ODU Physics 854: Homework 4

Due date: Tuesday October 1, 2019

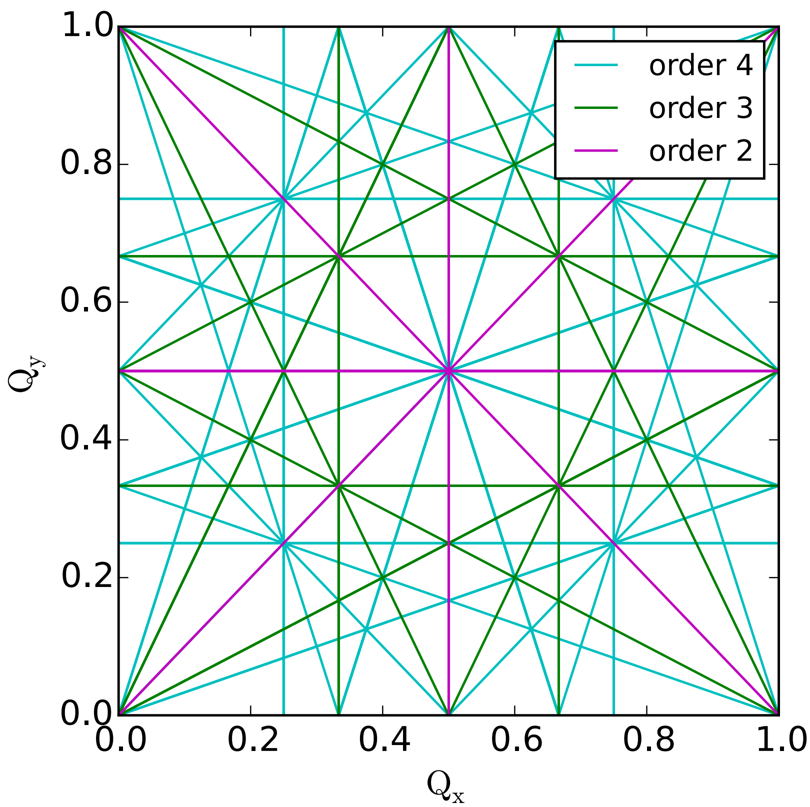
1 Peggs-Satogata 8.1

Consider a unit square in the tune plane (Q_x, Q_y) with corners at (n, n), $(n+1, n)$, $(n, n+1)$, and $(n+1, n+1)$.

(a) On graph paper or with a computer program, draw the lines representing all sum resonances $p = q\,Q_x + r\,Q_y$ through fourth order – for positive integer values of q and r, with $q + r \leq 4$. **Solution:** See figure below.

(b) Plot all difference resonances $p = q\,Q_x - r\,Q_y$ through fourth order. **Solution:** See figure.

(c) Where are the largest areas of tune space that are resonance-free? **Solution:** The largest areas free of low order resonances are near the corners, near the diagonal(s), and near the center of the tune plane at $(Q_x, Q_y) = (0.5, 0.5)$.



2 Peggs-Satogata 8.6

The interaction region quadrupole Q2 in RHIC has a focal length of about 3.0 m, at a location where the β-function is about 1400 m in collision optics with $\beta^* = 1$ m.

(a) How accurately must the strength of this magnet be known and set, if the strength error must be guaranteed to generate a β-wave amplitude of less than 1%? **Solution:** The beta-wave perturbation launched by a quadrupole strength error is

$$\frac{\Delta\beta}{\beta} = \frac{-\Delta q \beta_0}{2\sin(2\pi Q)} \cos(2|\psi - \psi_0| - 2\pi Q) \tag{2.1}$$

so that the amplitude of the β-wave

$$a_\beta = \frac{|\Delta q|}{q} \frac{q\beta_0}{|2\sin(2\pi Q)|} \tag{2.2}$$

depends on the value of Q through the resonance denominator. Roughly approximating

$$|2\sin(2\pi Q)| \approx 1 \tag{2.3}$$

then we require

$$\left|\frac{\Delta q}{q}\right| < \frac{0.01}{\beta |q|} = \frac{0.01 \times 3.0}{1400} \approx 2 \times 10^{-5} \tag{2.4}$$

to avoid launching β-waves at the 1% level.

(b) What tune shift is generated at this level of error? **Solution:** The tune shift generated by such an error

$$\Delta Q = \frac{\beta\,\Delta q}{4\pi} = \frac{\Delta q}{q}\frac{q\beta}{4\pi} = 2 \times 10^{-5} \frac{1400}{3 \times 4\pi} \approx 8 \times 10^{-4} \tag{2.5}$$

is quite small.

3 Peggs-Satogata 9.4

Consider the equilateral triangle in (x, x') normalised phase space predicted by Equations 9.27 and 9.28.

(a) What is the radius of the largest circle that can be inscribed inside the triangle? **Solution:** An inscribed circle just touches each side of the equilateral triangle, including the straight line

$$x = \frac{2\mu}{g}$$

The radius

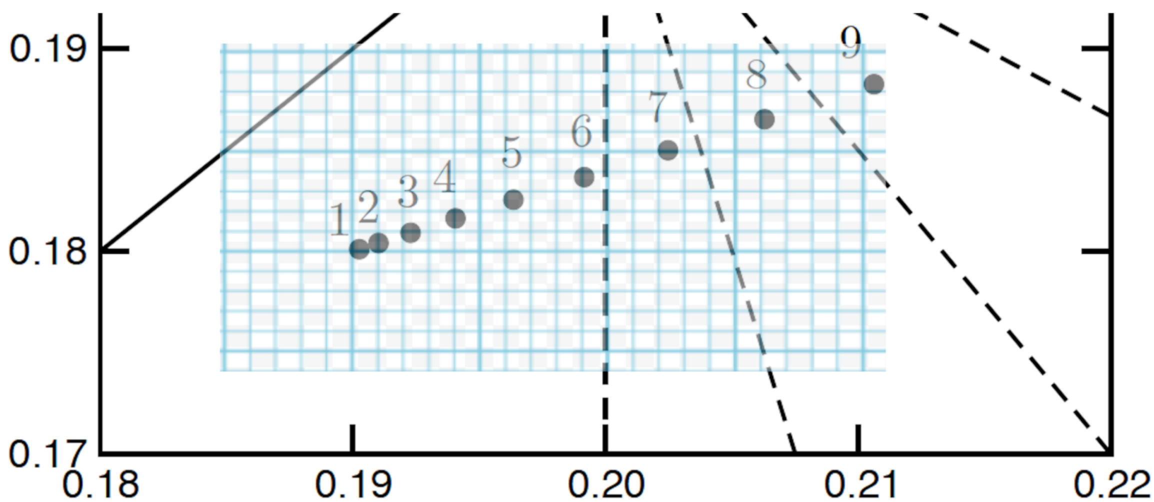
$$r = \left|\frac{2\mu}{g}\right| = \left|\frac{4\pi(Q - 1/3)}{g}\right|$$

is independent of the sign of μ (and g). For example, when $g = -1$ and $Q = 0.324$ then the radius is predicted to be $r = 0.117$, consistent with the results shown at the top left of Figure 9.3.

(b) What is the orientation of the triangle? **Solution:** When $g = -1$ and $Q = 0.324$ then μ/g is positive and the side of the triangle with $x =$ constant is predicted to be above the origin, in agreement with the top left of Figure 9.3.

(c) What happens to the area and the orientation of the triangle as the tune Q is (slowly) swept through the value of 1/3? **Solution:** Since the radius $r \sim \mu$ then the area of the triangle scales like $\mu^2 \sim (\delta Q)^2$. The orientation of the triangle flips when Q crosses 1/3 and μ changes sign.

4 Peggs-Satogata 10.1



(Modified from Peggs/Satogata problem 10.1) You have simulated the RHIC accelerator with a set of nine particles launched with design momentum ($\delta = 0$), $x' = 0$, and initial x offsets of 1, 2, ... 9 mm at a location with horizontal beta function $\beta_x = 40$ m. You "measure" the fractional tunes of these particles from the plot shown above to be:

x [mm]	Q_x	Q_y
1	0.1903	0.1800
2	0.1910	0.1802
3	0.1923	0.1809
4	0.1941	0.1816
5	0.1963	0.1825
6	0.1991	0.1837
7	0.2024	0.1851
8	0.2061	0.1866
9	0.2105	0.1884

(a) plot Q_x and Q_y vs. J_x from the above table. **Solution:** The action J_x of a particle with coordinates $(x, x' = 0)$ is $J_x = x^2/\beta_x$, so we can tabulate and plot:

(b) What is the simplest fit to the tune vs. action data? **Solution:** This plot is about as linear as it gets.

(c) What is the simplest and most likely dominant nonlinearity? **Solution:** Chapter 10 notes that the change in tune depending on action J is linear for octupoles, e.g. equation 10.9. The simplest and most likely dominant nonlinearity in this lattice is not sextupoles, but octupoles. As mentioned in class, octupoles drive this detuning to first order in octupole strength, while sextupoles drive this detuning to second order in sextupole strength.

J_x [mm]	Q_x	Q_y
0.025	0.1903	0.1800
0.100	0.1910	0.1802
0.225	0.1923	0.1809
0.400	0.1941	0.1816
0.625	0.1963	0.1825
0.900	0.1991	0.1837
1.225	0.2024	0.1851
1.600	0.2061	0.1866
2.025	0.2105	0.1884

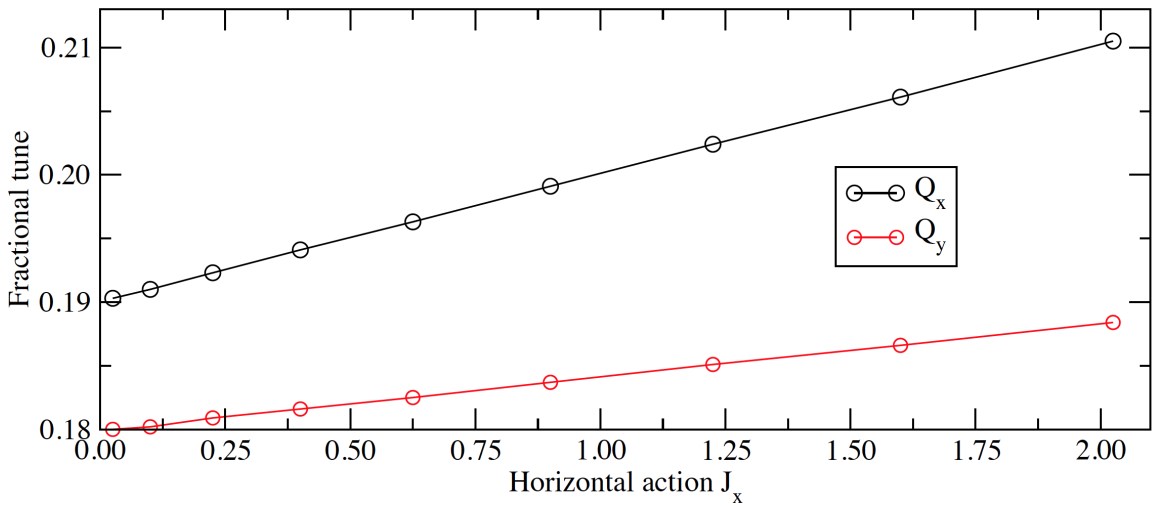


Figure 1: Plot of measured tunes (Q_x, Q_y) vs action J_x.

5 Peggs-Satogata 10.6

Consider the electrostatic and magnetic septa sketched in Figure 10.6.

- What are typical realistic values of E and B? **Solution:** Typical largest magnetic fields created by iron magnets are about 1 Tesla, while typical electric fields that can be created are on the order of 50–60 kV/cm.

- What is a typical ratio of electromagnetic forces, $E/(cB)$, for fully relativistic particles? **Solution:** Here we use $v \approx c$ for fully relativistic particles. Then the ratio of electric force in the electrostatic septum to magnetic force in the magnetic septum is about $(50\text{ kV/cm})/(c(1\text{ Tesla})) \approx 0.017 \approx 1/60$. The magnetic septum is about 60 times more effective at applying a transverse force than the electrostatic septum.

- How small must the kinetic energy of a proton (or an electron) be, in order for electrostatic optics to be competitive with magnetic optics? **Solution:** For the electrostatic and magnetic forces to be comparable, the particle velocity must be $\approx c/60$, or $\beta \approx 1/60$. This gives $\gamma \approx 1.00014$. The kinetic energy of the particle therefore is about 1.4×10^{-4} of its rest mass! This is why electrostatic fields are typically used at very low energies in most accelerators, or in areas where the septa (and integrated field strength) can be quite long.