Electromagnetism I

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Lecture 19

Linear Media

- For many materials stressed by small magnetic fields, the magnetization is proportional to ***B*** (and ***H***!) Such materials are called linear paramagnetic or diamagnetic materials.
- By custom, the proportionality constant is referenced to ***H***

$$\boldsymbol{M} = \chi_m \boldsymbol{H}.$$

χ_m is know as the magnetic susceptibility.

- In this case, ***B*** is proportional to ***H***,

$$\boldsymbol{B} = \mu_0 \left(1 + \chi_m\right) \boldsymbol{H} \equiv \mu H$$

and the proportionality constant μ is known as the material permeability, the ratio $\mu = \mu/\mu_0$ is known as the relative permeability of the material. The relationship(s) $\boldsymbol{B} = \mu \boldsymbol{H}$ is/are known as the constitutive relation(s).

Magnetic Susceptibility

- Magnetic susceptibility is positive for paramagnetic materials and negative for diamagnetic materials
- When the relative permeability of a material is constant,

$$\boldsymbol{J}_b = \nabla \times \boldsymbol{M} = \nabla \times \left(\chi_m \boldsymbol{H} \right) = \chi_m \nabla \times \boldsymbol{H} = \chi_m \boldsymbol{J}_f$$

$$\nabla \cdot \boldsymbol{J}_b = \nabla \cdot \left(\nabla \times \boldsymbol{M} \right) = 0 \qquad \text{(automatically)}$$

- In particular, unless free current actually flows through the material, all the bound current will be at the surface
- For electromagnet design, need to consider only surface currents.
- The value is usually small, see Griffiths' table

Ferromagnetism

- Permanent magnets, having a fixed magnetization in the material, exist. Usually, the material consists of an alloy of iron, and so the phenomenon is called ferromagnetism
- Material tends to consist of small domains, each of which is magnetized in a certain direction
- Impressed magnetic fields, through magnetic torque, can change the orientation of the domains and make net magnetization
- But the domains can also be “frozen” in aligned states
- Magnetization and net magnetic field depends on history.
- Hysteresis loop and Hysteresis losses

Example 6.2

- Find the magnetic field ***H*** inside and outside a copper wire carrying a current *I*. Assume the current uniformly distributed in the wire.

$$\oint \boldsymbol{H}\cdot d\boldsymbol{l} = H\,2\pi s = \begin{cases} \dfrac{I\pi s^2}{\pi R^2} & r<R \\ I & r>R \end{cases}$$

$$\boldsymbol{H} = H\hat{\phi} = \begin{cases} \dfrac{Is}{2\pi R^2}\hat{\phi} & r<R \\ \dfrac{I}{2\pi s}\hat{\phi} & r>R \end{cases}$$

- Then ***B*** is

$$\boldsymbol{B} = \begin{cases} \dfrac{\mu_0\left(1+\chi_m\right)Is}{2\pi R^2}\hat{\phi} & r<R \\ \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & r>R \end{cases}$$

Example 6.3

- What is the field in an infinite solenoid (n turns per unit length) filled with a linear material with susceptibility χ_m?

$$\oint \boldsymbol{H} \cdot dl = Hl + 0 + 0 + 0 = nIl$$

$$\boldsymbol{H} = H\hat{z} = nI\hat{z}$$

- By the definition

$$\boldsymbol{B} = \mu_0 \left(1 + \chi_m\right) \boldsymbol{H} = \mu_0 \left(1 + \chi_m\right) nI\hat{z}$$

- If material paramagnetic, field is enhanced, if diamagnetic, the field is reduced. Adjusted field due to surface current (only)

$$\boldsymbol{K}_b = \boldsymbol{M} \times \hat{n} = \chi_m \boldsymbol{H} \times \hat{n} = \chi_m nI\hat{\phi}$$