Betatron Motion with Coupling of Horizontal and Vertical Degrees of Freedom – Part II

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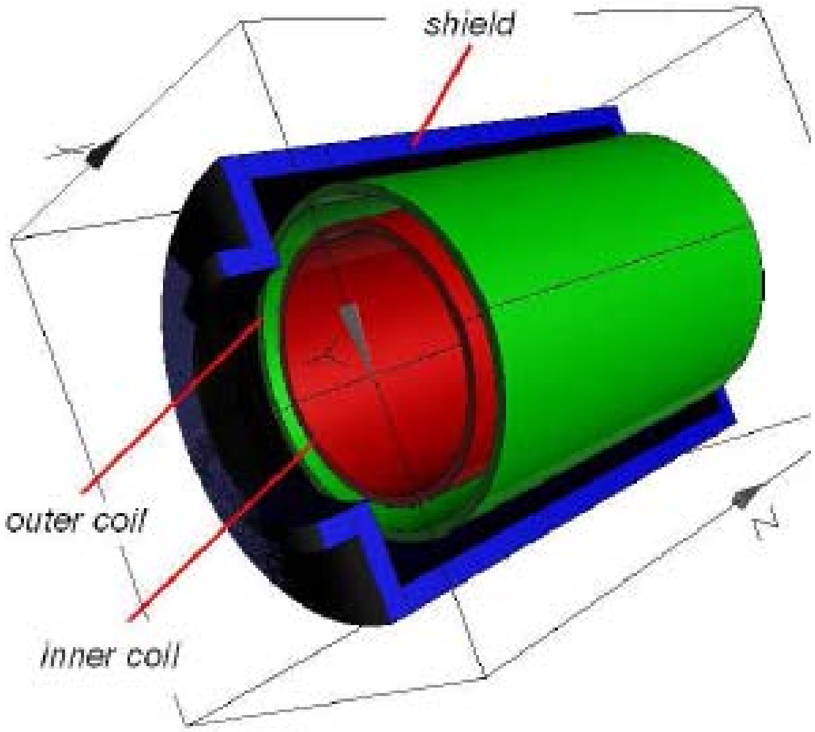
Outline

- Practical Examples:
  - Soft-edge Solenoid model
  - Vertex-to-plane adapter for electron cooling (Fermilab)
  - Spin Rotator for Figure-8 Collider ring
  - Ionization cooling channel for Neutrino Factory and Muon Collider
  - Generalized vertex-to-plane transformer insert

    - V. Lebedev, A. Bogacz, 'Betatron Motion with Coupling of Horizontal and Vertical Degrees of Freedom', 2000, http://dx.doi.org/10.1088/1748-0221/5/10/P10010

'Hard-edge' Solenoid



$$B_z(s) = B_0\left[\theta_h\left(s+L/2\right)-\theta_h\left(s-L/2\right)\right]$$

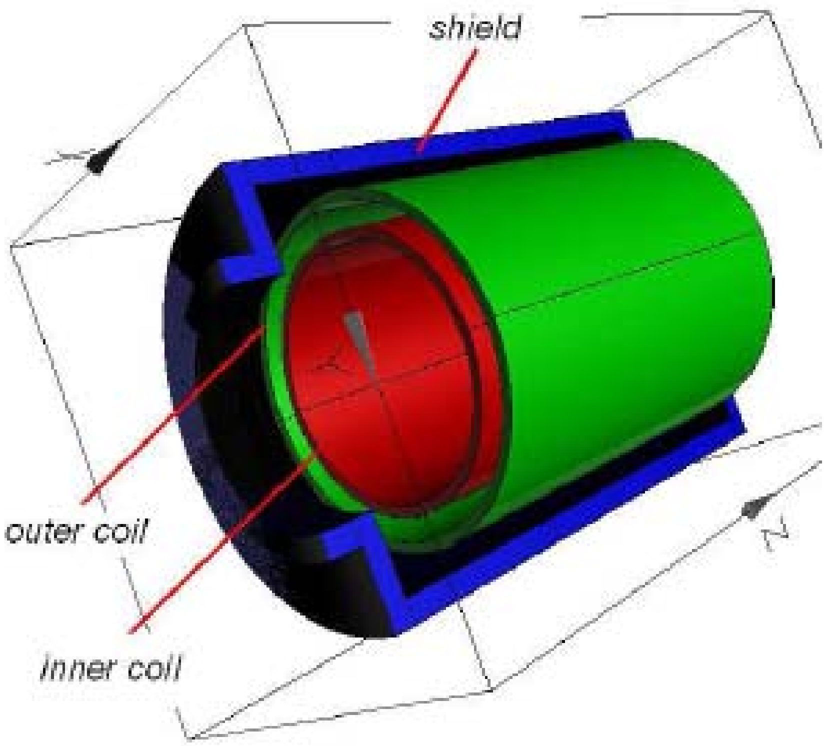
Solenoid – 'Hard-edge' Model

- Linear Transfer Matrix for infinitely long solenoid : $a << L$

$$\mathbf{M}_{sol} = \begin{bmatrix} \frac{1+\cos(kL)}{2} & \frac{\sin(kL)}{k} & \frac{\sin(kL)}{2} & \frac{1-\cos(kL)}{k} \\ -\frac{k\sin(kL)}{4} & \frac{1+\cos(kL)}{2} & -k\frac{1-\cos(kL)}{4} & \frac{\sin(kL)}{2} \\ -\frac{\sin(kL)}{2} & -\frac{1-\cos(kL)}{k} & \frac{1+\cos(kL)}{2} & \frac{\sin(kL)}{k} \\ k\frac{1-\cos(kL)}{4} & -\frac{\sin(kL)}{2} & -\frac{k\sin(kL)}{4} & \frac{1+\cos(kL)}{2} \end{bmatrix} \qquad k = \frac{e}{pc}B_0$$

$$\Phi_{sol} = \left(\frac{e}{pc}\right)^2 \int_{-\infty}^{\infty} B_z^2(s)\,ds = \left(\frac{e}{pc}\right)^2 B_0^2 L = \frac{1}{f_{sol}}$$

'Soft-edge' Solenoid



$$B_z(s) = \frac{1}{2}B_0\left[1 - \tanh\left(\frac{s - L/2}{a}\right)\right]$$

Solenoid – 'Soft-edge' Model

- Non-zero aperture - correction due to the finite length of the edge: $a \approx L$
  - It introduces axially symmetric edge focusing at each solenoid end:

$$\mathbf{M}_{\text{soft sol}} = \mathbf{M}_{\text{edge}}\,\mathbf{M}_{\text{sol}}\,\mathbf{M}_{\text{edge}}$$

$$\mathbf{M}_{\text{edge}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\Phi_{\text{edge}} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\Phi_{\text{edge}} & 1 \end{bmatrix}$$

$$\Phi_{\text{edge}} = \frac{1}{2}\left(\frac{e}{pc}\right)^2 \left(\int_{-\infty}^{\infty} B_z^2(s)\,ds - B_0^2\,L\right) = -\frac{k^2 a}{8}$$

$$k = \frac{e}{pc} B_0$$

$$\mathbf{M}_{\text{sol}} = \begin{bmatrix} \frac{1+\cos(kL)}{2} & \frac{\sin(kL)}{k} & \frac{\sin(kL)}{2} & \frac{1-\cos(kL)}{k} \\ -\frac{k\sin(kL)}{4} & \frac{1+\cos(kL)}{2} & -k\frac{1-\cos(kL)}{4} & \frac{\sin(kL)}{2} \\ -\frac{\sin(kL)}{2} & -\frac{1-\cos(kL)}{k} & \frac{1+\cos(kL)}{2} & \frac{\sin(kL)}{k} \\ k\frac{1-\cos(kL)}{4} & -\frac{\sin(kL)}{2} & -\frac{k\sin(kL)}{4} & \frac{1+\cos(kL)}{2} \end{bmatrix}$$

‘Soft-edge’ Solenoid – Off-axis Fields

- Nonlinear focusing term $\Delta F \sim \mathcal{O}(r^2)$ follows from the scalar potential:

$$\phi(r,z) = \phi_0 + B_z \cdot z + \left(\frac{d}{dz}B_z\right)\cdot\frac{2\cdot z^2 - r^2}{4} + \left(\frac{d^2}{dz^2}B_z\right)\cdot\frac{z\cdot\left(2\cdot z^2 - 3\cdot r^2\right)}{12} + \left(\frac{d^3}{dz^3}B_z\right)\cdot\frac{8\cdot z^4 - 24\cdot z^2\cdot r^2 + 3\cdot r^4}{192}$$

  - Solenoid B-fields

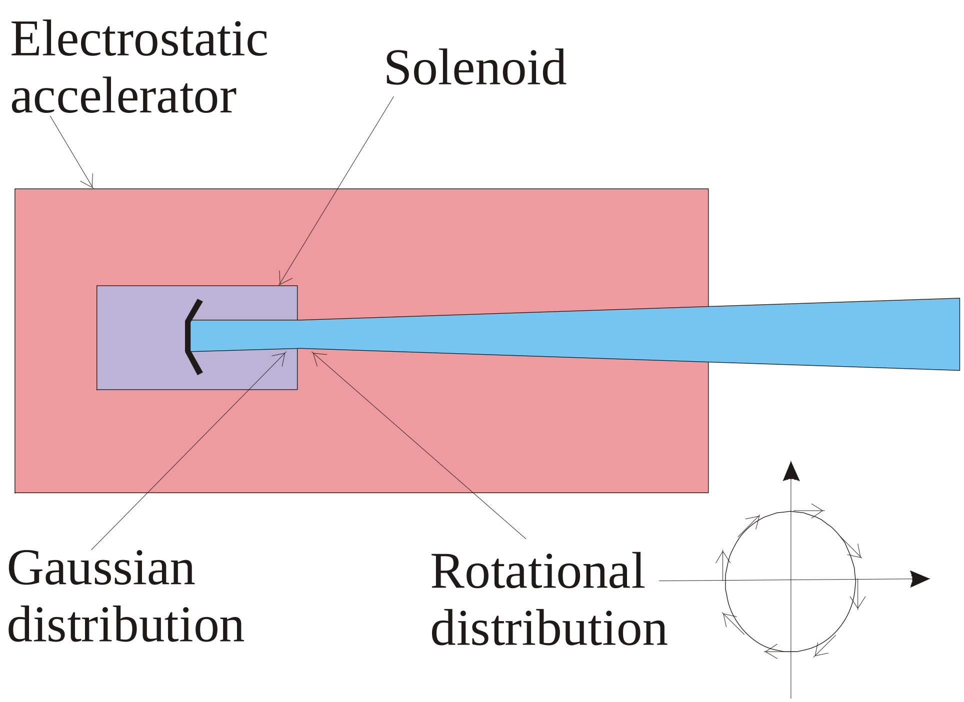
$$B_z(r,z) = B_z - \left(\frac{d^2}{dz^2}B_z\right)\cdot\frac{r^2}{4}$$

$$B_r(r,z) = -\frac{r}{2}\cdot\left(\frac{d}{dz}B_z\right) + \frac{r^3}{16}\cdot\left(\frac{d^3}{dz^3}B_z\right)$$

- Nonlinear focusing included in particle tracking

Axisymmetric Rotational Distribution

- Fermilab electron cooling



The electron beam distribution is axially symmetric, and uncoupled at the cathode:

$$\mathbf{\Xi}_B = \frac{1}{\varepsilon_T}\begin{bmatrix} \gamma_0 & \alpha_0 & 0 & 0 \\ \alpha_0 & \beta_0 & 0 & 0 \\ 0 & 0 & \gamma_0 & \alpha_0 \\ 0 & 0 & \alpha_0 & \beta_0 \end{bmatrix}$$

where $\varepsilon_T = r_c\sqrt{mkT_c}\,/\,P_0$ is the thermal emittance of the beam

Axisymmetric Rotational Distribution

- At the exit of the solenoid the electron beam distribution is still axially symmetric

$$\boldsymbol{\Xi}_{in} = \boldsymbol{\Phi}^T \ \boldsymbol{\Xi}_B \ \boldsymbol{\Phi} = \frac{1}{\varepsilon_T}\begin{bmatrix} \gamma_0 + \Phi^2\beta_0 & \alpha_0 & 0 & -\Phi\beta_0 \\ \alpha_0 & \beta_0 & \Phi\beta_0 & 0 \\ 0 & \Phi\beta_0 & \gamma_0 + \Phi^2\beta_0 & \alpha_0 \\ -\Phi\beta_0 & 0 & \alpha_0 & \beta_0 \end{bmatrix}$$

where

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \Phi & 0 \\ 0 & 0 & 1 & 0 \\ -\Phi & 0 & 0 & 1 \end{bmatrix}$$

  - $\Phi = eB/2P_0c$ is the rotational focusing strength of the solenoid
  - *B* is the solenoid magnetic field.

Axisymmetric Rotational Distribution

- The eigen-vectors of the rotational distribution:

$$\hat{\mathbf{v}}_1 = \begin{bmatrix} \sqrt{\beta} \\ -\dfrac{i+2\alpha}{2\sqrt{\beta}} \\ i\sqrt{\beta} \\ -i\dfrac{i+2\alpha}{2\sqrt{\beta}} \end{bmatrix} \quad , \quad \hat{\mathbf{v}}_2 = \begin{bmatrix} i\sqrt{\beta} \\ -i\dfrac{i+2\alpha}{2\sqrt{\beta}} \\ \sqrt{\beta} \\ -\dfrac{i+2\alpha}{2\sqrt{\beta}} \end{bmatrix}$$

  - It corresponds to *u* = 1/2, $\nu_1 = \nu_2 = \pi / 2$

- Then, the matrix $\hat{\mathbf{V}}$ is

$$\hat{\mathbf{V}} = \begin{bmatrix} \sqrt{\beta} & 0 & 0 & -\sqrt{\beta} \\ -\dfrac{\alpha}{\sqrt{\beta}} & \dfrac{1}{2\sqrt{\beta}} & \dfrac{1}{2\sqrt{\beta}} & \dfrac{\alpha}{\sqrt{\beta}} \\ 0 & -\sqrt{\beta} & \sqrt{\beta} & 0 \\ \dfrac{1}{2\sqrt{\beta}} & \dfrac{\alpha}{\sqrt{\beta}} & -\dfrac{\alpha}{\sqrt{\beta}} & \dfrac{1}{2\sqrt{\beta}} \end{bmatrix}$$

Axisymmetric Rotational Distribution

- Comparing left and right hand sides of the equation

$$\hat{\mathbf{\Xi}}_{in} = \mathbf{U}\hat{\mathbf{V}}\begin{bmatrix} 1/\varepsilon_1 & 0 & 0 & 0 \\ 0 & 1/\varepsilon_1 & 0 & 0 \\ 0 & 0 & 1/\varepsilon_2 & 0 \\ 0 & 0 & 0 & 1/\varepsilon_2 \end{bmatrix}\hat{\mathbf{V}}^T\mathbf{U}^T$$

- One obtains

$$\beta = \frac{\beta_0}{2\sqrt{1+\Phi^2\beta_0^{\,2}}} \quad ,$$

$$\alpha = \frac{\alpha_0}{2\sqrt{1+\Phi^2\beta_0^{\,2}}} \quad ,$$

$$\varepsilon_1 = \frac{\varepsilon_T}{\sqrt{1+\Phi^2\beta_0^{\,2}} - \Phi\beta_0} \xrightarrow{\Phi\beta_0 >>1} 2\Phi\beta_0\varepsilon_T \quad ,$$

$$\varepsilon_2 = \frac{\varepsilon_T}{\sqrt{1+\Phi^2\beta_0^{\,2}} + \Phi\beta_0} \xrightarrow{\Phi\beta_0 >>1} \frac{\varepsilon_T}{2\Phi\beta_0} \quad .$$

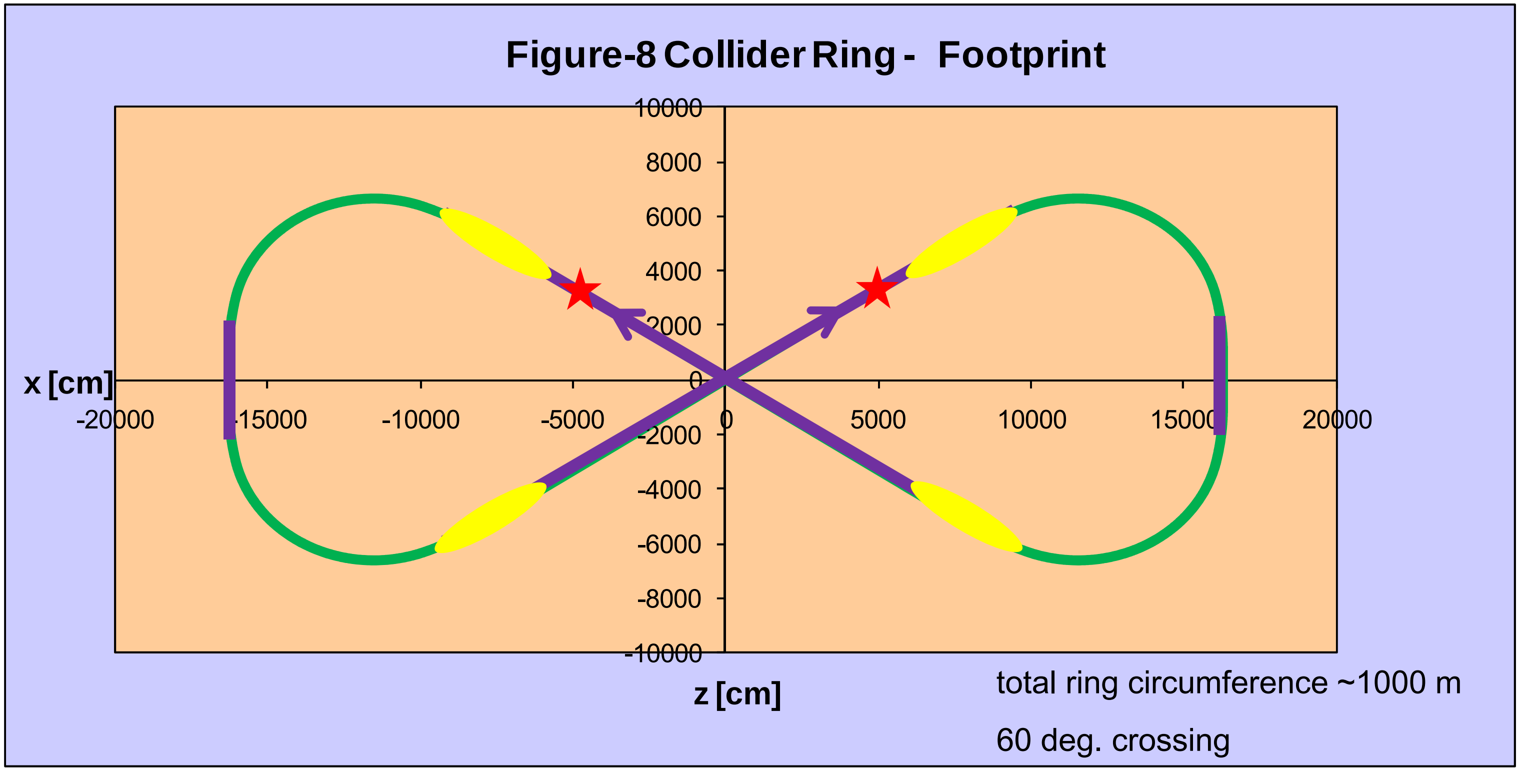
- 4D-emmitance conservation:

$$\varepsilon_1\varepsilon_2 = \varepsilon_T^{\,2}$$

- Rotational emittance estimate

$$\varepsilon_{rot} = r\theta = r\left(r\Phi\right) = r^2\Phi = \left(\varepsilon_T\beta_0\right)\Phi$$

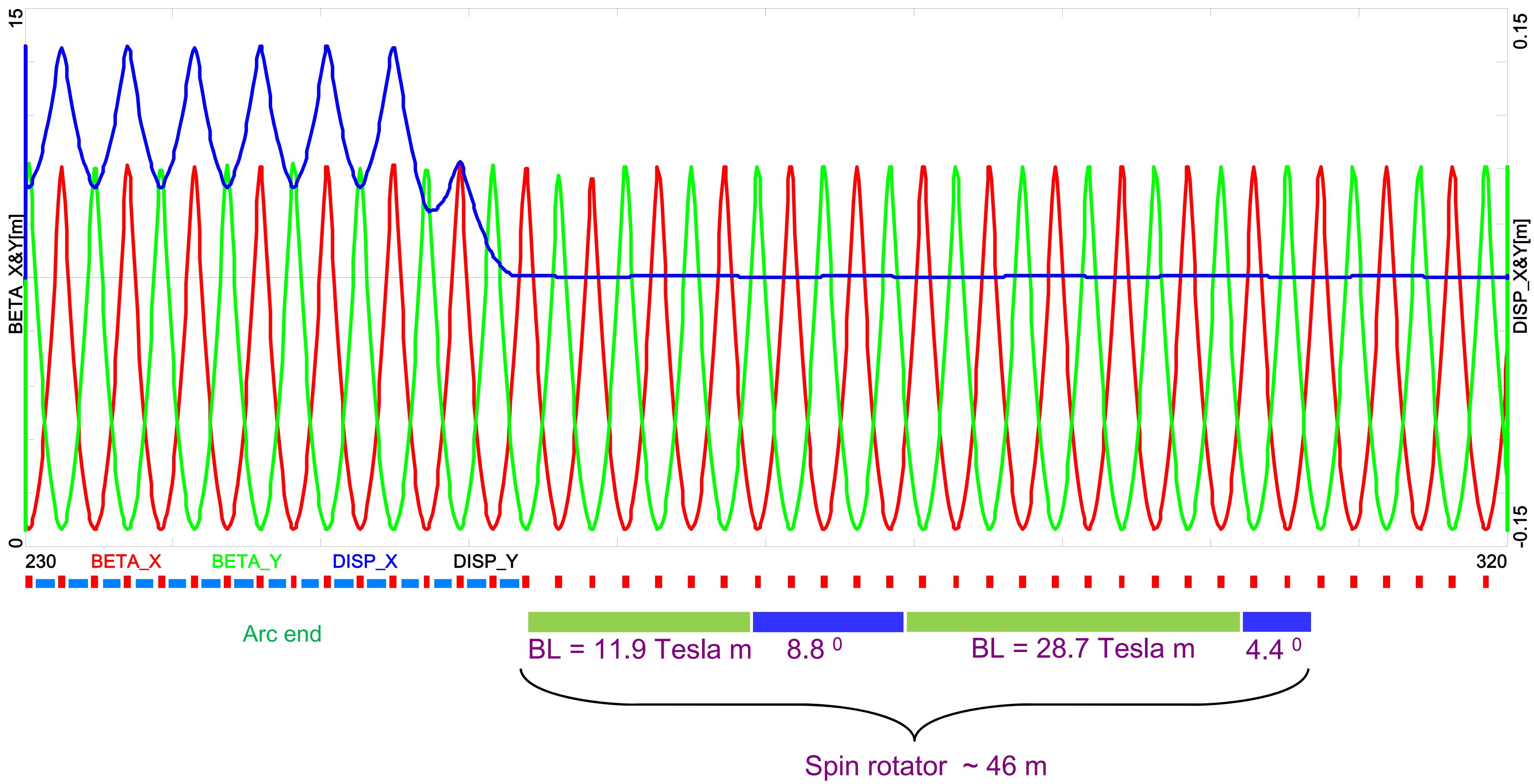
Spin Rotators for Figure-8 Collider Ring



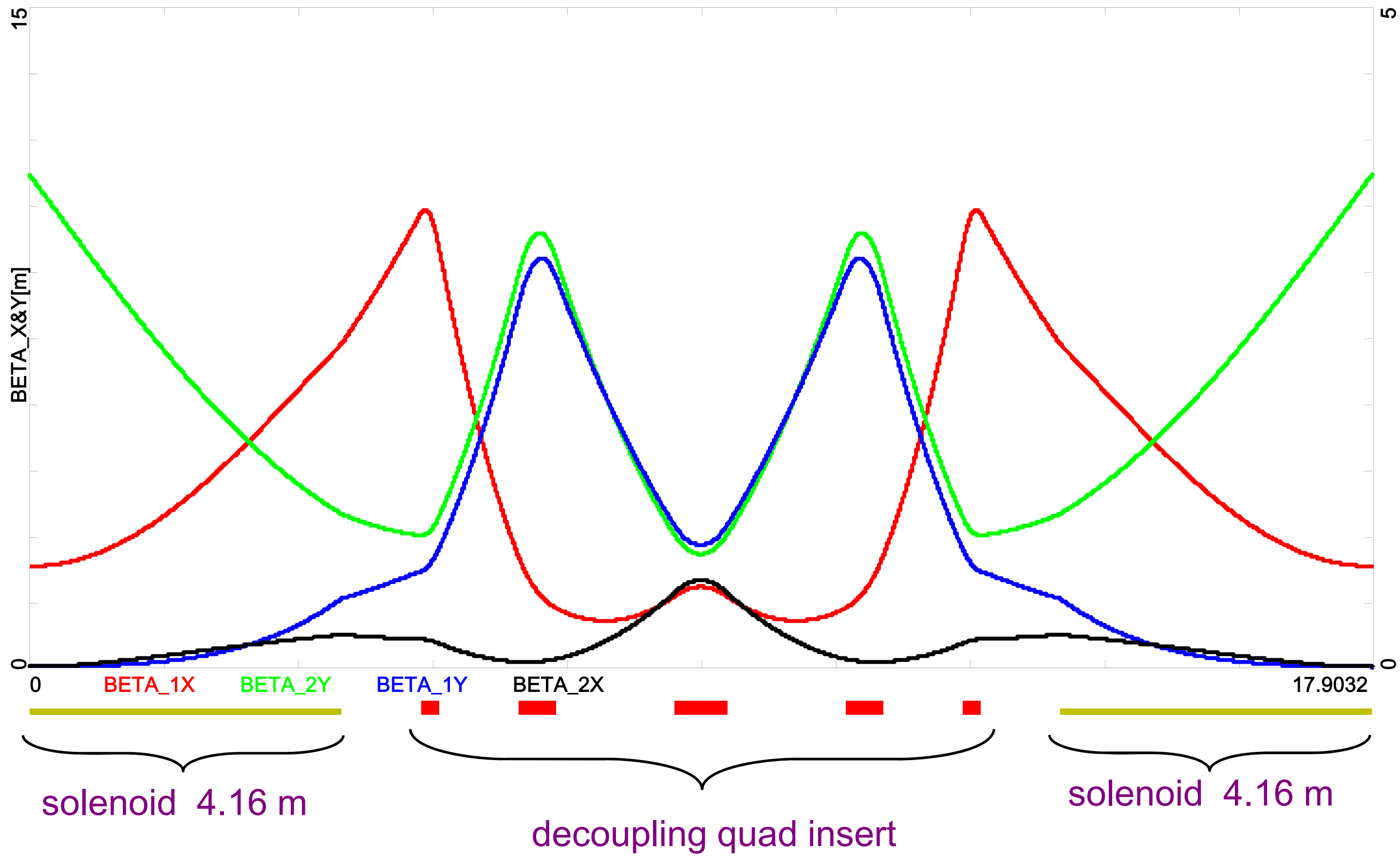
total ring circumference ~1000 m

60 deg. crossing

Spin Rotator – Ingredients…



Locally Decoupled Solenoid Pair



$$M = \begin{pmatrix} C & 0 \\ 0 & -C \end{pmatrix}$$

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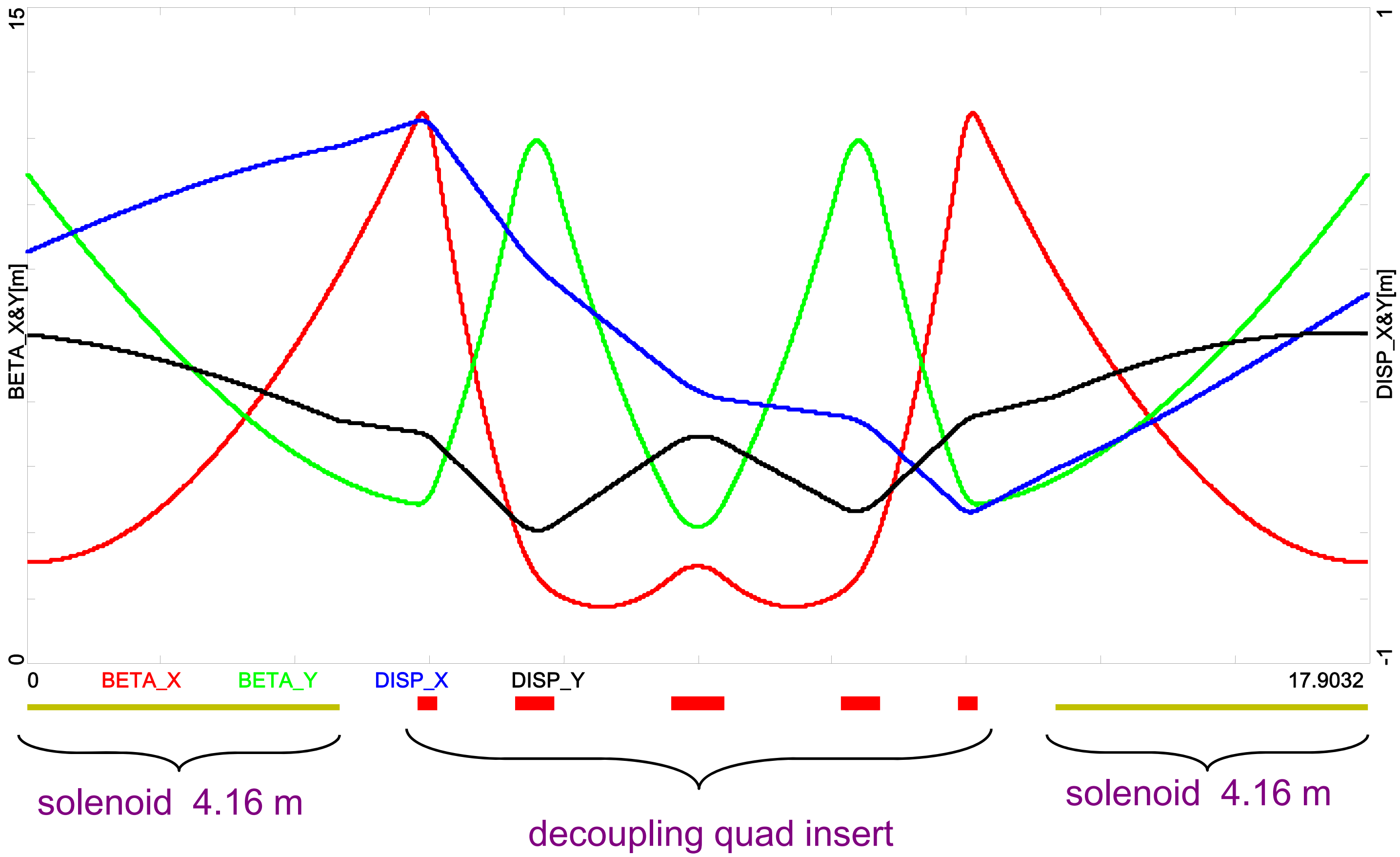
ODU, 2011

Locally Decoupled Solenoid Pair



$$M = \begin{pmatrix} C & 0 \\ 0 & -C \end{pmatrix}$$

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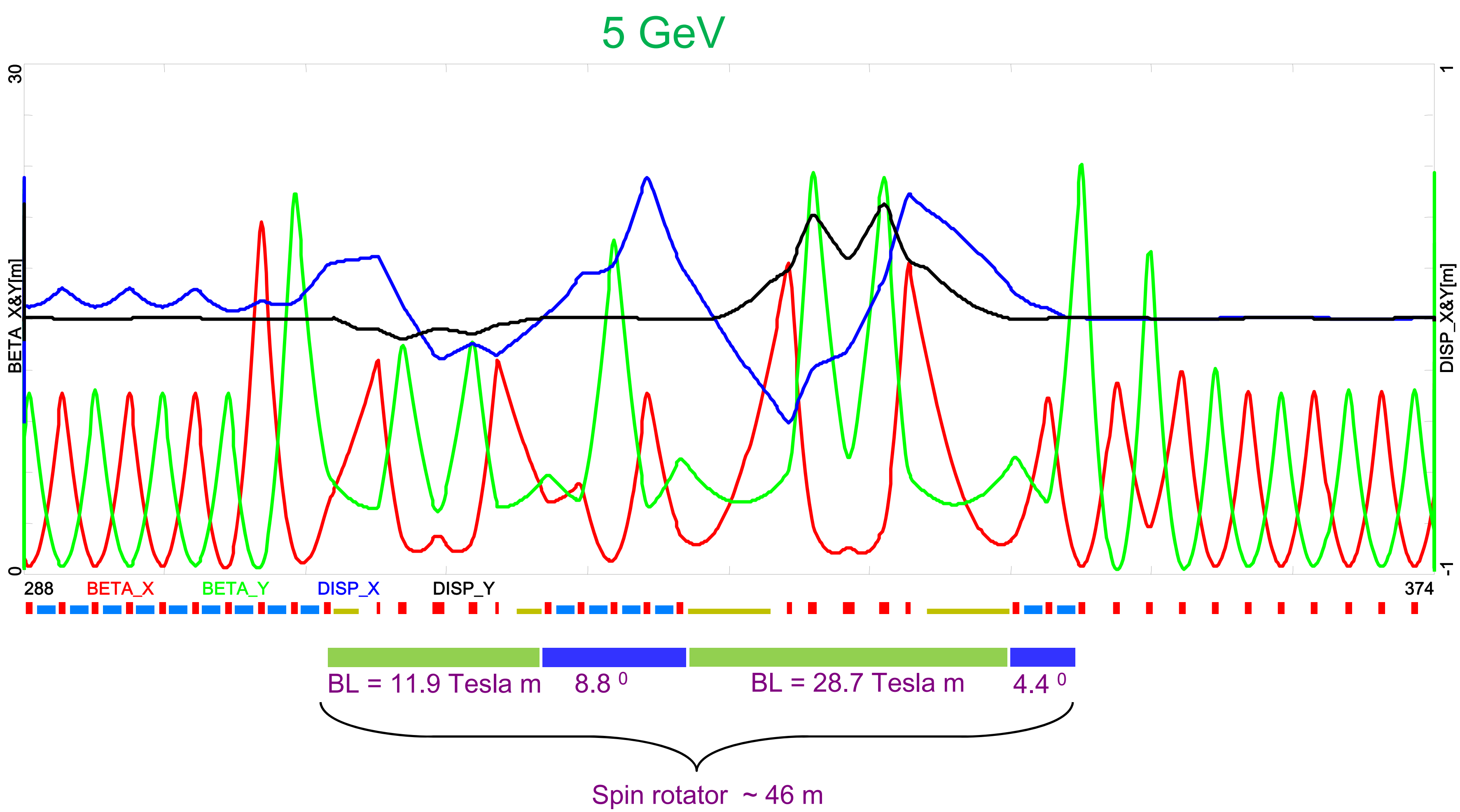
Universal Spin Rotator - Optics



Ionization Cooling in an Axially Symmetric Channel

❖ A single-particle phase-space trajectory along the beam orbit can be expressed as:

$$\hat{\mathbf{x}}(s) = \mathrm{Re}\left(\sqrt{\varepsilon_1}\hat{\mathbf{v}}_1(s)e^{-i(\psi_1+\mu_1(s))} + \sqrt{\varepsilon_2}\hat{\mathbf{v}}_2(s)e^{-i(\psi_2+\mu_2(s))}\right) \quad ,$$

- One can rewrite the above equations in the following compact form

$$\hat{\mathbf{x}}(s) = \hat{\mathbf{V}}(s)\mathbf{a}(s)$$

where

$$\hat{\mathbf{V}}(s) = \left[\hat{\mathbf{v}}_1{}'(s), -\hat{\mathbf{v}}_1{}''(s), \hat{\mathbf{v}}_2{}'(s), -\hat{\mathbf{v}}_2{}''(s)\right] \qquad \mathbf{a}(s) = \begin{bmatrix} \sqrt{\varepsilon_1}\cos(\psi_1+\mu_1(s)) \\ \sqrt{\varepsilon_1}\sin(\psi_1+\mu_1(s)) \\ \sqrt{\varepsilon_2}\cos(\psi_2+\mu_2(s)) \\ \sqrt{\varepsilon_2}\sin(\psi_2+\mu_2(s)) \end{bmatrix}$$

Ionization Cooling in an Axially Symmetric Channel

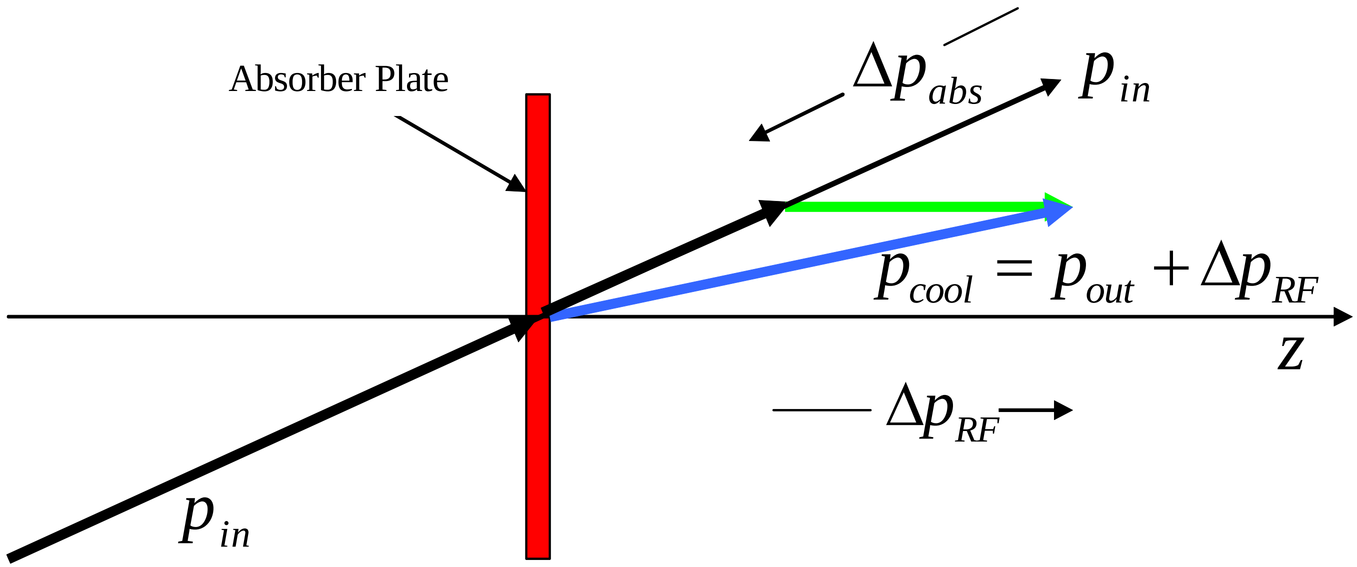
- In the case of axially symmetric focusing the eigen-vectors reduce to

$$\hat{\mathbf{v}}_1 = \begin{bmatrix} \sqrt{\beta} \\ -\dfrac{i+2\alpha}{2\sqrt{\beta}} \\ i\sqrt{\beta} \\ -i\dfrac{i+2\alpha}{2\sqrt{\beta}} \end{bmatrix} \quad , \qquad \hat{\mathbf{v}}_2 = \begin{bmatrix} i\sqrt{\beta} \\ -i\dfrac{i+2\alpha}{2\sqrt{\beta}} \\ \sqrt{\beta} \\ -\dfrac{i+2\alpha}{2\sqrt{\beta}} \end{bmatrix}$$

$$\hat{\mathbf{V}} = \begin{bmatrix} \sqrt{\beta} & 0 & 0 & -\sqrt{\beta} \\ -\dfrac{\alpha}{\sqrt{\beta}} & \dfrac{1}{2\sqrt{\beta}} & \dfrac{1}{2\sqrt{\beta}} & \dfrac{\alpha}{\sqrt{\beta}} \\ 0 & -\sqrt{\beta} & \sqrt{\beta} & 0 \\ \dfrac{1}{2\sqrt{\beta}} & \dfrac{\alpha}{\sqrt{\beta}} & -\dfrac{\alpha}{\sqrt{\beta}} & \dfrac{1}{2\sqrt{\beta}} \end{bmatrix}$$

- here we used that u = 1/2, $\nu_1 = \nu_2 = \pi/2$

Principle of Ionization Cooling



- Each particle loses momentum by ionizing a low-Z absorber
- Only the longitudinal momentum is restored by RF cavities
- The angular divergence is reduced until limited by multiple scattering

Ionization Cooling in an Axially Symmetric Channel

❖ Cooling Description

- Ionization cooling due to energy loss in a thin absorber can be described as:

$$\Delta\theta_\perp = -\theta_\perp \frac{\Delta p}{p} \equiv -\theta_\perp \delta \ ,$$

  - here the longitudinal energy restoration by immediate re-acceleration is assumed

- Using canonical variables the above cooling equation can be written as:

$$\hat{\mathbf{x}}_{out} = \mathbf{R} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\delta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\delta \end{bmatrix} \mathbf{R}^{-1} \hat{\mathbf{x}}_{in}$$

Canonical vs Geometric Variables

- Canonical variables

$$p_x = x' - \frac{R}{2} y,$$

$$p_y = y' + \frac{R}{2} x.$$

$R = eB_s / Pc$ - longitudinal magnetic field

- Relation between geometrical and canonical variables

$$\hat{\mathbf{x}} = \mathbf{R}\mathbf{x} \quad ,$$

where

$$\hat{\mathbf{x}} \equiv \begin{bmatrix} x \\ p_x \\ y \\ p_y \end{bmatrix} \quad , \quad \mathbf{x} \equiv \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \end{bmatrix} \quad , \quad \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -R/2 & 0 \\ 0 & 0 & 1 & 0 \\ R/2 & 0 & 0 & 1 \end{bmatrix} \quad ,$$

A ‘cap’ denotes transfer matrices and vectors related to the canonical variables.

Ionization Cooling in an Axially Symmetric Channel

- Employing amplitude vector representation: $\hat{\mathbf{x}}(s) = \hat{\mathbf{V}}(s)\mathbf{a}(s)$, one can rewrite the cooling equation as:

$$\hat{\mathbf{V}}\mathbf{a}_{out} = \mathbf{R}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\delta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\delta \end{bmatrix}\mathbf{R}^{-1}\hat{\mathbf{V}}\mathbf{a}_{in}$$

$$\mathbf{a}(s) = \begin{bmatrix} \sqrt{\varepsilon_1}\cos(\chi_1 + \mu_1(s)) \\ \sqrt{\varepsilon_1}\sin(\chi_1 + \mu_1(s)) \\ \sqrt{\varepsilon_2}\cos(\chi_2 + \mu_2(s)) \\ \sqrt{\varepsilon_2}\sin(\chi_2 + \mu_2(s)) \end{bmatrix}$$

and finally

$$\mathbf{a}_{out} = \hat{\mathbf{V}}^{-1}\mathbf{R}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\delta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\delta \end{bmatrix}\mathbf{R}^{-1}\hat{\mathbf{V}}\mathbf{a}_{in}$$

Ionization Cooling in an Axially Symmetric Channel

- Carrying out the above calculation explicitly one obtains:

$$\mathbf{a}_{out} = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \delta \begin{bmatrix} \frac{1-R\beta}{2} & \alpha & -\alpha & \frac{1+R\beta}{2} \\ -\alpha & \frac{1-R\beta}{2} & \frac{1+R\beta}{2} & \alpha \\ -\alpha & \frac{1-R\beta}{2} & \frac{1+R\beta}{2} & \alpha \\ \frac{1-R\beta}{2} & \alpha & -\alpha & \frac{1+R\beta}{2} \end{bmatrix} \right\} \mathbf{a}_{in}$$

- 2D emittances after cooling are given by the following formula:

$$\varepsilon_1' \equiv a_{out_1}{}^2 + a_{out_2}{}^2 = \varepsilon_1 \left[1 - (1 - \beta R)\delta\right] + \sqrt{\varepsilon_1 \varepsilon_2}\left[2\alpha\cos\phi - (1+\beta R)\sin\phi\right]\delta + O(\delta^2)$$

$$\varepsilon_2' \equiv a_{out_3}{}^2 + a_{out_4}{}^2 = \varepsilon_2 \left[1 - (1 + \beta R)\delta\right] + \sqrt{\varepsilon_1 \varepsilon_2}\left[2\alpha\cos\phi - (1-\beta R)\sin\phi\right]\delta + O(\delta^2)$$

where $\phi = \mu_1 + \mu_2 + \psi_1 + \psi_2$

Ionization Cooling in an Axially Symmetric Channel

- If cooling effect of one absorber is sufficiently small one can perform averaging over betatron phases. That yields

$$\Delta\varepsilon_1 \approx -\varepsilon_1(1-\beta R)\delta$$
$$\Delta\varepsilon_2 \approx -\varepsilon_2(1+\beta R)\delta$$

$$\Rightarrow$$

$$\frac{1}{\varepsilon_1}\frac{d\varepsilon_1}{ds} \approx -\frac{1-\beta R}{p_0}\frac{dp}{ds}$$
$$\frac{1}{\varepsilon_2}\frac{d\varepsilon_2}{ds} \approx -\frac{1+\beta R}{p_0}\frac{dp}{ds}$$

Ionization Cooling in an Axially Symmetric Channel

- Canonical momentum of a single particle

$$\mathbf{M} = xp_y - yp_x = \hat{\mathbf{x}}^T \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \hat{\mathbf{x}} = \left(\hat{\mathbf{V}}\mathbf{a}\right)^T \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \hat{\mathbf{V}}\mathbf{a} = \frac{\varepsilon_1 - \varepsilon_2}{2}$$

Ionization Cooling in an Axially Symmetric Channel

- Second order moments of the Gaussian distribution

(Note that for a single particle - $\varepsilon_{rms} = \varepsilon / 2$ and we use rms. emittances below)

$$\langle x^2 \rangle = \langle y^2 \rangle = \beta(\varepsilon_1 + \varepsilon_2) \quad ,$$

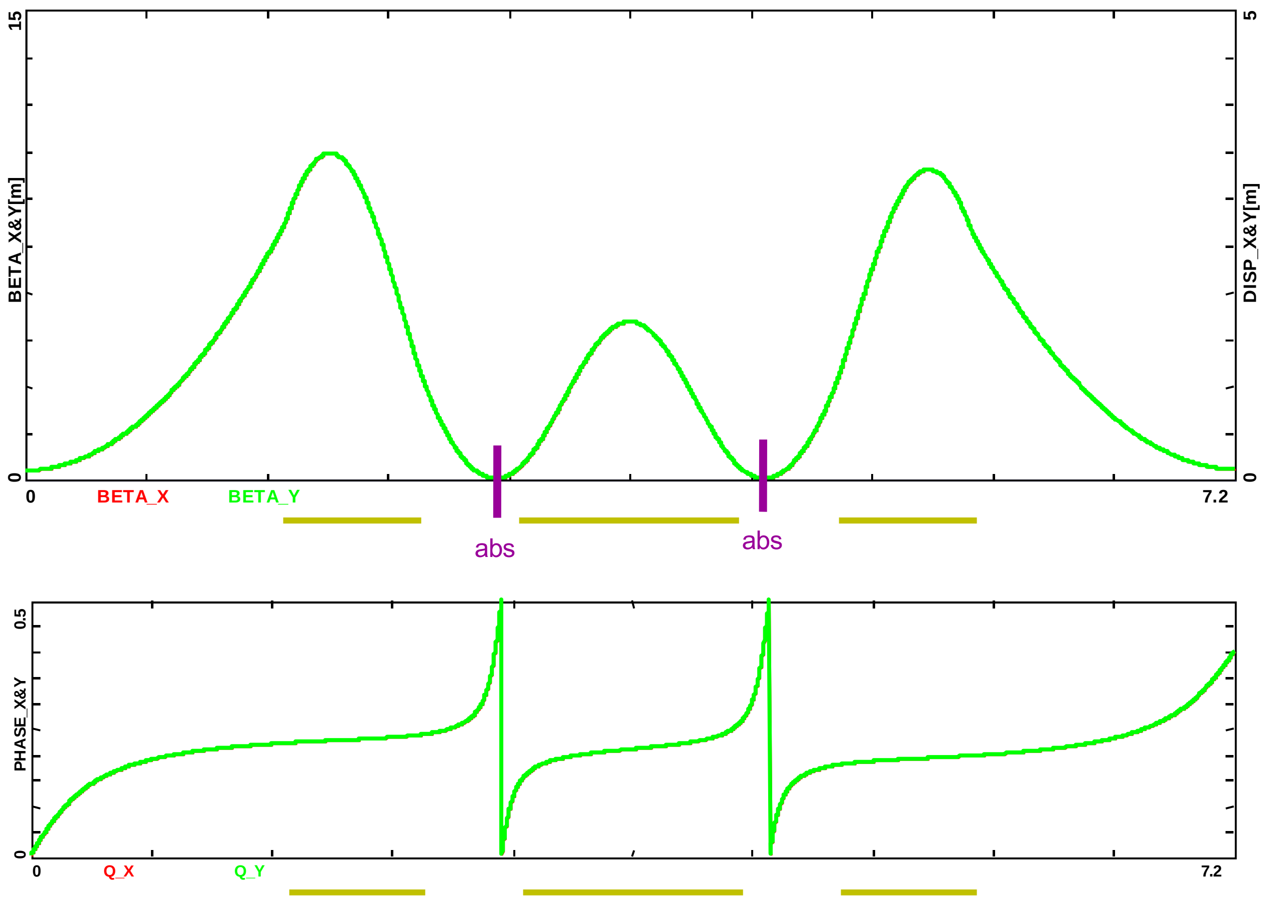
$$\langle x p_x \rangle = \langle y p_y \rangle = -\alpha(\varepsilon_1 + \varepsilon_2) \quad ,$$

$$\langle {p_x}^2 \rangle = \langle {p_y}^2 \rangle = \frac{1 + 4\alpha^2}{4\beta}(\varepsilon_1 + \varepsilon_2) \quad ,$$

$$\langle x p_y \rangle = -\langle y p_x \rangle = \frac{\varepsilon_1 - \varepsilon_2}{2} \quad , \quad \Rightarrow \quad \langle M \rangle = \varepsilon_1 - \varepsilon_2$$

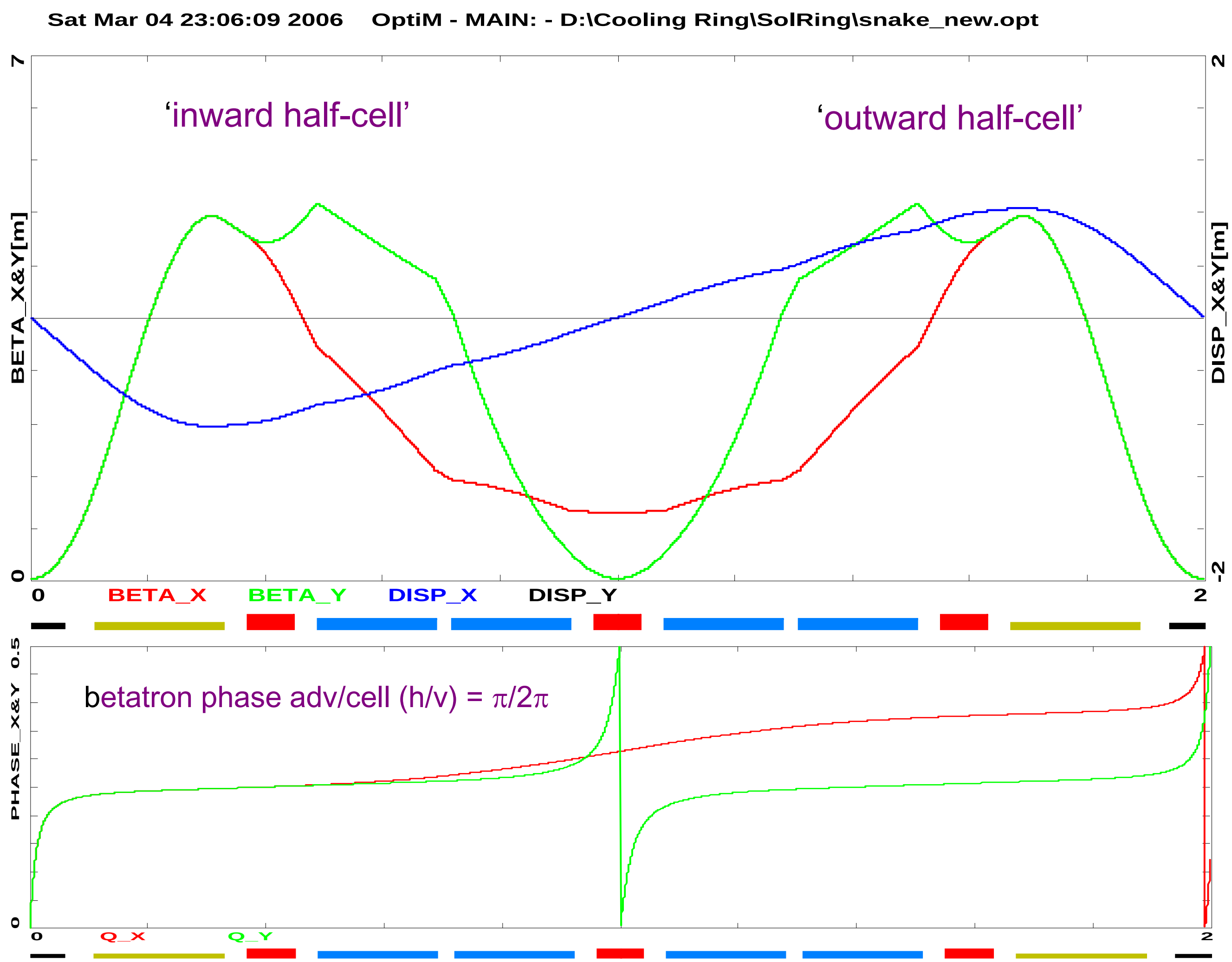
$$\langle xy \rangle = \langle p_x p_y \rangle = 0$$

Axially Symmetric FOFO Cell



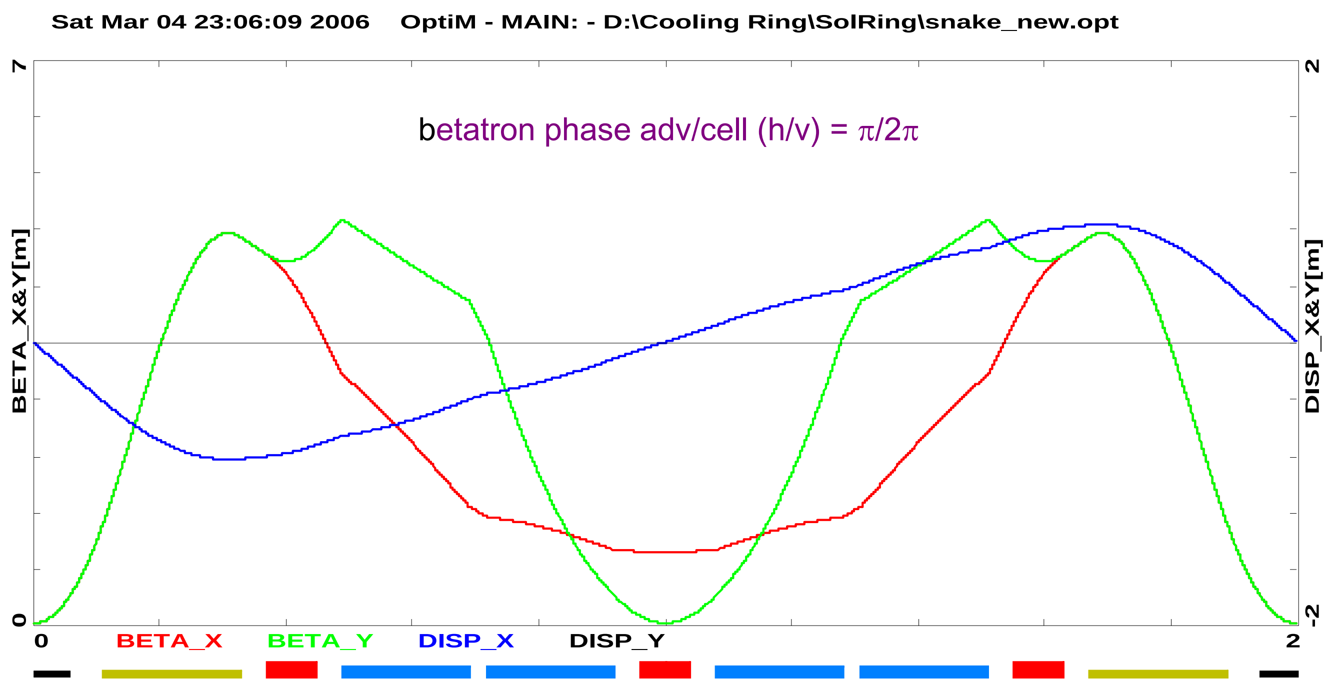
c1	L[cm]=130	B[kG]=38.1	Aperture[cm]=10
c2	L[cm]=80	B[kG]=-34.3	Aperture[cm]=10

Periodic Cell - Optics



Periodic Cell – Magnets

‘inward half-cell’ ‘outward half-cell’



solenoids:

L[cm]	B[kG]
22	105
22	105

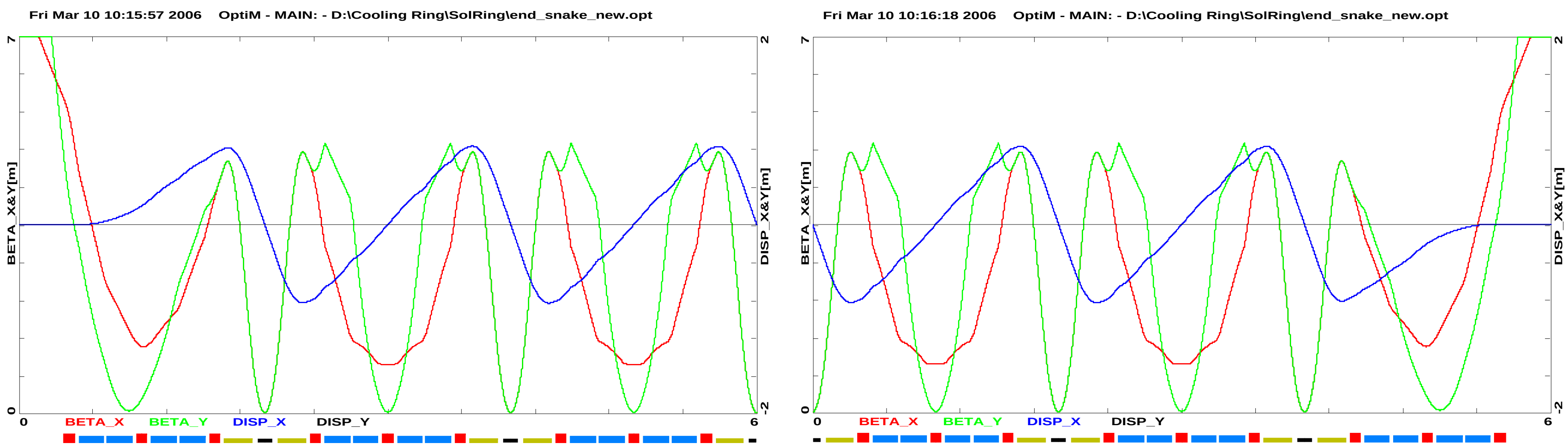
quadrupoles:

L[cm]	G[kG/cm]
8	1.79754
8	-0.3325
8	1.79754

dipoles:

$L=20; => 20 cm
$B= 25; => 25 kGauss
$Ang=$L*$B/$Hr; => 0.4996 rad
$ang=$Ang*180/$PI; => 28.628 deg

‘Snake’ Cooling Channel

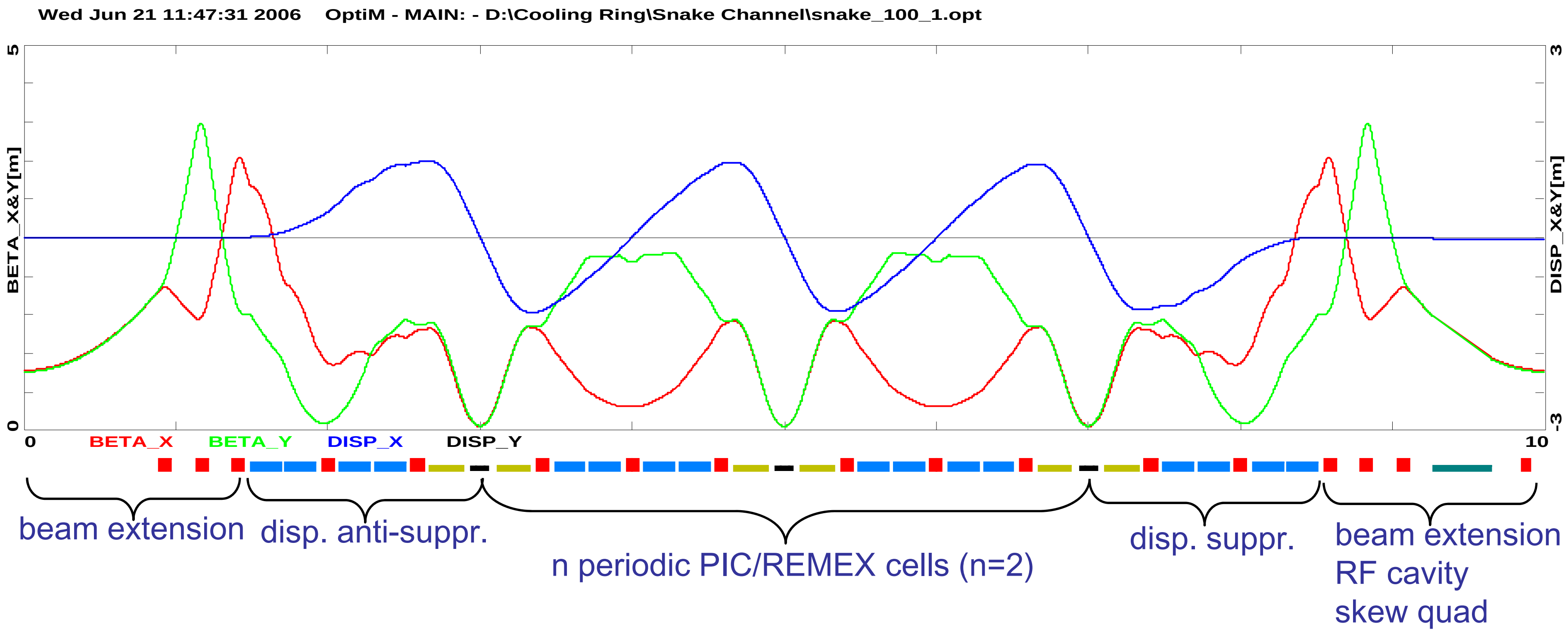


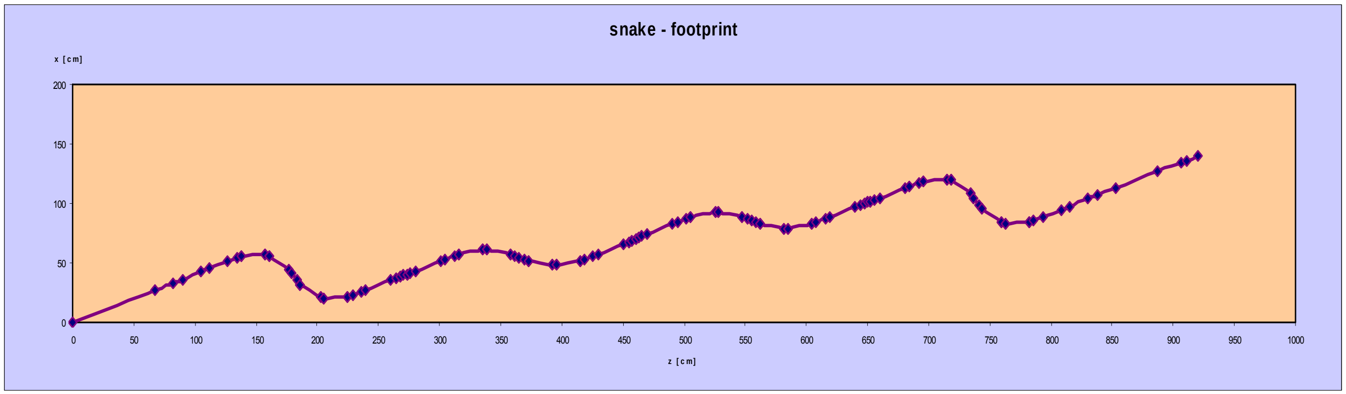
entrance cell

periodic cells

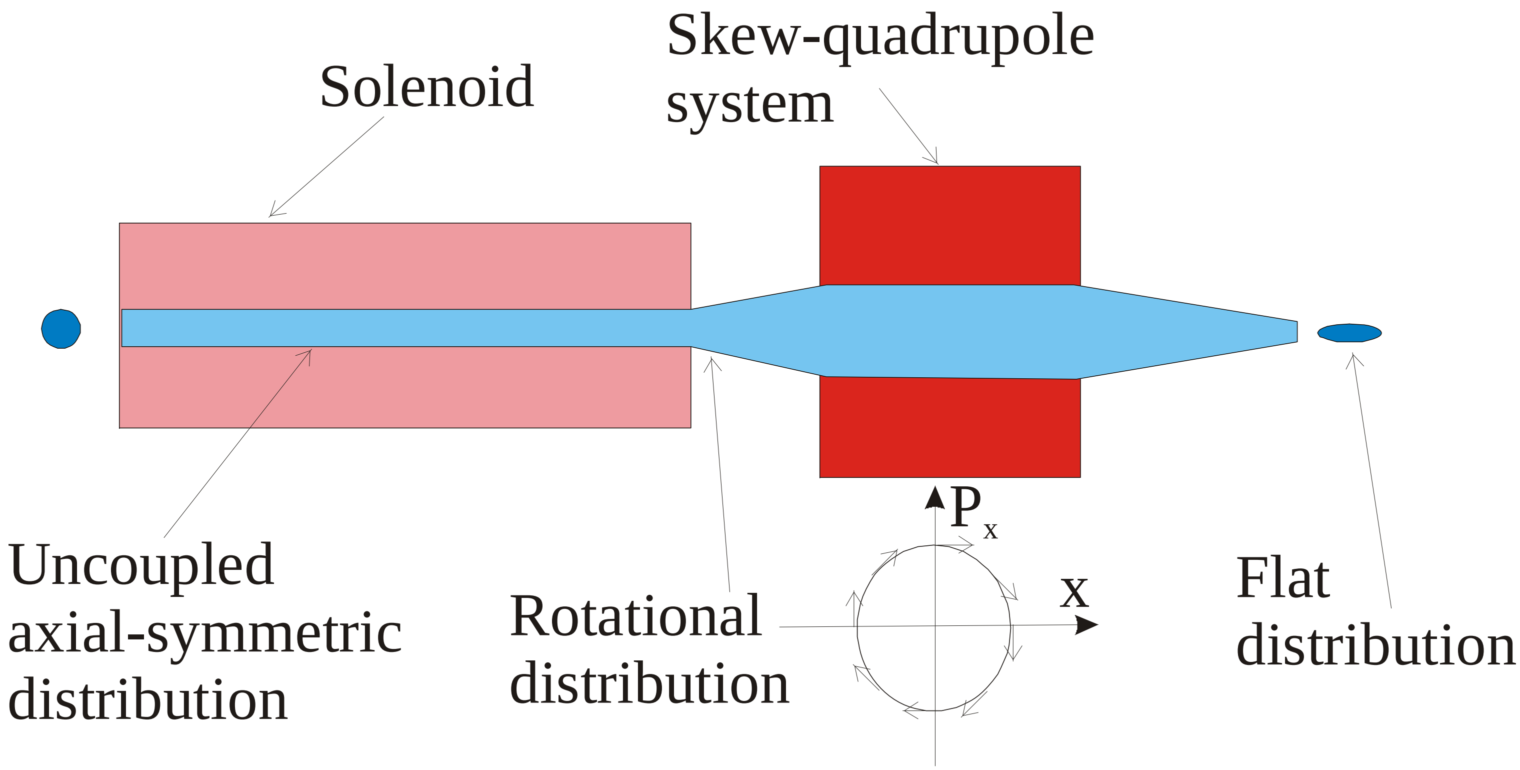
exit cell

Muon Cooling Channel – Optics





Vertex-to-plane Transformer Insert



Vertex-to-plane Transformer Insert

- Eigen-vectors of the decoupled motion in the coordinate system rotated by 45^0

$$\begin{bmatrix} \sqrt{\beta_1} \\ -\dfrac{i+\alpha_1}{\sqrt{\beta_1}} \\ 0 \\ 0 \end{bmatrix} \xrightarrow[\text{rotation}]{\text{45deg.}} \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{\beta_1} \\ -\dfrac{i+\alpha_1}{\sqrt{\beta_1}} \\ \sqrt{\beta_1} \\ -\dfrac{i+\alpha_1}{\sqrt{\beta_1}} \end{bmatrix} \xrightarrow[\alpha_1=2\alpha]{\beta_1=2\beta} \begin{bmatrix} \sqrt{\beta} \\ -\dfrac{i+2\alpha}{2\sqrt{\beta}} \\ \sqrt{\beta} \\ -\dfrac{i+2\alpha}{2\sqrt{\beta}} \end{bmatrix} \equiv \begin{bmatrix} \mathbf{F}_2 \\ \mathbf{F}_2 \end{bmatrix}$$

  - Rotational eigen-vectors

$$\begin{bmatrix} \mathbf{F}_2 \\ i\mathbf{F}_2 \end{bmatrix} \qquad \begin{bmatrix} i\mathbf{F}_2 \\ \mathbf{F}_2 \end{bmatrix}$$

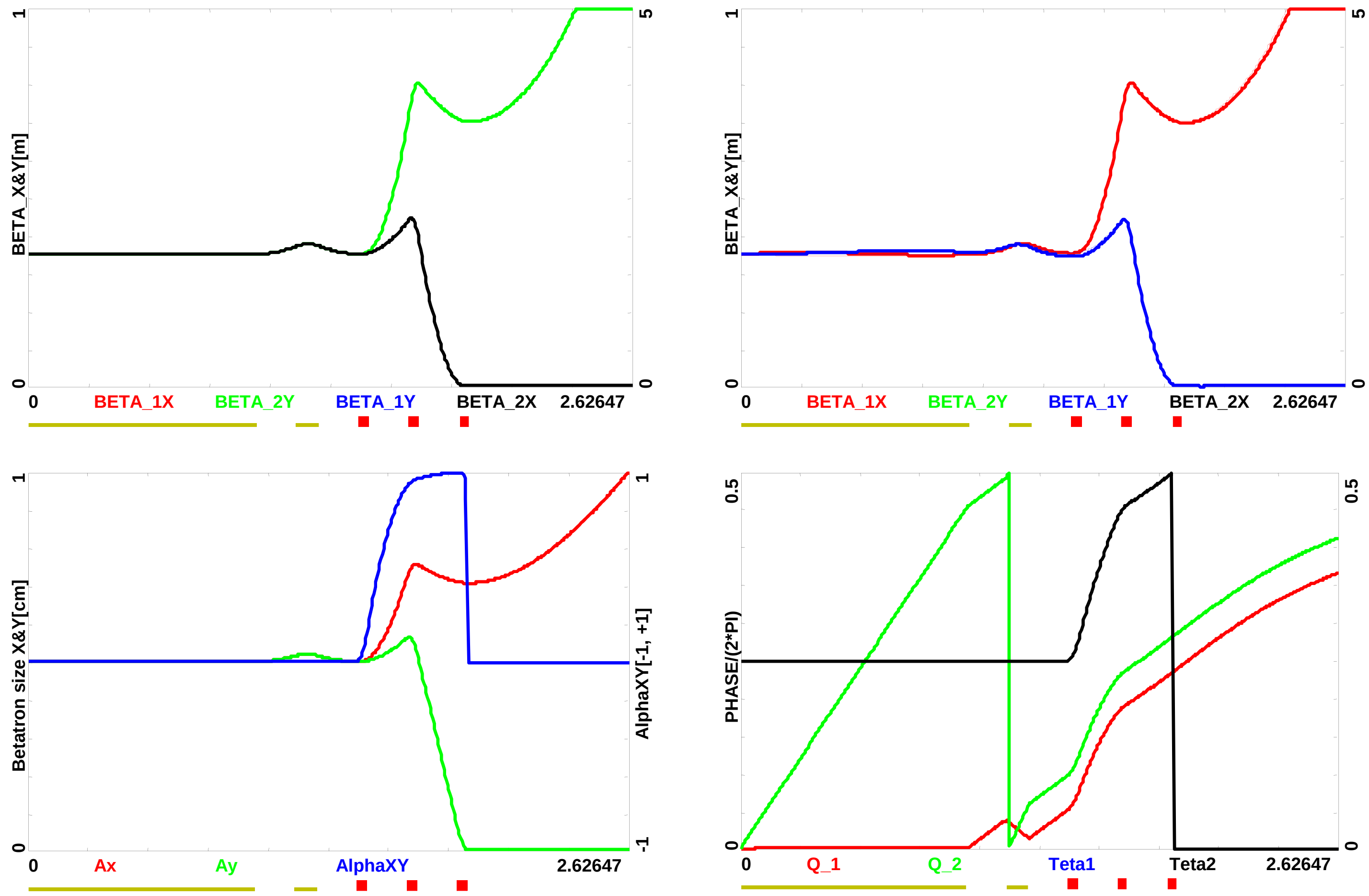
Vertex-to-plane Transformer Insert

- Focusing system with 45^0 difference between the horizontal and vertical betatron phase advances will transform the initial vertex distribution into the flat one
- The resulting 2D emittances are as follows

$$\varepsilon_1 = \frac{\varepsilon_T}{\sqrt{1+\Phi^2\beta_0^{\;2}} - \Phi\beta_0} \quad , \qquad \varepsilon_2 = \frac{\varepsilon_T}{\sqrt{1+\Phi^2\beta_0^{\;2}} + \Phi\beta_0} \quad .$$

- Lattice implementation – Twiss functions, beam sizes etc.

Vertex-to-plane Transformer Insert



E_{kin} = 10 MeV, T_c = 0.2 eV, R_c = 0.5 cm, B_{sol} = 1 kG,

$\Rightarrow \varepsilon_1 = 7.14 \cdot 10^{-3}$ cm, $\varepsilon_2 = 3.24 \cdot 10^{-8}$ cm

Summary

- Relationships between the eigen-vectors, beam emittances and the beam ellipsoid in 4D phase space
  - From the beam ellipsoid to the eigen-vectors (equivalence of both pictures)
- New parametrization of eigen-vectors in terms of generalized Twiss functions
  - Complete Weyl-like representation
    - 10 independent parameters to fully describe the motion
    - transport line ambiguities resolved
  - Developed software based on this representation allows effective analysis of coupled betatron motion for both circular accelerators and transfer lines (OptiM).