Mid-term Exam

Physics 425/525

Multiple Choice (circle or otherwise indicate the correct answer)

1. (5pts) Two 1 C positive charges are place 1 km apart in vacuum. The force on the charges is
   a. 9 Nt repulsive along the line between the charges
   b. 9 Nt attractive along the line between the charges
   c. 9000 Nt repulsive along the line between the charges
   d. 9000 Nt attractive along the line between the charges

2. (5pts) What is $\nabla \cdot \hat{\boldsymbol{\mathscr{r}}}$ for $\boldsymbol{r} - \boldsymbol{r}' \neq 0$?
   a. $1/|\boldsymbol{\mathscr{r}}|^2$
   b. $2/|\boldsymbol{\mathscr{r}}|$
   c. 0
   d. $-1/|\boldsymbol{\mathscr{r}}|^3$

3. (5pts) Which is NOT a property of a conductor
   a. The electric potential function varies inside the conductor
   b. Excess charge resides on the surface of a conductor
   c. The electric field vanishes inside the conductor
   d. Unbound electrons are free to move in a conductor

4. (5pts) A conducting sphere of radius *R* is charged with a charge *Q*. The electric potential (referenced to 0 at ∞) is
   a. $Qr/\left(4\pi\varepsilon_0 R^2\right) \;\; r<R; \;\; QR/\left(4\pi\varepsilon_0 r^2\right) \;\; r>R$
   b. $Q/\left(4\pi\varepsilon_0 r\right) \;\; r<R; \;\; Q/\left(4\pi\varepsilon_0 R\right) \;\; r>R$
   c. $Qr^2/\left(4\pi\varepsilon_0 R^3\right) \;\; r<R; \;\; QR^2/\left(4\pi\varepsilon_0 r^3\right) \;\; r>R$
   d. $Q/\left(4\pi\varepsilon_0 R\right) \;\; r<R; \;\; Q/\left(4\pi\varepsilon_0 r\right) \;\; r>R$

5. (5pts) Suppose $\boldsymbol{E} = \left[E_0\sigma_s\left(1-\exp\left(-s^2/2\sigma_s^2\right)\right)/s\right]\hat{s}$ where *s* is the cylindrical radial coordinate. What is $\rho(s)$?
   a. $\left(\varepsilon_0 E_0/\sigma_s\right)^2 \exp\left(-s^2/2\sigma_s^2\right)$
   b. $\left(\varepsilon_0 E_0/\sigma_s\right)\exp\left(-s^2/2\sigma_s^2\right)$
   c. $\left(\varepsilon_0 E_0/\sigma_s\right)\left(1-\exp\left(-s^2/2\sigma_s^2\right)\right)$
   d. $\left(\varepsilon_0 E_0 s/\sigma_2^2\right)\exp\left(-s^2/2\sigma_s^2\right)$

Problems

6. (20 pts) For each vector field, determine whether it can be described by a scalar potential, a vector potential, or whether it needs both for a complete description. Circle or otherwise indicate one answer for each field.
   a. $\boldsymbol{v} = x\hat{x} + y\hat{y} + z\hat{z}$ (scalar potential, vector potential, both needed)
   b. $\boldsymbol{v} = xy\hat{x} + yz\hat{y} + zx\hat{z}$ (scalar potential, vector potential, both needed)
   c. $\boldsymbol{v} = \left(x\hat{x} + y\hat{y} + z\hat{z}\right)\left(x^2 + y^2 + z^2\right)$ (scalar potential, vector potential, both needed)
   d. $\boldsymbol{v} = yz\hat{x} - zx\hat{y} + xy\hat{z}$ (scalar potential, vector potential, both needed)

   Extra credit: (10 pts) Record a scalar potential for those fields that can be so described

7. (25 pts) A conducting sphere of radius *R* is charged with a charge *Q*. It is surrounded by an UNCHARGED spherical conducting shell of inner radius *a* and outer radius *b*.
   a. What is the electric field in each of the regions $r < R$, $R < r < a$, $a < r < b$, and $r > b$?
   b. What is the value of the potential of the inner sphere assuming the electric potential function vanishes as $r \to \infty$?

8. (30 pts) Solve the potential for a conducting sphere in a uniform field by the method of images. Follow the steps indicated.
   a. Place a point charge of magnitude $-q$ at $z = a$ and a point charge of magnitude q at $z = -a$ on the *z*-axis. Show that near the origin, the electric field is uniform and
   $$E_z \approx \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}$$
   when $a \to \infty$.
   b. Place a conducting sphere of radius R centered at the origin. By Example 3.2 which was used in a homework problem, what is the magnitude of two image charges and how should they be placed so that the potential from all 4 charges vanishes on the sphere?
   c. Show the dipole moment of the two image charges is
   $$\boldsymbol{p} = \frac{2R^3}{a^2}q\hat{z} \to 4\pi\varepsilon_0 E_z R^3\hat{z},$$
   when $a \to \infty$ holding $2q/a^2$ constant.
   d. In this same limit, what is the dipole potential (in spherical coordinates) for $r > R$ for the two image charges?
   e. By the uniqueness theorem, this same potential must govern the problem of a sphere in a uniform field. Find the total potential and compare to the answer for the potential of a sphere in a uniform field in the book or lectures.