Physics 696
Topics in Advanced Accelerator Design I

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Energy Units

• When a particle is accelerated, i.e., its energy is changed by an electromagnetic field, it must have fallen through an Electric Field (we show later by very general arguments that Magnetic Fields cannot change particle energy). For electrostatic accelerating fields the energy change is

\[ \Delta E = q \Delta \Phi = q \left( \Phi_a - \Phi_b \right) \]

\( q \) charge, \( \Phi \), the electrostatic potentials before and after the motion through the electric field. Therefore, particle energy can be conveniently expressed in units of the “equivalent” electrostatic potential change needed to accelerate the particle to the given energy. Definition: 1 eV, or 1 electron volt, is the energy acquired by 1 electron falling through a one volt potential difference.
Energy Units

\[ 1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J} \]

\[ 1 \text{ MeV} = 10^6 \text{ eV} = 1.6 \times 10^{-13} \text{ J} \]

To convert rest mass to eV use Einstein relation

\[ E_0 = mc^2 \]

where \( m \) is the rest mass. For electrons

\[ E_{\text{electron},0} = 9.1 \times 10^{-31} \text{ kg} \left( 3 \times 10^8 \text{ m/sec} \right)^2 = 81.9 \times 10^{-15} \text{ J} \]

\[ = 0.512 \text{ MeV} \]

Recent “best fit” value 0.51099906 MeV
Methods of Acceleration

• Acceleration by Static Electric Fields (DC) Acceleration
  – Cockcroft-Walton
  – van de Graaf Accelerators
  – Limited by voltage breakdowns to potentials of under a million volts in 1930, and presently to potentials of tens of millions of volts (in modern van de Graaf accelerators). Not enough to do nuclear physics at the time.

• Radio Frequency (RF) Acceleration
  – Main means to accelerate in most present day accelerators because one can get to 10-100 MV in a meter these days. Reason: alternating fields don’t cause breakdown (if you are careful!) until much higher field levels than DC.
  – Ideas started with Ising and Wideröe
Cockcroft-Walton

Proton Source at Fermilab, Beam Energy 750 keV
van de Graaf Accelerator

Brookhaven Tandem van de Graaf ~ 15 MV

Generator

Tandem trick multiplies the output energy
Ising’s Linac Idea

Prinzip einer Methode zur Herstellung von Kanalstrahlen hoher Voltzahl’ (in German), Arkiv för matematik o. fysik, 18, Nr. 30, 1-4 (1924).
Drift Tube Linac Proposal

Idea Shown in Wideröe Thesis
Über ein neues Prinzip zur Herstellung hoher Spannungen, *Archiv für Elektrotechnik* 21, 387 (1928)

(On a new principle for the production of higher voltages)
The Production of Heavy High Speed Ions without the Use of High Voltages
Alvarez Drift Tube Linac

- The first large proton drift tube linac built by Luis Alvarez and Panofsky after WW II
  - (1945-1955) Alvarez Proton Linac

Fig. 2. Linear accelerator produced by introducing drift tubes into cavity excited as in Fig. 1. Division into unit cells.

not being able to read

German easily, I merely
looked at the diagrams and
photographs of Windies
apparatus and from the
various figures in the article
readily understood
his general approach to the
problem — i.e., the multiple
acceleration of the positive ions
approximates
by application of radio
frequency oscillating voltages
to a series of cylindrical electrodes

*Stated in
E. O. Lawrence
Nobel Lecture
Lawrence’s Question

• Can you re-use “the same” accelerating gap many times?

\[ \ddot{F} = m\ddot{a} = q\vec{v} \times \vec{B} \]

\[ \frac{d^2 x}{dt^2} = \frac{qB}{m} v_y \rightarrow \frac{d^2 v_x}{dt^2} + \Omega_c^2 v_x = 0 \]

\[ \frac{d^2 y}{dt^2} = -\frac{qB}{m} v_x \rightarrow \frac{d^2 v_y}{dt^2} + \Omega_c^2 v_y = 0 \]

\[ \frac{d}{dt} \left( v_x^2 + v_y^2 \right) = \frac{qB}{m} \left( v_x v_y - v_y v_x \right) = 0 \]

\[ v_0 = \sqrt{v_x^2(t) + v_y^2(t)} \] is a constant of the motion
Cyclotron Frequency

\[ v_x(t) = v_0 \cos(\Omega_c t + \delta); \quad v_y(t) = -v_0 \sin(\Omega_c t + \delta) \]

\[ x(t) = x_0 + \frac{v_0}{\Omega_c} \sin(\Omega_c t + \delta); \quad y(t) = y_0 + \frac{v_0}{\Omega_c} \cos(\Omega_c t + \delta) \]

The radius of the oscillation \( r = \frac{v_0}{\Omega_c} \) is proportional to the velocity after the gap. Therefore, the particle takes the same amount of time to come around to the gap, independent of the actual particle energy!!!! (only in the non-relativistic approximation). Establish a resonance (equality!) between RF frequency and particle transverse oscillation frequency, also known as the Cyclotron Frequency

\[ f_{rf} = f_c = \frac{\Omega_c}{2\pi} = \frac{qB}{2\pi m} \]
What Correspond to Drift Tubes?

- Dee’s!
U. S. Patent Diagram
Magnet for 27 Inch Cyclotron (LHS)
Lawrence and “His Boys”

1-4: Jack Livingood, Frank Knox, M.S.Livingston, David Sloan, R.O.Lawrence, Milton White; Wesley Coates, L.Jackson Laselett and Commander T. Lucci - 1933
And Then!

Nov. 13
Bloodcount
Mon. 9 AM
Kneugler
Coan
Tues. 9 AM
Alvany
Rehersal
Livinston
Wed. 9 AM
Wright
Backus
Helmheit
Fri. 9 AM
Salisbury
Cook

Nov. 9 '39
Assoc'd Press
Unconfirmed
E.O.L. has
Nobel Prize

Confirmed
Beam Extracted from a Cyclotron

Radiation Laboratory 60 Inch Cyclotron, circa 1939
88 Inch Cyclotron at Berkeley Lab
Relativistic Corrections

When include relativistic effects (you’ll see in the HW!) the “effective” mass to compute the oscillation frequency is the relativistic mass $\gamma m$

$$f_c = \Omega_c / 2\pi = \frac{qB}{2\pi \gamma m}$$

where $\gamma$ is Einstein’s relativistic $\gamma$, most usefully expressed as

$$\gamma = \frac{E_{tot}}{E_0} = \frac{E_0 + E_{kin}}{E_0} = \frac{mc^2 + E_{kin}}{mc^2}$$

$m$ particle rest mass, $E_{kin}$ particle kinetic energy
Cyclotrons for Radiation Therapy
Betatrons

25 MeV electron accelerator with its inventor: Don Kerst. The earliest electron accelerators for medical uses were betatrons.
Transformer

Primary winding
$N_p$ turns

Primary current $I_p$
Primary voltage $V_p$

Magnetic Flux, $\Phi$

Transformer Core

Secondary winding
$N_s$ turns

Secondary current $I_s$
Secondary voltage $V_s$
Generic Modern Synchrotron

Focusing

RF Acceleration

Bending

Spokes are user stations for this X-ray ring source
Edwin McMillan discovered phase stability independently of Veksler and used the idea to design first large electron synchrotron.

$$h = \frac{L f_{RF}}{\beta c}$$

Harmonic number: # of RF oscillations in a revolution
Transition Energy

Beam energy where speed increment effect balances path length change effect on accelerator revolution frequency. Revolution frequency independent of beam energy to linear order. We will calculate in a few weeks.

- Below Transition Energy: Particles arriving EARLY get less acceleration and speed increment, and arrive later, with respect to the center of the bunch, on the next pass. Applies to heavy particle synchrotrons during first part of acceleration when the beam is non-relativistic and accelerations still produce velocity changes.

- Above Transition Energy: Particles arriving EARLY get more energy, have a longer path, and arrive later on the next pass. Applies for electron synchrotrons and heavy particle synchrotrons when approach relativistic velocities. As seen before, Microtrons operate here.
Ed McMillan

Vacuum chamber for electron synchrotron being packed for shipment to Smithsonian
Cosmotron (First GeV Accelerator)
BNL Cosmotron and Shielding
Cosmotron Magnet

The Cosmotron magnet
Cosmotron People

E. Courant - Lattice Designer

Stan Livingston - Boss
Snyder - theorist

Christofilos - inventor
Bevatron

Designed to discover the antiproton; Largest Weak Focusing Synchrotron
Strong Focusing

• Betatron oscillation work has showed us that, apart from bend plane focusing, a shaped field that focuses in one transverse direction, defocuses in the other.

• Question: is it possible to develop a system that focuses in both directions simultaneously?

• Strong focusing: alternate the signs of focusing and defocusing: get net focusing!!

Order doesn’t matter
Linear Magnetic Lenses: Quadrupoles

Source: Danfysik Web site
First Strong-Focusing Synchrotron

Cornell 1 GeV Electron Synchrotron (LEPP-AP Home Page)
Alternating Gradient Synchrotron (AGS)
CERN PS

25 GeV Proton Synchrotron
Eventually 400 GeV protons and antiprotons
First TeV-scale accelerator; Large Superconducting Benders
LEP Tunnel (Now LHC!)

Empty

LHC
Storage Rings

• Some modern accelerators are designed not to “accelerate” much at all, but to “store” beams for long periods of time that can be usefully used by experimental users.
  – Colliders for High Energy Physics. Accelerated beam-accelerated beam collisions are much more energetic than accelerated beam-target collisions. To get to the highest beam energy for a given acceleration system design a collider
Eventually became leading synchrotron radiation machine
Cornell 10 GeV ES and CESR
SLAC’s PEP II B-factory
ALADDIN at Univ. of Wisconsin
VUV Ring at NSLS

VUV ring “uncovered”
Berkeley’s ALS
Argonne APS