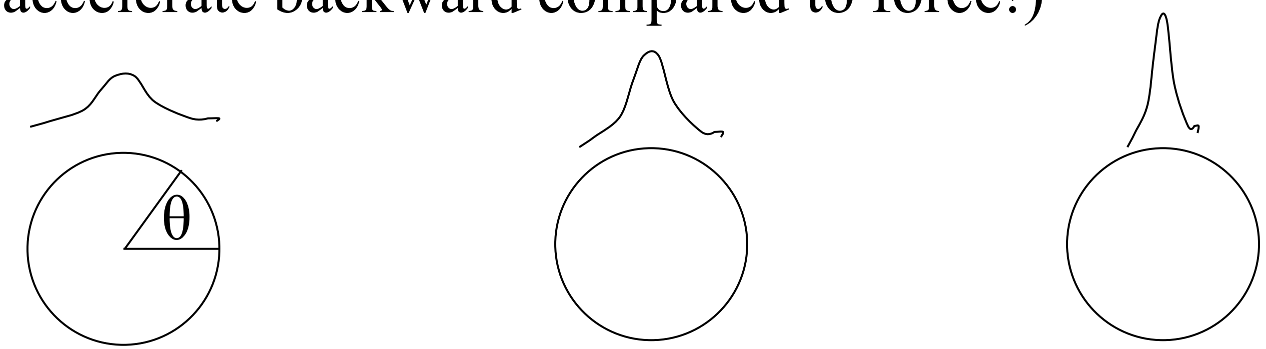
Lecture 15

Negative Mass Instability

- Simplified argument: assume longitudinal clump on otherwise uniform beam
- Particles pushed away from clump centroid
- If above transition, come back LATER if ahead of clump center and EARLIER if behind it
- The clump is therefore enhanced!
- INSTABILITY; particles act as if they have negative mass (they accelerate backward compared to force!)



Longitudinal Impedance

$W_{||}$ longitudinal wake function

ξ distance between exciting charge q and test charge

$$W_{||}(\xi) \equiv \frac{1}{q} \int_{ring} E_z\left(z, t_{q\ arrival} + \xi / \beta c\right) dz$$ units V/C

trailing particle (singly charged) picks up voltage per turn of

$$\Delta V(\bar{z}) = -e \int_{\bar{z}}^{"\infty"} \lambda(z) W_{||}(z - \bar{z})\, dz$$

total energy loss

$$\Delta U = -\int_{-\infty}^{\infty} e\lambda(\bar{z})\, d\bar{z} \int_{\bar{z}}^{"\infty"} e\lambda(z) W_{||}(z - \bar{z})\, dz$$

Frequency Domain

$I(\bar{z},t) = \beta c \lambda(\bar{z},t)$ note the coordinate $\bar{z}$ moves with beam

$$\Delta V(\bar{z},t) = -\frac{1}{\beta c}\int_{\bar{z}}^{"\infty"} I\left(z, t+\frac{z-\bar{z}}{\beta c}\right) W_{\parallel}(z-\bar{z})\,dz$$

Fourier Transform

$$\Delta V(\omega) = -I(\omega)\frac{1}{\beta c}\int_{\bar{z}}^{"\infty"} e^{-i\omega\xi/\beta c} W_{\parallel}(\xi)\,dz \equiv -Z_{\parallel}(\omega) I(\omega)$$

$$Z_{\parallel}(\omega) = \frac{1}{\beta c}\int_{-\infty}^{\infty} e^{-i\omega\xi/\beta c} W_{\parallel}(\xi)\,d\xi$$

$$W_{\parallel}(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega z/\beta c} Z_{\parallel}(\omega)\,d\omega$$

Loss factor

$$k = \frac{\Delta U}{q^2} = \frac{2}{q^2}\int_0^{\infty} \mathrm{Re}\left[Z(\omega)\right] |I|^2(\omega)\,d\omega$$

NMI Simple Analysis

ω revolution frequency of particle

$$\frac{d\omega}{dt} = \frac{\partial\omega}{\partial t} + \frac{\partial\omega}{\partial\theta}\frac{\partial\theta}{\partial t}$$

$$\frac{d\omega}{dt} = \frac{d\omega}{dE}\frac{dE}{dt} = \frac{\eta_c\omega_0}{\beta^2 E_0}\frac{dE}{dt}$$

$$\frac{dE}{dt} = qV_{zn}\frac{\omega_0}{2\pi} = -qZ_{\|}I_n e^{i(n\theta-\Omega t)}\frac{\omega_0}{2\pi}$$

$$\omega = \omega_0 + \omega_n e^{i(n\theta-\Omega t)}$$ Ω oscillation frequency of disturbance

$$\omega_n\left(\Omega - n\omega_0\right) = -i\frac{q\eta_c\omega_0^2}{2\pi\beta^2}\frac{Z_{\|}I_n}{E_0}$$

Linearized Continuity Equation

$$I = v_z \rho \pi r_b^2 = v_z \lambda$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(v_z \rho) = 0$$

$$\frac{\partial \lambda}{\partial t} + \frac{1}{R}\frac{\partial}{\partial \theta}(v_z \lambda) =$$

$$\frac{\partial \delta\lambda}{\partial t} + \omega_0 \frac{\partial \delta\lambda}{\partial \theta} + \lambda_0 \frac{\partial \delta\omega}{\partial \theta} = 0$$

$$(\Omega - n\omega_0) I_n = \omega_n n I_0$$

Oscillation Frequency

$$\Delta\Omega^2 = \left(\Omega - n\omega_0\right)^2 = -i\,\frac{nq\eta_c\omega_0^2 I_0}{2\pi\beta^2 E_0} Z_{||}$$

$\mathrm{Re}\, Z_{||} \neq 0 \rightarrow 1$ mode has positive imaginary part

$\rightarrow$ instability

Resistive impedance has positive real part

"Resistive wall instability"

If $\mathrm{Re}\, Z_{||} = 0$ (e.g. space charge impedance at long wavelengths)

stability/instability depends on sign of RHS

$\mathrm{Im}\, Z_{||} < 0$ (inductive, stable if $\eta_c < 0$, unstable if $\eta_c > 0$)

$\mathrm{Im}\, Z_{||} > 0$ (capacitive, space charge is this way,

stable if $\eta_c > 0$, unstable if $\eta_c < 0$)

Later case is negative mass instability