Electromagnetism I

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Lecture 21

Flux Change Version

- Define the unambiguous magnetic flux through the loop (any surface will do! Why?)

$$\Phi_B = \int_S \boldsymbol{B} \cdot \hat{n} da$$

- For rectangular loop

$$\Phi_B = BLx$$

- The rate of change of flux is

$$\frac{d\Phi_B}{dt} = BL\frac{dx}{dt} = -BLv$$

- emf in this case is

$$\mathcal{E} = vBL = -\frac{d\Phi_B}{dt}$$

General Argument

- We have shown the motional emf is equal to the negative time derivative of the magnetic flux in a special case. It is more generally true!
  - The loop doesn't have to have straight wires or perpendicular orientation
  - The velocity can be in arbitrary directions
  - Even the size of the loop in different directions can change!
- Proof: consider the loop at two slightly different times t and $t + \Delta t$. For constant $\boldsymbol{B}$

$$d\Phi_B = \Phi_B\left(t+\Delta t\right) - \Phi_B\left(t+\Delta t\right) = \Phi_{B,ribbon} = \int_{ribbon} \boldsymbol{B} \cdot \hat{n} da$$

$$da = (\boldsymbol{v} \times d\boldsymbol{l})\, dt$$

- The velocity of the electrons is $\boldsymbol{w} = \boldsymbol{v} + \boldsymbol{v}_{drift}$. But $\boldsymbol{v}_{drift}$ is along $d\boldsymbol{l}$

$$\frac{d\Phi_B}{dt} = \oint_L \boldsymbol{B} \cdot (\boldsymbol{v} \times d\boldsymbol{l}) = \oint_L \boldsymbol{B} \cdot (\boldsymbol{w} \times d\boldsymbol{l})$$

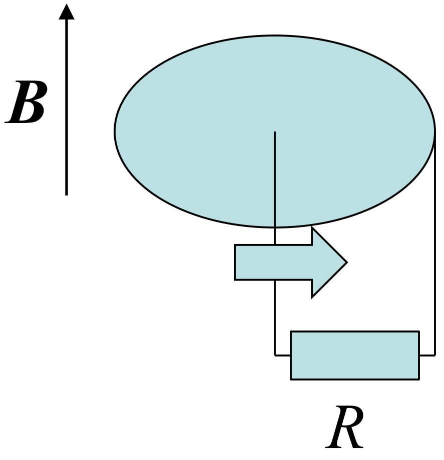
$$\boldsymbol{B} \cdot (\boldsymbol{w} \times d\boldsymbol{l}) = (\boldsymbol{B} \times \boldsymbol{w}) \cdot d\boldsymbol{l} = -(\boldsymbol{w} \times \boldsymbol{B}) \cdot d\boldsymbol{l}$$

$$\frac{d\Phi_B}{dt} = -\oint_L (\boldsymbol{w} \times \boldsymbol{B}) \cdot d\boldsymbol{l} = -\oint_L \boldsymbol{f}_{mag} \cdot d\boldsymbol{l} = -\mathcal{E}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Example 7.4 Faraday Disk

- Not all motional emf computed via flux rule. An example is the Faraday disk



$$v = \omega s$$

$$\mathcal{E} = \int_0^a \omega s B ds = \omega B \frac{s^2}{2}\Bigg|_0^a$$

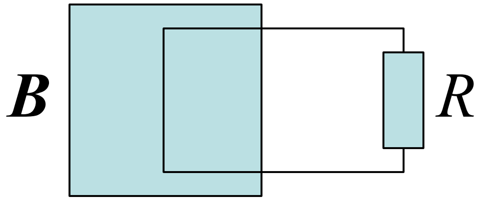
$$= \omega B \frac{a^2}{2}\Bigg|$$

- Current is

$$I = \frac{\mathcal{E}}{R} = \frac{\omega B a^2}{2R}$$

Faraday’s Experiments

- In 1831, Michael Faraday did three experiments



  - Pulled the loop through a magnetic field. A current is observed.
  - Moved the magnet keeping the coil fixed. A current is observed! There must be ***E***
  - With both magnet and coil fixed, changed the magnetic field strength. A current is observed!! ***E*** again
- Ingenious *induction (pun intended)*: a changing magnetic field from whatever cause, induces an electric field. Faraday’s Law

Faraday's Law

- Experimentally, in all three cases

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- Induced $\boldsymbol{E}$ in experiments 2 and 3

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \boldsymbol{B} \cdot \hat{n} da$$

$$\int \nabla \times \boldsymbol{E} \cdot \hat{n} da = -\int \frac{\partial \boldsymbol{B}}{\partial t} \cdot \hat{n} da$$

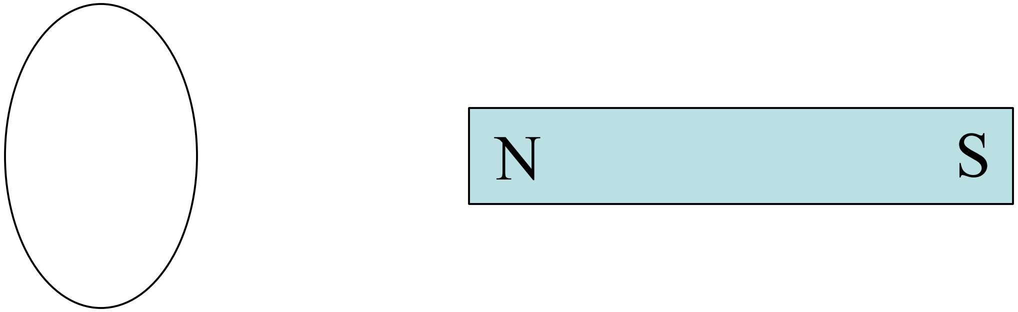
- Faraday's Law in differential form

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

- Whenever the flux changes, an emf is produced in a loop

Lenz's Law

- Magnet experiment



- What are $\Phi_B(t)$ and $I(t)$?

- Handy rule: nature resists a change in flux (Lenz's Law)

Induced pure "Faraday" Field

- If electric field produced only by changing magnetic field

$$\nabla \cdot \boldsymbol{E} = 0 \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

- Biot-Savart for ***E***

$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{1}{4\pi}\int \frac{\partial \boldsymbol{B}/\partial t(\boldsymbol{r}') \times \hat{\boldsymbol{\mathscr{r}}}}{|\boldsymbol{\mathscr{r}}|^2} dV'$$

$$= -\frac{1}{4\pi}\frac{\partial}{\partial t}\int \frac{\boldsymbol{B}(\boldsymbol{r}') \times \hat{\boldsymbol{\mathscr{r}}}}{|\boldsymbol{\mathscr{r}}|^2} dV'$$

- The "Ampere Law" equivalent is automatically

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi_B}{dt}$$

Examples 7.7 and 7.8