Homework Problems IV
Accelerator Physics

1. Show the following properties of the unit symplectic matrix **U**:

$$\mathbf{U}^T = -\mathbf{U} \qquad \mathbf{U} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\mathbf{UU} = -\mathbf{I}$$

$$\mathbf{UU}^T = \mathbf{I}$$

2. A commonly applied focusing system in accelerators is the so-called FODO system. For a thin lens approximation to this system, the one period transfer matrix starting from the middle of the focusing lens is

$$M = \begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix}.$$

where f is the lens focal length and L is the distance between lenses. Evaluate the total transfer matrix.

What is the result of a similar calculation to obtain the one period transfer map starting at the middle of the defocusing lens? (Hint: You don't have to perform the whole matrix multiplication again. Change a relevant parameter in the solution you've already obtained!).

Compare the matrix traces of the two results you have obtained. How must one choose the ratio L/f to obtain a phase advance of 60 degrees? In this case, what are the beta-functions and alpha-functions for the periodic solutions in the middle of the focusing lens and in the middle of the defocusing lens.

3. Show explicitly that the matrix representation

$$M_{s',s} = \begin{pmatrix} \sqrt{\dfrac{\beta(s')}{\beta(s)}}\left(\cos\Delta\mu_{s',s} + \alpha(s)\sin\Delta\mu_{s',s}\right) & \sqrt{\beta(s')\beta(s)}\sin\Delta\mu_{s',s} \\ -\dfrac{1}{\sqrt{\beta(s')\beta(s)}}\begin{bmatrix} \left(1+\alpha(s')\alpha(s)\right)\sin\Delta\mu_{s',s} \\ +\left(\alpha(s')-\alpha(s)\right)\cos\Delta\mu_{s',s} \end{bmatrix} & \sqrt{\dfrac{\beta(s)}{\beta(s')}}\left(\cos\Delta\mu_{s',s} - \alpha(s')\sin\Delta\mu_{s',s}\right) \end{pmatrix}.$$

satisfies the composition formula $M_{s'',s} = M_{s'',s'}M_{s',s}$. Also, show from this representation that

$$\tan \Delta\mu_{s',s} = \frac{\left(M_{s',s}\right)_{12}}{\beta(s)\left(M_{s',s}\right)_{11} - \alpha(s)\left(M_{s',s}\right)_{12}}.$$