Homework Problems VI
Accelerator Physics

1. Assuming a charged particle moves only in the x-direction as in the beginning of Lecture 10, verify the "Jackson" formula

$$\frac{d^2E}{d\omega d\Omega} = \frac{e^2\omega^2}{32\pi^3\varepsilon_0 c}\left|\int_{-\infty}^{\infty} \vec{n}\times\left(\vec{n}\times\vec{\beta}(t)\right)e^{i\omega\left(t-\vec{n}\cdot\vec{r}(t)/c\right)}dt\right|^2$$

leads directly to the result

$$\frac{d^2E}{d\omega d\Omega} = \frac{e^2\omega^4}{32\pi^3\varepsilon_0 c^3}\left|\tilde{d}(\omega)\right|^2 \sin^2\Theta$$

(the units in Lecture 10 are CGS) for non-relativistic motions. The angle Θ is the angle the emission direction makes with the x-axis. Note also the formula for general 3-dimensional motions is correct by the same argument.

2. Assuming no microphonics, plot β_{opt} and $P^{opt}{}_g$ as function of b (beam loading), b = -5 to 5, and explain the results.

   How do the results change if microphonics is present?

3. Using the Jacobian formalism show that the quadratic Hennon's twist map (defined below) is conservative.

$$\begin{bmatrix} q_{n+1} \\ p_{n+1} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\epsilon q_n & 1 \end{bmatrix} \begin{bmatrix} q_n \\ p_n \end{bmatrix}$$