

Accelerator Physics Particle Acceleration

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Some Fundamental Cavity Parameters

• Energy Gain

$$\frac{d\left(\gamma mc^{2}\right)}{dt} = -e\vec{E}\left(\vec{x}\left(t\right),t\right)\cdot\vec{v}$$

• For standing wave RF fields and velocity of light particles

$$\vec{E}(\vec{x},t) = \vec{E}(\vec{x})\cos(\omega_{RF}t + \delta) \rightarrow \Delta(\gamma mc^{2}) \approx -e\int_{-\infty}^{\infty} E_{z}(0,0,z)\cos(2\pi z / \lambda_{RF} + \delta)dz$$
$$= \frac{e\tilde{E}_{z}(2\pi / \lambda_{RF})e^{-i\delta} + \text{c.c.}}{2} \qquad V_{c} \equiv \left|e\tilde{E}_{z}(2\pi / \lambda_{RF})\right|$$

• Normalize by the cavity length *L* for gradient

$$\mathbf{E}_{\mathrm{acc}}\left(\mathbf{M}\mathbf{V}/\mathbf{m}\right) = \frac{V_c}{L}$$



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Shunt Impedance *R*/Q



• Ratio between the square of the maximum voltage delivered by a cavity and the product of ω_{RF} and the energy stored in a cavity (using "accelerator" definition)

$$\frac{R}{Q} \equiv \frac{V_c^2}{\omega_{RF} \text{ (stored energy)}}$$

- Depends only on the cavity geometry, independent of frequency when uniformly scale structure in 3D
- Piel's rule: $R/Q \sim 100 \Omega/cell$

CEBAF 5 Cell	480 Ω
CEBAF 7 Cell	760 Ω
DESY 9 Cell	1051 Ω





Unloaded Quality Factor

 As is usual in damped harmonic motion define a quality factor by

$$Q = \frac{2\pi (\text{energy stored in oscillation})}{\text{energy dissipated in 1 cycle}}$$

• Unloaded Quality Factor Q₀ of a cavity

$$Q_0 \equiv \frac{\omega_{RF} \text{(stored energy)}}{\text{heating power in walls}}$$

 Quantifies heat flow directly into cavity walls from AC resistance of superconductor, and wall heating from other sources.





Loaded Quality Factor

• When add the *input* coupling port, must account for the energy loss through the port on the oscillation

$$\frac{1}{Q_{tot}} \equiv \frac{1}{Q_L} = \frac{\text{total power lost}}{\omega_{RF} \text{ (stored energy)}} = \frac{1}{Q_{ext}} + \frac{1}{Q_0}$$

• Coupling Factor

$$\beta \equiv \frac{Q_0}{Q_{ext}} \gg 1$$
 for present day SRF cavities, $Q_L = \frac{Q_0}{1+\beta}$

- It's the loaded quality factor that gives the effective resonance width that the RF system, and its controls, seen from the superconducting cavity
- Chosen to minimize operating RF power: current matching (CEBAF, FEL), rf control performance and microphonics (SNS, ERLs)





Q_0 vs. Gradient for Several 1300 MHZ $\dot{q}\dot{q}$





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E_{acc} vs. time





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RF Cavity Equations

- Introduction
- Cavity Fundamental Parameters
- RF Cavity as a Parallel LCR Circuit
- Coupling of Cavity to an rf Generator
- Equivalent Circuit for a Cavity with Beam Loading
 - On Crest and on Resonance Operation
 - Off Crest and off Resonance Operation
 - Optimum Tuning
 - Optimum Coupling
- RF cavity with Beam and Microphonics
- Q_{ext} Optimization under Beam Loading and Microphonics
- RF Modeling

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Conclusions







Introduction

- Goal: Ability to predict rf cavity's steady-state response and develop a differential equation for the transient response
- We will construct an equivalent circuit and analyze it
- We will write the quantities that characterize an rf cavity and relate them to the circuit parameters, for
 - a) a cavity
 - b) a cavity coupled to an rf generator
 - c) a cavity with beam
 - d) include microphonics





RF Cavity Fundamental Quantities

Quality Factor Q_0 :

 $Q_0 = \frac{\omega_0 W}{P_{diss}} = \frac{\text{Energy stored in cavity}}{\text{Energy dissipated in cavity walls per radian}}$

Shunt impedance R_a (accelerator definition);

$$R_a \equiv \frac{V_c^2}{P_{diss}}$$

Note: Voltages and currents will be represented as complex quantities, denoted by a tilde. For example:

$$V_{c}(t) = \operatorname{Re}\left\{\tilde{V}_{c}(t)e^{i\omega t}\right\} \qquad \tilde{V}_{c}(t) = V_{c}e^{i\phi(t)}$$

where $V_c = |\tilde{V}_c|$ is the magnitude of \tilde{V}_c and ϕ is a slowly varying phase.



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Equivalent Circuit for an RF Cavity

Simple LC circuit representing an accelerating resonator.

Metamorphosis of the LC circuit into an accelerating cavity cell.

Chain of weakly coupled pillbox cavities representing an accelerating cavity. Chain of coupled pendula as its mechanical analogue.

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pillbox variety (0.2<8<0.5)

Chain of weakly-coupled pillbox

cavities representing an accele-

rating module

Chain of coupled pendula as a mechanical analogue to Fig. 6a





• Stored energy W: $W = \frac{1}{2}CV_c^2$

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Equivalent Circuit for an RF Cavity

• Average power dissipated in resistor *R*:

$$P_{diss} = \frac{V_c^2}{2R}$$

From definition of shunt impedance

$$R_a \equiv \frac{V_c^2}{P_{diss}} \qquad \qquad \therefore \ R_a = 2R$$

Quality factor of resonator:

$$Q_0 \equiv \frac{\omega_0 W}{P_{diss}} = \omega_0 CR$$

Note:
$$\tilde{Z} = R \left[1 + iQ_0 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right]^{-1} \quad \tilde{Z} \approx R \left[1 + 2iQ_0 \left(\frac{\omega - \omega_0}{\omega_0} \right) \right]^{-1} \quad \text{Wiedemann}$$

16.13



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Cavity with External Coupling

- Consider a cavity connected to an rf source
- A coaxial cable carries power from an rf source to the cavity
- The strength of the input coupler is adjusted by changing the penetration of the center conductor
- There is a fixed output coupler, the transmitted power probe, which picks up power transmitted through the cavity







Cavity with External Coupling



Consider the rf cavity after the rf is turned off. Stored energy W satisfies the equation WV / dV

Stored energy *W* satisfies the equation: $dW / dt = -P_{tot}$

Total power being lost, P_{tot} , is: $P_{tot} = P_{diss} + P_e + P_t$ P_e is the power leaking back out the input coupler. P_t is the power coming out the transmitted power coupler. Typically P_t is very small $\Rightarrow P_{tot} \approx P_{diss} + P_e$ Recall



Energy in the cavity decays exponentially with time constant:

$$\tau_L = Q_L / \omega_0$$



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Decay rate equation

$$\frac{P_{tot}}{\omega_0 W} = \frac{P_{diss} + P_e}{\omega_0 W}$$

suggests that we can assign a quality factor to each loss mechanism, such that

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e}$$

where, by definition,

$$Q_e \equiv \frac{\omega_0 W}{P_e}$$

Typical values for CEBAF 7-cell cavities: $Q_0 = 1 \times 10^{10}$, $Q_e \approx Q_L = 2 \times 10^{7}$.



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Have defined "coupling parameter":

$$\beta = \frac{P_e}{P_{diss}} = \frac{Q_0}{Q_e}$$

and therefore

$$\frac{1}{Q_L} = \frac{(1+\beta)}{Q_0}$$

Wiedemann 16.9

It tells us how strongly the couplers interact with the cavity. Large β implies that the power leaking out of the coupler is large compared to the power dissipated in the cavity walls.





Cavity Coupled to an RF Source

• The system we want to model. A generator producing power P_g transmits power to cavity through transmission line with characteristic impedance Z_0



 Between the rf generator and the cavity is an isolator – a circulator connected to a load. Circulator ensures that signals reflected from the cavity are terminated in a matched load.



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Transmission Lines



Inductor Impedance and Current Conservation

$$V_{n-1} - V_n = i\omega LI_{n-1}$$

$$I_{n-1} - I_n = i\omega CV_n$$



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