Electromagnetism I

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Lecture 20

Time Dependences

- Up to now we've dealt with static (solutions not depending on time) cases
- When time dependence is allowed, essentially two adjustments must be made to our equations to capture electrodynamic effects (order historical).
  - Faraday's Law of Induction (what electromagnetic field is generated by a changing magnetic field?)
  - Maxwell's Displacement Current (what electromagnetic field is generated by a changing electric field?)
- Most of the solutions that we've built up still apply
- Additional solutions, including electromagnetic wave solutions, will be possible in the "final" Maxwell set.

Conductivity

Electromagnetic currents are "driven" (created) by very tiny electric fields inside conductors. Wire conductors are very good, but only some are "perfect" at low temperatures (superconductors). Inside an imperfect conductor

$$\boldsymbol{J} = \sigma \boldsymbol{E}.$$

σ is called the material conductivity. [σ] is A/(Vm). It quantifies the Joule heat dissipated in the conductor by having it transport the current.

- A resistor, is a piece of low conductivity material. Let's assume it is a cylinder
- When a battery drives a resistor

$$\frac{\Delta V}{L} = \frac{J}{\sigma} = \frac{I}{\sigma \pi r^2}$$

Resistance

- In this special case, the electromotive force (emf) is

$$\Delta V = IR \qquad R = \frac{L}{\sigma \pi r^2}$$

where R is the resistance of the resistor [R] is Ω (V/A) and [emf] is V.

- There are tiny Joule losses (and voltage changes) in the wire too, but usually they are negligible compared to the losses in the resistor.
- Joule heating in a resistor

$$P = IV = I^2 R$$

Electromotive force

- More generally, for any circuit (wire loop!), compute the line integral of the electric field (again, usually neglecting the small field within the wire itself!)

$$\mathcal{E} = \oint \boldsymbol{E} \cdot d\boldsymbol{l}$$

- In battery case, if include line integral through battery must get zero. The emf produced by the battery (voltage) is only that portion of the line integral outside of the terminals of the battery.

Motional emf

- What happens when a wire moves in a magnetic field?
- The free electrons in the wire experience a force

$$\boldsymbol{F} = \boldsymbol{v} \times \boldsymbol{B}$$

which can drive electrons down the wire just as effectively as an electric field

$$\mathcal{E} = \oint \boldsymbol{v} \times \boldsymbol{B} \cdot \boldsymbol{dl}$$

- In particular, for wire of length *L* in a perpendicular magnetic field

$$\mathcal{E} = vBL$$

- Work is done by the agency moving the wire, not the magnetic field

Flux Change Version

- Define the unambiguous magnetic flux through the loop (any surface will do! Why?)

$$\Phi_B = \int_S \boldsymbol{B} \cdot \hat{n} da$$

- For rectangular loop

$$\Phi_B = BLx$$

- The rate of change of flux is

$$\frac{d\Phi_B}{dt} = BL\frac{dx}{dt} = -BLv$$

- emf in this case is

$$\mathcal{E} = vBL = -\frac{d\Phi_B}{dt}$$