Electromagnetism I

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Lecture 17

Magnetic Multipoles

- Recall multipole expansion and apply to $\boldsymbol{A}$.

$$\frac{1}{|\boldsymbol{\mathscr{r}}|} = \frac{1}{\sqrt{r^2 + r'^2 - 2rr'\cos\alpha}} = \frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{r'}{r}\right)^l P_l(\cos\alpha)$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} I \oint \frac{1}{|\boldsymbol{\mathscr{r}}|} d\boldsymbol{l}' = \frac{\mu_0 I}{4\pi}\left[\begin{array}{l} \frac{1}{r}\oint d\boldsymbol{l}' + \frac{1}{r^2}\oint r'\cos\alpha\, d\boldsymbol{l}' + \\ \frac{1}{r^3}\oint r'^2\left(\frac{3}{2}\cos^2\alpha - \frac{1}{2}\right) d\boldsymbol{l}' + \cdots \end{array}\right]$$

- Magnetic monopole vanishes (no magnetic charges!)

$$\oint dl' = \int \frac{dx}{d\sigma} d\sigma \hat{x} + \int \frac{dy}{d\sigma} d\sigma \hat{y} + \int \frac{dz}{d\sigma} d\sigma \hat{z} = 0 + 0 + 0$$

Magnetic Dipole

- Magnet dipole term in ***A***

$$\boldsymbol{A}_{dip}(\boldsymbol{r}) = \frac{\mu_0 I}{4\pi r^2}\oint r'\cos\alpha\, d\boldsymbol{l}' = \frac{\mu_0 I}{4\pi r^2}\oint \boldsymbol{r}'\cdot\hat{r}\, d\boldsymbol{l}'$$

$$\oint \boldsymbol{r}'\cdot\hat{r}\, d\boldsymbol{l}' = -\hat{r}\times\int d\boldsymbol{a}'$$

$$\boldsymbol{A}_{dip}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\hat{r}}{r^2}$$

- ***m*** is called the magnetic dipole moment

$$\boldsymbol{m} = I\int d\boldsymbol{a}' = I\boldsymbol{a}$$

Field for Magnetic Dipole

- Vector potential

$$\boldsymbol{A}_{dip}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$$

- Magnetic field

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right]$$

$$= \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\boldsymbol{m}\cdot\hat{r})\hat{r} - \boldsymbol{m}\right]$$

Magnetization

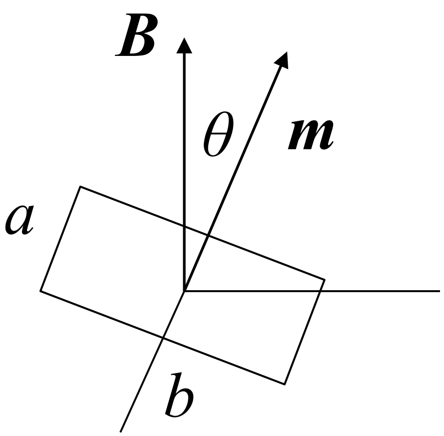
- Apply magnetic field to a material. Can get
  - Induced field parallel to applied field (paramagnetism)
  - Induced field opposite to applied field (diamagnetism)
- Toque on magnetic dipoles

$$N = aF \sin\theta \hat{x}$$

$$F = IbB$$

$$N = IabB \sin\theta \hat{x}$$

$$\boldsymbol{N} = \boldsymbol{m} \times \boldsymbol{B}$$



- Tend to align moment and field

Gradient Force on Dipole

- For uniform magnetic field total force 0

$$\boldsymbol{F} = I\oint\left(d\boldsymbol{l}\times\boldsymbol{B}\right) = I\left(\oint d\boldsymbol{l}\right)\times\boldsymbol{B} = 0$$

- For non-uniform field have a gradient force

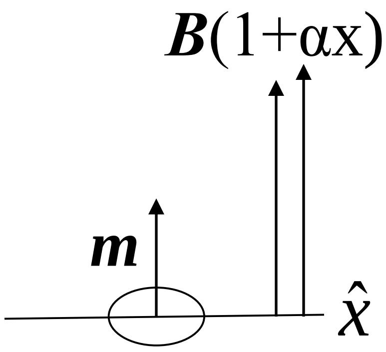
$$\boldsymbol{F} = I\oint\left(d\boldsymbol{l}\times\boldsymbol{B}\right)$$

$$x = R\cos\theta \qquad y = R\sin\theta$$

$$\boldsymbol{F} = I\int_0^{2\pi}\left(-R\sin\theta\hat{x} + R\cos\theta\hat{y}\right)\times B\hat{z}\left(1+\alpha R\cos\theta\right)d\theta$$

$$= I(-0\hat{y} + RB\alpha R\hat{x})\int_0^{2\pi}\cos^2\theta d\theta$$

$$= \alpha IR^2B\frac{1}{2}(2\pi) = \alpha\left(\pi R^2\right)B$$



- Example of general formula

$$\boldsymbol{F} = \nabla\left(\boldsymbol{m}\cdot\boldsymbol{B}\right)$$

Source of Diamagnetism

- Compute effect of magnetic field on atomic orbits assuming the atomic structure doesn't change much (Quantum Mechanics)!

$$I = \frac{-e}{T} = \frac{-ev}{2\pi R}$$

$$|\boldsymbol{m}| = I\pi R^2 = \frac{-evR}{2}$$

- Without $\boldsymbol{B}$, outer electrons see shielded Coulomb field of nucleus. Force equilibrium is

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2} = m_e\frac{v^2}{R}$$

- With a magnetic field

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}+e\bar{v}B=m_e\frac{\bar{v}^2}{R}$$

$$e\bar{v}B=m_e\left[\frac{\bar{v}^2}{R}-\frac{v^2}{R}\right]=\frac{m_e}{R}(\bar{v}+v)(\bar{v}-v)$$

$$(\bar{v}-v)\approx\frac{eRB}{2m_e}$$

- Electron speeds up! Moment change (magnitude gets bigger!) is

$$\Delta m=-\frac{e}{2}\Delta vR$$

$$=-\frac{e}{2}\frac{eRB}{2m_e}R=-\frac{e^2R^2}{4m_e}B,$$

- But opposite the magnetic field! Change in ***m*** negative!!!