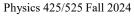


Electromagnetism I

G. A. Krafft, V. Ziemann Jefferson Lab Old Dominion University Lecture 14









Dielectric Energy

• Simple consideration of simple capacitor

$$W = \frac{1}{2}CV^2$$

• With dielectric inside

$$C = \varepsilon_r C_{vacuum}$$

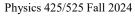
• Without dielectric

$$W = \frac{\varepsilon_0}{2} \int \boldsymbol{E}^2 dV$$

• Suggests with dielectric

$$W = \frac{1}{2} \int \boldsymbol{D} \cdot \boldsymbol{E} dV$$









Atomic model again

• Assume the "atomic springs" have constant *k*

$$E_{spring} = \frac{1}{2} k \left(x_{+} - x_{-} \right)^{2} = \frac{1}{2} k \frac{q^{2} E^{2}}{k^{2}}$$

- In terms of individual atomic dipole moments $p = q(x_{+} - x_{-}) = \frac{q^{2}E}{k}$
- When *n* is the number of dipole moments per volume, the energy/volume in the springs is

$$\frac{U_{spring}}{Volume} = \frac{1}{2} (np) E = \frac{1}{2} P \times E$$

• The total energy in the springs generalizes to

$$U_{spring} = \frac{1}{2} \int \boldsymbol{P} \cdot \boldsymbol{E} dV$$





Total Energy

- **(()***P* **ODU**
- Electromagnetic field energy plus spring energy for a linear dielectric is

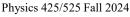
$$W = U_{em} + U_{spring} = \frac{\varepsilon_0}{2} \int \boldsymbol{E} \cdot \boldsymbol{E} dV + \frac{1}{2} \int \varepsilon_0 \chi_e \boldsymbol{E} \cdot \boldsymbol{E} dV = \frac{1}{2} \int \boldsymbol{D} \cdot \boldsymbol{E} dV$$

• Griffiths "add free charge" derivation. Start with 2.43 version of the work

$$W = \frac{1}{2} \int \rho \phi dV$$

where the potential ϕ is the total potential including any material polarization effects. Generally, for a given free charge distribution the total potential is proportional (but will have a different space distribution $\phi = \alpha(\mathbf{r})\rho_f$). Adding free charge $\Delta \rho_f$









$$\Delta W = \frac{1}{2} \int \alpha \left(\rho_f + \Delta \rho_f \right)^2 dV = \int \left(\Delta \rho_f \right) \phi dV = \int \nabla \cdot \left(\Delta D \right) \phi dV$$

• "Integrating by parts"

$$\nabla \cdot (\phi \Delta D) = \nabla \phi \cdot \Delta D + \phi \nabla \cdot (\Delta D)$$
$$\Delta W = \int \left[-\nabla \phi \cdot \Delta D + \nabla \cdot (\phi \Delta D) \right] dV$$
$$= \int E \cdot \Delta D dV + \int \phi \Delta D \cdot \hat{n} da$$

Boundary integral vanishes at large radii

• For linear dielectric

$$\frac{1}{2}\Delta(\boldsymbol{D}\cdot\boldsymbol{E}) = \frac{1}{2}\Delta(\boldsymbol{\varepsilon}\boldsymbol{E}\cdot\boldsymbol{E}) = \boldsymbol{\varepsilon}\Delta\boldsymbol{E}\cdot\boldsymbol{E} = \Delta\boldsymbol{D}\cdot\boldsymbol{E}$$
$$W = \frac{1}{2}\int\boldsymbol{D}\cdot\boldsymbol{E}dV$$



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Magnetic Fields



- Static magnetic fields (magnetostatics) produced by
 - Aligned atomic electron currents (permanent magnets)
 - Steady currents in wires (electromagnets)
- Unfortunate terminology problem

B is uniformly the notation for a (pseudo)vector field called by Griffiths the "magnetic field" and by other authors the "magnetic induction field". In this latter case, *H*, which will be defined in Chapter 6, is called the "magnetic field". MKSA Units

$$[B] = \text{Tesla}(T) = \frac{\text{Weber}}{\text{m}^2} = \frac{\text{Vsec}}{\text{m}^2} = \frac{\text{Nt}}{\text{A m}}$$

 $[H] = \frac{\text{A}}{\text{m}}$



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Lorentz Force



• Total electromagnetic force on a charged particle

$$\boldsymbol{F} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

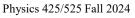
- Force perpendicular to both velocity and magnetic field direction
- Magnetic fields can do no work on charged particles

$$\Delta T = \int \boldsymbol{F} \cdot d\boldsymbol{l}$$
$$\dot{T} = \boldsymbol{F} \cdot \boldsymbol{v}$$

• For magnetic forces

$$\dot{T} = q\left(\boldsymbol{v} \times \boldsymbol{B}\right) \cdot \boldsymbol{v} = 0$$







Charged Particle Motion

• For a uniform magnetic field (say in *z*-direction)

$$\boldsymbol{F}_{\boldsymbol{B}} = \boldsymbol{m}\boldsymbol{\dot{v}} = \boldsymbol{q}\boldsymbol{v} \times \boldsymbol{B} = \boldsymbol{q} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_{x} & v_{y} & v_{z} \\ 0 & 0 & B_{z} \end{vmatrix} = \boldsymbol{q}B_{z}v_{y}\hat{x} - \boldsymbol{q}B_{z}v_{x}\hat{y}$$

• Equations of motion

$$\dot{v}_{x} = \frac{qB_{z}}{m}v_{y}$$
$$\dot{v}_{y} = -\frac{qB_{z}}{m}v_{x}$$
$$\dot{v}_{z} = 0$$









Circles!

- OD
- Simple harmonic motion in each degree of freedom

$$\frac{d}{dt} \left(v_x^2 + v_y^2 \right) = 2 \frac{qB_z}{m} \left(v_x v_y - v_y v_x \right) = 0$$
$$\ddot{v}_x = \frac{qB_z}{m} \dot{v}_y = -\left(\frac{qB_z}{m}\right)^2 v_x$$
$$\ddot{v}_y = -\frac{qB_z}{m} \dot{v}_x = -\left(\frac{qB_z}{m}\right)^2 v_x$$
$$\dot{v}_z = 0$$

• Cyclotron (angular) frequency

$$\omega_c = \frac{qB_z}{m} \qquad \left[\omega_c\right] = \frac{\text{C V sec}}{\text{kg m}^2} = \frac{1}{\text{sec}}$$

• Radius of orbit

$$\omega_c R = \sqrt{v_x^2 + v_y^2}$$



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