Homework Problems VII
Accelerator Physics

1. Normalize, and compute the *rms* emittance of the following distributions:

Gaussian $$f\left(x,x'\right)=A\exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{x'^2}{2\sigma_{x'}^2}\right)$$

Waterbag $$f\left(x,x'\right)=A\Theta\left(1-\frac{x^2}{\Delta x^2}-\frac{x'^2}{\Delta x'^2}\right)$$

K-V, or microcanonical $$f\left(x,x'\right)=A\delta\left(1-\frac{x^2}{\Delta x^2}-\frac{x'^2}{\Delta x'^2}\right)$$

Klimontovich $$f\left(x,x'\right)=A\sum_{i=1}^{N}\delta\left(x-x_i\right)\delta\left(x'-x'_i\right)$$

Treat σ_x, $\sigma_{x'}$, Δx, $\Delta x'$, x_i, x'_i as parameters. Θ Unit step, δ Dirac's delta

For distributions (1)-(3), what does the projected distribution, e.g., $p(x)=\int f(x,x')dx'$ look like?

2. Normalize the Gaussian-elliptical phase space distribution
$$\rho\left(x,x'\right)=A\exp\left(-\left(\gamma x^2+2\alpha xx'+\beta x'^2\right)/2\varepsilon\right)$$
assuming $\beta\gamma-\alpha^2=1$. Show the statistical average definitions of $\alpha,\beta,$ and γ evaluate to exactly the correct values for this distribution, and $\varepsilon_{rms}=\varepsilon$.

3. Write down the general solution for the K-V envelope equation for a drift region when $a=b$. Show that for solutions initially converging that the beam size at maximum compression is bigger than if there is no space-charge term.