Electromagnetism I

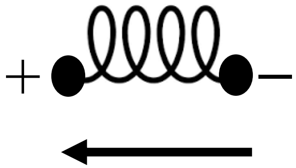
G. A. Krafft, V. Ziemann
Jefferson Lab
Old Dominion University

Lecture 11

Dielectrics

- In contrast to conductors, most everyday materials are insulators or dielectrics (Griffiths uses these terms interchangeably). Fundamental electromagnetic property: individual electrons tied to individual atoms
- Simple model: electromagnetic force within the atom "looks like" a spring binding the electron to the nucleus

No Electric Field



With Electric Field

$$PE = Ex_- - Ex_+ + k\frac{(x_+ - x_-)^2}{2}$$

$$\frac{\partial H}{\partial x_-} = 0 \rightarrow E - k(x_+ - x_-) = 0 \qquad \frac{\partial H}{\partial x_+} = 0 \rightarrow -E + k(x_+ - x_-) = 0$$

$$x_+ - x_- = qE/k$$

- Induced dipole moment proportional to electric field
- Energy stored in spring proportional to E^2

Polar Molecules

- Many materials (water the most common example!) have a permanent dipole moment. In a glass of water at room temperature the individual dipoles are randomly oriented and there is no net dipole moment.
- If an electric field present there is a tendency for the permanent dipoles to align with the field (can be calculated knowing the interaction energy and statistical physics)
- As above, the average dipole moment generated tends to be proportional to the electric field

- Both situations characterized by the polarization vector

$$\boldsymbol{P} \equiv \text{"average dipole moment/volume"} \quad [\boldsymbol{P}] = \frac{Cm}{m^3} = \frac{C}{m^2}$$

[Induced] Field from Polarization

- From Chapter 3, the field away from an individual dipole at $\boldsymbol{r}'$ is

$$\phi_1(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\hat{\boldsymbol{\mathscr{r}}}\cdot\boldsymbol{p}}{\mathscr{r}^2}$$

- Integrated over a volume of polarized material is

$$\phi(\boldsymbol{r}) = \int_V \frac{1}{4\pi\varepsilon_0}\frac{\hat{\boldsymbol{\mathscr{r}}}\cdot\boldsymbol{P}(\boldsymbol{r}')}{\mathscr{r}^2}dV'$$

- Differentiating with respect to *source* coordinates

$$\nabla'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$$

- Obtain

$$\phi(\boldsymbol{r}) = \int_V \frac{1}{4\pi\varepsilon_0} \nabla' \left(\frac{1}{\mathscr{r}} \right) \cdot \boldsymbol{P}(\boldsymbol{r}') \, dV'$$

- Integrating by parts

$$\phi(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \left[\int_V \nabla' \cdot \left(\frac{\boldsymbol{P}(\boldsymbol{r}')}{\mathscr{r}} \right) dV' - \int_V \left(\frac{1}{\mathscr{r}} \right) \nabla' \cdot \boldsymbol{P}(\boldsymbol{r}') dV' \right]$$

- Divergence theorem

$$\phi(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \left[\int_{S=\partial V} \frac{\boldsymbol{P}(\boldsymbol{r}')}{\mathscr{r}} \cdot \hat{n} \, da' - \int_V \left(\frac{1}{\mathscr{r}} \right) \nabla' \cdot \boldsymbol{P}(\boldsymbol{r}') dV' \right]$$

Bound Volume and Surface Charge

- Bound surface charge

$$\sigma_b = \boldsymbol{P} \cdot \hat{n}$$

- Bound volume charge

$$\rho_b = -\nabla \cdot \boldsymbol{P}$$

- Potential

$$\phi(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\left[\int_{S=\partial V} \frac{\sigma_b}{\mathscr{r}} da' - \int_V \left(\frac{\rho_b}{\mathscr{r}}\right) dV'\right]$$

- Field same as that generated by the volume charge density ρ_b and the surface charge density σ_b

Uniformly Polarized Dielectric Sphere

- If $\sigma_b(\theta) = P \cos \theta = P\, P_1 (\cos \theta)$, the potential is

$$\phi(r,\theta) = \frac{P}{3\varepsilon_0} r \cos\theta \qquad r \le R$$

$$= \frac{PR^2}{3\varepsilon_0 r^2} \cos\theta \qquad r \ge R$$

- Field inside sphere is uniform

$$\boldsymbol{E} = -\frac{\boldsymbol{P}}{3\varepsilon_0} \qquad r \le R$$

- Field outside is that of a perfect dipole with moment

$$\boldsymbol{p} = \frac{4\pi R^3}{3} \boldsymbol{P}$$