Graduate Accelerator Physics

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Lecture 1

Course Outline

- Introduction to Accelerators and Linear Dynamics (Krafft)
    - Relativity and E&M
    - Transverse Stability and Betatron Motion
    - Linear Optics
    - Synchrotron Motion
- Advanced Linear Dynamics (Krafft)
    - Solenoids
    - Coupled Motion
    - Multiple Energy Rings

- Magnets (Satogata)
  - Normal and Skew
  - Multipoles
  - Iron and Conductor-dominated Magnets
- RF Cavities (Satogata)
  - Waveguides
  - Transverse Modes
  - Pill-box Model
- Linear and Non-linear Errors and Their Correction (Satogata)
  - Closed Orbit Distortion and Correction
  - Resonance and Resonance Theory
  - Chromaticity and Its Correction
  - Slow Extraction

- Linacs (Satogata)
  - Proton and Ion
  - Electron
  - Energy Recovery
  - BBU
- Synchrotron Radiation (Krafft)
  - Synchrotron Radiation Distributions
  - Radiation Damping
  - Damped Beam Properties
- Collective Effects (Krafft)
  - Luminosity
  - Negative Mass Instability

Energy Units

- When a particle is accelerated, i.e., its energy is changed by an electromagnetic field, it must have fallen through an Electric Field (we show later by very general arguments that Magnetic Fields cannot change particle energy). For electrostatic accelerating fields the energy change is

$$\Delta E = q\Delta\Phi = q\left(\Phi_a - \Phi_b\right)$$

q charge, Φ, the electrostatic potentials before and after the motion through the electric field. Therefore, particle energy can be conveniently expressed in units of the "equivalent" electrostatic potential change needed to accelerate the particle to the given energy. Definition: 1 eV, or 1 electron volt, is the energy acquired by 1 electron falling through a one volt potential difference.

Energy Units

$$1\text{ eV} = 1.6\times10^{-19}\text{ C} \times 1\text{ V} = 1.6\times10^{-19}\text{ J}$$

$$1\text{ MeV} = 10^{6}\text{ eV} = 1.6\times10^{-13}\text{ J}$$

To convert rest mass to eV use Einstein relation

$$E_0 = mc^2$$

where m is the rest mass. For electrons

$$E_{electron,0} = 9.1\times10^{-31}\text{ kg}\left(3\times10^{8}\text{ m/sec}\right)^2 = 81.9\times10^{-15}\text{ J}$$

$$= 0.512\text{ MeV}$$

Recent “best fit” value 0.51099906 MeV

Some Needed Relativity

Accelerator Physics is "Applied Special Relativity"
Following Maxwell Equations, which exhibit this symmetry, assume all Laws of Physics must be of form to guarantee the invariance of the space-time interval

$$(ct')^2 - x'^2 - y'^2 - z'^2 = (ct)^2 - x^2 - y^2 - z^2$$

Coordinate transformations that leave interval unchanged are the usual rotations and Lorentz Transformations, e.g. the *z* boost

$$ct' = \gamma(ct - \beta z)$$

$$x' = x$$

$$y' = y$$

$$z' = \gamma(z - \beta ct)$$

Relativistic Factors

Following Einstein define the relativistic factors

$$\vec{\beta} = \frac{\vec{\mathrm{v}}}{c} \qquad \beta = \frac{|\vec{\mathrm{v}}|}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Easy way to accomplish task of defining a *Relativistic Mechanics*: write all laws of physics in terms of 4-vectors and 4-tensors, i.e., quantities that transform under Lorentz transformations in the same way as the coordinate differentials.

Four-vectors

Four-vector transformation under *z* boost Lorentz Transformation

$$v^0{}' = \gamma\left(v^0 - \beta v^3\right)$$

$$v^1{}' = v^1$$

$$v^2{}' = v^2$$

$$v^3{}' = \gamma\left(v^3 - \beta v^0\right)$$

Important example: Four-velocity. Note that interval

$$d\tau \equiv \sqrt{1-\beta^2}\,dt$$

Lorentz invariant. So the following is a 4-vector

$$cu^\alpha \equiv \left(\frac{dct}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right) = c\gamma\left(1, \beta_x, \beta_y, \beta_z\right)$$

4-Momentum

Single particle mechanics must be defined in terms of Four-momentum

$$p^\alpha \equiv mcu^\alpha = mc\gamma\left(1, \beta_x, \beta_y, \beta_z\right) = \left(E/c, p_x, p_y, p_z\right)$$

Norms, which must be Lorentz invariant, are ($g_{\mu\nu}=g^{\mu\nu}=(1,-1,-1,-1)$)

$$\sqrt{u_\alpha u^\alpha} \equiv 1, \sqrt{p_\alpha p^\alpha} \equiv mc$$

What happens to Newton's Law $\vec{F} = m\vec{a} = d\vec{p}/dt$?

$$\frac{dp^\alpha}{d\tau} \equiv F^\alpha$$

But need a Four-force on the RHS!!!

Electromagnetic Field

Described by the Four-vector potential

$$A^{\mu} = \left(\phi, cA_x, cA_y, cA_z\right)$$

Field Tensor

$$F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

$$= \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{pmatrix}$$

Electromagnetic (Lorentz Force)

Non-relativistic

$$\vec{F} = q\left(\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right)$$

Relativistic Generalization (ν summation implied)

$$F^{\alpha} = qF^{\alpha}{}_{\nu}u^{\nu}$$

Electromagnetic Field with lower second index

$$F^{\alpha}{}_{\nu} \equiv F^{\alpha\beta}g_{\beta\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix}$$

Relativistic Mechanics in E-M Field

Energy Exchange Equation (Note: no magnetic field!)

$$\frac{d\gamma}{dt} = \frac{q\vec{E} \cdot \vec{\mathrm{v}}}{mc^2}$$

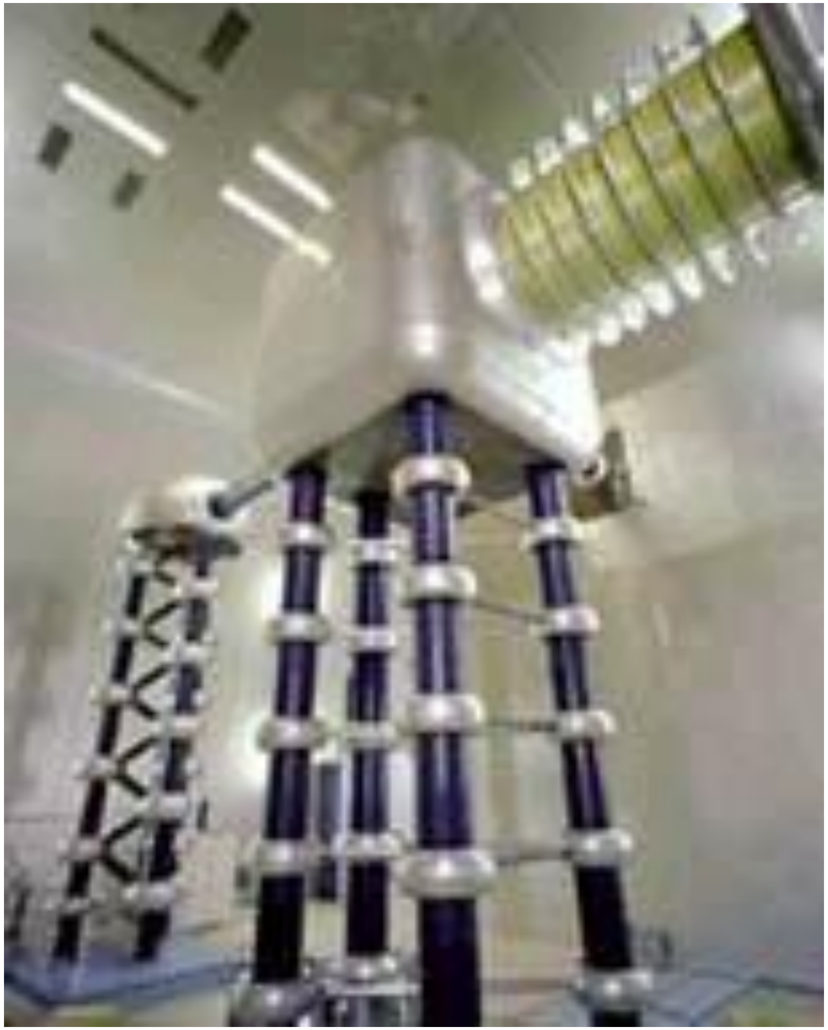
Relativistic Lorentz Force Equation (you verify in HW!)

$$\frac{d\left(\gamma m\vec{\mathrm{v}}\right)}{dt} = q\left(\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right)$$

Methods of Acceleration

- Acceleration by Static Electric Fields (DC) Acceleration
  - Cockcroft-Walton
  - van de Graaf Accelerators
  - Limited by voltage breakdowns to potentials of under a million volts in 1930, and presently to potentials of tens of millions of volts (in modern van de Graaf accelerators). Not enough to do nuclear physics at the time.
- Radio Frequency (RF) Acceleration
  - Main means to accelerate in most present day accelerators because one can get to 10-100 MV in a meter these days. Reason: alternating fields don't cause breakdown (if you are careful!) until much higher field levels than DC.
  - Ideas started with Ising and Wideröe

Cockcroft-Walton



Proton Source at Fermilab, Beam Energy 750 keV

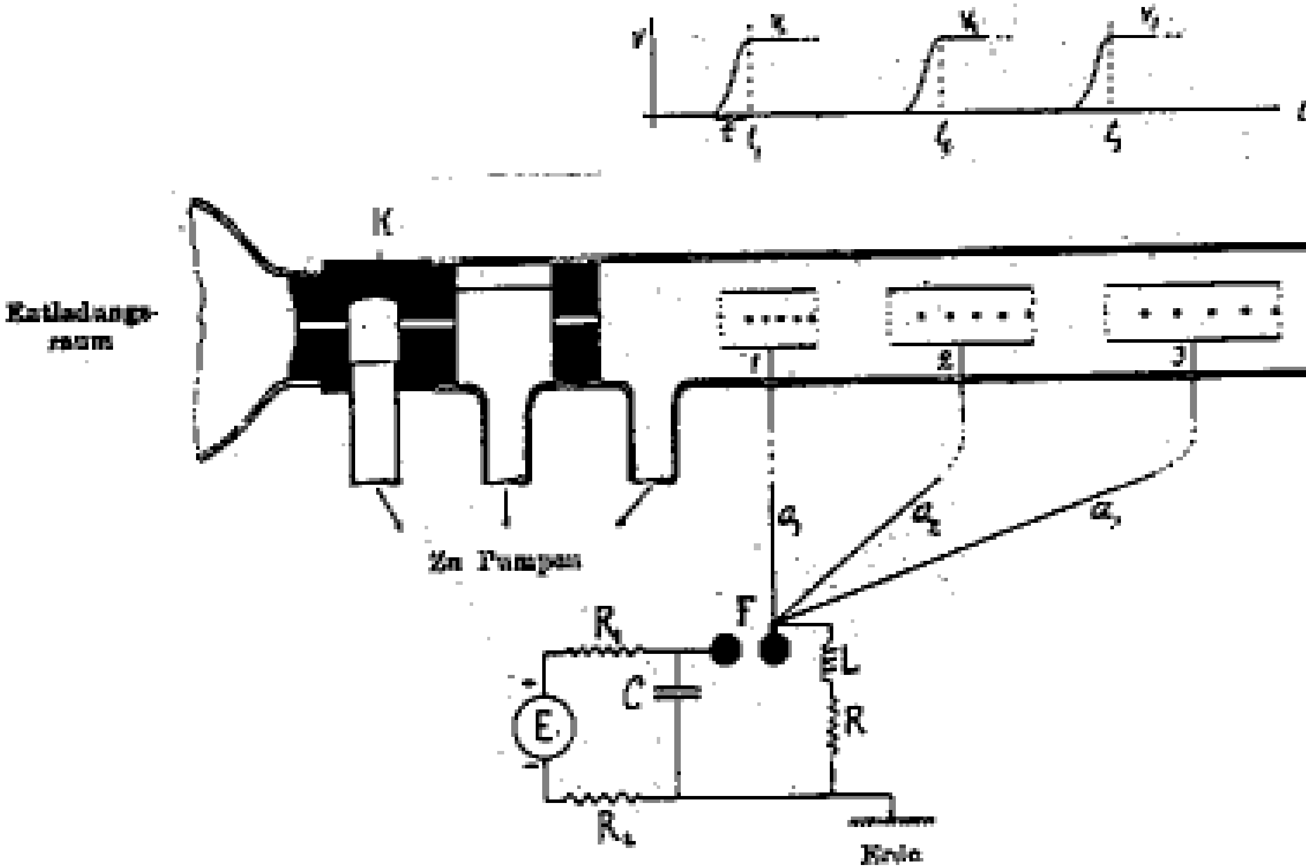
van de Graaf Accelerator

Generator

Brookhaven Tandem van de Graaf ~ 15 MV

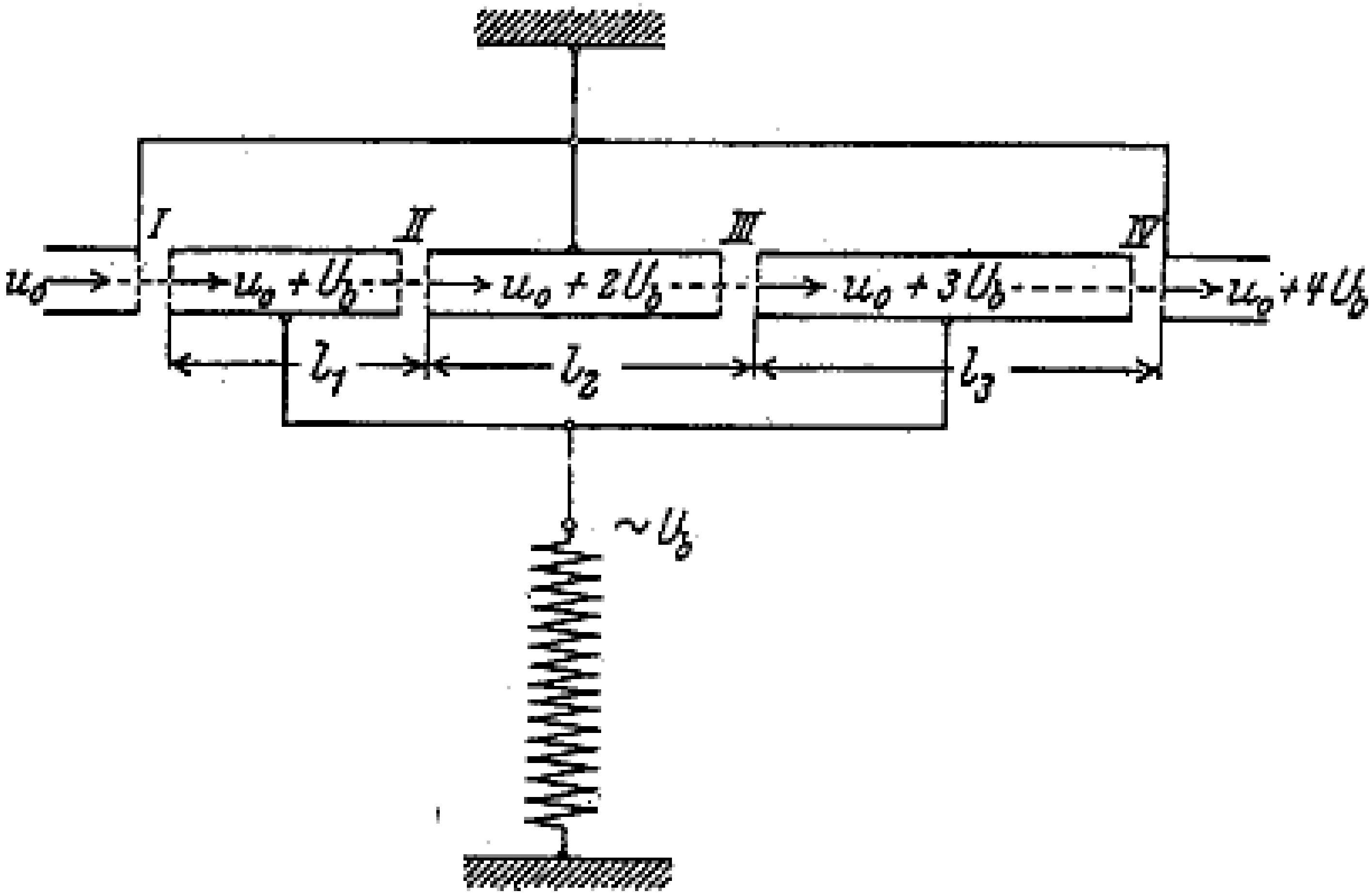
Tandem trick multiplies the output energy

Ising's Linac Idea



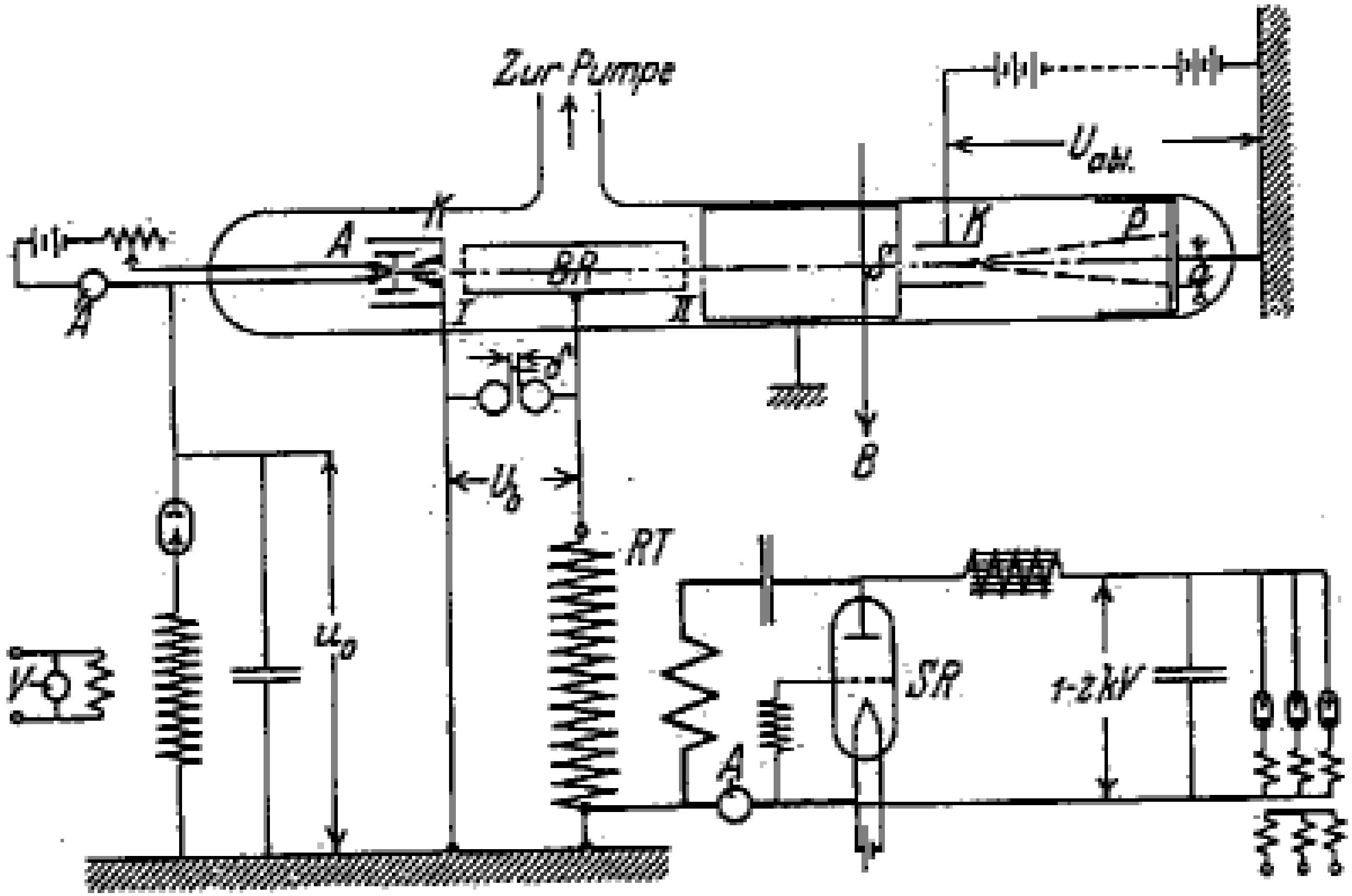
Prinzip einer Methode zur Herstellung von Kanalstrahlen hoher Voltzahl' (in German), Arkiv för matematik o. fysik, 18, Nr. 30, 1-4 (1924).

Drift Tube Linac Proposal



Idea Shown in Wideröe Thesis

Wideröe Thesis Experiment



Über ein neues Prinzip zur Herstellung hoher Spannungen, *Archiv für Elektrotechnik* **21**, 387 (1928)

(On a new principle for the production of higher voltages)

Sloan-Lawrence Heavy Ion Linac

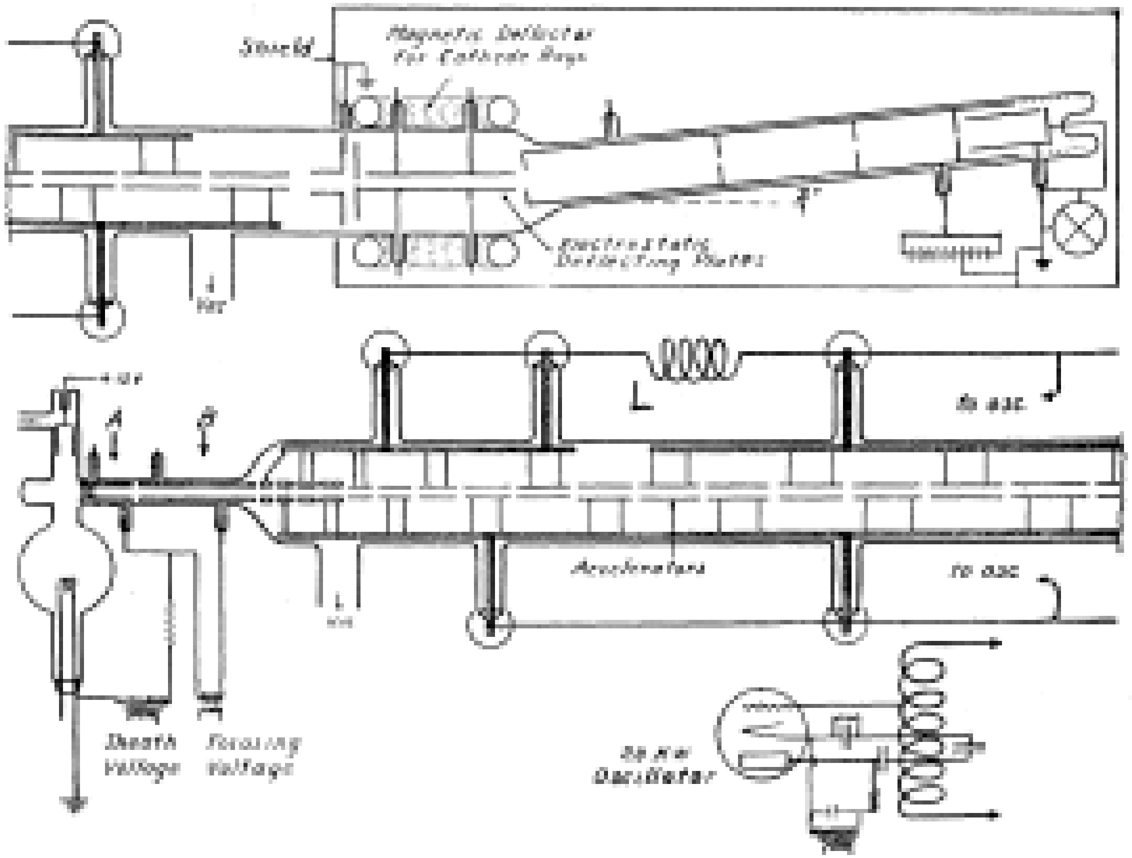


Fig. 1. Diagram of apparatus.

The Production of Heavy High Speed Ions without the Use of High Voltages
David H. Sloan and Ernest O. Lawrence Phys. Rev. **38**, 2021 (1931)

Alvarez Drift Tube Linac

- The first large proton drift tube linac built by Luis Alvarez and Panofsky after WW II

. (1945-1955) Alvarez Proton Linac

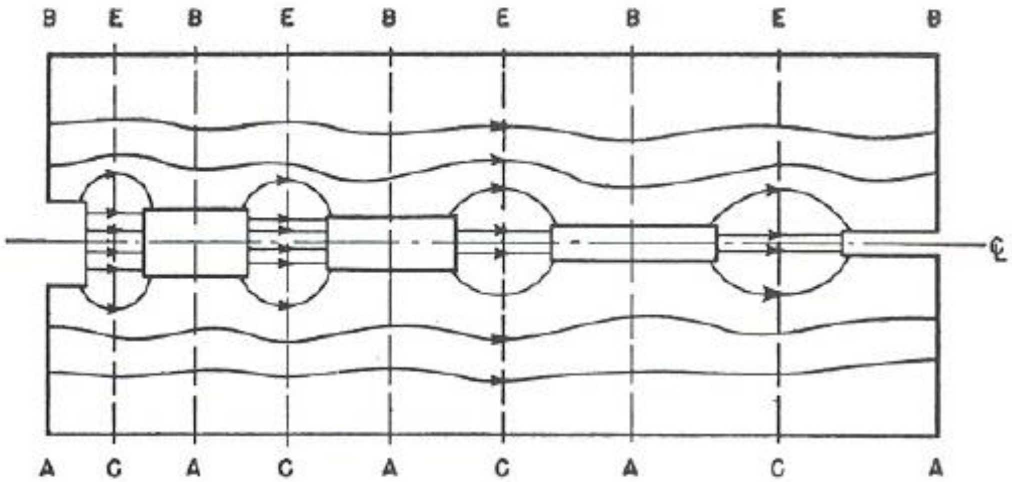
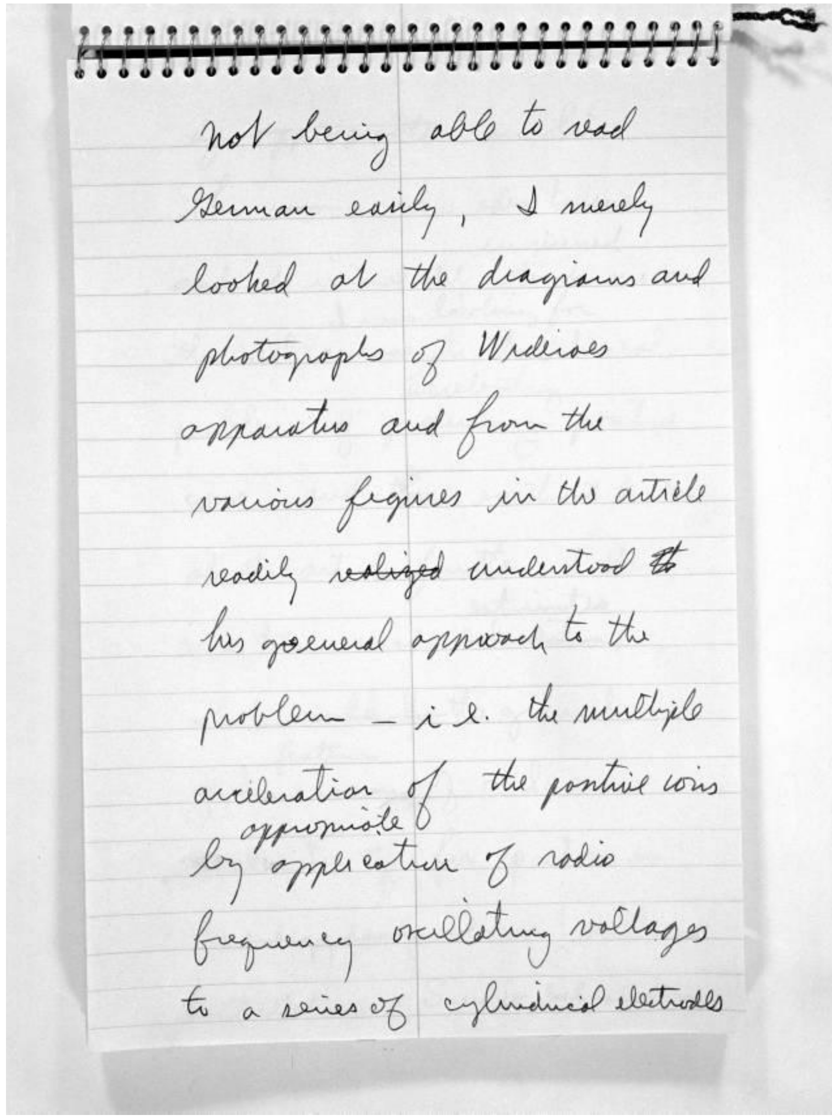


FIG. 2. Linear accelerator produced by introducing drift tubes into cavity excited as in Fig. 1. Division into unit cells.

Alvarez, Bradner, Frank, Gordon, Gow, Marshal, F. Oppenheimer, Panofsky, Richman, and Woodyard, Rev. Sci. Instrum., **26**, 111-133, (1955)

Earnest Orlando Lawrence

Germ of Idea*



*Stated in
E. O. Lawrence
Nobel Lecture

Lawrence's Question

- Can you re-use "the same" accelerating gap many times?

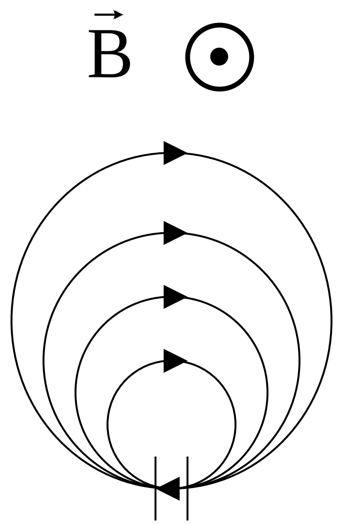
$$\vec{F} = m\vec{a} = q\vec{\mathrm{v}} \times \vec{B}$$

$$\frac{d^2x}{dt^2} = \frac{qB}{m}\mathrm{v}_y \quad \rightarrow \frac{d^2\mathrm{v}_x}{dt^2} + \Omega_c^2 \mathrm{v}_x = 0$$

$$\frac{d^2y}{dt^2} = -\frac{qB}{m}\mathrm{v}_x \rightarrow \frac{d^2\mathrm{v}_y}{dt^2} + \Omega_c^2 \mathrm{v}_y = 0$$

$$\frac{d}{dt}\left(\mathrm{v}_x^2 + \mathrm{v}_y^2\right) = \frac{qB}{m}\left(\mathrm{v}_x\mathrm{v}_y - \mathrm{v}_y\mathrm{v}_x\right) = 0$$

$$\mathrm{v}_0 = \sqrt{\mathrm{v}_x^2(t) + \mathrm{v}_y^2(t)} \quad \text{is a constant of the motion}$$



Cyclotron Frequency

$$\mathrm{v}_x(t)=\mathrm{v}_0\cos(\Omega_c t+\delta);\mathrm{v}_y(t)=-\mathrm{v}_0\sin(\Omega_c t+\delta)$$

$$x(t)=x_0+\frac{\mathrm{v}_0}{\Omega_c}\sin(\Omega_c t+\delta);y(t)=y_0+\frac{\mathrm{v}_0}{\Omega_c}\cos(\Omega_c t+\delta)$$

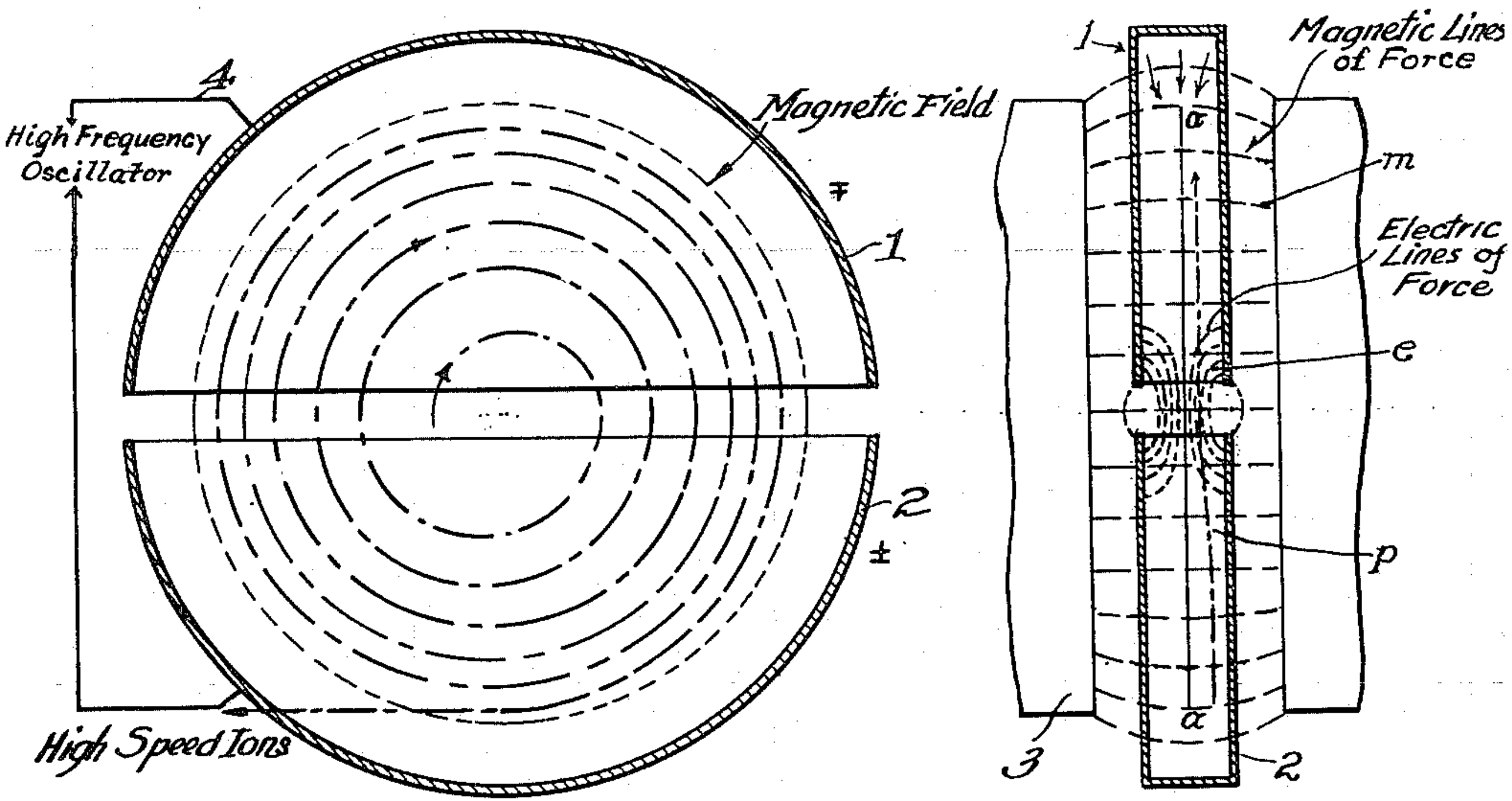
The radius of the oscillation $r = \mathrm{v}_0/\Omega_c$ is proportional to the velocity after the gap. Therefore, the particle takes the same amount of time to come around to the gap, independent of the actual particle energy!!!! (only in the non-relativistic approximation). Establish a resonance (equality!) between RF frequency and particle transverse oscillation frequency, also known as the Cyclotron Frequency

$$f_{rf}=f_c=\Omega_c/2\pi=\frac{qB}{2\pi m}$$

What Correspond to Drift Tubes?

- Dee's!

U. S. Patent Diagram

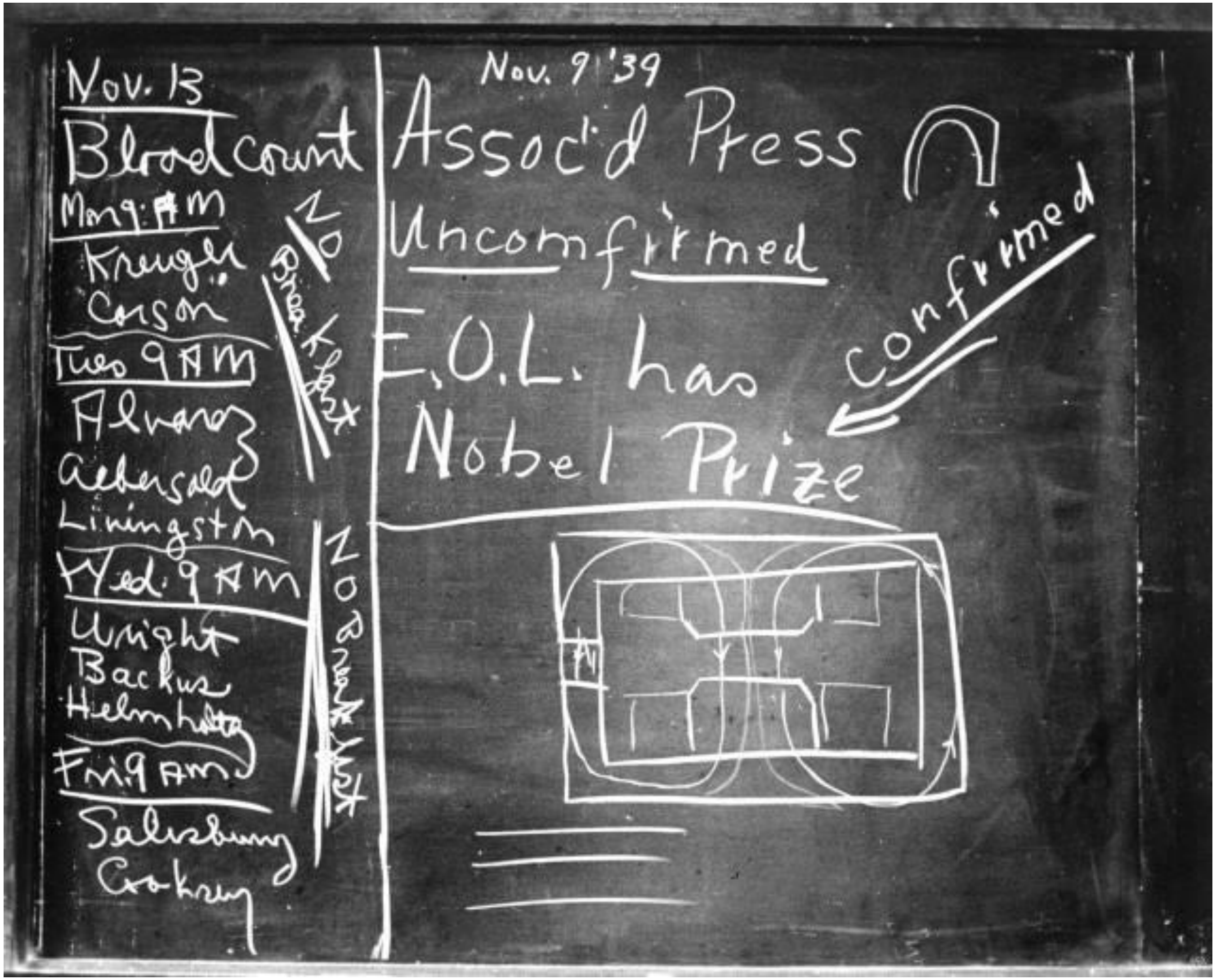


Magnet for 27 Inch Cyclotron (LHS)

Lawrence and “His Boys”

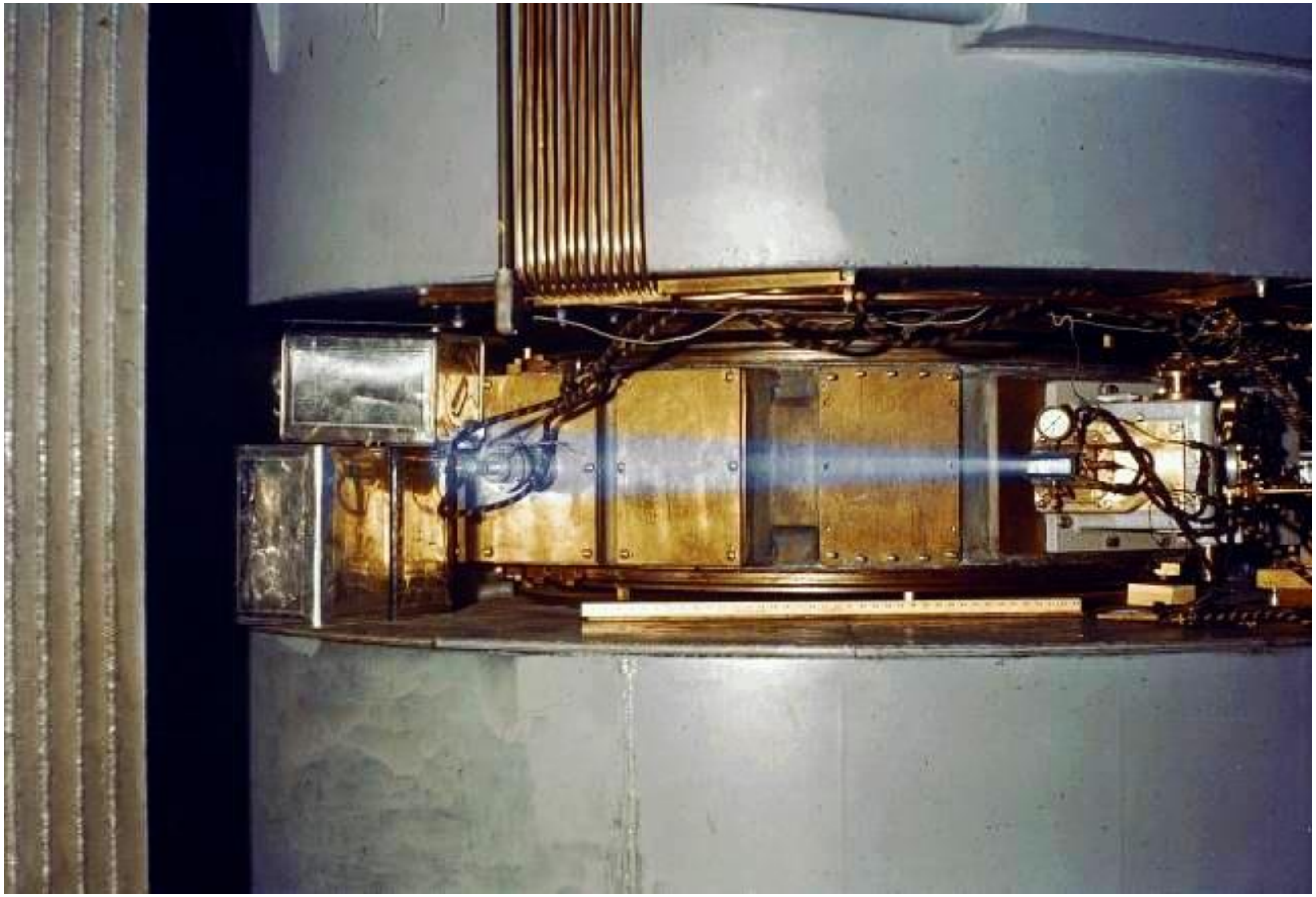
1-4: Jack Livingood, Frank Exner, M.S.Livingston, David Sloan, E.O.Lawrence, Milton White, Wesley Coates, L.Jackson Laslett and Commander T. Lucci - 1933

And Then!



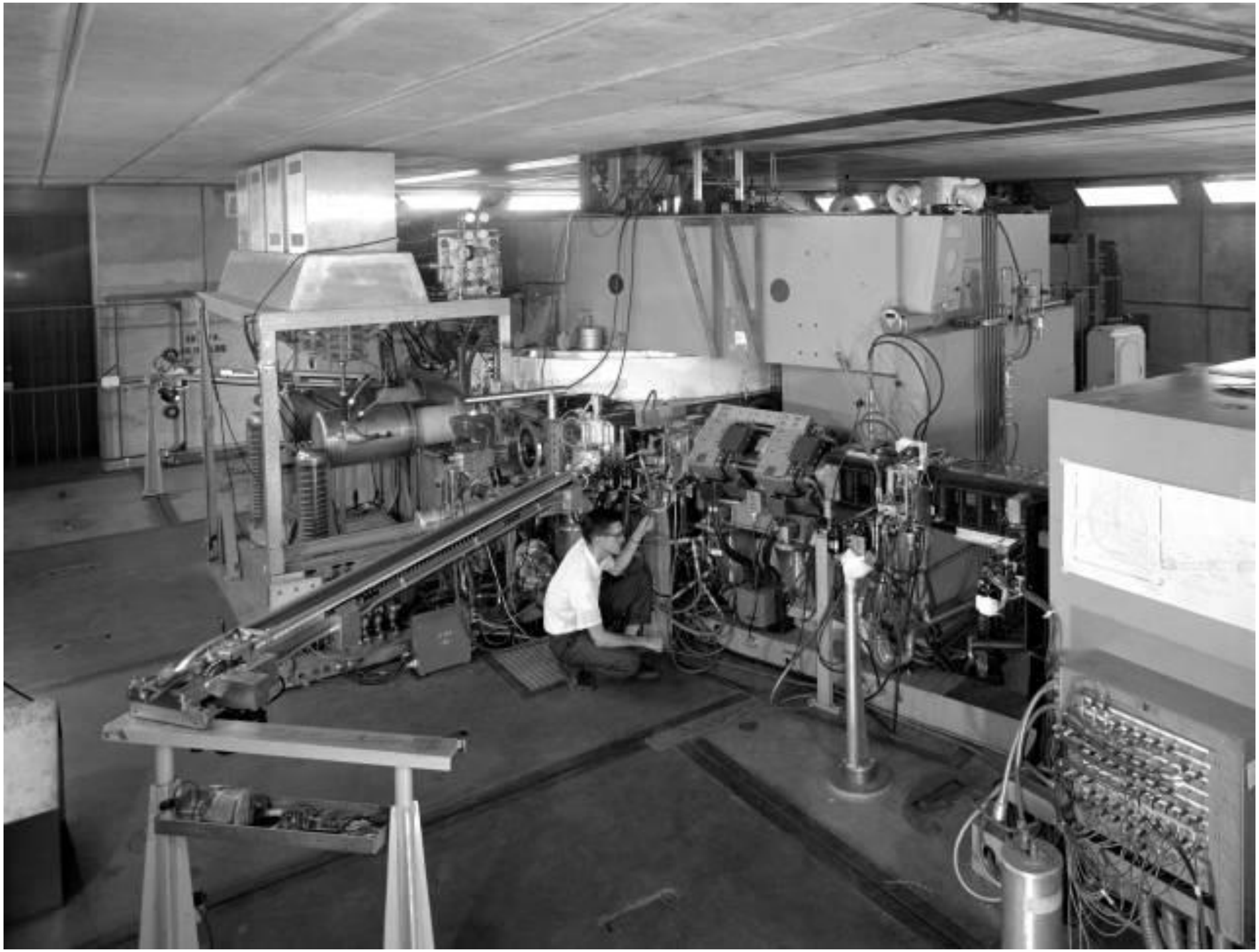
Nov. 13
Blood count
Mon 9 AM
Kruger
Carson
Tues 9 AM
Alvarez
Livingston
Wed. 9 AM
Wright
Backus
Helmholtz
Fri 9 AM
Salisbury
Nov. 9 '39
Assoc'd Press
Uncomfirmed
E.O.L. has
Nobel Prize
confirmed

Beam Extracted from a Cyclotron



Radiation Laboratory 60 Inch Cyclotron, circa 1939

88 Inch Cyclotron at Berkeley Lab



Relativistic Corrections

When include relativistic effects (you'll see in the HW!) the "effective" mass to compute the oscillation frequency is the relativistic mass γm

$$f_c = \Omega_c / 2\pi = \frac{qB}{2\pi \gamma m}$$

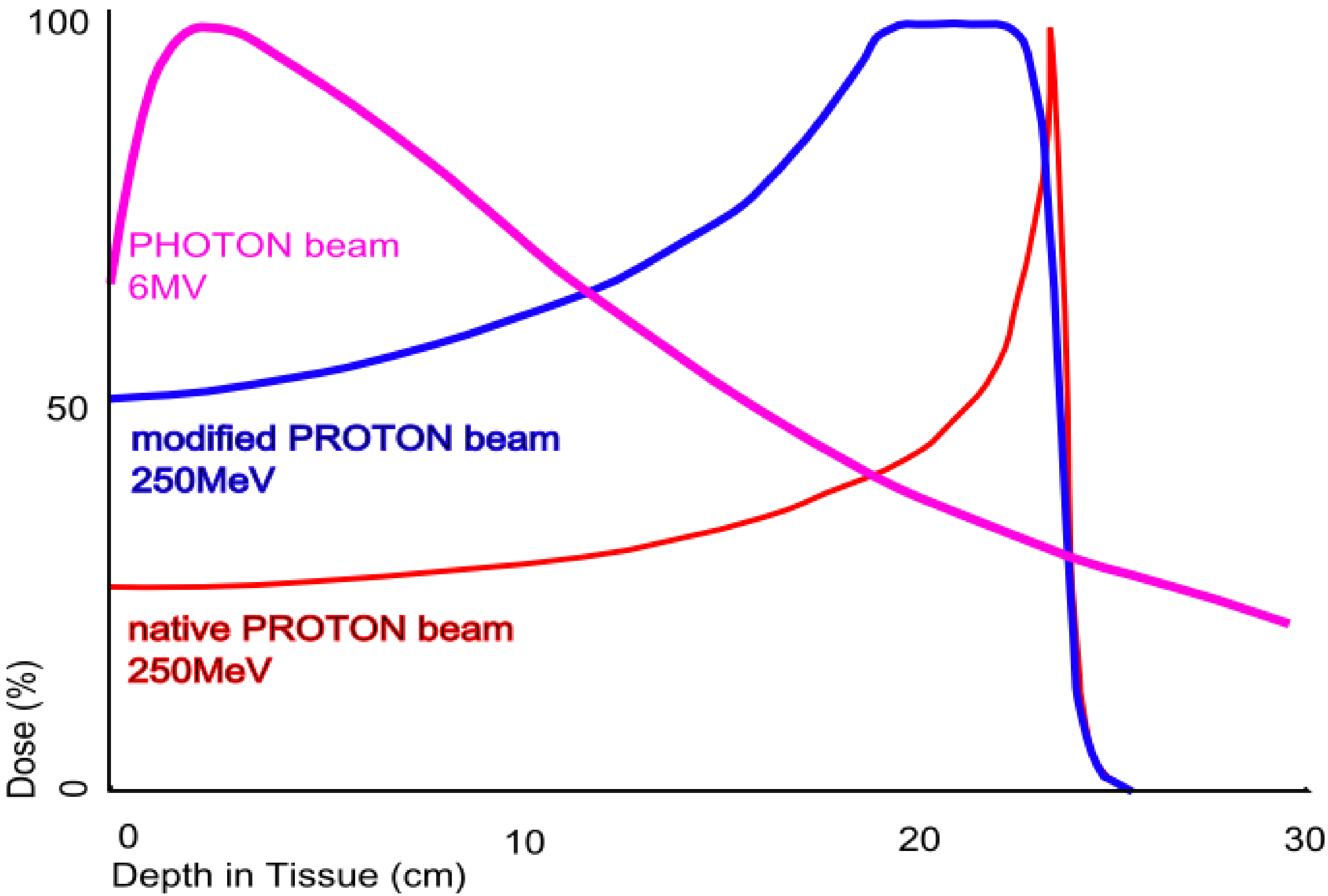
where γ is Einstein's relativistic γ, most usefully expressed as

$$\gamma = \frac{E_{tot}}{E_0} = \frac{E_0 + E_{kin}}{E_0} = \frac{mc^2 + E_{kin}}{mc^2}$$

m particle rest mass, E_{kin} particle kinetic energy

Cyclotrons for Radiation Therapy

Bragg Peak


100
50
0
Dose (%)
PHOTON beam
6MV
modified PROTON beam
250MeV
native PROTON beam
250MeV
0
10
20
30
Depth in Tissue (cm)

Lorentz Group: Linear Lie Group

- The group of Lorentz Transformations is a Linear (Matrix) Lie group. It can therefore be analyzed by standard methods. First find the generators

$$\Lambda = e^G \approx I + G + \cdots$$

$$\Lambda^t g \Lambda = g \to G^t g + gG = 0 \to \left(gG\right)^t = -gG$$

- General anti-symmetrical matrix yields

$$gG = \begin{pmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & -\varsigma_z & \varsigma_y \\ \omega_y & \varsigma_z & 0 & -\varsigma_x \\ \omega_z & -\varsigma_y & \varsigma_x & 0 \end{pmatrix} \to G = \begin{pmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ -\omega_x & 0 & \varsigma_z & -\varsigma_y \\ -\omega_y & -\varsigma_z & 0 & \varsigma_x \\ -\omega_z & \varsigma_y & -\varsigma_x & 0 \end{pmatrix}$$

Generators

Six Fundamental Generators

Rotations

$$S_{1,2,3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Boosts

$$K_{1,2,3} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Commutation Relations

$(S_{1,2,3})^2$ and $(K_{1,2,3})^2$ are straightforward to compute
$(\varepsilon\cdot S)^3 = -\,\varepsilon\cdot S$ and $(\varepsilon\cdot K)^3 = \varepsilon\cdot K$ for any unit three-vector ε

Commutation Relations are

$$\left[S_i, S_j\right] = \varepsilon_{ijk} S_k \qquad \left[S_i, K_j\right] = \varepsilon_{ijk} K_k \qquad \left[K_i, K_j\right] = -\varepsilon_{ijk} S_k$$

Arbitrary Lorentz transformation connected to the identity is

$$G = -\omega\cdot S - \varsigma\cdot K \qquad \Lambda = \exp\left(G\right)$$

$$mc\frac{du^\alpha}{d\tau} = qF^\alpha{}_\nu u^\nu \rightarrow u^\alpha\left(\tau\right) = \exp\left(qF\tau/mc\right)u\left(\tau = 0\right)$$

- Electric Fields give boosts
- Magnetic Fields give rotations