Introduction to polarized beams

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History

- polarized proton acceleration in high-energy circular accelerators at the Zero Gradient Synchrotron (ZGS) at Argonne 1970's.
- 1980's AGS at Brookhaven National Laboratory (BNL) 24 GeV.
- Siberian Snakes 1970s, opened the possibility to achieve higher energies. A test of principle of Siberian Snakes with a low-energy polarized proton beam was performed at the Indiana University Cyclotron Facility (IUCF) in 1989.
- 1999, the efficacy of partial Siberian Snakes and radio-frequency dipoles for maintaining polarization during acceleration in the AGS was demonstrated.
- The next big advance came with the acceleration of polarized protons to 100 GeV & 2005 to 205 GeV with 30% polarization

Spin

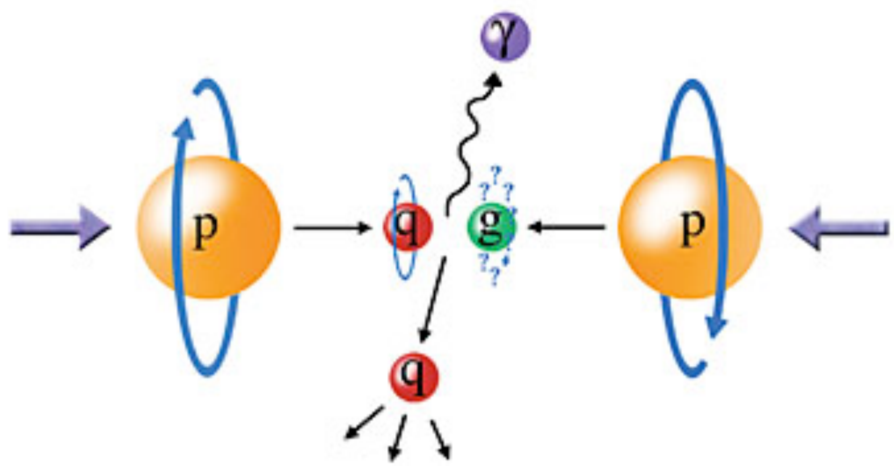
- The spin magnetic moment

$$\vec{\mu} = g\frac{e}{2m}\vec{s}$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.2740154x10^{-24} J/T = 5.7883826x10^{-5} eV/T$$

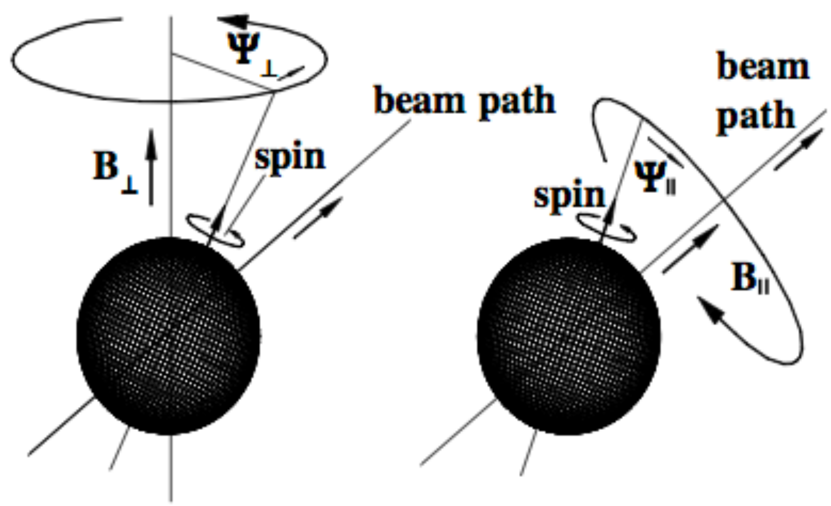
Bohr magneton

Magnetic moment of electron 2000 times of the proton



Polarized Beams

- The magnetic moment vector of a particle rotates about a magnetic field vector
- longitudinally polarized electron traversing vertical dipole field experience rotation of the longitudinal polarization about the vertical axis.
- Vertical polarization not affected by a horizontally bending magnet.
- longitudinal polarization is not affected by a solenoid field



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Polarized Beams

- Electron and positron beams circulating for a long time in a storage ring can become polarized due to the reaction of continuous emission of transversely polarized synchrotron radiation.
- polarization time

$$\frac{1}{\tau_{\text{pol}}} = \frac{5\sqrt{3}}{8}\frac{e^2\hbar\gamma^5}{m^2c^2\rho^3}$$

- Theoretically maximum achievable polarization of 92.38%
- The polarization time is a strong function of beam energy and is very long for low energies.
- At energies of several GeV polarization time becomes short compared to the storage time of an electron beam in a storage ring.
- Polarization build up is counteracted by nonlinear magnetic field errors which cause precession of the spin depending on the betatron amplitude and energy of the particle thus destroying polarization

Polarized Beams

- To rotate the spin by a magnetic field → need a finite angle between spin direction and magnetic field.
- spin rotation angle about axis of transverse field depends on
  - angle between spin direction σ_s and magnetic field $B_\perp$

$$\psi_\perp = C_\perp (1 + \frac{1}{\gamma}) |\sigma_s \times B_\perp| l$$

$$C_\perp = \frac{e\eta_g}{mc^2} = 0.0068033 (T^{-1} m^{-1})$$

$$\eta_{g-electron} = \frac{g-2}{2} = 0.00115965$$

The spin rotation is independent of the energy

Spin component normal to the field direction can be rotated by 90^o while passing though a magnetic field of 2.309 T (~at any energy)

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Polarized Beams

- In flat storage ring (horizontal bending only) unless the polarization of the incoming beam is strictly vertical. Any horizontal or longitudinal polarization component would precess while the beam circulates in the storage ring.
- As long as this spin is the same for all particles polarization is preserved.
- The small energy dependence of the precession angle and the finite energy spread in the beam would wash out the polarization.
- The vertical polarization of a particle beam is preserved in an ideal storage ring.
- Field errors may introduce a depolarization effect.
  - Horizontal field errors from misalignments of magnets rotate the vertical spin.
  - The integral of all horizontal field components in a storage ring is always zero along the closed orbit and the net effect on the vertical polarization is zero.
  - Nonlinear fields do not cancel and must be minimized to preserve the polarization.

Polarized Beams

- To rotate the spin by a magnetic field → need a finite angle between spin direction and magnetic field.
  - A transverse spin can be rotated about the longitudinal axis of a solenoid field

$$\psi_{\parallel} = \frac{e}{E}(1 + \eta_g \frac{\gamma}{1+\gamma})|\sigma_s \times B_{\parallel}|l$$

Spin rotation in a longitudinal field is energy dependent and such spin rotations should therefore be done at low energies if possible.
Rotate a horizontal polarization into a vertical polarization or vice versa

Spin Equation in particle's rest frame

$$\hbar \frac{d\vec{S_R}}{dt} = \vec{\mu} \times \vec{B}$$

g → gyromagnetic factor
S → spin vector in particles rest frame
$\vec{\Omega_R}$ → Angular velocity

$$\hbar \frac{d\vec{S_R}}{dt} = g \frac{e\hbar}{2m} \vec{S_R} \times \vec{B} = \hbar \vec{S_R} \times \vec{\Omega_R}$$

$$\hbar \frac{d\vec{S_R}}{dt} = \hbar \vec{S_R} \times \vec{\Omega_R}$$

Consider spin particle moving in a circular orbit with transverse magnetic fields in the lab frame

The 4-velocity of charged spin particle and position

$$V^i = (\gamma c, \gamma \vec{v})$$
$$X^i = (ct, \vec{x})$$

$$\frac{X}{d\tau} = V$$
$$d\tau = \frac{dt}{\gamma}$$

- Equation of motion of charged particle in presence of electromagnetic fields.

$$mc^2\frac{d\gamma}{dt} = e\vec{E}\cdot\vec{v}$$

$$m\frac{d\gamma\vec{v}}{dt} = e[\vec{E} + \vec{v}\times\vec{B}]$$

The spin 4-vector $$S^i = (S_0, \vec{S})$$

In particle rest frame $$S_R^i = (0, \vec{S_R})$$

Where R refers to the rest frame

Decomposing spin vector into components parallel and perpendicular to the velocity of the particle

$$\vec{S_R} = \vec{S_{R||}} + \vec{S_{R\perp}}$$
$$= \frac{1}{\beta^2}\vec{\beta}(\vec{\beta}\cdot\vec{S_R}) + [\vec{S_R} - \frac{1}{\beta^2}\vec{\beta}(\vec{\beta}\cdot\vec{S_R})]$$

In Lorentz boosted frame the spin 4-vector becomes

$$S^i = (S_{0L}, \vec{S_L})$$
$$= (\gamma\vec{\beta}\cdot\vec{S_R}, \vec{S_R} + \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{S_R}))$$

This equation relates the spin vector in lab frame with the spin vector in particle's rest frame.

The scalar product of S & V is zero at the lab frame

$$S_R^i \cdot V_0^i = 0$$

Since it is invariant under Lorentz transformation so it zero everywhere

$$S \cdot V = \gamma c(S_0 - \vec{S} \cdot \vec{\beta}) = 0$$

Taking the derivative of the above we see that the time evolution of S becomes

$$\frac{dS}{dt}V + S\frac{dV}{dt} = 0$$

$$\frac{dS}{dt}V = -S\frac{dV}{dt}$$

The spin evolution equation in rest frame

$$\frac{dS}{dt}\mid_R = (\frac{e}{mc}\vec{S}\cdot\vec{E}, g\frac{e}{2m}\vec{S}\times\vec{B})\mid_R$$

In rest frame S_0 =0 but it's derivative is not

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

g is the metric tensor

Spin equation of motion must be linear in S & F

$$\frac{dS}{d\tau} = -aSF - bV(SFV)$$

Where a & b parameters determined from the rest fame reduction

$$-SF = (\frac{1}{c}\vec{S} \cdot \vec{E}, \frac{1}{c}S_0\vec{E} + \vec{S} \times \vec{B})$$

$$-V(SFV) = V(\gamma\vec{S} \cdot \vec{E} - \frac{\gamma}{c}S_0\vec{E} \cdot \vec{v} - \gamma(\vec{S} \times \vec{B}) \cdot \vec{v})$$

The equation of spin motion in any reference frame (lab frame) becomes

$$\frac{dS}{d\tau} = -\frac{e}{m}\left[\frac{g}{2}SF - \frac{g-2}{2c^2}V(SFV)\right]$$

$$\frac{d\vec{S}}{d\tau} = \frac{e}{2m\gamma}[g\vec{S}\times\vec{B}+(g-2)\gamma^2\vec{\beta}(\vec{S}\times\vec{B}\cdot\vec{\beta})+\frac{g}{c}S_0\vec{E}-(g-2)\gamma^2(\vec{S}\cdot\vec{E}-S_0(\vec{E}\cdot\vec{\beta}))]$$

$$\frac{dS_0}{d\tau} = \frac{e(g-2)}{2m}\gamma(\vec{S}\times\vec{B}\cdot\vec{\beta}) + \frac{g}{2m\gamma c}\vec{S}\cdot\vec{E} - \frac{e\gamma(g-2)}{2mc}[\vec{S}\cdot\vec{E} - S_0\vec{E}\cdot\vec{\beta}]$$

Thomas-BMT equation

The equation of motion for the spin vector defined in the rest frame of the particle in a synchrotron

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\vec{S} \times [(1 + G\gamma)\vec{B}_\perp + (1 + G)\vec{B}_\| + (G\gamma + \frac{\gamma}{\gamma + 1})\frac{E \times \vec{\beta}}{c}]$$

$\vec{S}$ Spin vector in particle rest frame

$\vec{B}_\perp$ $\vec{B}_\|$ Transverse & longitudinal magnetic fields in lab frame w.r.t particle's velocity $\vec{\beta}$

Thomas-BMT equation

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\vec{S} \times [(1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_\parallel + (G\gamma + \frac{\gamma}{\gamma+1})\frac{E \times \vec{\beta}}{c}]$$

Quadrupoles & sextupoles (with no dipole fields or misalignment) have no effect on spin.
In a solenoid magnet, $B_{||}$ produces a spin rotation around the longitudinal direction.
In dipoles the equation of spin motion relative to the particle motion shows that spin rotation relative to the orbit motion is independent of energy.

$$\frac{d\vec{s}_p}{dt} = -\frac{q}{m}G\vec{B}_\perp \times \vec{s}_b$$

Spin precession angle around vertical bending field B

$$\phi_s \approx \frac{Gq}{mcv}(Bs)$$

Protons with speed close to c needs 5.48 Tm to spin rotation of π.
Electrons with speed close to c needs 4.62 Tm to spin rotation of π

Thomas-BMT equation

If the orbit is deflected by an angle ϕ in a transverse magnetic field, then the spin is rotated by an angle $G\gamma\phi$ relative to the orbit

$$\phi_s = G\gamma\phi$$

1 mrad orbit kick for a 920 GeV proton produces 100° of spin rotation.
1 mrad orbit kick for 27.5 GeV electrons produces 3.6° of spin rotation.
The spin precess at $G\gamma$ precession turns per orbital revolution (i.e. spin tune = $G\gamma$)

Spin motion in terms of particle coordinates

- Following Frenet-Serret coordinate system (s,z,x)

The bedning angle θ defined by $d\theta = \frac{ds}{\rho}$

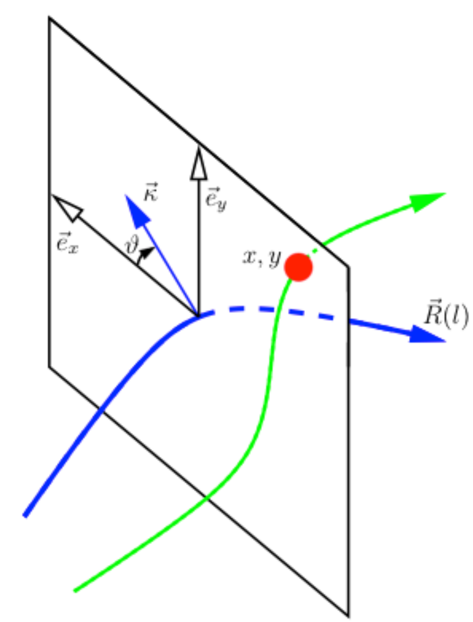
Representation of magnetic field in terms of Frenet-Serret coordinates

$$\vec{F} = F_1\hat{x} + F_2\hat{s} + F_3\hat{z}$$

Horizontal field (F_1)

Longitudinal field (F_2)

Vertical field (F_3)



$$F_1 = -\rho z''(1 + G\gamma)$$
$$F_2 = (1 + G\gamma)z' - \rho(1 + G)(\frac{z}{\rho})'$$
$$F_3 = -(1 + G\gamma) + (1 + G\gamma)\rho x''$$

$$\vec{S} = S_1\hat{x} + S_2\hat{s} + S_3\hat{z}$$

Thomas-BMT Equation $\quad \frac{d\vec{S}}{d\theta} = \vec{S} \times \vec{F}$

$$\frac{dS_1}{d\theta} = (1 + F_3)S_2 - F_2S_3$$

$$\frac{dS_2}{d\theta} = -(1 + F_3)S_1 + F_1S_3$$

$$\frac{dS_3}{d\theta} = F_2S_1 - F_1S_2$$

$$S_\pm = S_1 \pm iS_2$$
$$F_\pm = F_1 \pm iF_2$$

$$\frac{dS_\pm}{d\theta} = \pm iG\gamma S_\pm \pm iF_\pm S_3$$
$$\frac{dS_3}{d\theta} = \frac{i}{2}(F_- S_+ - F_+ S_-)$$

Horizontal dipole case

$$F_1 = F_2 = 0$$
$$S_\pm = e^{\pm iG\gamma\theta} S_{\pm 0}$$
$$S_3 = S_{30}$$

→ The spin precess about the vertical axis at GY precessions per orbital revolution

Spinor equation of motion

$$\frac{d\vec{S}}{d\theta} = \vec{n} \times \vec{S}$$
$$\vec{n} = G\gamma\hat{z} - F_1\hat{x} - F_2\hat{s}$$

Define a two-component spinor

$$\Psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$S_1 = u^*d + ud^*$$
$$S_2 = -i(u^*d + ud^*)$$
$$S_3 = |u|^2 - |d|^2$$

Spinor equation of motion

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}\begin{pmatrix} G\gamma & -\xi \\ -\xi^* & -G\gamma \end{pmatrix}\Psi$$

Spin precession kernel

$$\xi(\theta) \equiv F_1 - iF_2$$

Characterize spin depolarization kick by coupling the up & down components of spinor wave function

Spin Transfer Matrix "STM"

IF spin precessing kernel is constant "STM"

$$\Psi(\theta_2) = t(\theta_2, \theta_1)\Psi(\theta_1)$$

$$t = t_0 I - it_1\sigma_1 - it_2\sigma_2 - it_3\sigma_3$$

Parameterized in Pauli matrices

Spin motion in perfect accelerator where $\xi = 0$

$$\Psi(\theta) = e^{-i\frac{G\gamma}{2}\theta\sigma_3}\Psi(\theta_0)$$

In case of local field error

$$M = e^{-i\frac{G\gamma}{2}(2\pi-\theta)\sigma_3} M_{error} e^{-i\frac{G\gamma}{2}\theta\sigma_3}$$

$$M_{error} = e^{\frac{-i}{2}\psi\vec{\sigma}\cdot\hat{n}_e}$$

$$= cos\frac{\psi}{2} - i\vec{\sigma}\cdot\hat{n}_e sin\frac{\psi}{2}$$

$$\hat{n}_e = cos\chi_1\hat{x} + cos\chi_2\hat{s} + cos\chi_3\hat{z}$$

ψ → Precession angle due to error field

Derbenev & Kondratenko Siberian Snake

Spin Closed Orbit

$$\hat{n}_c = cos\Phi_1\hat{x} + cos\Phi_2\hat{s} + cos\Phi_3\hat{z}$$

Spin tune and spin closed orbit are modified by the error field
When the error field rotates the spin $\psi=\pi$ with respect to an axis on the horizontal plane, then the spin tune is equal ½ and is independent of energy.

$$\begin{aligned}
cos\Phi_3 &= 0 \\
cos\Phi_1 &= cos[G\gamma(\pi-\theta)+\chi_1] \\
cos\Phi_2 &= sin[G\gamma(\pi-\theta)+\chi_1]
\end{aligned}$$

Spin closed orbit vector lies on the horizontal plane. Its orientation depends on Gy and its location in the ring, except at the symmetry point $\theta=\pi$ where the spin closed orbit vector is independent of Gy and lies on the axis of the error field

Siberian snakes

Types of Siberian snakes

Type I $\psi=\pi$ about longitudinal axis
Type II $\psi=\pi$ about radial axis
Type III $\psi=\pi$ about vertical axis

longitudinal snake

$$\int B_{\parallel}\ dl = \frac{mc\beta\gamma}{(1+G)e}\psi$$

depends linearly on γ
best suited for low energy beams

transverse snake

$$\int B_{\perp}\ dl = \frac{mc\beta}{Ge}\psi$$

Independent of γ so fixed-field magnets may be used
dipole produces an orbital deflection angle of $\psi/G\gamma$ which is large at low beam energies
therefore best suited for high energy beams

Siberian snakes force ν_0=1/2 (full snake) so the resonance condition is never satisfied at any energy
snake designs generally are optically transparent. The choice of solenoidal or dipole snakes depends on the beam energy

Spin Motion in Circular Accelerators

Case I: Closed orbit with no distortions

- *The design trajectory of the accelerator: No field errors, misaligned elements, or energy deviations are present*.
- After particles with different spins travel one turn along the closed orbit from azimuth θ_0 *to azimuth* $\theta_0 + 2\pi$, *all spins* have rotated around some effective unit rotation axis n_0 (θ_0) *by a rotation* angle $2\pi\nu_0$.
- ν_0 *Closed orbit* spin tune *independent on the azimuth* θ_0 *at which* n_0 *is* determined.
- In a flat accelerator without solenoids and field errors and misaligned elements, the closed orbit is in the horizontal plane and passes only through vertical magnetic fields.

$$n_0 \text{ is vertical } \& \ \nu_0 = G\gamma$$

- When ν_0 *is close to an integer "imperfection resonance", the rotation matrix is close to the identity* and spin directions have hardly changed after one turn.

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Spin Motion in Circular Accelerators

Case II: Closed orbit with distortions

- Misalignments distort the closed orbit of a flat ring and create horizontal field components on it, which produce spin precessions away from the vertical direction.
- For small misalignments, these rotations around the horizontal might be very small but they can still dominate spin motion when the main fields hardly produce any

net spin rotation during one turn, i.e., close to integer values of ν_0.*The* rotation axis $\boldsymbol{n_0}$ ***for spins is almost vertical away from imperfection resonances*** but it can be nearly horizontal in their vicinity.

RIHC

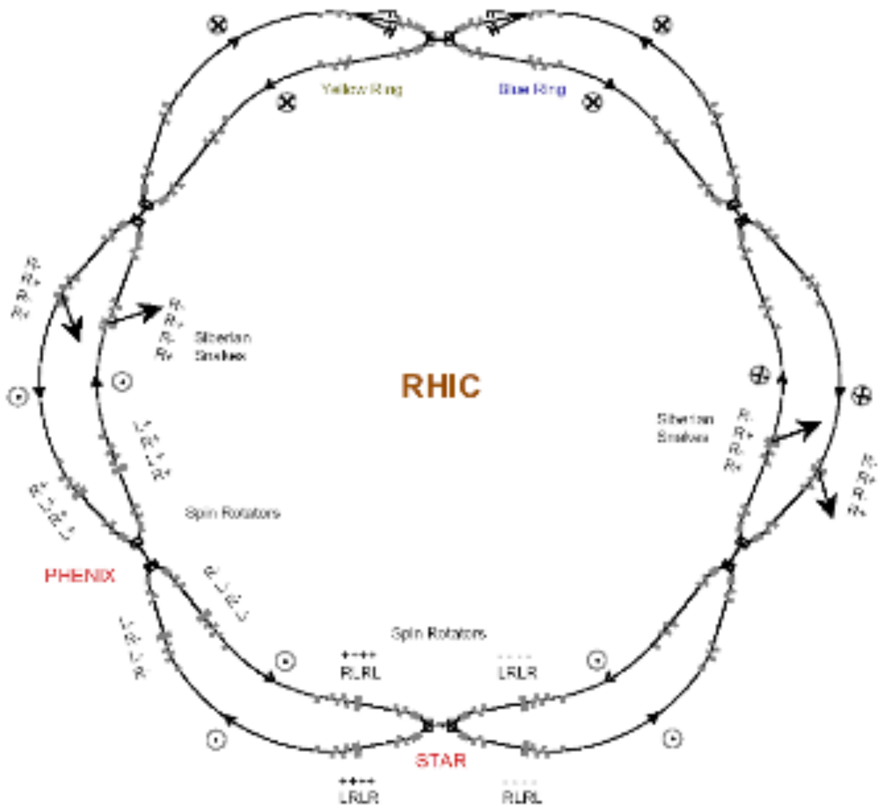
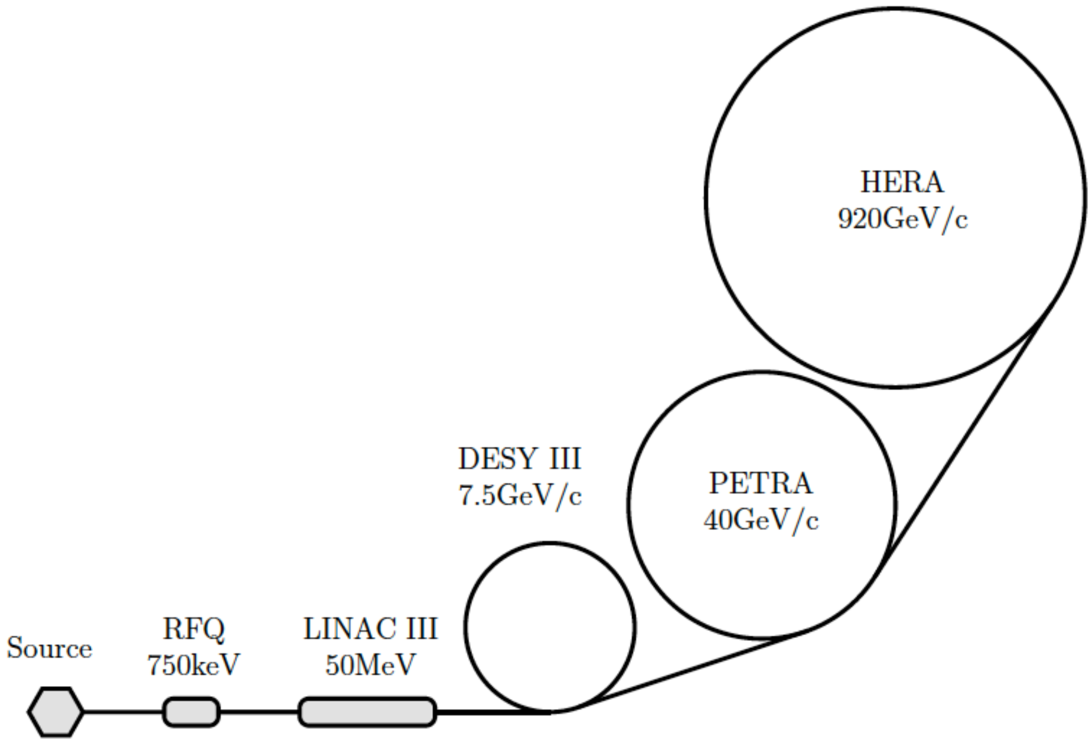


Fig. 5.1. Schematic layout of RHIC at BNL. The two Siberian Snakes per ring and their snake axes are shown.

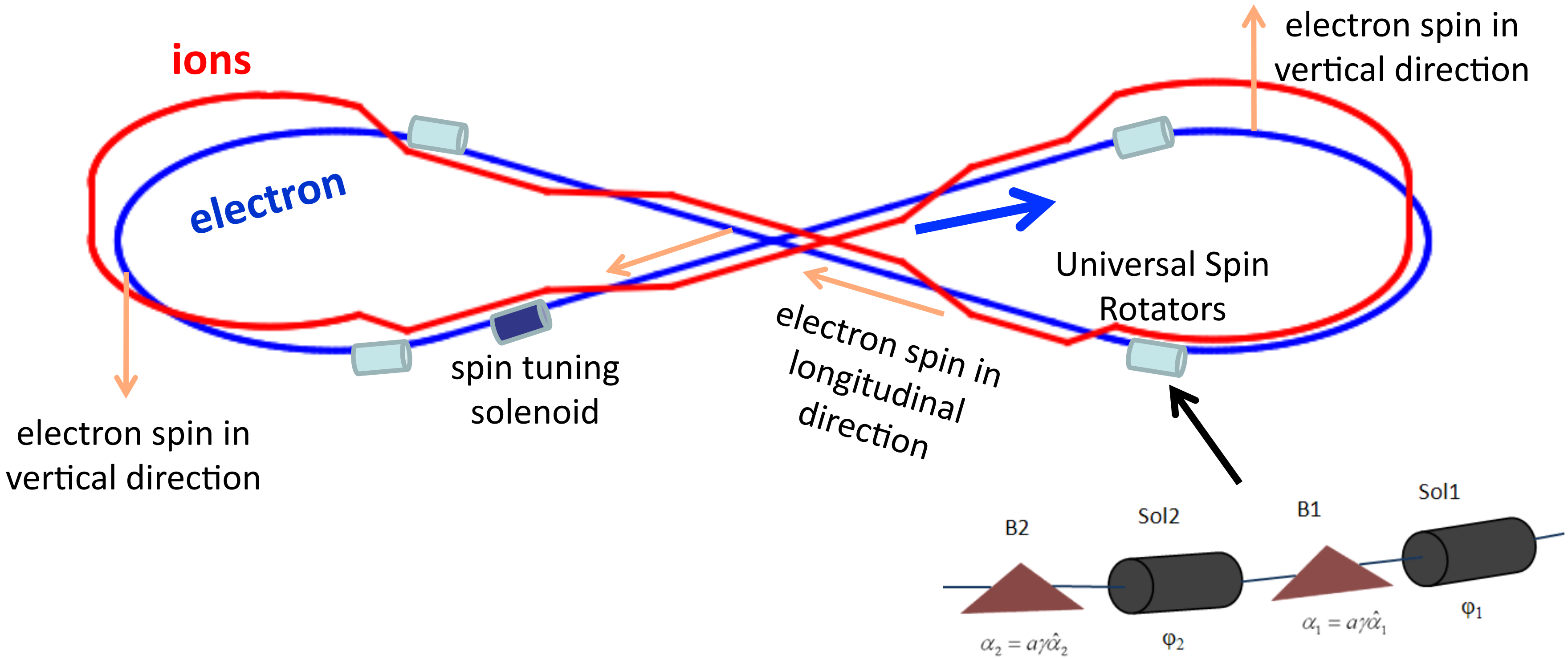
Courtesy: High Energy Polarized Proton Beams G. Hoffstaetter

HERA



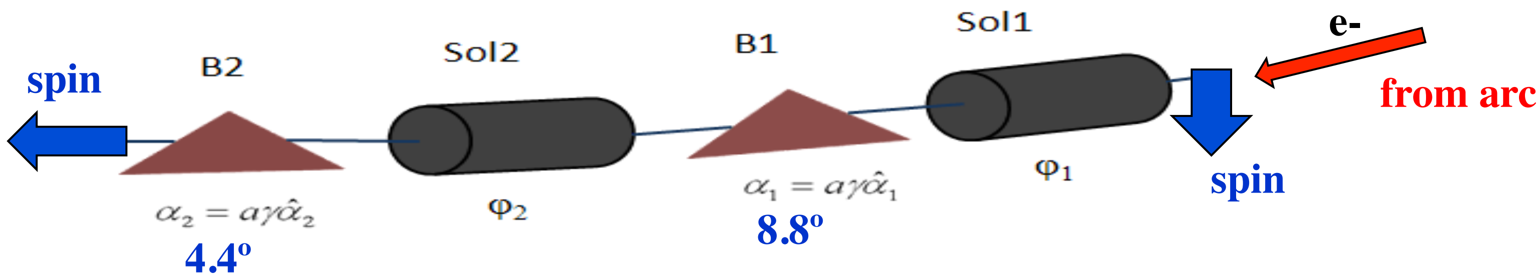
Courtesy: High Energy Polarized Proton Beams G. Hoffstaetter

Electron Polarization in Figure-8 Ring



- Polarized electron beam is injected at full energy from 12 GeV CEBAF
- Electron spin is in vertical direction in the figure-8 ring, taking advantage of self-polarization effect
- Spin rotators will rotate spin to longitudinal direction for collision at IP, than back to vertical direction in the other half of the ring

Universal Spin Rotator



- The last two arc dipole sections interleave with two solenoids
- The rotator works by adjusting spin rotation angles in solenoids depending on the beam energy.
- X-Y betatron coupling introduced by solenoids must be compensated

E	Solenoid 1		Solenoid 2		Spin rotation	
	spin rot.	BDL	spin rot.	BDL	arc bend 1	src bend 2
GeV	rad	T m	rad	T m	rad	rad
3	π/2	15.7	0	0	π/3	π/6
4.5	π/4	11.8	π/2	23.6	π/2	π/4
6	0.63	12.3	π-1.23	38.2	2π/3	π/3
9	π/6	15.7	2π/3	62.8	π	π/2
12	0.62	24.6	π-1.23	76.4	4π/3	2π/3

Optics Coupling Compensation

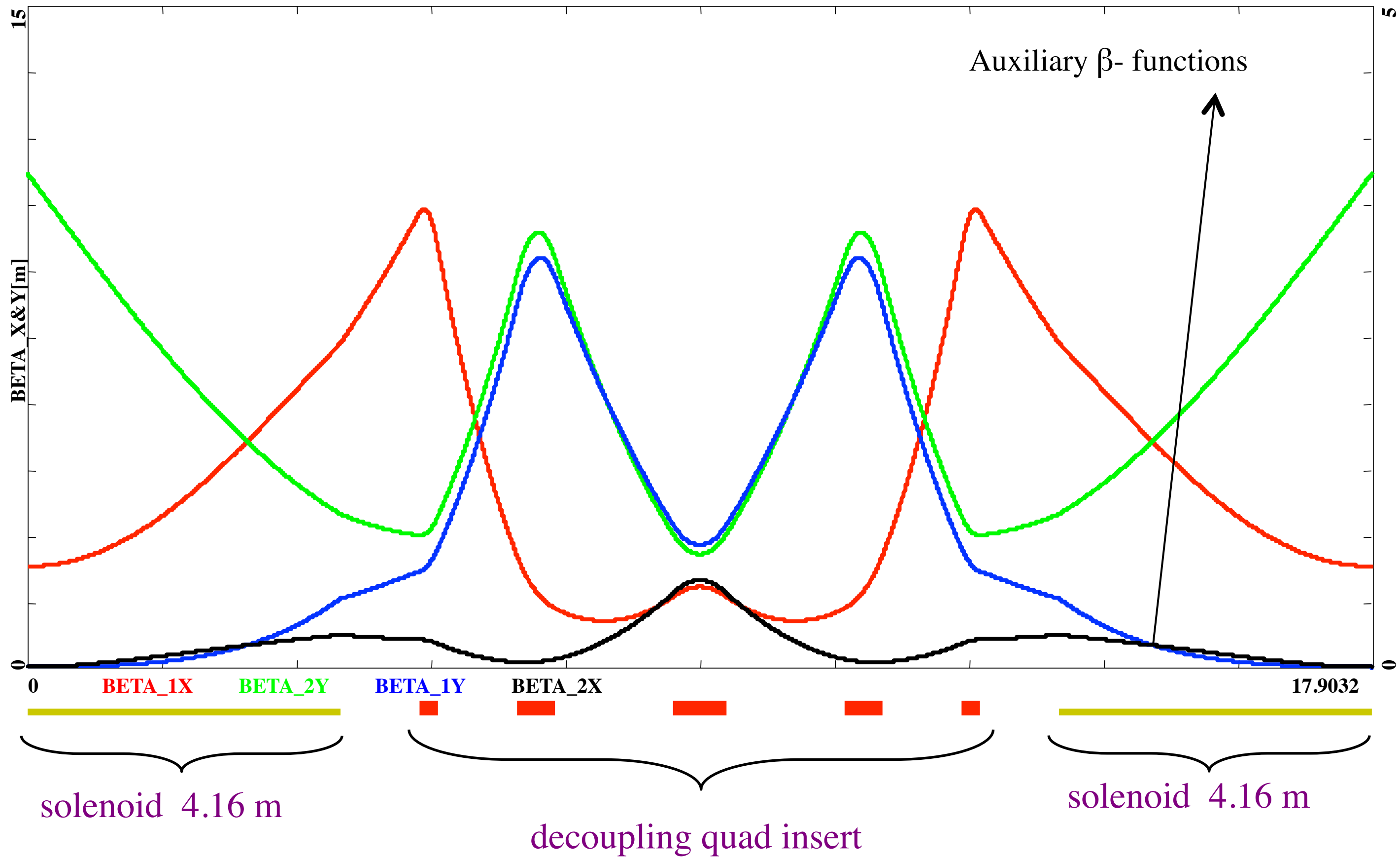
- X-Y beam coupling introduced by solenoids is compensated locally
- Each solenoid is divided into two equal parts and a set of quadrupoles is inserted between them to cancel coupling
  - Emma rotator
  - More general solution (*Litvinenko*)

$$M_{sol} = \begin{pmatrix} \cos^2\Phi & \frac{\sin 2\Phi}{S} & \frac{\sin 2\Phi}{2} & \frac{2\sin^2\Phi}{S} \\ \frac{-S\sin 2\Phi}{4} & \cos^2\Phi & \frac{-S\sin^2\Phi}{2} & \frac{\sin 2\Phi}{2} \\ \frac{-\sin 2\Phi}{2} & \frac{-2\sin^2\Phi}{S} & \cos^2\Phi & \frac{\sin 2\Phi}{S} \\ \frac{S\sin^2\Phi}{2} & \frac{-\sin 2\Phi}{2} & \frac{-S\sin 2\Phi}{4} & \cos^2\Phi \end{pmatrix}$$

Φ=B L/2
B solenoid field strength
L length respectively

Challenges:
1. Work at all energies
2. Independent of solenoid strength
3. Space economy (we need at least four of USR)
4. Modular to be easily matched and implemented at different places along the ring

Locally decoupled solenoid



$$M = \begin{pmatrix} C & 0 \\ 0 & -C \end{pmatrix}$$

References

- **Spin Dynamics and Snakes in Synchrotrons** S. Y. Lee
- **High Energy Polarized Proton Beams: A Modern View (Springer Tracts in Modern Physics) G. Hoffstaetter**
- **Particle Accelerator Physics Wiedemann**
- **SPIN ROTATOR OPTICS FOR MEIC H. Sayed et al, International Particle Accelerator Conference IPAC 2010**

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