HW 1 Solution

Problem 4.8 In this problem there is really only on degree of freedom. Place the square so that in equilibrium each point mass is at $x = \pm L/\sqrt{2}$ and $y = \pm L/\sqrt{2}$. Let dx be the displacement of the mass from $x_1 = L/\sqrt{2}$. Then $y_1 = L/\sqrt{2} - dx$, $x_1 = -L/\sqrt{2} - dx$, $y_2 = -L/\sqrt{2} + dx$ by the small angle approximation of the Pythagorean Theorem. For example

$$\sqrt{\left(L/\sqrt{2}+dx\right)^2+\left(L/\sqrt{2}+dy\right)^2} = L^2 \rightarrow Ldx/\sqrt{2} + Ldy/\sqrt{2} = 0 \rightarrow dy = -dx.$$

The other expressions follow similarly. Therefore

$$L = 4\frac{m}{2}d\dot{x}^2 - \left[k\frac{(2dx)^2}{2} + k\frac{(2dx)^2}{2}\right] = 2md\dot{x}^2 - 4kdx^2$$

The Euler-Lagrange equation of motion is

$$\frac{d}{dt}4md\dot{x} = 4md\ddot{x} = -8kdx$$

$$\omega = \sqrt{\frac{2k}{m}}$$