Homework Problems
Physics 451/551
Due October 16, 2018

Solve and submit 2 of these problems on Arnold's material

1. By expanding out the definitions in terms of the basic one-forms show

$$\omega^1_{\vec{f}} \wedge \omega^1_{\vec{g}} = \omega^2_{\vec{f}\times\vec{g}}$$

$$\omega^1_{\vec{f}} \wedge \omega^2_{\vec{g}} = \left(\vec{f} \cdot \vec{g}\right) dx \wedge dy \wedge dz$$

2. Show, using coordinate expressions or otherwise, that the Poisson Bracket solves the Leibniz-like product rule

$$\left[F_1 F_2, F_3\right] = F_1\left[F_2, F_3\right] + F_2\left[F_1, F_3\right]$$

for any three functions F_1, F_2 and F_3.

3. Define the matrix commutator by $\left[M_1, M_2\right] = M_1 M_2 - M_2 M_1$. Show that the commutator operation satisfies the Jacobi identity

$$\left[M_1,\left[M_2,M_3\right]\right] + \left[M_2,\left[M_3,M_1\right]\right] + \left[M_3,\left[M_1,M_2\right]\right] = 0.$$

4. In three dimensions let $\omega^1 = xdx + ydy + zdz$.
   a. What is $d\omega^1$?
   b. What does the Generalized Stoke's Theorem say about the value of

   $$\int_C \omega^1$$

   for any closed curve *C*?
   c. Suppose a curve starts at $(0,0,0)$ and ends at (x, y, z). What is

   $$\int_C \omega^1 ?$$

   (The hard way is to do the line integral. The easy way is to find a function f with $\omega^1 = df$ and used the Generalized Stoke's Theorem.)